Syllabus:
Bayesian methodology and advanced Monte Carlo Simulations with applications to finance and network data

Professor Antonietta Mira

March 15, 2024

The goal of this course is to introduce students to the basics of Bayesian statistical inference and to simulation algorithms such as Monte Carlo integration, Importance sampling, Markov chain Monte Carlo (MCMC), adaptive MCMC, and likelihood free Monte Carlo simulation such as Approximate Bayesian Computation.

Examples will be used to illustrate to theory and the code to run such algorithms will be either provided by the instructor or developed by the students.

Course Main Topics:

- "Probability does not exist": the Bayesian philosophy
- An historical introduction and a review of Bayes Theorem
- The Bayesian paradigm: priors and posteriors. Some examples
- Conjugate Bayesian inference. Some examples
- Prior specification. Jeffreys priors
- Credible intervals and hypothesis testing
- Prediction and model comparison
- Bayesian Neural Networks
- Monte Carlo integration: definition and properties
- Review of Markov chain theory for MCMC purposes
  - transition probabilities (for finite and general state spaces)

- stationarity
- reversibility
- aperiodicity
- recurrence
- ergodicity
- The Law of Large Numbers
- The Central Limit Theorem
- Geometric and uniform ergodicity

- Markov chain Monte Carlo (MCMC) methods
  - the Metropolis algorithm
  - the Hastings algorithm
  - the Gibbs sampler
  - the Independence sampler
  - the Random Walk algorithm
- Burn-in, convergence diagnostics, starting point of the simulation
- The asymptotic variance in MCMC and its estimators
- Algorithm performance comparison
- Monte Carlo variance reduction techniques
- Adaptive MCMC algorithms
- Approximate Bayesian Computation

Software: The R statistical software will be used and can be downloaded at www.r-project.org Most of the routines and the code illustrated during the lectures is also available in Matlab and Python.
The lab sessions will be in Python

Evaluation: Course evaluation is based on a final project to be presented in front of the class.
This project is worth 100% of the final grade. Class participation will also be considered.

Prerequisites: Basic knowledge of a statistically oriented software (such as R or Matlab or Python) is strongly recommended.

Reference books:

- Alicia A. Johnson, Miles Q. Ott, Mine Dogucu, Bayes Rules! An Introduction to Applied Bayesian Modeling, ISBN 9780367255398 by Chapman & Hall, December 2021
- J. Liu, *Monte Carlo Strategies in Scientific Computing*, Springer, 2001
- C.P. Robert and G. Casella, *Monte Carlo Statistical Methods*, New York: Springer-Verlag, 2004
- W. R. Gilks and S. Richardson and D. J. Spiegelhalter, *Markov chain Monte Carlo in practice*, London: Chapman and Hall, 1996
- C.P. Robert and G. Casella, *Introducing Monte Carlo Methods with R*, Springer-Verlag, 2009

Reference papers (in order of suggested reading):

- Special section on Bayesian computations in the 21st Century, Vol. 39(1), Statistical Science, 2024
- L. Tierney, *Markov Chains for Exploring Posterior Distributions*, The Annals of Statistics, Vol. 22, n. 4., pp. 1701-1728, 1994
- G.O. Roberts and J.S. Rosenthal, *Optimal Scaling for Various Metropolis-Hastings Algorithms*, Statist. Sci. Vol. 16, n. 4, 351-367, 2001
- G.L. Jones, J.P. Hobert, *Honest Exploration of Intractable Probability Distributions via Markov Chain Monte Carlo*, Statistical Science, Vol. 16, n. 4, pp. 312-334, 2001
- G.L. Jones, *On the Markov Chain Central Limit Theorem*, Probab. Surveys, Vol. 1, 299-321, 2004
- C. Andrieu and J. Thoms, *A tutorial on adaptive MCMC*, Statistics and Computing, 18, pp. 343-373, 2008
- J. Rosenthal and G.O. Roberts, *Examples of adaptive MCMC*, Journal of Computational and Graphical Statistics, 18(2), pp. 349-367, 2009
- J.M. Marin JM, P. Pudlo, C.P. Robert, R.J. Ryder, *Approximate Bayesian computational methods*, Statistics and Computing, 22(6), pp. 1167-1180, 2012