Conditional alphas and realized betas

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Abstract: This paper proposes a two-step procedure to back out the conditional alpha of a given stock using high-frequency data. We first estimate the realized factor loadings of the stocks, and then retrieve their conditional alphas by estimating the conditional expectation of their risk-adjusted returns. We start with the underlying continuous-time stochastic process that governs the dynamics of every stock price and then derive the conditions under which we may consistently estimate the daily factor loadings and the resulting conditional alphas. We also contribute empirically to the conditional CAPM literature by examining the main drivers of the conditional alphas of the S&P 100 index constituents from January 2001 to December 2008. In addition, to confirm whether these conditional alphas indeed relate to pricing errors, we assess the performance of both cross-sectional and time-series momentum strategies based on the conditional alpha estimates. The findings are very promising in that these strategies not only seem to perform pretty well both in absolute and relative terms, but also exhibit virtually no systematic exposure to the usual risk factors (namely, market, size, value and momentum portfolios).

Keywords: asset pricing, conditional CAPM, pricing errors, realized beta, risk-adjusted performance.

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1 Introduction

The unconditional CAPM does not provide a good description of equity markets. It fails to explain important market anomalies, such as the size effect, the value premium and momentum. In addition, the absence of pricing errors does not suffice to ensure a zero unconditional alpha because market betas may change over time in tandem with market volatility and/or with risk premia. For instance, allowing for time-varying betas helps explain most of the (unconditional) value premium given that value stocks are riskiest precisely when risk premium is higher, namely, in recession times (Petkova and Zhang, 2005; Zhang, 2005).

The usual fix is to assume a conditional CAPM framework in which alphas and betas are affine on pre-determined predictor variables, e.g., stock characteristics, interest rates and spreads as well as other business-cycle indicators.[1] Ferson, Simin and Sarkissian (2008) discuss three stylized factors that emerge from this literature. First, market betas do vary over time in a significant manner (Shanken, 1990; Cochrane, 1996; Ferson and Schadt, 1996; Jagannathan and Wang, 1996; Ferson and Harvey, 1999; Lettau and Ludvigson, 2001; Santos and Veronesi, 2006). Second, the intercept of the conditional alpha term is smaller than the unconditional alpha. This means that conditional asset pricing models entail on average smaller pricing errors than their unconditional versions. Third, despite of their better fit, conditional asset pricing models still fail in view that conditional alphas are not only nonzero, but also time-varying. See, among others, Christopherson, Ferson and Glassman (1998), Wang (2002), Ang and Chen (2007), and Adrian and Franzoni (2009).

This paper goes after these pricing errors. We build on the realized beta technology (Barndorff-Nielsen and Shephard, 2004; Andersen, Bollerslev, Diebold and Wu, 2006) to come up with a novel two-step procedure to estimate conditional alphas. In the first stage, we employ high-frequency data to retrieve a stock's realized beta or, in general, any other risk factor loading. The second step then backs out the conditional alpha by estimating the conditional expectation of the risk-adjusted return (i.e., the return in excess over the realized risk premia) at a lower frequency. The resulting estimator is nonparametric, enjoying much more flexibility and robustness than the usual parametric estimators. In particular, it does not require conditional alphas and betas to depend linearly on

[1] Note however that, if the conditional betas are affine, the conditional alpha is necessarily quadratic in the absence of arbitrage opportunities (Gagliardini, Ossola and Scaillet, 2013).

the conditioning state variables, reducing misspecification risks in a substantial manner. Although we focus on the conditional CAPM in the empirical analysis, the framework is general enough to handle any asset pricing model with tradeable factors. The conditional CAPM with higher-order moments ensues as one includes additional powers of the S&P 500 index returns, whereas adding exchange-traded funds (ETFs) based on size and book-to-market considerations would boil down to a conditional Fama-French model. Alternatively, we could also think of continuous and discontinuous market betas as in Todorov and Bollerslev (2010).

Integrating observations at different sampling frequencies is now new to finance. Merton (1980) notes that one can accurately estimate the variance over a fixed interval of time by summing squared returns at a sufficiently high sampling frequency. French, Schwert and Stambaugh (1987) and Ghysels, Santa Clara and Valkanov (2005) accordingly exploit daily (squared) returns to estimate the monthly volatility in their search for the risk-return tradeoff, whereas the realized measure literature employs intraday returns to compute daily realized variances, covariances, and market betas (Barndorff-Nielsen and Shephard, 2004; Andersen et al., 2006; Ait-Sahalia and Mykland, 2009; Andersen, Bollerslev and Diebold, 2009; Ait-Sahalia, Fan and Xiu, 2011) as well as to conduct statistical inference for *parametric* continuous-time stochastic volatility models (Bollerslev and Zhou, 2002; Corradi and Distaso, 2006; Todorov, 2009). More recently, Chang, Kim and Park (2009) combine low- and high-frequency observations to estimate continuous-time factor pricing models with constant factor loadings.

Traditionally, the focus is on testing for the absence of systematic pricing errors as in Gibbons, Ross and Shanken (1989). More recently, Grundy and Martin (2001), Lewellen and Nagel (2006), Li and Yang (2011) and Ang and Kristensen (2012) consider more general settings for testing conditional asset pricing models with time-varying alphas and betas. They advocate for the use of daily data to estimate monthly factor loadings within short local windows. We take this idea to the limit by employing ultra-high frequency data to estimate daily betas, but then change the focus from *assessing the magnitude of pricing errors* to *how could we take advantage of them.* This is in line with Hansen and Richard's (1987) and Wheatley's (1989) point that it is impossible to test a conditional factor pricing model without observing the investors' information sets. In addition, we also differ from previous nonparametric approaches by assuming that both alphas and betas are measurable functions of conditioning state variables and hence predictable. This allows us to carry out a more-than-descriptive analysis of pricing errors. Apart from looking at the main features of the time-varying alphas (e.g., persistence), we can also ask all sort of interesting questions about alpha portability and about the cross-sectional variation in the partial effect of each instrument.

Along these lines, we examine the main forces driving the pricing errors in the S&P 100 index constituents from January 2001 to December 2008. We first estimate the daily market betas using a multivariate realized kernel with refresh time as in Barndorff-Nielsen, Hansen, Lunde and Shephard (2011), so as to account for market microstructure noise and nonsynchronicity issues. The realized betas consistently estimate the daily conditional betas as long as the latter are constant within a day. Accordingly, we assume a continuous-time process for the stock prices in which the drift and diffusive parameters are measurable functions of conditioning state variables that evolve only at the daily frequency. This ensures that the conditional alphas and betas vary over time on a daily basis, but remain constant at any higher (intraday) frequency.

Given the realized market betas, we next compute risk-adjusted returns by subtracting the realized risk premia. By definition, the latter is the sum of the conditional alpha and the idiosyncratic innovation. The key to identify the pricing errors is that the conditional expectation of the idiosyncratic term is zero. We thus estimate the conditional expectation of the risk-adjusted returns using a nonparametric approach, so as to minimize misspecification and overconditioning risks (Boguth, Carlson, Fisher and Simutin, 2011). Differently from Welch and Goyal (2008), we work at the daily frequency, ruling out many of the usual suspects for the conditioning state variables. Accordingly, we employ as lagged predictors various interest rates and spreads, market liquidity and volatility measures as well as characteristic-based portfolios based on momentum, short- and long-term reversals, size, and value effects. By conditioning on firm-specific attributes, we take the view of Graham and Dodd (1934), Lakonishok, Shleifer and Vishny (1994), Haugen and Baker (1996), and Daniel and Titman (1997) that these variables are perhaps useful to spot systematic mispricings in equity markets. And, by taking all possible candidates available at the daily frequency as instruments, we also reduce the risk of underconditioning (Ghysels, 1998; Harvey, 2001).

We are also particularly attentive to data snooping and spurious regression biases that may affect the estimation of conditional alphas (Ferson et al., 2008). Due to their local nature, kernel estimators are relatively less prone to the spurious regression problem that may arise in the presence of persistent regressors. We also control to some extent for data snooping by considering only a few principal components of the various instruments we entertain. Further, conditioning on principal components also helps reduce the dimensionality of the nonparametric regression without having to assume from the start that the conditional alphas are nonconstant as other dimension reduction techniques would require (see, among others, Li, 1991; Ichimura, 1993; Huang, Horowitz and Wei, 2010). It does not seem fair after all to presuppose nonzero conditional alphas if the aim is to uncover them.

Given the conditional alpha estimates, the next step is to assess whether it is profitable to trade them away. We first show that the nonparametric alpha estimates convey information about future stock returns as one would expect if they indeed relate to pricing errors. We then implement simple self-financed trading strategies that take long positions in stocks with positive conditional alphas, while shorting stocks with negative conditional alphas. We determine whether a given conditional alpha is large/low enough to justify a long/short position by conditioning either on cross-sectional or time-series quantiles. This gives way to cross-sectional and time-series momentum strategies based on conditional alphas rather than on raw returns as in Jegadeesh and Titman (1993) and Moskowitz, Ooi and Pedersen (2012), respectively. Due to the time variation in the alphas, we would have in principle to rebalance our long-short portfolio every day, raising the issue of alpha portability after transaction fees. We thus entertain different holding horizons so as to effectively reduce the portfolio turnover of the trading strategies.

We find that the cross-sectional and time-series momentum strategies based on nonparametric alphas easily outperform not only the S&P 500 index, but also the equal-weight portfolio of the S&P 100 index constituents, even after controlling for transaction costs. In particular, the best performances are from the alpha-based cross-section strategy with a holding period of 3 days and the time-series strategies with holding periods of 10 and 22 days. The same does not apply, however, to similar strategies based on affine alphas. There is a striking difference in performance once one move from affine to nonparametric alphas. Finally, one may wonder whether it is possible to achieve a comparable performance by means of simpler passive trading strategies. A traditional multifactor analysis reveals that the answer is negative. Although they entail positive unconditional alphas, the alpha-based strategies display very little exposure to the market, size, value and momentum risk factors. This is very reassuring given our conditional CAPM assumption.

The rest of this paper is as follows. Section 2 spells out the assumptions we make on the continuous-time multivariate process that governs the dynamics of stock prices. Note that we do not start from the continuous-time version of the CAPM as in Mykland and Zhang (2006), for otherwise the CAPM would not hold in discrete time (Longstaff, 1989). Section 3 develops the asymptotic justification of the conditional alpha estimator, controlling for the fact that we only observe the realized beta and not the true conditional beta. Section 4 examines whether there are pricing errors in the S&P 100 index constituents and whether it is profitable to trade them away. Section 5 offers some concluding remarks. Appendix A collects all technical proofs for the realized beta estimator, whereas Appendix B outlines the extension to the multivariate realized kernel estimator that accounts for both nonsynchronicity and market microstructure noise.

2 Conditional factor pricing model

This section proposes a consistent two-step procedure to identify and estimate conditional alphas. We first estimate daily conditional betas using a realized beta approach that takes advantage of intraday observations. This yields consistent estimates of the conditional betas as long as the sampling interval shrinks to zero (i.e., infill asymptotics). We then carry out a nonparametric regression of the resulting daily risk-adjusted returns on conditioning state variables to estimate daily pricing errors. Asymptotic validity of this second step relies in turn on a long-span asymptotic theory, that is to say, on a large enough number of trading days. Given the interplay between infill and long-span asymptotics, we must pay special attention not only to the underlying continuous-time process, but also to the rates at which the number of intraday observations and the number of days in the sample go to infinity.

In the following, we derive a discrete-time conditional multifactor asset pricing model from the exact discretization of a conditional semimartingale process in continuous time. This is important because we must ensure that the probability limit of the realized betas is indeed the vector of factor loadings in the discrete-time multifactor model. Our discretization results complement well those in Longstaff (1989) and Chang et al. (2009). The former shows that temporally aggregating the continuous-time CAPM results in a multifactor model in discrete time, whereas the latter considers a continuous-time model that is also consistent with a discrete-time multifactor model. The main difference is that our setting delivers conditional alphas and betas that evolve in discrete time.

2.1 From continuous to discrete time

Let $P_i(s)$ and $\boldsymbol{F}(s)$ respectively denote the log-prices at time s of the i-th asset ($i = 1, \ldots, N$) and of k_F portfolios mimicking the common risk factors that drive assets' excess returns. For instance, the CAPM considers the market portfolio as a single factor ($k_F = 1$), whereas Fama and French (1992) advocate for a three-factor model that controls for size and book-to-market effects ($k_F = 3$). We assume that both $P_i(s)$ and $\boldsymbol{F}(s)$ follow continuous-time diffusion processes, with drift and volatility parameters evolving in discrete time as measurable functions of k_C conditioning instruments $\boldsymbol{C}_t$. One may think of the latter either as state variables that reflect changes in the future investment opportunity set as in Merton's (1973) ICAPM. In particular, all predetermined instruments that help predict future discounted returns are natural candidates. More precisely, for any $t \leq s < t+1$ and $i \in \{1, \ldots, N\}$,

$$\mathrm{d}P_i(s) = \mu_{i,t}\ \mathrm{d}s + \boldsymbol{\Sigma}'_{i,t}\ \mathrm{d}\boldsymbol{W}_F(s) + \sigma_{i,t}\ \mathrm{d}W_i(s) \tag{1}$$

$$\mathrm{d}\boldsymbol{F}(s) = \boldsymbol{\mu}_{F,t}\ \mathrm{d}s + \boldsymbol{\Sigma}_{F,t}\ \mathrm{d}\boldsymbol{W}_F(s), \tag{2}$$

where $\mu_{i,t} \equiv \mu_i(\boldsymbol{C}_t)$ and $\boldsymbol{\mu}_{F,t} \equiv \boldsymbol{\mu}_F(\boldsymbol{C}_t)$ are drift parameters, $\boldsymbol{\Sigma}_{i,t} \equiv \boldsymbol{\Sigma}_i(\boldsymbol{C}_t)$ is a $k_F \times 1$ vector that determines the exposure of asset i to each risk factor, $\boldsymbol{\Sigma}_{F,t} \equiv \boldsymbol{\Sigma}_F(\boldsymbol{C}_t)$ is the $k_F \times k_F$ covariance matrix of the common risk factors, $\boldsymbol{W}_F(s)$ is a k_F-dimensional standard Brownian motion, and $W_i(s)$ is a standard Brownian motion independent of $\boldsymbol{W}_F(s)$. We next document under which conditions asset and factor prices are continuous-time semimartingale processes. This is crucial because semimartingale prices are not only consistent with no-arbitrage considerations, but also necessary in the infill asymptotic theory we use to justify the realized beta measures.

Lemma 1: Let $\boldsymbol{X}_i(s) = (P_i(s), \boldsymbol{F}(s))$ evolve as in (1) and (2). Let also $\boldsymbol{C}(s) = \boldsymbol{C}_t$ for any $s \in [t, t+1)$ and define the filtration $\mathcal{F}_C(s) = \sigma(\boldsymbol{C}(\tau),\ \tau \leq s)$ for $s > 0$. If $\boldsymbol{C}(s)$ is independent of both $W_i(s)$ and $\boldsymbol{W}_F(s)$, then $\boldsymbol{X}_i(s)$ is a conditional semimartingale with independent increments given $\mathcal{F}_C(s)$.

The assumption that $\boldsymbol{C}(s)$ is independent of $W_i(s)$ and $\boldsymbol{W}_F(s)$ implies that $\mathbb{E}\left[\boldsymbol{\Sigma}_{i,t}' W_i(s)\right] = 0$ and that $\mathbb{E}\left[\boldsymbol{\Sigma}_{F,t}\, \boldsymbol{W}_F(s)\right] = 0$. This does not imply however that $\boldsymbol{F}(s)$ and $\boldsymbol{C}(s)$ are independent. In fact, the common risk factors $\boldsymbol{F}(s)$ depend on the conditioning state variables $\boldsymbol{C}_t$ for any $s \in [t, t+1)$ through the drift and diffusion parameters. In addition, it follows from Lemma 1 that, given the value of $\boldsymbol{C}_t$, the continuous-time process $\boldsymbol{X}_i(s)$ has independent increments for any $s \in [t, t+1)$. This means that market microstructure effects are responsible for any autocorrelation pattern within the interval $[t, t+1)$, say, a day. In contrast, due to the dependence on $\boldsymbol{C}_t$, daily increments $\boldsymbol{x}_{i,t} = \int_t^{t+1} \mathrm{d}\boldsymbol{X}_i(s)$ may display genuine autocorrelation.

Suppose that we have M equidistant observations within a day starting at time t, namely, $P_{i,t+\ell/M}$ and $\boldsymbol{F}_{t+\ell/M}$ with $j = 0, \ldots, M-1$. We then define the vector of realized betas as

$$\widehat{\boldsymbol{\beta}}_{i,t,M} = \left[\sum_{\ell=0}^{M-1} (\boldsymbol{F}_{t+\frac{\ell+1}{M}} - \boldsymbol{F}_{t+\frac{\ell}{M}})(\boldsymbol{F}_{t+\frac{\ell+1}{M}} - \boldsymbol{F}_{t+\frac{\ell}{M}})'\right]^{-1} \sum_{\ell=0}^{M-1} (\boldsymbol{F}_{t+\frac{\ell+1}{M}} - \boldsymbol{F}_{t+\frac{\ell}{M}})(P_{i,t+\frac{\ell+1}{M}} - P_{i,t+\frac{\ell}{M}}). \tag{3}$$

Barndorff-Nielsen and Shephard (2004) show that, under very mild regularity conditions, for all t,

$$\operatorname*{plim}_{M\to\infty} \widehat{\boldsymbol{\beta}}_{i,t,M} = \boldsymbol{\Sigma}_{FF,t}^{-1}\, \boldsymbol{\Sigma}_{F,t}\, \boldsymbol{\Sigma}_{i,t} \equiv \boldsymbol{\beta}_{i,t}, \tag{4}$$

where $\boldsymbol{\Sigma}_{FF,t} = \boldsymbol{\Sigma}_{F,t}\boldsymbol{\Sigma}_{F,t}'$. Note also that the conditioning state variables $\boldsymbol{C}_t$ are the only drivers of the daily factor loadings in that $\boldsymbol{\beta}_{i,t} \equiv \boldsymbol{\beta}_i(\boldsymbol{C}_t)$, and hence betas are constant within a day. This is much milder than Lewellen and Nagel's (2006) assumption of constant monthly/quarterly betas. In addition, it also ensures that realized betas converge to conditional betas without any need for further conditioning.

For simplicity, we henceforth assume without loss of generality that $\boldsymbol{F}_{t+j/M}$ consists of orthogonal risk factors, so that $\boldsymbol{\Sigma}_{FF,t}$ is diagonal.[2] Further note that the estimator in (3) assumes that we observe prices and factors without measurement error. However, there is ample evidence that market microstructure noise is present in high-frequency transaction data. As a remedy, one should employ a realized beta estimator that is robust to such a contamination. This is the route we take in the empirical part, using of Barndorff-Nielsen et al.'s (2011) multivariate realized kernel approach. For notational simplicity, we relegate the case of microstructure noise robust estimators to Appendix B.

[2] In fact, we can always make the factors orthogonal via a rotation matrix $\boldsymbol{B}_t$ such that $\breve{\boldsymbol{F}}_{t+\ell/M} = \boldsymbol{B}_t \boldsymbol{F}_{t+\ell/M}$ and $\breve{\boldsymbol{F}}_{t+\ell/M}' \breve{\boldsymbol{\beta}}_{i,t,M} = \boldsymbol{F}_{t+\ell/M}' \widehat{\boldsymbol{\beta}}_{i,t,M}$, where $\breve{\boldsymbol{\beta}}_{i,t,M}$ denote the realized betas associated with the orthogonal factors.

We now move to discrete time by letting $r_{i,t+1} \equiv \int_t^{t+1} \mathrm{d}P_i(s)$ and $\boldsymbol{f}_{t+1} \equiv \int_t^{t+1} \mathrm{d}\boldsymbol{F}(s)$ denote continuously-compounded returns over the time interval $[t, t+1)$. It then follows from (1) and (2) that

$$r_{i,t+1} = \mu_{i,t} + \sigma_{i,t} \int_t^{t+1} \mathrm{d}W_i(s) + \boldsymbol{\Sigma}'_{i,t} \int_t^{t+1} \mathrm{d}\boldsymbol{W}_F(s) \tag{5}$$

$$\boldsymbol{f}_{t+1} = \boldsymbol{\mu}_{F,t} + \boldsymbol{\Sigma}_{F,t} \int_t^{t+1} \mathrm{d}\boldsymbol{W}_F(s), \qquad t = 1, \ldots, T. \tag{6}$$

This means that

$$\left(r_{i,t+1}, \boldsymbol{f}_{t+1}\right) | \boldsymbol{C}_t \sim \mathcal{N}\left(\begin{pmatrix} \mu_{i,t} \\ \boldsymbol{\mu}_{F,t} \end{pmatrix}, \begin{pmatrix} \sigma^2_{i,t} + \boldsymbol{\Sigma}'_{i,t}\,\boldsymbol{\Sigma}_{i,t} & \boldsymbol{\Sigma}'_{i,t}\,\boldsymbol{\Sigma}_{F,t} \\ \boldsymbol{\Sigma}_{F,t}\,\boldsymbol{\Sigma}_{i,t} & \boldsymbol{\Sigma}_{FF,t} \end{pmatrix} \right),$$

and so further conditioning on factor returns entails

$$r_{i,t+1} | (\boldsymbol{f}_{t+1}, \boldsymbol{C}_t) \sim \mathcal{N}\left(\mu_{i,t} + (\boldsymbol{f}_{t+1} - \boldsymbol{\mu}_{F,t})' \boldsymbol{\Sigma}^{-1}_{FF,t} \boldsymbol{\Sigma}_{F,t}\,\boldsymbol{\Sigma}_{i,t},\ \sigma^2_{i,t} + \boldsymbol{\Sigma}'_{i,t}\boldsymbol{\Sigma}_{i,t} - \boldsymbol{\Sigma}'_{i,t}\boldsymbol{\Sigma}^{-1}_{FF,t}\boldsymbol{\Sigma}_{i,t} \right).$$

Using the definition of the betas in (4) then yields the following discrete-time factor model

$$r_{i,t+1} = \alpha_{i,t} + \boldsymbol{f}'_{t+1}\boldsymbol{\beta}_{i,t} + \epsilon_{i,t+1}, \tag{7}$$

with $\alpha_{i,t} = \mu_{i,t} - \boldsymbol{\mu}'_{F,t}\boldsymbol{\beta}_{i,t}$ and $\epsilon_{i,t+1} | (\boldsymbol{f}_{t+1}, \boldsymbol{C}_t) \sim \mathcal{N}\left(0, \sigma^2_{i,t} + \boldsymbol{\Sigma}'_{i,t}\boldsymbol{\Sigma}_{i,t} - \boldsymbol{\Sigma}'_{i,t}\boldsymbol{\Sigma}^{-1}_{FF,t}\boldsymbol{\Sigma}_{i,t}\right)$.

It is now easy to appreciate why we assume that the drift and diffusive parameters in (1) and (2) evolve in discrete time. If the conditioning state variables were evolving in continuous time, (5) and (6) would then represent just a discrete-time approximation. As a result, the probability limit of the realized betas would differ from the true integrated beta given by the integral of the ratio:

$$\operatorname*{plim}_{M \to \infty} \widehat{\boldsymbol{\beta}}_{i,t,M} = \left(\int_t^{t+1} \boldsymbol{\Sigma}_{FF,s}\, \mathrm{d}s \right)^{-1} \int_t^{t+1} \boldsymbol{\Sigma}_{F,s}\boldsymbol{\Sigma}_{i,s}\, \mathrm{d}s.$$

It is worth stressing however that this will only hurt our identification strategy if the discrete-time approximation is such that $\mathbb{E}\left(\epsilon_{i,t+1} | \boldsymbol{C}_t\right)$ differs from zero.

2.2 Retrieving conditional alphas from realized betas

Let $Z_{i,t+1} \equiv r_{i,t+1} - \boldsymbol{f}'_{t+1}\boldsymbol{\beta}_{i,t}$ denote the daily risk-adjusted return at time $t+1$. It then follows from (7) that $Z_{i,t+1} = \alpha_{i,t} + \epsilon_{i,t+1}$. Identification of the conditional alpha stems from the facts that the conditional expectation of $\epsilon_{i,t+1}$ is zero and that $\alpha_{i,t}$ is measurable in the information set. Altogether, this means that $\alpha_{i,t} \equiv \mathbb{E}(Z_{i,t+1} | \boldsymbol{C}_t)$ and hence it would suffice to estimate the conditional expectation of the risk-adjusted returns to back out the conditional alphas. Unfortunately, this is infeasible because we do not directly observe the conditional betas and so the risk-adjusted returns.

To obtain a feasible estimator, we proceed in two steps. First, we adjust asset returns for risk using a realized beta approach, namely,

$$\begin{aligned}\widehat{Z}_{i,t+1,M} &= r_{i,t+1} - \boldsymbol{f}'_{t+1}\widehat{\boldsymbol{\beta}}_{i,t,M} = r_{i,t+1} - \boldsymbol{f}'_{t+1}\boldsymbol{\beta}_{i,t} - \boldsymbol{f}'_{t+1}\left(\widehat{\boldsymbol{\beta}}_{i,t,M} - \boldsymbol{\beta}_{i,t}\right) \\ &= \alpha_{i,t} + \epsilon_{i,t+1} - \boldsymbol{f}'_{t+1}\left(\widehat{\boldsymbol{\beta}}_{i,t,M} - \boldsymbol{\beta}_{i,t}\right) = Z_{i,t+1} - \boldsymbol{f}'_{t+1}\left(\widehat{\boldsymbol{\beta}}_{i,t,M} - \boldsymbol{\beta}_{i,t}\right).\end{aligned}$$

Second, we estimate the conditional expectation $\mathbb{E}(\widehat{Z}_{i,t+1,M}|\boldsymbol{C}_t)$ of the realized risk-adjusted returns using nonparametric kernel methods. In the next section, we establish under mild conditions that the difference between the feasible and infeasible estimators shrinks to zero as the number of intraday observations M increases.

The fact that we employ a nonparametric regression is in stark contrast with the usual practice of imposing an affine structure for the conditional alpha. The motivation is not only to capture nonlinear effects in a more efficient manner, but also to alleviate any spurious regression issue that may arise from the use of persistent, though still weak dependent, conditioning instruments (Ferson et al., 2008). In particular, Robinson (1986) shows that the local nature of kernel regressions breaks some of the persistence of mixing regressors for large enough bandwidths. The price we pay for more robust alpha estimates comes in the guise of the curse of dimensionality. Typically, the dimension of the vector $\boldsymbol{C}_t$ is relatively high so as to avoid underconditioning bias. However, nonparametric rates of convergence decrease with the dimensionality of the conditioning state vector. We thus condition the realized risk-adjusted returns on the k largest principal components of $\boldsymbol{C}_t$ (rather than on the full vector).[3] This allows us to consider a large number of conditioning state variables without incurring into the usual data mining biases that arise in the search for predictor variables (Ferson et al., 2008).

Let then $\boldsymbol{PC}_t = (PC_{1,t}, \ldots, PC_{k,t})'$ denote the $k < k_C$ principal components of $\boldsymbol{C}_t$, where k_C is the dimension of $\boldsymbol{C}_t$. In the sequel, we assume that $\mathbb{E}(\epsilon_{i,t+1}|\boldsymbol{PC}_t) = 0$ for all i and, with a slightly

[3] This is obviously a suboptimal dimension-reduction technique given that it boils down to a particular case of the multiple index model in which we take the linear combinations as given by the principal component analysis (Park, Sriram and Yin, 2010). Indeed, there are several more efficient methods for dimension reduction in a nonparametric setting: e.g., sliced inverse regression (Li, 1991) and group-Lasso-type criteria (Huang et al., 2010). However, identification of these estimators require a nonconstant conditional expectation. While we conjecture that alpha is not constant for most stocks, it does not seem fair to rule out constant or even zero alphas beforehand.

abuse of notation, we define $\alpha_{i,t} = \mathbb{E}(Z_{i,t+1}|\boldsymbol{PC}_t)$. While the condition $\mathbb{E}(\epsilon_{i,t+1}|\boldsymbol{C}_t) = 0$ does not necessarily imply $\mathbb{E}(\epsilon_{i,t+1}|\boldsymbol{PC}_t) = 0$, their difference will nevertheless shrink to zero as $k \to k_C$. This means that, if we condition on enough principal components to capture most of the variation in $Z_{i,t+1}$, $\mathbb{E}(\epsilon_{i,t+1}|\boldsymbol{PC}_t)$ will get very close to zero. The idea of summarizing the predictive content of many variables into few principal components is exactly what explains the popularity of factor augmented predictive models (see, e.g., Bai and Ng, 2006).

The conditional alpha estimator then is

$$\widehat{\alpha}_{i,t} = \widehat{m}_{i,T,M}(\boldsymbol{PC}_t) = \frac{\frac{1}{Th_T^k}\sum_{\ell=1}^{T-1}\widehat{Z}_{i,\ell+1,M}\boldsymbol{K}\left(\frac{\boldsymbol{PC}_\ell - \boldsymbol{PC}_t}{h_T}\right)}{\widehat{g}_T(\boldsymbol{PC}_t)}, \tag{8}$$

where $\widehat{g}_T(\boldsymbol{PC}_t) = \frac{1}{Th_T^k}\sum_{\ell=1}^{T-1}\boldsymbol{K}\left(\frac{\boldsymbol{PC}_\ell - \boldsymbol{PC}_t}{h_T}\right)$ is the kernel estimator of the density g of the vector of principal components. Note that we must establish a uniform result over the support of the principal components because they are stochastic processes. To circumvent the difficulty of estimating $m_i(\boldsymbol{pc})$ over regions of low density, we consider a trimmed version of $\widehat{m}_{i,T,M}(\boldsymbol{pc})$ as in Andrews (1995):

$$\text{tr}_T\{\widehat{m}_{i,T,M}(\boldsymbol{pc})\} = \widehat{m}_{i,T,M}(c)\,\mathbf{1}\left\{c \in \widehat{G}_T(\boldsymbol{pc})\right\}, \tag{9}$$

where $\widehat{G}_T(\boldsymbol{pc}) = \{c : \widehat{g}_T(\boldsymbol{pc}) > d_T\}$. Section 3 establishes the uniform consistency of $\widehat{m}_{i,T,M}(\boldsymbol{PC}_t)$ by showing that

$$\left(\int_{\mathbb{R}^k} |\text{tr}_T\left(\widehat{m}_{i,T,M}(\boldsymbol{pc})\right) - m_i(\boldsymbol{pc})|^Q\; g(\boldsymbol{pc})\,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} = o_p(1) \tag{10}$$

for some $Q > 0$ as $d_T \to 0$ at an appropriate rate.

Apart from the conditional alpha, we also consider Treynor and Black's (1973) appraisal ratio:

$$A_{i,t} = A_i(\boldsymbol{C}_t) = \frac{\mathbb{E}(Z_{i,t+1}|\boldsymbol{C}_t)}{\sqrt{\text{Var}(Z_{i,t+1}|\boldsymbol{C}_t)}} = \frac{\alpha_{i,t}}{\sqrt{\mathbb{E}(\epsilon_{i,t+1}^2|\boldsymbol{C}_t)}},$$

where $\epsilon_{i,t} = Z_{i,t} - \alpha_{i,t}$. By comparing the conditional alpha to the conditional idiosyncratic volatility, it gauges the extra units of unsystematic risk investors have to bear for deviating from a passive management strategy. We estimate the conditional appraisal ratio by means of

$$\widehat{A}_{i,t,M} = \widehat{A}_{i,T,M}(\boldsymbol{PC}_t) = \frac{\widehat{m}_{i,T,M}(\boldsymbol{PC}_t)}{\sqrt{\widehat{m}_{i,T,M}^{(2)}(\boldsymbol{PC}_t) - \widehat{m}_{i,T,M}^2(\boldsymbol{PC}_t)}}, \tag{11}$$

where $\widehat{m}_{i,T,M}(\boldsymbol{PC}_t)$ is given by (8) and

$$\widehat{m}_{i,T,M}^{(2)}(\boldsymbol{PC}_t) = \frac{\frac{1}{Th_T^k}\sum_{\ell=1}^{T-1}\widehat{Z}_{i,\ell+1,M}^2\boldsymbol{K}\left(\frac{PC_\ell - \boldsymbol{PC}_t}{h}\right)}{\widehat{g}_T(\boldsymbol{PC}_t)}.$$

The advantage of using appraisal ratios instead of alphas to pick individual stocks is that it puts some discipline on the portfolio selection process by focusing on the magnitude of the mispricing relative to the noise level.

3 Asymptotic Theory

In this section, we derive the consistency of the nonparametric estimators we propose for the conditional alpha and appraisal ratio. We first establish the asymptotic consistency of the infeasible estimator based on unobservable risk-adjusted returns:

$$\widetilde{\alpha}_{i,t+1} = \widetilde{m}_{i,T}(\boldsymbol{PC}_t) = \frac{\frac{1}{Th_T^k}\sum_{\ell=1}^{T-1} Z_{i,\ell+1}\boldsymbol{K}\left(\frac{\boldsymbol{PC}_\ell - \boldsymbol{PC}_t}{h_T}\right)}{\widehat{g}_T(\boldsymbol{PC}_t)}$$

and then show that proxying for the latter by the realized risk-adjusted returns has no asymptotic cost. For the consistency of the conditional alpha estimator, we require the following conditions to hold.

Assumption A

(i) For $i = 1,\ldots,N$, $\mu_{i,t}$, $\sigma_{i,t}$, and the elements of $\boldsymbol{\Sigma}_{i,t}$, $\mu_{F,t}$ and $\boldsymbol{\Sigma}_{F,t}$ in (1) and (2) are $\mathcal{F}_C(t)-$measurable and $\delta-$dominated, with $\delta > 2$.[4]

(ii) For $i = 1,\ldots,N$, $\mathbb{E}\left|Z_{i,t}\right|^{\delta} < \infty$ with the same $\delta > 2$ as in (i), and $\{Z_{i,t+1}, \boldsymbol{C}_t\}_{t=1}^{T-1}$ is strictly stationary and $\alpha-$mixing with coefficients α_j such that $\sum_{j=1}^{\infty}\alpha_j^{1-2/\delta} < \infty$.

(iii) The distribution of $\boldsymbol{C}_t$ with respect to the Lebesgue measure on $\mathbb{R}^{k_C}$ is absolutely continuous, with a bounded density ϕ that is twice continuously differentiable with bounded derivatives.

(iv) The multivariate kernel function $\boldsymbol{K}$ is a k-dimensional product kernel with marginal bounded density K, such that $\int_{\mathbb{R}} xK(x)\,\mathrm{d}x = 0$ and $\int_{\mathbb{R}} x^2K(x)\,\mathrm{d}x < \infty$. In addition, the univariate kernel K has an absolutely integrable characteristic function $\Psi(u) = \int_{\mathbb{R}}\exp(iux)K(x)\,\mathrm{d}x$ such that $\int_{\mathbb{R}}|\Psi(u)|\,\mathrm{d}u < \infty$.

(v) Let $m_i(\boldsymbol{pc}) = \mathbb{E}(Z_{i,t+1}|\boldsymbol{PC}_t = \boldsymbol{pc})$ for $i = 1,\ldots,N$. The product $m_i(\boldsymbol{pc})\,g(\boldsymbol{pc})$ is bounded and twice continuously differentiable on $\mathbb{R}^k$ with bounded derivatives for $i = 1,\ldots,N$.

[4] Recall that X_t is $\delta-$dominated if $|X_t| \leq D_t$ for some D_t such that $\mathbb{E}(D_t^{\delta}) < \infty$.

(vi) $\sup_{\boldsymbol{pc}\in\mathbb{R}^k} |m_i(\boldsymbol{pc})| < \infty$ for $i = 1, \ldots, N$ and $\int_{\mathbb{R}^k} g(\boldsymbol{pc})^{1-a}\, \mathrm{d}\boldsymbol{pc} < \infty$ for some $0 < a < 1$.

Assumptions A$(ii)-(iii)$ relate to the conditioning state variables $\boldsymbol{C}_t$ rather than to their first k principal components as the statistic in (10). Because each principal component is a linear combination of the $k_C < \infty$ elements of $\boldsymbol{C}_t$, it follows straightforwardly that $\boldsymbol{PC}_t$ is also $\alpha-$mixing, with the same mixing size as $\boldsymbol{C}_t$. This means that A(ii) also holds for $\boldsymbol{PC}_t$. The marginal density of each individual principal component is absolutely continuous because they all result from the direct convolution of the k_C absolutely continuous marginals of the conditioning instruments. As principal components are mutually orthogonal, their joint density is absolutely continuous on $\mathbb{R}^k$. Assumption A(iii) thus ensures that $\boldsymbol{PC}_t$ has a absolutely continuous density on $\mathbb{R}^k$. Assumption A(iv) is standard, holding for instance for the multivariate standard normal kernel we use in the empirical application. Assumptions A$(v)-(vi)$ regulate the smoothness and the boundedness of the regression function as well as the thickness of the tails of the regressors. For example, if $g(\boldsymbol{pc})$ is multivariate normal, $\int_{\mathbb{R}^k} g(\boldsymbol{pc})^{1-a}\, \mathrm{d}\boldsymbol{pc} < \infty$ holds for any a arbitrarily close to 1.

We now ready to document our first result that there is no asymptotic difference between the feasible and infeasible estimators.

Proposition 1: Let the conditions of Lemma 1 as well as Assumption A hold. If $T, M, h_T^{-1} \to \infty$ and $d_T^{-1}\left(M^{-1}h_T^{-k} + M^{-1/2}T^{-1/2}h_T^{-k}\right) \to 0$, then $\mathrm{tr}_T\{\widetilde{m}_{i,T}(\boldsymbol{pc})\} - \mathrm{tr}_T\{\widehat{m}_{i,T,M}(\boldsymbol{pc})\}$ is of order $o_p(1)$ uniformly in $\boldsymbol{pc} \in G_T(\boldsymbol{pc})$.

It is immediate to see that we allow M to grow at a slower rate than T. This is empirically very important. In addition, the conditions on the rate of growth of M are much weaker than in Corradi, Distaso and Swanson's (2009) Theorem 1. This happens because the estimation error in the betas affects only the dependent variable of the nonparametric regression, and hence it does not enter in the kernel function.

Proposition 2: Let the conditions of Lemma 1 as well as Assumption A hold. It then follows for a as in Assumption A(vi) and $0 < Q < \infty$ that

$$\left[\int_{\mathbb{R}^k} \left|\mathrm{tr}_T\left(\widehat{m}_{i,T,M}(\boldsymbol{pc})\right) - m_i(\boldsymbol{pc})\right|^Q g(\boldsymbol{pc})\, \mathrm{d}\boldsymbol{pc}\right]^{\frac{1}{Q}} = O_p(T^{-1/2}M^{-1/2}h_T^{-k} + M^{-k}h_T^{-k}) + O(h_T^2 d_T^{-2}) + O_p(d_T^{-2}T^{-1/2}h_T^{-k}) + O_p(d_T^{a/Q}).$$

In the sequel, we establish the consistency of the appraisal ratio estimator. To this end, we have to strengthen Assumption A so as to deal with the second moment of the risk-adjusted return, namely, on $m_i^{(2)}(\boldsymbol{pc}) = \mathbb{E}(Z_{i,t+1}^2 | \boldsymbol{PC}_t = \boldsymbol{pc})$.

Assumption B

(i) For $i = 1, \ldots, N$, $\mathbb{E}\,|Z_{i,t}|^{2\delta} < \infty$ with $\delta > 2$ and $\{Z_{i,t+1}, \boldsymbol{C}_t\}_{t=1}^{T-1}$ is strictly stationary and $\alpha-$mixing with coefficients α_j such that $\sum_{j=1}^{\infty} \alpha_j^{\frac{\beta-2}{\beta}} < \infty$.

(ii) For $i = 1, \ldots, N$, $m_i^{(2)}(\boldsymbol{pc})\, g(\boldsymbol{pc})$ is bounded and twice continuously differentiable on $\mathbb{R}^k$ and with bounded derivatives.

(iii) For $i = 1, \ldots, N$ and for some $0 < a < 1$, $\sup_{pc \in \mathbb{R}^k} \left| m_i^{(2)}(\boldsymbol{pc}) \right| < \infty$ and $\int_{\mathbb{R}^k} g(\boldsymbol{pc})^{1-a}\, \mathrm{d}\boldsymbol{pc} < \infty$.

Define $\mathrm{tr}_T\left(\widehat{A}_{i,T,M}(\boldsymbol{pc})\right) = \widehat{A}_{i,T,M}(\boldsymbol{pc})\, \mathbf{1}\left\{\boldsymbol{pc} \in \widehat{G}_T(\boldsymbol{pc})\right\}$. The next result ascertains the consistency of the trimmed nonparametric estimator of the appraisal ratio.

Proposition 3: Let the conditions of Lemma 1 as well as Assumptions A and B hold. It then follows for $0 < Q < \infty$ that

$$\left(\int_{\mathbb{R}^k} \left| \mathrm{tr}_T\left(\widehat{A}_{i,T,M}(\boldsymbol{pc})\right) - A_i(\boldsymbol{pc}) \right|^Q g(\boldsymbol{pc})\, \mathrm{dc}\right)^{\frac{1}{Q}} = O_p(T^{-1/2} M^{-1/2} h_T^{-k} + M^{-k} h_T^{-k}) + O_p(T^{-1/2} h_T^{-k} d_T^{-2}) + O(h_T^2 d_T^{-2}) + O_p(d_T^{a/Q}),$$

with a as in Assumption B(iii).

In the next section, we assess the performance of our estimators using real equity data. The idea is that, if our conditional alpha estimates are indeed uncovering pricing errors, they should predict to some extent future stock returns. To this end, we proceed in four steps. First, we compute daily factor loadings for the S&P 100 index constituents. Second, we estimate their conditional alphas and appraisal ratios by conditioning the corresponding risk-adjusted returns on a number of instruments. Third, we form portfolios that take long positions on undervalued stocks and short positions on overvalued stocks. Finally, we assess their performances to examine the economic relevance of the conditional alpha estimates.

4 Mispricing in the S&P 100 index constituents

To estimate the daily factor loadings, we must first define the risk factors we will consider. We employ a single-factor model using the S&P 500 index as proxy for the market portfolio. The main advantage is the ease of interpretation given that the conditional CAPM is a well-established model. In addition, it is much easier to find intraday data for a proxy of the market portfolio than for some of the other risk factors in the literature. We thus employ tick-by-tick data for the S&P 500 index as well as for each of the S&P 100 index constituents to estimate daily realized market betas. The sample period runs from 2 January 2001 to 30 December 2008, amounting to 1,915 trading days. We exploit as much as possible the richness of the tick-by-tick information by using a realized kernel approach with refreshing time. The latter is very convenient because it is robust to market microstructure noise as well as to nonsynchronous trading (see Appendix B for details).

Figure 1 displays the time series behavior of the cross-sectional quantiles of the daily betas we estimate. Cross-sectional dispersion seems fairly constant over time, even if the median beta increases significantly throughout 2002. There is also some evidence of co-movement among betas, with the first three principal components explaining over a third of the overall variation.

We next compute daily risk-adjusted returns for each stock in the sample. To obtain the pricing errors, we estimate the conditional expectation of the risk-adjusted returns using a kernel regression approach. In particular, we employ a multivariate standard Gaussian kernel with rule-of-thumb bandwidths. As for the amount of trimming in (9), uniform consistency requires $d_T = O(h_T^{k\zeta})$ with $0 < \zeta < 1/4$. Note that, because $h_T \to 0$ as $T \to \infty$, trimming becomes more aggressive as ζ declines. We set ζ to 0.20 in order to enforce only a mild trimming, though we also experiment with other values of ζ in Section 5.3. The daily frequency of the realized betas dictates the choice of the instruments, ruling out most macroeconomic variables as well as the predictors that Welch and Goyal (2008) use for the risk premium. In particular, we proxy the state variables using the following instruments: the changes in the VIX index and in the Fed Fund rate, the volatility risk premium (namely, difference between the VIX index and realized volatility), size and value factors, short- and long-term reversal factors, the credit and term spreads, and the momentum factor.

Figure 2 displays the time series behavior of the cross-sectional quantiles of the daily pricing errors. In contrast to the pattern we observe for the realized market betas, the cross-sectional dis-

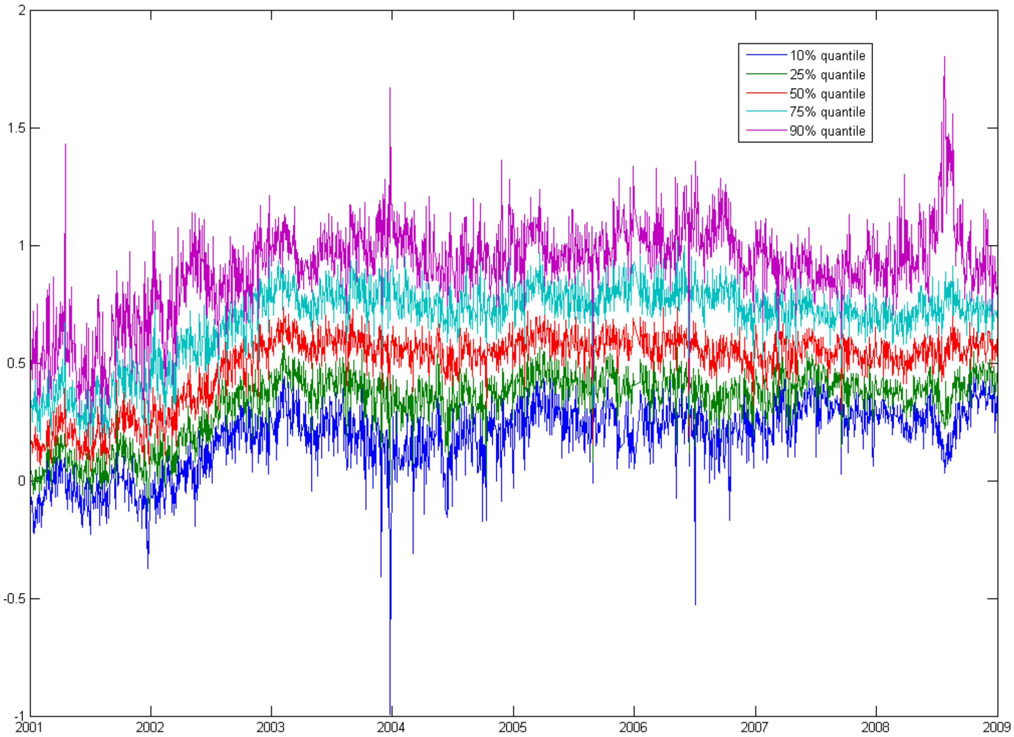


Figure 1: The cross-sectional quantiles of the daily realized betas

persion of the conditional alpha estimates is much more erratic, exhibiting a great deal of volatility clustering. In particular, cross-sectional dispersion is relatively much higher in the beginning and in the end of the sample period. Finally, there is also some moderate evidence of co-movement among alphas, with the first three principal components explaining over 45% of the overall variation. A similar picture arises if one considers affine alphas as opposed to nonparametric alphas. The only significant difference is that affine alphas co-move much more strongly, with the first three principal components responding for over 87% of the total variation.

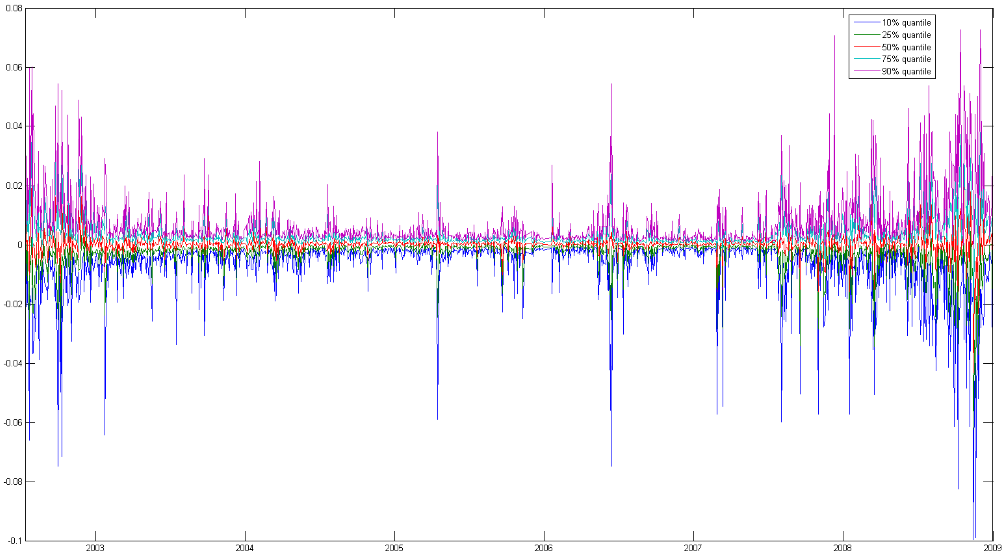
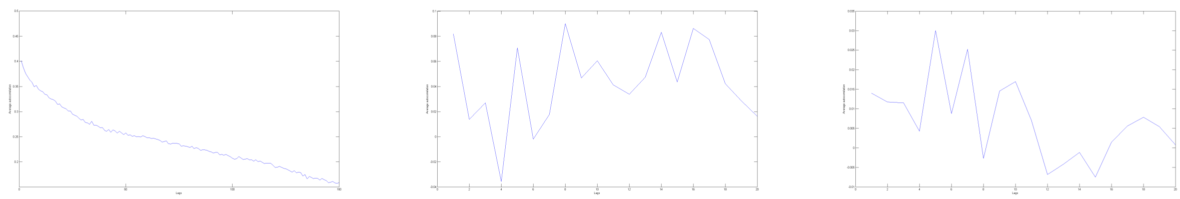


Figure 2: The cross-sectional quantiles of the nonparametric alpha estimates

Interestingly, daily realized betas feature a lot of persistence, displaying to some extent long range dependence. Figure 3(a) shows a slow decay in the average autocorrelation function, definitely slower than exponential. In contrast, Figure 3 shows that the average autocorrelation functions are quite low for both affine and nonparametric alpha estimates, especially for the latter.



(a) realized beta

(b) affine alpha

(c) nonparametric alpha

Figure 3: Average autocorrelation function of the daily realized betas and alpha estimates

We next examine whether the conditional alpha estimates are able to predict future returns by looking at average correlations at different horizons. Figure 4 reveals that the nonparametric alphas have a much better predictive power than their affine counterparts, even though they are less persistent. Although the average correlation of the affine alphas with contemporaneous returns are about threefold that of the nonparametric alphas, the latter entails much more predictive power. Their average correlations with future returns are around 2% up to 4 days ahead, whereas the average correlations for the affine alphas are virtually zero even at the one-day horizon.

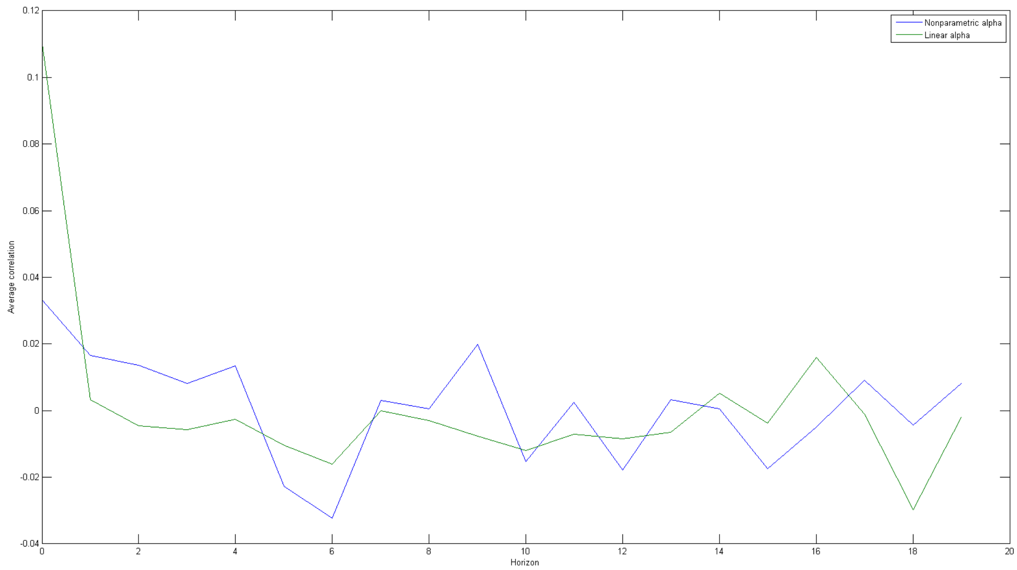


Figure 4: Average correlations between alphas and future returns across different horizons

Even if the nonparametric estimates of the conditional alphas have some significant predictive power in the statistical sense, it is not necessarily the case that the latter translates into economic significance. Apart from limits to arbitrage, the analysis so far is in sample, whereas a feasible trading strategy must rely exclusively on real-time information. In the next section, we develop a simple trading strategy that exploits the pricing errors we uncover by taking long positions on stocks with relatively high positive alphas and short positions on stocks with relatively high negative alphas.

5 Is it profitable to trade these pricing errors away?

If the conditional alpha estimates indeed correspond to pricing errors, we should be able to arbitrage them away. To this end, we may exploit either the cross-section or the time series information in the alphas. We start with the former by sorting stocks cross-sectionally according to alphas. We then take long positions on stocks within some upper quantile and short stocks within some lower quantile. This is similar in spirit to Jegadeesh and Titman's (1993) momentum strategy. There are two key differences, though. First, the signal is given by the alpha estimates rather than past raw returns. Second, we assume a much shorter holding period given the preliminary correlation analysis in Figure 4.

Alternatively, we may compare conditional alphas to their time-series quantiles. This would yield a sort of alpha-based counterpart to Moskowitz et al.'s (2012) time-series momentum strategy. In particular, we long any stock with alpha currently at some top quantile of the historic distribution and short any stock with alpha currently at some bottom quantile. Given that we observe some persistence in the nonparametric alphas, we expect the time-series momentum strategy to perform better than the cross-sectional for longer holding periods.

To ensure feasibility in practice, we form these long-short portfolios based on real-time estimates. To this effect, we estimate the pricing errors using an expanding window with an initial subsample of 375 daily observations. This means that the precision of the conditional alpha estimates increases over time as the sample size grows. In addition, we also consider the corresponding long-short strategies based on appraisal ratios. Finally, we contemplate variations of the alpha-based strategies in which we correct the alpha estimates by means of a risk-free rate adjustment.

5.1 Exploiting the cross-sectional information

In this section, we first describe in details the alpha-based strategies based on cross-sectional quantiles and then report their performance. The trading algorithm is very simple. We first nonparametrically estimate the daily conditional alphas of each stock using past data. We then daily sort the stocks by the value of their alphas so as to compute daily cross-sectional quintiles. We next form a long-short portfolio by purchasing the top quintile stocks and selling the bottom quintile stocks. We keep the portfolio for a given holding period, varying from 1 to 5 days, and then liquidate the position. To assess performance in a realistic fashion, we impose a transaction cost of 4 basis points to each trade. This is slightly conservative given that the S&P 100 index constituents are all actively traded large caps. Note also that, to ensure feasibility in practice, we employ only historic information in the estimation of the conditional alphas. In particular, we use an expanding window and hence the precision of the alpha estimates increases over time as the sample size grows.

Table 1 reports the main descriptive statistics. In line with the correlation analysis, the returns on the long-short cross-sectional momentum strategy seem to increase with the holding period until 3 days and then start declining. The same applies to the Sharpe ratio, which achieves its maximum of 0.90 at the 3-day holding period, even if the volatility of the strategy is monotonically increasing with the holding period. As expected, the long-short nature of the portfolio helps reduce the volatility of the strategy in a substantial manner. Individually, the long and short portfolios are much more volatile than the resulting long-short combination. Interestingly, the long-short strategy displays significant positive skewness for holding periods of up to 3 days, mostly due to the short positions on stocks with negative alpha.

Adjusting for the risk-free rate does not change qualitatively the big picture. Recall that we back out the conditional alphas from the daily risk-adjusted returns $Z_{i,t+1} \equiv r_{i,t+1} - \beta_{i,t}\, f_{t+1}$, where f_t denotes the returns on the S&P 500 index. Note that the returns are not in excess over the risk-free rate and hence, to obtain risk-adjusted excess returns, we would have to compute

$$\begin{aligned}\breve{Z}_{i,t+1} &\equiv r_{i,t+1} - r_{f,t+1} - \beta_{i,t}\,(f_{t+1} - r_{f,t+1}) = r_{i,t+1} - \beta_{i,t}\, f_{t+1} - (1-\beta_{i,t})\, r_{f,t+1} \\ &= Z_{i,t+1} - (1-\beta_{i,t})\, r_{f,t+1},\end{aligned}$$

where $r_{f,t+1}$ is the return on a risk-free investment from time t to $t+1$. The conditional expectation of the risk-adjusted excess returns is then equal to the unadjusted conditional alpha minus a predetermined correction term that depends both on the risk-free rate and on the market beta. We thus adjust our conditional alpha estimates for the risk-free rate by subtracting $(1-\widehat{\beta}_{i,t,M})r_{f,t+1}$, where $\widehat{\beta}_{i,t,M}$ is the realized market beta. To proxy for the risk-free rate $r_{f,t+1}$, we employ the 1-month Treasury bill rate available at Kenneth French's website.[5]

Because we are mostly interesting in the cross-sectional alpha quantiles (rather than on levels), the risk-free rate does not matter as much as the realized market beta given that only the latter

[5] See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

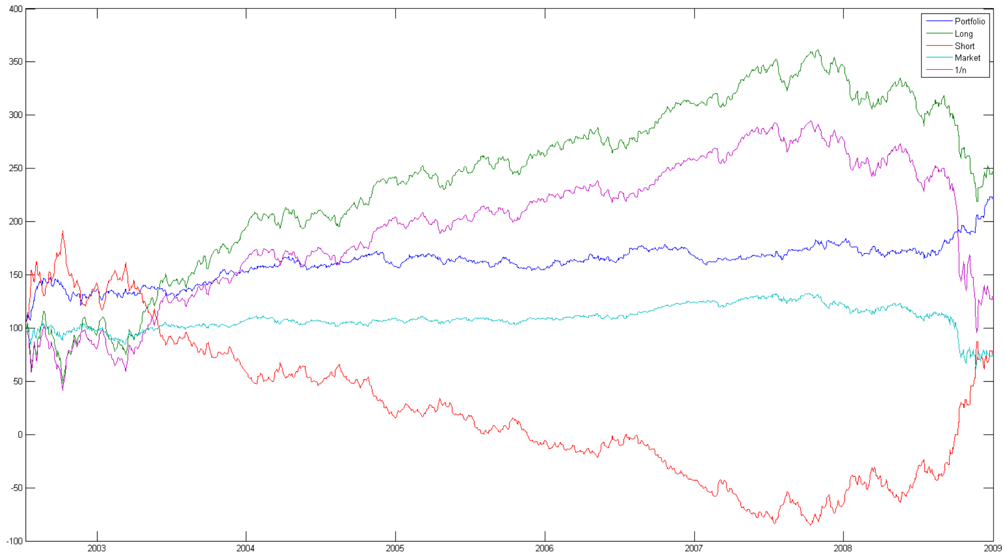


Figure 5: Cumulative returns to the cross-sectional momentum strategy with a 3-day holding period

does vary across stocks. However, the risk-adjusted return already reflects, even if only partially, the information on the market beta and hence it is not surprising that the qualitative results do not change if we focus on the risk-adjusted *excess* returns instead. Table 3 indeed reveals that the only difference the risk-free adjustment makes is that the realized returns become slightly lower.

As expected from Figure 4, putting together a long-short portfolio based on affine alphas (as opposed to nonparametric) does not yield as promising results. Tables 2 and 4) show a slight improvement in performance only for a holding period of one day, with nonparametric alphas clearly dominate otherwise. In particular, the cross-sectional momentum strategies based on an affine specification for the conditional alpha display higher volatility than their nonparametric counterparts.

To gauge the informational content in the conditional alphas, we next examine the performance of the cross-sectional momentum strategy relative to the S&P 100 index and to the equal-weight portfolio of all index constituents. Figure 5 plots the cumulative returns to the cross-sectional momentum strategy with a holding period of 3 days. It reveals the conditional alphas help select stocks with better future performance. The wedge between the cumulative returns of the long portfolio of the cross-sectional strategy and those of the equal-weight portfolio is indeed sizeable. The same pattern arises for different holding periods as well as for other variants of the strategy (e.g., with or without risk-free adjustment).

5.2 A time-series momentum strategy based on conditional alphas

We next engineer a trading strategy that exploits the time series properties of conditional alpha estimates. The motivation is to take benefit of the persistence of the nonparametric alphas by looking at the quantiles of the historical distribution of the alphas. So, what matters now is not whether a given stock currently has a very large alpha relative to the other stocks. We gauge the relevance of the pricing error relative to its past magnitudes. In particular, we go long in any stock whose alpha is currently at the top 25% of its historical distribution, and short any stock with alpha at the bottom quartile. Given that we expect more persistence in the time-series quantiles than in the cross-sectional quantiles, we now also contemplate longer holding periods before liquidating the position. We thus present results for holding periods of 1, 5, 10 and 22 days. Finally, we apply the same 4 basis points fee to each transaction.

Table 7 reports the main results. Both realized returns and volatility of the time-series strategy seem to increase with the holding period. The long-short strategies that rebalance their portfolios every 10 and 22 days achieve annualized average returns of 37% and 59%, with Sharpe ratios of 0.82 and 0.92, respectively. It is interesting to notice that skewness is once again always positive, whereas kurtosis decreases with the holding period. In general, the time-series momentum strategy based on conditional alphas entails higher returns than the cross-sectional strategy, probably because it captures some degree of persistence.

Differently from the cross-sectional strategy, correcting the conditional alpha estimates by the risk-free rate ameliorates the performance of the time-series momentum portfolio, essentially because it helps reduce the volatility of the strategy. As we are now looking at the quantiles of the historic distribution, the risk-free rate starts to matter as well as the dynamics of the realized market betas. Accordingly, the risk-free adjustment has a much higher impact than if we were restricting our attention to the cross-section of the pricing errors.

It is not surprising that the time-series momentum strategy yields a poorer performance if based on affine alphas given the lack of persistence in the latter. Tables 8 and 10 also reveal that the long portfolio of the strategy entails particularly low returns across all holding periods. In addition, as in the cross-sectional case, trading affine conditional alphas brings about much more volatility than

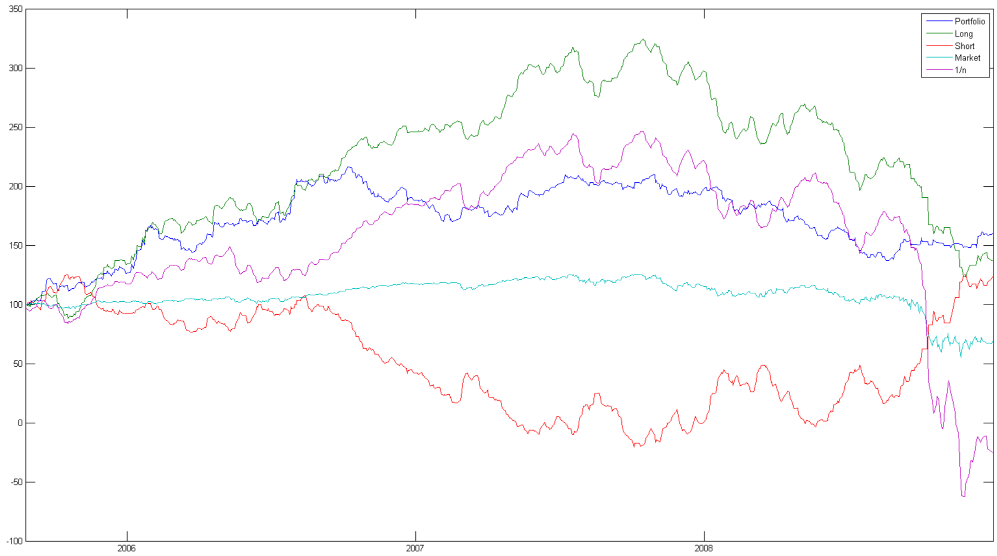


Figure 6: Cumulative returns of the time-series momentum strategy with a 5-day holding period

trading nonparametric alphas, evincing one more time the benefits of employing a more robust method to estimate the pricing errors.

All in all, the time-series information in the nonparametric conditional alphas helps identify the stocks that will outperform in the future. Figure 6 depicts the cumulative returns of the time-series momentum strategy with a 5-day holding period. Note that it starts much later than Figure 5 because we must reserve the beginning of the sample period in order to compute the quantiles of the historical distribution with a reasonable degree of precision. As in the cross-sectional strategy, it is striking how the short portfolio helps restrain the volatility of the overall strategy as well as safeguard the performance during the recent financial crisis. This is more than convenient given that the S&P 100 index and, especially, the equal-weight portfolio tank after the bankruptcy of Lehman Brothers in September 15, 2008.

5.3 Robustness checks

In this section, we further investigate our results through a number of robustness checks to make sure they are not an artifact due to, say, overfitting. Accordingly, we examine whether the findings are sensitive to changes in the holding periods, in the trimming of the nonparametric alpha estimator, and in the signal upon which we engineer the trading strategy.

We start with the holding period. In the absence of transaction costs, one could well rebalance the portfolio every day to fully exploit the daily changes in the conditional alphas. However, as soon as transaction costs are present, rebalancing too much would imply excessive turnover. This raises a tradeoff between shortening the holding period to trade current (rather than past) pricing errors and lengthening the holding period to mitigate transaction costs. Our choice of the holding periods to report for the cross-sectional and time-series momentum strategies follows from the preliminary correlation analysis between current and past alphas, and future returns. It is nonetheless important to stress that the trends we observe in the performance analyses remain valid for different holding periods.

As for the amount of trimming, it turns out that the performances of both cross-sectional and time-series momentum strategies hardly change if we implement a more aggressive trimming scheme (namely, $\zeta = 0.01, 0.10$). However, they worsen a bit if we employ no trimming in the nonparametric estimation of the conditional alphas. This happens because sometimes the conditional alpha estimates are large only because of a very low value of the denominator in (8). This is exactly the sort of situation that trimming avoids, for it sets the alpha to zero if there is too much uncertainty (i.e., very low density of the conditioning instruments). Accordingly, trimming puts some discipline in the conditional alpha estimates, improving the performance of the alpha-based strategies.

Alternatively, we may account for the uncertainty in the estimation of the conditional alphas by using appraisal ratios as a signal for the trading strategy. The appraisal ratio standardize the alpha by the level of idiosyncratic volatility as in (11. This means assigning more weight to large positive alphas in periods of low uncertainty. Tables 5 and 6 respectively report the results for the cross-sectional momentum strategies based on the nonparametric and affine appraisal ratios, whereas Tables 11 and 12 document the performance of their time-series counterparts.

We uncover some interesting evidence. Building a cross-sectional strategy on the basis of the appraisal ratios reduces both returns and volatility, with a net effect of a lower Sharpe ratio. Using affine appraisal ratios nonetheless seems to improve the short end of the strategy. Conversely, for the time-series momentum strategies, the reduction in the volatility is substantial enough to compensate the decline in the realized returns. In other words, the net effect is that Sharpe ratios do increase. In contrast to what happens with the nonparametric alphas, the absence of trimming does not affect much the strategies based on appraisal ratios for they already control to some extent for the uncertainty in the nonparametric estimation.

5.4 Multifactor analysis

Although the cross-sectional and time-series momentum strategies based on conditional alpha estimates perform well at first glance, it remains to show that it cannot be easily reproduced following a passive strategy, such as traditional momentum. In what follows, we check whether they have significant exposure to the usual risk factors, namely, the market, size, value and momentum factors also available at Kenneth French's website. To take into account the likely temporal dependence among daily returns due to overlapping trading periods for multiple days holding periods, we use Newey-West standard errors to compute robust t-ratios.

Tables 1 to 6 show in their Panel A that the cross-sectional momentum strategies yield positive unconditional alphas (i.e., intercepts of the multifactor regression). This does not mean much given that the risk factors are not tradeable and hence do not consider transaction costs (Berk and van Binsbergen, 2011)see, for instance, the excellent discussion in. In addition, these intercepts are significant only for the nonparametric approach and not always. The affine specification for the pricing errors are never significant, on the other hand. More importantly, the cross-sectional momentum strategy does not load on the Fama-French portfolios, with the exception perhaps to the positive market beta of the alpha-based strategy that corrects for the risk-free rate.

The story is not so simple for the momentum factor. This is not surprising given that our strategies are momentum-based, even if we do not sort the stocks by raw returns. The cross-sectional strategies based on affine specifications have always a significantly positive loading on the momentum portfolio, whereas the evidence is much milder for the nonparametric alphas and appraisal ratios. It is also worth stressing that the risk factors do not explain more than a tiny portion of the variation of the realized returns on the cross-sectional strategies.

Tables 7 to 12) reveal a few negative alphas, especially for short holding periods (reflecting transaction costs) and for affine signals. As the time-series momentum strategies exhibit much higher volatility, it is not surprising that the intercepts of the multifactor regression are insignificant. The same largely applies to the three Fama-French factors as well. Interestingly, the loadings on the momentum factor is markedly different from the previous case. The sign is not always positive and, most importantly, there are not many significant coefficient estimates. Accordingly, the fit of the multifactor models is even poorer than before.

All in all, the findings of the above multifactor analysis confirm the dominance of the trading strategies rooted in the nonparametric conditional alphas (as long as we employ some trimming). In particular, the stars are the cross-sectional strategy with a holding period of 3 days and the time-series momentum strategy with a holding period of 22 days. The former entails the highest annualized unconditional alpha as well as the highest appraisal, Sharpe and Sortino ratios among the cross-sectional strategies (0.90, 22.40, and 0.86, respectively). The only significant factor loading is for the momentum portfolio, with a coefficient of 0.15. The latter yields a striking annualized alpha of 60.93 (though insignificant), with a Sharpe ratio of 0.92, Sortino ratio of 21.75, and appraisal ratio of 0.80. In addition, it does not load on any of the usual risk factors in a significant manner.

6 Conclusion

This paper shows how to identify pricing errors within a multifactor asset pricing model. The procedure is in two steps. We first estimate the conditional factor loadings (e.g., the market beta in a conditional CAPM world) so as to control for the amount of risk. Once we risk-adjust the returns, we then regress them on the state variables to back out the conditional alpha or pricing error. Despite the simplicity of our two-stage estimator, it requires thinking very carefully about the joint dynamics of factors and stock prices. In particular, we start with a multivariate diffusion process in continuous time for which exact discretization yields the conditional multifactor asset pricing model of interest. To accomplish this, we must assume that the drift and diffusive parameters are measurable functions of state variables that change only in discrete time. Accordingly, the factor loadings are also constant over shorter periods of time and hence we may estimate them using high-frequency data by means of a realized approach. We provide conditions under which the error we make in the realized estimation of the factor loadings does not affect the consistency of the nonparametric regression we do to retrieve the conditional alpha.

To assess empirical relevance, we estimate daily conditional alphas for the S&P 100 index constituents within a conditional CAPM world. To proxy for state variables, we employ as instruments size, value, momentum and reversal mimicking portfolios as well as some interest-rate spreads and market volatility measures. Although the resulting conditional alpha estimates exhibit little persistence (even if they oscillate around zero), we nonetheless find that they correlate with future stock returns in a significant manner. We thus check how exploiting the information in the conditional alpha estimates would fair in practice. All in all, we find that momentum-type strategies based on the conditional alpha estimates perform pretty well. Returns are indeed very high as well as Sharpe and Sortino ratios, even if volatility is also high.

Interestingly, a traditional multifactor analysis reveals positive unconditional alphas as well as very little exposure to the market, size, value and momentum risk factors. The latter is a key result in that, if the true model were a conditional multifactor pricing model (rather than a conditional CAPM), our conditional alpha estimates would then capture the exposure to the risk factors other than the market portfolio. The fact that our alpha-based momentum strategies do not have significant exposure to size, value and momentum effects seems to attest that the conditional CAPM assumption is not so restrictive after all.

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Appendix A

Proof of Lemma 1: Define the filtered probability space $\mathcal{B} = \left(\Omega^B, \mathcal{F}^B, (\mathcal{F}_s^B)_{s\geq 0}, \mathcal{P}^B\right)$, where $\mathcal{F}_s^B = \sigma(\boldsymbol{C}_\tau, \tau \leq s)$, $s \in \mathbb{R}_+$, $\boldsymbol{C}_\tau = \boldsymbol{C}_t$ for $\tau \in [t, t+1)$. Also, define the filtered probability space $\mathcal{A}^i = \left(\Omega^{A,i}, \mathcal{F}^{A,i}, (\mathcal{F}_s^{A,i})_{s\geq 0}, \mathcal{P}^{A,i}\right)$, where $\mathcal{F}_s^{A,i} = \sigma(\boldsymbol{W}_\tau, \tau \leq s)$ with $\boldsymbol{W}_\tau = (W_{i,\tau}, \boldsymbol{W}_{F,\tau})$. Given the independence between $\boldsymbol{C}_\tau$ and $\boldsymbol{W}_\tau$, we can define the enlarged filtered probability space $\widetilde{\mathcal{B}} = \left(\widetilde{\Omega}, \widetilde{\mathcal{F}}, (\widetilde{\mathcal{F}}_s)_{s\geq 0}, \widetilde{\mathcal{P}}\right)$, where $\widetilde{\Omega} = \Omega^B \times \Omega^{A,i}$, $\widetilde{\mathcal{F}} = \mathcal{F}^B \otimes \mathcal{F}^{A,i}$, $\widetilde{\mathcal{F}}_s = \cap_{\tau > s} \mathcal{F}_\tau^B \otimes \mathcal{F}_\tau^{A,i}$, and $\widetilde{\mathcal{P}}\left(\mathrm{d}\omega^B \, \mathrm{d}\omega^{A,i}\right) = \mathcal{P}^B\left(\mathrm{d}\omega^B\right)\mathcal{P}^{A,i}\left(\mathrm{d}\omega^{A,i}\right)$, with $\omega^B \in \Omega^B$, $\omega^{A,i} \in \Omega^{A,i}$, and with $\otimes$ denoting the product measure. Now, $X_i(s) = (P_i(s), \boldsymbol{F}(s))$ can be defined on the enlarged filtered probability space $\widetilde{\mathcal{B}}$ and, in fact, $X_i(s)$ is $\widetilde{\mathcal{F}}_s-$measurable. Because $\boldsymbol{C}_\tau$ is independent of $\boldsymbol{W}_\tau$, it also follows that all measurable function of $\boldsymbol{C}_\tau$ are independent of $\boldsymbol{W}_\tau$. Let then $\boldsymbol{\Sigma}_{A,i,t} = (\mu_{i,t}, \mu_{F,t}, \sigma_{i,t}, \boldsymbol{\Sigma}_{i,t}, \boldsymbol{\Sigma}_{F,t})$ and define $\boldsymbol{\Sigma}_{A,i,\tau} = \boldsymbol{\Sigma}_{A,i,t}$ for $t \leq \tau < t+1$. Note also that $\boldsymbol{\Sigma}_{A,i,\tau}$ is also independent of W_τ and hence, for each $\omega^B \in \Omega^B$, except of a set of $\mathcal{P}^B-$zero probability, $X_i(s)$ is a conditional semimartingale with independent increments (Jacod, 1997). ■

Hereafter, for notational simplicity, we denote the orthogonal factors by $\boldsymbol{f}_{t+1}$ and the vector of corresponding realized betas by $\widehat{\boldsymbol{\beta}}_{t,M}$.

Proof of Proposition 1: Given A(iv), $\boldsymbol{K}$ is a bounded density function, and so

$$\left|\operatorname{tr}\left(\widetilde{m}_{i,T,M}(c)\right) - \operatorname{tr}\left(\widehat{m}_{i,T,M}(c)\right)\right| \leq c\, d_T^{-1} \left| \frac{1}{Th_T^k} \sum_{t=1}^{T-1} \boldsymbol{f}_{t+1}' \left(\widehat{\boldsymbol{\beta}}_{i,t,M} - \boldsymbol{\beta}_{i,t}\right) \right|,$$

where c is a positive constant. Letting $f_{1,t}$ denote the first element of $\boldsymbol{f}_t$, the first component of the vector $\widehat{\boldsymbol{\beta}}_{i,t,M}$ is given by

$$\widehat{\beta}_{i,t,M}^{(1)} = \frac{\sum_{j=0}^{M-1} \left(P_{i,t+(j+1)/M} - P_{i,t+j/M}\right)\left(f_{1,t+(j+1)/M} - f_{1,t+j/M}\right)}{\sum_{j=0}^{M-1} \left(f_{1,t+(j+1)/M} - f_{1,t+j/M}\right)^2}, \tag{12}$$

and, given that $\boldsymbol{\Sigma}_{F,t}$ is a diagonal matrix,

$$\beta_{i,t}^{(1)} = \frac{\boldsymbol{\Sigma}_{i,t}^{(1)} \boldsymbol{\Sigma}_{F,t}^{(1)}}{\boldsymbol{\Sigma}_{FF,t}^{(1,1)}}, \tag{13}$$

where $\boldsymbol{\Sigma}_{i,t}^{(1)}$ and $\boldsymbol{\Sigma}_{F,t}^{(1)}$ are respectively the first elements of $\boldsymbol{\Sigma}_{i,t}$ and $\boldsymbol{\Sigma}_{F,t}$, and $\boldsymbol{\Sigma}_{FF,t}^{(1,1)}$ is the first entry of $\boldsymbol{\Sigma}_{FF,t} = \boldsymbol{\Sigma}_{F,t}\boldsymbol{\Sigma}_{F,t}'$. It thus suffices to show that $\frac{1}{Th_T^k} \sum_{t=1}^{T-1} f_{1,t+1} \left(\widehat{\beta}_{i,t,M}^{(1)} - \beta_{i,t}^{(1)}\right) = o_p(d_T)$. To this end, note that

$$\frac{1}{Th_T^k}\sum_{t=1}^{T-1} f_{1,t+1}\left(\widehat{\beta}_{i,t,M}^{(1)}-\beta_{i,t}^{(1)}\right)=\frac{1}{Th_T^k}\sum_{t=1}^{T-1}\mu_{f_1}\left(\widehat{\beta}_{i,t,M}^{(1)}-\beta_{i,t}^{(1)}\right) \tag{14}$$

$$+\frac{1}{Th_T^k}\sum_{t=1}^{T-1}(f_{1,t+1}-\mu_{f_1})\left(\widehat{\beta}_{i,t,M}^{(1)}-\beta_{i,t}^{(1)}\right)$$

$$=I_{T,M,h}^{(1)}+I_{T,M,h}^{(2)}, \tag{15}$$

where $\mu_{f_1}=\mathbb{E}(f_{1,t})$. Given (12) and (13),

$$\widehat{\beta}_{i,t,M}^{(1)}-\beta_{i,t+1}^{(1)}=\frac{1}{\boldsymbol{\Sigma}_{FF,t}^{(1,1)}}\left(\sum_{j=0}^{M-1}\Delta P_{i,t+(j+1)/M}\Delta f_{1,t+(j+1)/M}-\boldsymbol{\Sigma}_{i,t}^{(1)}\boldsymbol{\Sigma}_{F,t}^{(1)}\right)$$

$$+\frac{\sum_{j=0}^{M-1}\left(\Delta^2 f_{1,t+(j+1)/M}-\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\right)}{\sum_{j=0}^{M-1}\Delta^2 f_{1,t+(j+1)/M}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}}\sum_{j=0}^{M-1}\Delta P_{i,t+(j+1)/M}\Delta f_{1,t+(j+1)/M}$$

$$=A_{t,M}+B_{t,M}, \tag{16}$$

where $\Delta f_{1,t+(j+1)/M}=f_{1,t+(j+1)/M}-f_{1,t+j/M}$, $\Delta^2 f_{1,t+(j+1)/M}=\left(f_{1,t+(j+1)/M}-f_{1,t+j/M}\right)^2$ and $\Delta P_{i,t+(j+1)/M}=P_{i,t+(j+1)/M}-P_{i,t+j/M}$. Using (1) and (2) then yields

$$\Delta P_{i,t+(j+1)/M}=\frac{1}{M}\mu_{i,t}+\sigma_{i,t}\Delta W_{i,t+(j+1)/M}+\boldsymbol{\Sigma}_{i,t}'\Delta\boldsymbol{W}_{F,t+(j+1)/M}$$

and

$$\Delta f_{1,t+(j+1)/M}=\frac{1}{M}\mu_{1,F,t}+\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\Delta\boldsymbol{W}_{F,t+(j+1)/M}(1)$$

with $\Delta\boldsymbol{W}_{F,t+(j+1)/M}(1)$ denoting the first element of $\Delta\boldsymbol{W}_{F,t+(j+1)/M}$. Assumption A($i$) then ensures that

$$\frac{1}{Th_T^k}\sum_{t=1}^{T-1}A_{t,M}=\frac{1}{Th_T^k}\sum_{t=1}^{T-1}\frac{1}{\boldsymbol{\Sigma}_{FF,t}^{(1,1)}}\sum_{j=0}^{M-1}\Bigg\{\sigma_{i,t}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}W_{i,s}\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}\boldsymbol{W}_{F,s}^{(1)}$$

$$+\boldsymbol{\Sigma}_{i,t}^{(1)}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\left[\left(\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}\boldsymbol{W}_{F,s}^{(1)}\right)^2\right]-\frac{1}{M}\Bigg\}[1+o_p(1)]+O_p(M^{-1}h_T^{-k})$$

$$=\frac{1}{Th_T^k}\sum_{t=1}^{T-1}\mu_{FF(1,1)}^{-1}\sum_{j=0}^{M-1}\Bigg\{\sigma_{i,t}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}W_{i,s}\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}\boldsymbol{W}_{F,s}^{(1)}$$

$$+\boldsymbol{\Sigma}_{i,t}^{(1)}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\left[\left(\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}\boldsymbol{W}_{F,s}^{(1)}\right)^2-\frac{1}{M}\right]\Bigg\}[1+o_p(1)]+O_p(M^{-1/2}h_T^{-k})$$

$$=\frac{1}{Th_T^k}\sum_{t=1}^{T-1}\widetilde{A}_{t,M}[1+o_p(1)]+O_p(M^{-1}h_T^{-k}),$$

where $\mu_{FF(1,1)} = \mathbb{E}\left[\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\right]$. Note that the $o_p(1)$ and $O_p\left(M^{-1}h_T^{-k}\right)$ terms capture the cross term and drift contributions, respectively. Now, define

$$u_{j,t,M} = \sigma_{i,t}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}W_i(s)\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}\boldsymbol{W}_{F,s}^{(1)} + \boldsymbol{\Sigma}_{i,t}^{(1)}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\left[\left(\int_{t+j/M}^{t+(j+1)/M}\mathrm{d}\boldsymbol{W}_{F,s}^{(1)}\right)^2 - \frac{1}{M}\right].$$

It is immediate to see that $\mathbb{E}(u_{j,t,M}) = 0$. By Lemma 1, for $t \neq \iota$ and/or $k \neq j$, $\mathbb{E}(u_{j,t,M}\, u_{k,\iota,M}) = 0$, whereas $\mathbb{E}(u_{j,t,M}^2) = \frac{1}{M^2}\left(\sigma_{i,t}\boldsymbol{\Sigma}_{FF,t}^{(1,1)} + \boldsymbol{\Sigma}_{i,t}^{(1)}\boldsymbol{\Sigma}_{FF,t}^{(1,1)}\right)^2$ by Assumption A(i). This implies that

$$\begin{aligned}\mathrm{Var}\left(\frac{1}{Th_T^k}\sum_{t=1}^{T-1}\widetilde{A}_{t,M}\right) &= \mu_{FF(1,1)}^{-2}\,\frac{1}{T^2h_T^{2k}}\sum_{t=1}^{T-1}\sum_{\iota=1}^{T-1}\mathbb{E}\left(\sum_{j=0}^{M-1}u_{j,t,M}\sum_{j=0}^{M-1}u_{j,\iota,M}\right)\\ &= \mu_{FF(1,1)}^{-2}\frac{1}{T^2h_T^{2k}}\sum_{t=1}^{T-1}\mathbb{E}\left(\sum_{j=0}^{M-1}u_{j,t,M}^2\right) = O\left(\frac{1}{TMh_T^{2k}}\right),\end{aligned}$$

and so $\frac{1}{Th_T^k}\sum_{t=1}^{T-1}A_{t,M} = O_p(T^{-1/2}M^{-1/2}h_T^{-k}) + O_p(M^{-1}h_T^{-k})$. It follows from a similar argument that $\frac{1}{Th_T^k}\sum_{t=1}^{T-1}B_{t,M} = O_p(T^{-1/2}M^{-1/2}h_T^{-k}) + O_p(M^{-1}h_T^{-k})$. The first term on the RHS of (15) then is $I_{T,M,h}^{(1)} = O_p(T^{-1/2}M^{-1/2}h_T^{-k}) + O_p\left(M^{-1}h_T^{-k}\right)$. To complete the proof, it suffices to appreciate that

$$I_{T,M,h}^{(2)} \leq \left[\frac{1}{Th_T^k}\sum_{t=1}^{T-1}(f_{1,t} - \mu_{f_1})^2\right]^{1/2}\left[\frac{1}{Th_T^k}\sum_{t=1}^{T-1}\left(\widehat{\beta}_{i,t,M}^{(1)} - \beta_{i,t}^{(1)}\right)^2\right]^{1/2} = O_p\left(T^{-1/2}M^{-1/2}h_T^{-k}\right). \quad \blacksquare$$

Proof of Proposition 2: We know from Proposition 1 that

$$\begin{aligned}\mathbb{E}^{\frac{1}{Q}}\left|\mathrm{tr}_T\left(\widehat{m}_{i,T,M}(\boldsymbol{pc})\right) - m_i(\boldsymbol{pc})\right|^Q - \mathbb{E}^{\frac{1}{Q}}\left|\mathrm{tr}_T\left(\widetilde{m}_{i,T,M}(\boldsymbol{pc})\right) - m_i(\boldsymbol{pc})\right|^Q &= O_p(T^{-1/2}M^{-1/2}h_T^{-k})\\ &\quad + O_p(M^{-k}h_T^{-k}).\end{aligned}$$

It thus remains to show that

$$\mathbb{E}^{\frac{1}{Q}}\left|\mathrm{tr}_T\left(\widetilde{m}_{i,T,M}(\boldsymbol{pc})\right) - m_i(\boldsymbol{pc})\right|^Q = O_p(T^{-1/2}h_T^{-k}d_T^{-2}) + O(h_T^2d_T^{-2}) + O_p(d_T^{a/Q}).$$

Note that, given (9),

$$\begin{aligned}\mathbb{E}^{\frac{1}{Q}}\left|\mathrm{tr}_T\left(\widetilde{m}_{i,T,M}(\boldsymbol{pc})\right) - m_i(\boldsymbol{pc})\right|^Q &\leq \left(\int_{\mathbb{R}^k}\mathbf{1}\left\{\widehat{g}_T(\boldsymbol{pc}) \geq d_T\right\}\left|\widetilde{m}_{i,T}(\boldsymbol{pc}) - m_i(\boldsymbol{pc})\right|^Q g(\boldsymbol{pc})\,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}}\\ &\quad + \left(\int_{\mathbb{R}^k}\mathbf{1}\left\{\widehat{g}_T(\boldsymbol{pc}) < d_T\right\}\left|m_i(\boldsymbol{pc})\right|^Q g(\boldsymbol{pc})\,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}}\\ &= \Delta_{1T} + \Delta_{2T}.\end{aligned} \tag{17}$$

The statement then follows from Theorem 1(b) and Corollary 1 in Andrews (1995) provided that his assumptions NP1-NP7 hold for $\eta = \infty$ and $d_{1,T} = d_{2,T} = d_T$. Assumption A(ii) implies NP1 with

$\eta = \infty$ given that $(Z_{i,t+1}, \boldsymbol{C}_t)$ is $\alpha-$mixing (rather than near epoch dependent on a mixing basis). Assumption A(iii) implies NP2, whereas A(iv) is equivalent to NP4 in a strong mixing context as we are not dealing with adaptive (random) bandwidths. Similarly, as we use the same deterministic bandwidth for both numerator and denominator of the conditional expectation estimator, it is not necessary to require NP5. Moreover, both cross-validation and plug-in bandwidths satisfy NP5 in a straightforward manner. Assumption A(v) is equivalent to NP3 for $\lambda = 0$ and $\omega = 2$. Assumptions A(ii) to A(v) thus ensure that the statement in Andrews's (1995) Theorem 1(b) hold and so $\Delta_{1T} = O_p(T^{-1/2}h_T^{-k}d_T^{-2}) + O(h_T^2 d_T^{-2})$. Further, the trimming device in (9) suffices for the validity of NP6, whereas Assumption A(vi) is equivalent to NP7. Andrews's (1995) Theorem 1(a) establishes that $\sup_{\boldsymbol{pc}\in\mathbb{R}^k} |\widehat{g}_T(\boldsymbol{pc}) - g(\boldsymbol{pc})| = o_p(d_T)$ as long as d_T is such that $\Delta_{1T} = o_p(1)$. This means that

$$\begin{aligned}\Delta_{2T} &\leq \sup_{\boldsymbol{pc}\in\mathbb{R}^k} |m_i(\boldsymbol{pc})|^Q \left(\int_{\mathbb{R}^k} d_T^a \frac{g(\boldsymbol{pc})}{\widehat{g}_T^a(\boldsymbol{pc})} \,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} \\ &\leq \sup_{\boldsymbol{pc}\in\mathbb{R}^k} |m_i(\boldsymbol{pc})|^Q \left(\int_{\mathbb{R}^k} d_T^a \,[g(\boldsymbol{pc})]^{1-a} \,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} \{1 + o_p(d_T)\} = O_p(d_T^{a/Q}),\end{aligned}$$

completing the proof. ■

Proof of Proposition 3: By the same argument as in Proposition 1, uniformly in $\boldsymbol{pc}$,

$$\mathrm{tr}_T\left(\widehat{A}_{i,T,M}(\boldsymbol{pc})\right) - \mathrm{tr}_T\left(\widetilde{A}_{i,T}(\boldsymbol{pc})\right) = O_p(T^{-1/2}M^{-1/2}h_T^{-k} + M^{-k}h_T^{-k}),$$

where $\widetilde{A}_{i,T}(\boldsymbol{pc})$ is the infeasible counterpart of $\widehat{A}_{i,T,M}(\boldsymbol{pc})$ that uses the true risk-adjusted return. Along the same lines of the proof of Proposition 2,

$$\begin{aligned}\mathbb{E}^{\frac{1}{Q}}\left|\mathrm{tr}_T\left(\widetilde{A}_{i,T}(\boldsymbol{pc})\right) - A_i(\boldsymbol{pc})\right|^Q &\leq \left(\int_{\mathbb{R}^k} \mathbf{1}\{\widehat{g}_T(\boldsymbol{pc}) \geq d_T\}\left|\widetilde{A}_{i,T}(\boldsymbol{pc}) - A_i(\boldsymbol{pc})\right|^Q g(\boldsymbol{pc})\,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} \\ &\quad + \left(\int_{\mathbb{R}^k} \mathbf{1}\{\widehat{g}_T(\boldsymbol{pc}) < d_T\}\,|A_i(\boldsymbol{pc})|^Q \,g(\boldsymbol{pc})\,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} \qquad (18)\end{aligned}$$

Given Assumption B, by Theorem 1(b) in Andrews (1995),

$$\sup_{\{\boldsymbol{pc}:\, \widehat{g}_T(\boldsymbol{pc}) > d_T\}} \left|\widetilde{m}_{i,T}^{(2)}(\boldsymbol{pc}) - m^{(2)}(\boldsymbol{pc})\right| = O_p(T^{-1/2}h_T^{-k}d_T^{-2}) + O(h_T^2 d_T^{-2}),$$

and so the first term on the RHS of (18) is $O_p(T^{-1/2}h_T^{-k}d_T^{-2}) + O(h_T^2 d_T^{-2})$. As for the second term, applying the same argument as in the proof of Proposition 2 yields

$$\begin{aligned}\left(\int_{\mathbb{R}^k} \mathbf{1}\{\widehat{g}_T(\boldsymbol{pc}) < d_T\}\,|A_i(\boldsymbol{pc})|^Q \,g(\boldsymbol{pc})\,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} &\leq \sup_{\boldsymbol{pc}\in\mathbb{R}^k} |A_i(\boldsymbol{pc})|^Q \left(\int_{\mathbb{R}^k} d_T^a \frac{g(\boldsymbol{pc})}{\widehat{g}_T^a(\boldsymbol{pc})} \,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} \\ &\leq \sup_{\boldsymbol{pc}\in\mathbb{R}^k} |A_i(\boldsymbol{pc})|^Q \left(\int_{\mathbb{R}^k} d_T^a \,[g(\boldsymbol{pc})]^{1-a} \,\mathrm{d}\boldsymbol{pc}\right)^{\frac{1}{Q}} \{1 + o_p(d_T)\},\end{aligned}$$

which is of order $O_p(d_T^{a/Q})$ given Assumption A(vi) and B(iii). This completes the proof. ■

Appendix B

The realized beta estimator in (3) implicitly assumes that, for each day t, we record all asset prices and factors at the same frequency j/M, with $j = 1, \ldots, M$. In practice, however, we observe asset prices and factors only up to a market microstructure noise, and also at time intervals of irregular lengths. The assumptions of synchronicity and absence of microstructure noise are perhaps reasonable for a low enough intraday frequency (say, 30 minutes), but not at the ultra-high frequency. To account for both nonsynchronous trading and microstructure noise, we use Barndorff-Nielsen et al.'s (2011) multivariate realised kernel approach in the empirical sections. In what follows, we briefly describe the realized kernel estimator for the factor loadings and then establish the conditions under which Propositions 1 to 3 remain valid.

We start with the notion of refresh time (Harris, McInish, Shoesmith and Wood, 1995; Barndorff-Nielsen et al., 2011). For each asset i, the first refresh time $\tau_{i,t,1} \in [t, t+1]$ is the time it takes to observe the opening trades of the asset and of all factors. We now define $\tau_{i,t,2} \in (\tau_{i,t,1}, t+1]$ as the minimum time it takes to observe at least one additional trade for the asset i as well as for all factors. We proceed computing subsequent refresh times $\tau_{i,t,\ell+1} \in (\tau_{i,t,\ell+1}, t+1]$ with $\ell = 2, 3, \ldots$ until we reach the end of day t. This results in a set of refresh times $(\tau_{i,t,1}, \tau_{i,t,2}, \ldots, \tau_{i,t,M_{i,t}}$, so that $M_{i,t}$ corresponds to the effective sample of intraday observations that we have to estimate the daily factor loadings. The crucial assumption is that the refresh times are independent of asset prices and returns.

Further, to allow for market microstructure noise, we consider $\widetilde{P}_{i,\tau_{i,t,\ell}} = P_{i,\tau_{i,t,\ell}} + \eta_{i,\tau_{i,t,\ell}}$ and $\widetilde{F}_{j,\tau_{i,t,\ell}} = F_{j,\tau_{i,t,\ell}} + \eta_{F_j,\tau_{i,t,\ell}}$ for $\ell = 1, \ldots, M_{i,t}$ and $j = 1, \ldots, k_F$. The error terms $\eta_{i,\tau_{i,t,\ell}}$ and $\eta_{F_j,\tau_{i,t,\ell}}$ may correlate respectively with $P_{i,\tau_{i,t,\ell}}$ and $F_{j,\tau_{i,t,\ell}}$ as well as exhibit some autocorrelation as in **?** Lemma 1(iii) or as in Barndorff-Nielsen et al. (2011) Assumption U. Although we further allow for correlation between $\eta_{i,\tau_{i,t,\ell}}$ and $\eta_{F_j,\tau_{i,t,\ell}}$, the orthogonality of the factors rule out any correlation between the factor noises, that is to say, $\eta_{F_j,\tau_{i,t,\ell}}$ and $\eta_{F_k,\tau_{i,t,\ell}}$ are orthogonal for $1 \leq j \neq k \leq k_F$.

We now define the realized kernel estimator of the j-th factor loading for asset i at dat t as

$$\widehat{\beta}^{(j)}_{i,t,M_{i,t}} = \frac{\sum_{h=1}^{H} \kappa\left(\frac{h-1}{h}\right)\left(\gamma_{t,h}^{\widetilde{P}_i,\widetilde{F}_j} + \gamma_{t,-h}^{\widetilde{P}_i,\widetilde{F}_j}\right)}{\sum_{h=1}^{H} \kappa\left(\frac{h-1}{h}\right)\left(\gamma_{t,h}^{\widetilde{F}_j} + \gamma_{t,-h}^{\widetilde{F}_j}\right)}, \qquad \text{for } j = 1, \ldots, k_F$$

where κ is a twice continuously differentiable kernel function such that $\kappa(0) = 0$ and $\kappa'(0) = 0$,[6]

$$\gamma_{t,h}^{\widetilde{P}_i,\widetilde{F}_j} = \sum_{\ell=H}^{M_{i,t}-H-1} \left(\widetilde{P}_{i,\tau_{i,t,\ell+1}} - \widetilde{P}_{i,\tau_{i,t,\ell}}\right)\left(\widetilde{F}_{j,\tau_{i,t,\ell+1-h}} - \widetilde{F}_{j,\tau_{i,t,\ell-h}}\right)$$

$$\gamma_{t,h}^{\widetilde{F}_j} = \sum_{\ell=H}^{M_{i,t}-H-1} \left(\widetilde{F}_{j,\tau_{i,t,\ell+1}} - \widetilde{F}_{j,\tau_{i,t,\ell}}\right)\left(\widetilde{F}_{j,\tau_{i,t,\ell+1-h}} - \widetilde{F}_{j,\tau_{i,t,\ell-h}}\right).$$

Assume now that $\mathbf{\Sigma}_{FF,t}^{(j,j)}$, $\mathbf{\Sigma}_{i,t}^{(j)}$, and $\mathbf{\Sigma}_{F,t}^{(j)}$ are bounded uniformly in t for $j = 1, \ldots, k_F$. It then follows from Theorem 3 and Corollary 1 in Barndorff-Nielsen et al. (2011) that

$$\widehat{\beta}_{i,t,M_{i,t}}^{(j)} - \beta_{i,t}^{(j)} = \frac{1}{\mathbf{\Sigma}_{FF,t}^{(j,j)}}\left[\sum_{h=1}^{H}\kappa\left(\frac{h-1}{h}\right)\left(\gamma_{t,h}^{\widetilde{P}_i,\widetilde{F}_j} + \gamma_{t,-h}^{\widetilde{P}_i,\widetilde{F}_j}\right) - \mathbf{\Sigma}_{i,t}^{(j)}\mathbf{\Sigma}_{F,t}^{(j)}\right]$$
$$+ \frac{\sum_{h=1}^{H}\kappa\left(\frac{h-1}{h}\right)\left(\gamma_{t,h}^{\widetilde{F}_j} + \gamma_{t,-h}^{\widetilde{F}_j}\right) - \mathbf{\Sigma}_{FF,t}^{(j,j)}}{\sum_{h=1}^{H}\kappa\left(\frac{h-1}{h}\right)\left(\gamma_{t,h}^{\widetilde{F}_j} + \gamma_{t,-h}^{\widetilde{F}_j}\right)\mathbf{\Sigma}_{FF,t}^{(j,j)}}\sum_{h=1}^{H}\kappa\left(\frac{h-1}{h}\right)\left(\gamma_{t,h}^{\widetilde{P}_i,\widetilde{F}_j} + \gamma_{t,-h}^{\widetilde{P}_i,\widetilde{F}_j}\right),$$

for $H = H_{i,t} \simeq cM_{i,t}^{3/5}$ and some positive constant c. This implies that $\mathbb{E}\left(\widehat{\beta}_{i,t,M_{i,t}}^{(j)} - \beta_{i,t}^{(j)}\right) = O_p\left(M_{i,t}^{-1/5}\right)$ and that $\mathrm{Var}\left(\widehat{\beta}_{i,t,M_{i,t}}^{(j)} - \beta_{i,t}^{(j)}\right) = O_p\left(M_{i,t}^{-2/5}\right)$. Alternatively, we could set $H = H_{i,t} \simeq cN_{i,t}^{1/2}$ for some positive constant c and choose a kernel such that $\kappa''(0) = 0$. This would entail, by **?** Lemma 1, an average realized beta estimation error of order $O_p\left(M_{i,t}^{-1/4}\right)$, with a variance of order $O_p\left(M_{i,t}^{-1/2}\right)$. However, the drawback is that the realized kernel estimator of the covariance matrix with $\kappa''(0) = 0$ is not necessarily positive definite.

Now, under the additional assumption that $\mu_{i,t}$ and $\mu_{F_j,t}$ are bounded uniformly in t, the contribution of the drift component to $\frac{1}{Th^k}\sum_{t=1}^{T-1} f_{j,t+1}\left(\widehat{\beta}_{i,t,M_{i,t}}^{(j)} - \beta_{i,t}^{(j)}\right)$ is of order $O\left([\inf_t M_{i,t}]^{-2/5}h_T^{-k}\right)$. It then follows along the same lines as in the proof of Proposition 1 that the estimation error becomes negligible. This means that Proposition 1 still holds as long as

$$d_T^{-1}\max\left\{[\inf_t M_{i,t}]^{-2/5}h_T^{-k}, [\inf_t M_{i,t}]^{-1/5}T^{-1/2}h_T^{-k}\right\}$$

shrinks to zero. In addition, Proposition 2 still holds, replacing M with $[\inf_t M_{i,t}]^{-2/5}$.

[6] Note that, for notational simplicity, we omit the jittering term correction by implicitly assuming $m = 1$ in Section 2.2 of Barndorff-Nielsen et al. (2011).

Table 1: Descriptive statistics for the cross-sectional momentum strategy based on nonparametric alphas

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 2 days			holding period: 3 days			holding period: 5 days		
annualized returns	3.29	7.31	-4.02	12.31	13.16	-0.85	18.00	24.13	-6.13	11.55	39.76	-28.21
annualized volatility	11.27	19.58	19.67	17.27	27.93	29.21	20.02	33.37	34.65	24.49	41.93	40.24
t-ratio (Newey-West)	0.02	0.03	-0.01	0.05	0.02	-0.00	0.05	0.03	-0.01	0.03	0.03	-0.02
Panel A: Multifactor Model												
annualized alpha	2.54	8.27	-5.73	11.64	13.69	-2.05	17.02	23.91	-6.89	10.16	39.43	-29.27
	(0.55)	(1.75)	(-1.38)	(1.70)	(1.42)	(-0.20)	(2.01)	(1.52)	(-0.42)	(0.96)	(1.46)	(-1.11)
market	-0.02	0.72	-0.74	-0.05	0.72	-0.77	0.02	0.64	-0.62	0.03	0.49	-0.46
	(-1.00)	(8.65)	(-9.82)	(-1.32)	(9.65)	(-10.85)	(0.41)	(7.62)	(-7.14)	(0.72)	(4.37)	(-4.16)
SMB	0.06	0.08	-0.02	0.17	0.07	0.10	0.10	0.14	-0.04	0.13	0.27	-0.14
	(0.90)	(0.91)	(-0.26)	(3.04)	(0.65)	(0.84)	(1.33)	(1.08)	(-0.32)	(1.52)	(1.21)	(-0.66)
HML	0.17	-0.20	0.36	-0.02	0.01	-0.03	-0.03	0.08	-0.11	0.16	-0.05	0.21
	(2.66)	(-1.59)	(3.82)	(-0.15)	(0.08)	(-0.14)	(-0.25)	(0.43)	(-0.50)	(1.22)	(-0.18)	(0.79)
momentum	0.03	-0.02	0.05	0.06	-0.10	0.15	0.15	-0.01	0.16	0.11	-0.02	0.13
	(1.05)	(-0.37)	(1.32)	(0.99)	(-1.51)	(2.02)	(2.82)	(-0.13)	(2.36)	(1.80)	(-0.19)	(1.23)
R^2	0.02	0.57	0.61	0.02	0.31	0.34	0.02	0.16	0.17	0.01	0.07	0.07
Panel B: Returns Distribution												
Sharpe ratio	0.29	0.37	-0.20	0.71	0.47	-0.03	0.90	0.72	-0.18	0.47	0.95	-0.70
Sortino ratio	7.16	7.69	-4.77	17.65	8.93	-0.70	22.40	14.14	-4.46	10.40	17.85	-17.02
annualized semivolatility	7.30	15.09	13.36	11.07	23.41	19.18	12.75	27.09	21.85	17.62	35.36	26.31
skewness	1.11	-0.20	0.52	2.07	-0.75	1.09	1.24	-0.48	1.23	-0.05	-1.02	0.53
kurtosis	15.77	8.14	10.13	30.13	12.12	18.53	14.41	9.24	17.38	7.28	11.56	9.85
maximum	7.61	7.73	8.85	14.79	11.97	17.81	12.33	12.93	21.76	8.57	13.24	20.11
minimum	-3.36	-7.48	-7.39	-5.69	-13.95	-15.55	-5.54	-13.39	-16.76	-8.50	-21.67	-14.96
% of positive returns	48.46	51.29	45.12	49.00	54.21	45.27	48.91	53.73	44.21	50.58	55.22	42.53
appraisal ratio	0.29	0.68	-0.45	0.74	0.56	-0.05	0.86	0.63	-0.16	0.44	0.59	-0.43

Notes: We estimate the nonparametric conditional alphas with a trimming parameter $\iota = 0.20$. We report annualized returns and standard deviations of the long-short strategy as well as individually for the long and short portfolios, across different holding periods. We report robust t-statistics, based on Newey-West standard errors calculated using the automatic choice for the bandwidth. Panel A contains the results of a factor analysis, with annualized alphas and robust t-ratios in parentheses. Finally, Panel B contains some statistics on the returns distribution.

Table 2: Descriptive statistics for the cross-sectional momentum strategy based on affine alphas

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 2 days			holding period: 3 days			holding period: 5 days		
annualized returns	5.69	8.69	-3.00	3.01	9.80	-6.79	13.99	19.44	-5.45	11.51	26.22	-14.70
annualized volatility	13.43	22.62	23.21	20.14	30.56	33.12	25.54	35.64	40.99	33.37	47.31	51.56
t-ratio (Newey-West)	0.02	0.03	-0.01	0.01	0.02	-0.01	0.03	0.02	-0.01	0.02	0.02	-0.01
Panel A: Multifactor Model												
annualized alpha	4.45	9.34	-4.88	1.38	9.64	-8.26	12.80	19.00	-6.20	10.24	25.22	-14.97
	(0.78)	(2.36)	(-1.04)	(0.18)	(0.99)	(-0.75)	(1.09)	(1.22)	(-0.32)	(0.68)	(0.85)	(-0.45)
market	0.01	0.97	-0.95	0.01	0.86	-0.85	-0.05	0.81	-0.86	0.07	0.84	-0.77
	(0.37)	(20.71)	(-22.05)	(0.16)	(15.65)	(-14.46)	(-0.76)	(11.52)	(-7.91)	(0.66)	(6.62)	(-4.49)
SMB	0.02	-0.11	0.12	-0.08	-0.02	-0.07	0.22	0.41	-0.19	-0.28	0.03	-0.31
	(0.25)	(-2.67)	(2.74)	(-0.94)	(-0.13)	(-0.48)	(1.80)	(1.90)	(-1.10)	(-1.85)	(0.12)	(-1.17)
HML	0.20	0.14	0.06	0.03	0.17	-0.14	-0.23	0.12	-0.35	0.08	0.52	-0.44
	(2.26)	(2.13)	(0.87)	(0.28)	(1.02)	(-0.78)	(-0.98)	(0.71)	(-1.07)	(0.44)	(1.75)	(-1.18)
momentum	0.14	0.02	0.12	0.35	0.02	0.33	0.22	-0.10	0.32	0.31	-0.05	0.36
	(2.30)	(0.40)	(4.22)	(6.44)	(0.21)	(4.12)	(2.44)	(-0.86)	(3.05)	(3.56)	(-0.46)	(2.75)
R^2	0.03	0.79	0.78	0.06	0.35	0.38	0.05	0.25	0.26	0.02	0.16	0.15
Panel B: Returns Distribution												
Sharpe ratio	0.42	0.38	-0.13	0.15	0.32	-0.21	0.55	0.55	-0.13	0.35	0.55	-0.29
Sortino ratio	9.75	7.42	-2.98	3.15	5.99	-4.62	12.26	10.50	-3.23	7.01	9.01	-7.24
annualized semivolatility	9.26	18.60	15.98	15.15	25.97	23.34	18.12	29.40	26.81	26.08	46.20	32.24
skewness	0.89	-0.34	0.55	0.12	-0.73	0.34	0.61	-0.17	0.99	-0.16	-2.22	1.29
kurtosis	16.73	18.91	12.69	13.57	14.16	15.68	16.20	13.60	15.98	14.41	21.41	14.92
maximum	8.16	12.61	10.73	11.38	15.13	17.27	14.62	21.89	19.16	15.53	13.66	26.85
minimum	-6.46	-12.75	-11.26	-9.72	-14.18	-19.01	-13.15	-12.65	-21.42	-16.50	-31.33	-17.62
% of positive returns	50.51	54.37	46.08	51.77	55.11	46.30	51.29	55.98	46.91	51.29	57.41	43.81
appraisal ratio	0.40	0.99	-0.32	0.14	0.42	-0.25	0.46	0.51	-0.12	0.29	0.36	-0.18

Notes: See Table 1 for further details.

Table 3: Descriptive statistics for the cross-sectional momentum strategy based on nonparametric alphas, with risk-free rate adjustment

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 2 days			holding period: 3 days			holding period: 5 days		
annualized returns	5.07	8.35	-3.27	12.71	14.03	-1.33	12.06	17.23	-5.17	6.47	22.42	-15.95
annualized volatility	15.28	25.40	21.12	20.96	33.86	30.35	23.87	41.14	35.98	30.26	53.53	44.36
t-ratio (Newey-West)	0.02	0.02	-0.01	0.04	0.02	-0.00	0.03	0.02	-0.01	0.01	0.01	-0.01
Panel A: Multifactor Model												
annualized alpha	4.03	9.04	-5.01	11.71	14.31	-2.60	11.35	16.93	-5.58	5.03	21.35	-16.32
	(0.69)	(1.77)	(-1.43)	(1.54)	(1.42)	(-0.27)	(1.15)	(0.98)	(-0.34)	(0.37)	(0.65)	(-0.56)
market	0.09	1.00	-0.91	-0.00	0.84	-0.85	0.14	0.91	-0.76	0.17	0.85	-0.69
	(1.89)	(15.51)	(-24.88)	(-0.13)	(13.96)	(-18.24)	(1.88)	(8.47)	(-7.89)	(2.22)	(4.87)	(-5.22)
SMB	0.01	-0.11	0.12	0.27	0.18	0.10	0.21	0.41	-0.20	0.17	0.32	-0.15
	(0.13)	(-1.82)	(4.01)	(2.80)	(1.23)	(0.87)	(2.03)	(2.00)	(-1.24)	(1.44)	(1.24)	(-0.64)
HML	0.60	0.40	0.20	0.25	0.31	-0.06	0.18	0.38	-0.20	0.53	0.71	-0.18
	(5.29)	(3.99)	(4.55)	(2.31)	(1.68)	(-0.35)	(1.15)	(1.25)	(-0.91)	(2.20)	(1.65)	(-0.66)
momentum	-0.07	-0.10	0.04	-0.05	-0.23	0.18	-0.04	-0.24	0.20	-0.06	-0.25	0.19
	(-1.28)	(-2.55)	(1.78)	(-0.85)	(-3.10)	(2.31)	(-0.37)	(-1.66)	(2.72)	(-0.80)	(-1.96)	(1.93)
R^2	0.16	0.77	0.79	0.03	0.34	0.39	0.03	0.27	0.24	0.05	0.16	0.13
Panel B: Returns Distribution												
Sharpe ratio	0.33	0.33	-0.15	0.61	0.41	-0.04	0.51	0.42	-0.14	0.21	0.42	-0.36
Sortino ratio	7.45	6.31	-3.57	15.36	8.08	-1.06	11.38	7.55	-3.57	4.29	6.88	-9.11
annualized semivolatility	10.81	21.01	14.57	13.13	27.56	19.91	16.83	36.21	23.00	23.95	51.76	27.79
skewness	1.70	-0.26	0.30	2.81	0.11	0.93	1.31	-0.86	0.77	-0.26	-1.95	0.86
kurtosis	41.03	21.88	10.62	35.51	15.70	16.38	24.43	19.43	12.05	13.02	19.14	10.66
maximum	11.68	14.74	8.85	14.86	17.73	17.81	18.32	23.32	15.93	12.95	17.40	20.11
minimum	-9.94	-15.53	-9.89	-6.04	-14.86	-15.55	-9.07	-24.47	-16.76	-13.03	-30.12	-14.96
% of positive returns	49.87	53.41	46.40	50.42	55.69	47.33	50.84	55.66	45.82	52.13	57.02	44.07
appraisal ratio	0.35	0.71	-0.45	0.69	0.56	-0.07	0.50	0.40	-0.13	0.19	0.27	-0.22

Notes: See Table 1 for further details.

Table 4: Descriptive statistics for the cross-sectional momentum strategy based on affine alphas, with risk-free rate adjustment

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 2 days			holding period: 3 days			holding period: 5 days		
annualized returns	5.90	8.93	-3.03	4.06	10.89	-6.83	15.64	20.80	-5.16	11.90	26.82	-14.92
annualized volatility	13.39	22.69	23.11	20.09	30.63	32.98	25.46	35.73	40.80	33.22	47.37	51.40
t-ratio (Newey-West)	0.03	0.03	-0.01	0.01	0.02	-0.01	0.03	0.03	-0.01	0.02	0.02	-0.01
Panel A: Multifactor Model												
annualized alpha	4.68	9.58	-4.90	2.39	10.70	-8.31	14.45	20.33	-5.89	10.68	25.83	-15.14
	(0.82)	(2.42)	(-1.05)	(0.31)	(1.10)	(-0.76)	(1.24)	(1.31)	(-0.31)	(0.71)	(0.87)	(-0.45)
market	0.02	0.97	-0.95	0.02	0.87	-0.85	-0.04	0.82	-0.86	0.07	0.83	-0.76
	(0.62)	(20.67)	(-22.22)	(0.33)	(15.61)	(-14.56)	(-0.63)	(11.52)	(-7.91)	(0.68)	(6.60)	(-4.48)
SMB	0.02	-0.11	0.13	-0.08	-0.02	-0.06	0.23	0.42	-0.19	-0.28	0.03	-0.31
	(0.34)	(-2.69)	(2.95)	(-0.92)	(-0.14)	(-0.45)	(1.85)	(1.94)	(-1.12)	(-1.86)	(0.14)	(-1.18)
HML	0.20	0.14	0.06	0.04	0.18	-0.14	-0.22	0.12	-0.34	0.08	0.52	-0.44
	(2.29)	(2.17)	(0.86)	(0.37)	(1.07)	(-0.78)	(-0.93)	(0.71)	(-1.03)	(0.45)	(1.75)	(-1.18)
momentum	0.14	0.02	0.12	0.35	0.02	0.33	0.21	-0.10	0.31	0.30	-0.05	0.35
	(2.28)	(0.41)	(4.16)	(6.47)	(0.26)	(4.11)	(2.38)	(-0.85)	(2.98)	(3.50)	(-0.49)	(2.72)
R^2	0.03	0.79	0.78	0.06	0.35	0.38	0.04	0.26	0.26	0.02	0.16	0.15
Panel B: Returns Distribution												
Sharpe ratio	0.44	0.39	-0.13	0.20	0.36	-0.21	0.61	0.58	-0.13	0.36	0.57	-0.29
Sortino ratio	10.22	7.59	-3.02	4.27	6.62	-4.66	13.73	11.20	-3.07	7.27	9.20	-7.36
annualized semivolatility	9.17	18.67	15.89	15.10	26.11	23.26	18.08	29.48	26.68	25.98	46.30	32.15
skewness	0.92	-0.34	0.56	0.10	-0.75	0.34	0.58	-0.18	0.98	-0.19	-2.22	1.29
kurtosis	16.84	18.71	12.87	13.64	14.11	15.83	16.03	13.50	15.95	14.20	21.36	14.99
maximum	8.16	12.61	10.73	11.38	15.13	17.27	14.62	21.89	19.16	14.71	13.66	26.85
minimum	-6.46	-12.75	-11.26	-9.72	-14.18	-19.01	-13.15	-12.65	-21.42	-16.50	-31.33	-17.62
% of positive returns	50.32	54.69	45.82	51.77	55.24	46.56	51.87	56.37	46.59	51.61	57.80	44.01
appraisal ratio	0.42	1.02	-0.32	0.19	0.46	-0.25	0.53	0.55	-0.11	0.30	0.37	-0.18

Notes: See Table 1 for further details.

Table 5: Descriptive statistics for the cross-sectional momentum strategy based on nonparametric appraisal ratios

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 2 days			holding period: 3 days			holding period: 5 days		
annualized returns	1.00	6.41	-5.41	6.32	9.75	-3.43	12.26	20.17	-7.91	2.77	30.93	-28.16
annualized volatility	10.27	18.17	18.78	15.84	25.83	27.44	18.75	30.81	33.01	22.34	37.92	38.53
t-ratio (Newey-West)	0.01	0.02	-0.02	0.03	0.02	-0.01	0.04	0.03	-0.01	0.01	0.03	-0.03
Panel A: Multifactor Model												
annualized alpha	0.07	7.24	-7.17	5.64	10.18	-4.54	11.30	20.19	-8.89	1.53	30.80	-29.26
	(0.02)	(1.63)	(-1.82)	(0.91)	(1.11)	(-0.49)	(1.48)	(1.38)	(-0.58)	(0.15)	(1.25)	(-1.17)
market	-0.04	0.68	-0.72	-0.04	0.70	-0.74	-0.03	0.62	-0.64	0.03	0.49	-0.45
	(-2.29)	(9.17)	(-9.94)	(-1.15)	(11.32)	(-11.00)	(-0.80)	(7.78)	(-8.35)	(0.79)	(4.91)	(-4.17)
SMB	0.04	0.05	-0.01	0.08	0.01	0.07	0.11	0.09	0.02	0.11	0.18	-0.07
	(0.67)	(0.66)	(-0.16)	(1.39)	(0.07)	(0.63)	(1.46)	(0.71)	(0.13)	(1.48)	(0.87)	(-0.32)
HML	0.19	-0.22	0.40	0.01	-0.02	0.03	-0.06	-0.04	-0.02	0.05	-0.17	0.22
	(3.52)	(-2.01)	(4.19)	(0.09)	(-0.14)	(0.17)	(-0.62)	(-0.23)	(-0.09)	(0.49)	(-0.70)	(0.87)
momentum	0.07	0.02	0.05	0.09	-0.03	0.13	0.15	0.02	0.13	0.14	0.04	0.10
	(2.21)	(0.52)	(1.13)	(1.72)	(-0.55)	(1.70)	(3.50)	(0.33)	(2.11)	(2.55)	(0.37)	(0.95)
R^2	0.03	0.57	0.62	0.02	0.32	0.34	0.03	0.17	0.19	0.01	0.07	0.07
Panel B: Returns Distribution												
Sharpe ratio	0.10	0.35	-0.29	0.40	0.38	-0.12	0.65	0.65	-0.24	0.12	0.82	-0.73
Sortino ratio	2.42	7.50	-6.52	9.52	7.06	-2.98	16.00	12.48	-6.03	2.75	15.56	-18.03
annualized semivolatility	6.58	13.56	13.18	10.54	21.92	18.27	12.17	25.66	20.82	16.02	31.56	24.79
skewness	1.11	-0.06	0.14	2.22	-1.03	0.68	1.33	-0.79	1.22	-0.13	-1.08	0.61
kurtosis	15.22	8.08	8.82	39.00	13.34	15.25	17.91	9.92	18.55	6.82	11.31	10.43
maximum	6.68	7.26	7.11	14.79	10.91	13.60	12.33	11.54	21.76	8.74	11.21	20.52
minimum	-2.74	-7.17	-7.66	-5.94	-13.75	-14.99	-6.42	-13.24	-16.09	-8.50	-22.15	-14.23
% of positive returns	47.88	51.74	45.05	49.00	52.73	44.24	48.58	53.02	44.02	49.81	54.06	41.95
appraisal ratio	0.10	0.64	-0.61	0.42	0.44	-0.16	0.64	0.56	-0.22	0.11	0.51	-0.45

Notes: See Table 1 for further details.

Table 6: Descriptive statistics for the cross-sectional momentum strategy based on affine appraisal ratios

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 2 days			holding period: 3 days			holding period: 5 days		
annualized returns	6.64	6.61	0.03	2.75	6.83	-4.08	12.96	14.78	-1.81	8.74	20.58	-11.84
annualized volatility	12.61	20.78	22.40	18.80	28.39	31.74	23.90	32.88	38.99	30.92	43.96	48.79
t-ratio (Newey-West)	0.03	0.02	0.00	0.01	0.01	-0.01	0.03	0.02	-0.00	0.01	0.02	-0.01
Panel A: Multifactor Model												
annualized alpha	5.45	7.23	-1.79	1.30	6.74	-5.45	12.24	14.47	-2.24	7.67	19.76	-12.09
	(1.04)	(2.06)	(-0.39)	(0.17)	(0.74)	(-0.52)	(1.12)	(0.98)	(-0.12)	(0.53)	(0.70)	(-0.38)
market	-0.02	0.91	-0.93	-0.01	0.84	-0.85	-0.07	0.77	-0.84	0.04	0.80	-0.76
	(-0.47)	(23.36)	(-22.55)	(-0.19)	(17.98)	(-14.53)	(-1.37)	(10.91)	(-8.23)	(0.40)	(6.40)	(-4.72)
SMB	0.04	-0.10	0.14	-0.08	-0.04	-0.04	0.21	0.37	-0.16	-0.25	0.02	-0.28
	(0.60)	(-2.29)	(3.02)	(-0.84)	(-0.36)	(-0.30)	(1.93)	(1.91)	(-0.97)	(-1.95)	(0.11)	(-1.10)
HML	0.13	0.02	0.10	-0.02	0.07	-0.09	-0.30	0.04	-0.34	0.01	0.36	-0.36
	(1.42)	(0.32)	(1.64)	(-0.20)	(0.49)	(-0.51)	(-1.25)	(0.24)	(-1.00)	(0.04)	(1.29)	(-1.04)
momentum	0.15	0.06	0.09	0.34	0.06	0.28	0.16	-0.07	0.24	0.30	-0.00	0.30
	(2.75)	(1.69)	(2.99)	(6.61)	(0.79)	(3.44)	(2.35)	(-0.74)	(2.54)	(4.10)	(-0.03)	(2.58)
R^2	0.03	0.80	0.77	0.07	0.36	0.39	0.05	0.26	0.26	0.02	0.15	0.15
Panel B: Returns Distribution												
Sharpe ratio	0.53	0.32	0.00	0.15	0.24	-0.13	0.54	0.45	-0.05	0.28	0.47	-0.24
Sortino ratio	11.80	6.15	0.03	3.00	4.40	-2.92	11.96	8.46	-1.15	5.79	7.60	-6.34
annualized semivolatility	8.93	17.08	15.24	14.55	24.65	22.21	17.21	27.73	25.10	23.96	43.00	29.64
skewness	0.53	-0.56	0.57	-0.27	-1.06	0.41	0.52	-0.55	1.03	-0.04	-2.26	1.42
kurtosis	16.30	15.50	13.14	12.91	13.27	16.38	16.80	10.94	16.24	14.36	20.36	15.43
maximum	6.48	9.66	10.88	7.97	10.58	17.27	11.59	16.84	18.81	13.96	11.73	26.85
minimum	-6.78	-12.80	-11.12	-10.43	-13.60	-18.58	-13.49	-12.19	-20.62	-12.96	-27.60	-16.02
% of positive returns	51.41	54.82	46.79	52.03	54.79	47.85	52.32	55.53	47.04	51.68	57.09	44.14
appraisal ratio	0.49	0.84	-0.07	0.13	0.31	-0.16	0.46	0.41	-0.05	0.23	0.30	-0.15

Notes: See Table 1 for further details.

Table 7: Descriptive statistics for the time-series momentum strategy based on nonparametric alphas

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 5 days			holding period: 10 days			holding period: 22 days		
annualized returns	1.91	-2.11	4.02	18.97	11.59	7.39	37.28	25.06	12.22	58.55	-1.17	59.72
annualized volatility	14.63	20.57	20.08	31.18	40.43	37.02	45.45	59.66	56.90	63.77	91.67	88.52
t-ratio (Newey-West)	0.01	-0.01	0.01	0.03	0.01	0.01	0.04	0.01	0.01	0.05	-0.00	0.02
Panel A: Multifactor Model												
annualized alpha	1.04	2.95	-1.91	18.22	13.63	4.59	34.87	31.76	3.12	60.93	4.97	55.96
	(0.13)	(0.37)	(-0.25)	(0.86)	(0.41)	(0.15)	(1.04)	(0.52)	(0.05)	(1.43)	(0.04)	(0.49)
market	-0.02	0.55	-0.57	-0.09	0.27	-0.36	0.05	0.68	-0.63	0.18	0.90	-0.72
	(-0.91)	(7.70)	(-8.11)	(-1.69)	(2.63)	(-3.25)	(0.87)	(6.02)	(-5.87)	(1.34)	(2.88)	(-2.25)
SMB	0.03	0.17	-0.14	0.01	-0.03	0.05	-0.01	0.26	-0.26	-0.60	-0.46	-0.14
	(0.49)	(2.39)	(-1.54)	(0.11)	(-0.09)	(0.13)	(-0.06)	(0.83)	(-0.84)	(-2.20)	(-0.64)	(-0.18)
HML	0.19	-0.03	0.22	0.16	-0.19	0.35	-0.01	0.43	-0.44	0.05	-0.07	0.12
	(2.52)	(-0.18)	(1.66)	(1.01)	(-0.52)	(1.06)	(-0.04)	(0.70)	(-0.71)	(0.17)	(-0.14)	(0.24)
momentum	0.04	-0.02	0.07	-0.01	0.04	-0.04	0.22	-0.06	0.28	-0.01	0.19	-0.21
	(1.10)	(-0.44)	(1.47)	(-0.09)	(0.25)	(-0.37)	(1.66)	(-0.25)	(1.49)	(-0.07)	(0.56)	(-0.66)
R^2	0.01	0.42	0.45	0.01	0.02	0.04	0.01	0.10	0.13	0.02	0.05	0.03
Panel B: Returns Distribution												
Sharpe ratio	0.13	-0.10	0.20	0.61	0.29	0.20	0.82	0.42	0.21	0.92	-0.01	0.67
Sortino ratio	3.20	-2.03	4.69	14.54	5.15	5.40	18.76	6.84	6.46	21.75	-0.20	23.49
annualized semivolatility	9.48	16.48	13.62	20.71	35.68	21.73	31.56	58.17	30.04	42.73	92.14	40.35
skewness	2.30	0.08	0.64	0.53	-1.29	1.20	0.40	-1.68	2.20	0.19	-1.92	2.14
kurtosis	32.14	10.64	9.35	7.83	13.17	12.22	8.56	17.75	19.38	6.06	12.05	13.05
maximum	11.17	9.77	7.80	12.49	10.37	20.32	19.14	22.21	30.69	17.51	20.80	38.38
minimum	-5.24	-7.12	-6.53	-8.41	-22.93	-9.81	-14.02	-32.50	-20.65	-16.87	-40.42	-15.40
% of positive returns	46.15	45.04	44.04	48.50	49.88	42.39	48.43	52.20	40.65	46.50	51.85	41.27
appraisal ratio	0.13	0.20	-0.07	0.48	0.23	0.09	0.61	0.30	0.05	0.80	0.03	0.27

Notes: See Table 1 for further details.

Table 8: Descriptive statistics for the time-series momentum strategy based on affine alphas

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 5 days			holding period: 10 days			holding period: 22 days		
annualized returns	-2.81	-4.70	1.88	10.17	-30.83	41.00	14.35	-68.05	82.40	106.18	-149.76	255.94
annualized volatility	16.59	24.08	27.20	40.34	48.52	59.25	55.15	68.65	77.76	86.49	97.50	118.92
t-ratio (Newey-West)	-0.01	-0.01	0.00	0.01	-0.02	0.02	0.01	-0.03	0.03	0.04	-0.03	0.05
Panel A: Multifactor Model												
annualized alpha	-4.20	3.74	-7.95	6.42	-23.89	30.32	12.59	-55.84	68.43	102.19	-146.44	248.63
	(-0.42)	(0.70)	(-0.88)	(0.25)	(-0.57)	(0.58)	(0.30)	(-0.67)	(0.73)	(1.27)	(-0.99)	(1.44)
market	0.02	0.87	-0.85	-0.01	0.75	-0.76	0.07	0.90	-0.83	-0.40	0.71	-1.11
	(0.69)	(15.13)	(-15.12)	(-0.08)	(4.46)	(-3.54)	(0.37)	(6.36)	(-3.33)	(-2.72)	(2.35)	(-2.95)
SMB	0.13	-0.09	0.22	-0.36	0.01	-0.36	-0.16	-0.37	0.22	0.14	0.13	0.01
	(1.80)	(-1.98)	(3.75)	(-1.71)	(0.02)	(-0.70)	(-0.48)	(-0.88)	(0.37)	(0.37)	(0.18)	(0.01)
HML	-0.00	0.11	-0.11	0.34	0.28	0.06	0.22	0.03	0.19	0.27	0.46	-0.19
	(-0.04)	(1.40)	(-0.99)	(1.25)	(0.70)	(0.11)	(0.60)	(0.05)	(0.32)	(0.62)	(0.62)	(-0.32)
momentum	0.11	-0.05	0.16	0.29	-0.03	0.32	0.19	-0.29	0.48	0.02	0.24	-0.22
	(1.64)	(-1.28)	(3.32)	(2.65)	(-0.21)	(1.78)	(0.97)	(-1.15)	(1.70)	(0.07)	(0.77)	(-0.61)
R^2	0.02	0.82	0.68	0.02	0.16	0.13	0.00	0.14	0.10	0.02	0.04	0.06
Panel B: Returns Distribution												
Sharpe ratio	-0.17	-0.19	0.07	0.25	-0.64	0.69	0.26	-0.99	1.06	1.23	-1.54	2.15
Sortino ratio	-3.47	-3.69	1.51	5.31	-9.93	18.40	5.44	-14.24	32.16	30.40	-23.34	87.50
annualized semivolatility	12.87	20.18	19.81	30.38	49.27	35.37	41.91	75.86	40.68	55.46	101.84	46.44
skewness	-0.51	-0.25	0.27	-0.21	-2.33	1.36	-0.20	-3.23	1.90	0.55	-2.35	1.94
kurtosis	12.71	13.21	10.98	6.68	16.81	12.89	7.46	24.41	14.80	6.99	11.98	9.84
maximum	5.09	10.28	9.53	11.74	11.20	26.02	16.55	15.71	39.19	29.77	11.80	50.06
minimum	-8.94	-9.98	-10.77	-11.85	-24.77	-15.48	-18.80	-41.43	-27.14	-23.87	-41.64	-19.95
% of positive returns	50.00	51.99	46.65	51.75	55.61	46.38	53.07	57.47	45.17	52.99	56.56	47.39
appraisal ratio	-0.17	0.35	-0.39	0.20	-0.32	0.35	0.19	-0.40	0.44	0.73	-0.55	0.81

Notes: See Table 1 for further details.

Table 9: Descriptive statistics for the time-series momentum strategy based on nonparametric alphas, with risk-free rate adjustment

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 5 days			holding period: 10 days			holding period: 22 days		
annualized returns	2.33	-1.97	4.30	16.80	12.33	4.47	34.84	25.25	9.59	64.41	2.15	62.26
annualized volatility	14.64	20.56	19.93	30.75	40.51	36.80	44.94	59.58	56.49	62.98	91.89	87.70
t-ratio (Newey-West)	0.01	-0.01	0.01	0.03	0.01	0.00	0.04	0.01	0.01	0.05	0.00	0.02
Panel A: Multifactor Model												
annualized alpha	1.58	3.19	-1.60	15.75	14.19	1.56	32.56	32.00	0.56	66.72	7.92	58.80
	(0.19)	(0.40)	(-0.21)	(0.75)	(0.43)	(0.05)	(0.99)	(0.53)	(0.01)	(1.57)	(0.07)	(0.52)
market	-0.02	0.55	-0.57	-0.09	0.28	-0.37	0.04	0.67	-0.63	0.21	0.92	-0.71
	(-1.17)	(7.73)	(-8.12)	(-1.72)	(2.67)	(-3.27)	(0.79)	(5.98)	(-5.90)	(1.60)	(2.94)	(-2.21)
SMB	0.03	0.16	-0.13	0.04	-0.01	0.05	-0.01	0.26	-0.28	-0.57	-0.46	-0.11
	(0.44)	(2.22)	(-1.46)	(0.32)	(-0.03)	(0.13)	(-0.10)	(0.85)	(-0.88)	(-2.04)	(-0.65)	(-0.13)
HML	0.18	-0.04	0.22	0.19	-0.18	0.37	0.07	0.50	-0.43	-0.03	-0.07	0.04
	(2.40)	(-0.27)	(1.69)	(1.17)	(-0.49)	(1.12)	(0.35)	(0.83)	(-0.71)	(-0.09)	(-0.14)	(0.07)
momentum	0.03	-0.03	0.06	0.01	0.05	-0.04	0.21	-0.06	0.27	0.01	0.24	-0.23
	(0.80)	(-0.60)	(1.40)	(0.15)	(0.35)	(-0.34)	(1.64)	(-0.30)	(1.47)	(0.08)	(0.68)	(-0.73)
R^2	0.01	0.41	0.45	0.01	0.02	0.05	0.01	0.11	0.13	0.02	0.05	0.03
Panel B: Returns Distribution												
Sharpe ratio	0.16	-0.10	0.22	0.55	0.30	0.12	0.78	0.42	0.17	1.02	0.02	0.71
Sortino ratio	3.90	-1.89	5.02	13.12	5.46	3.29	18.00	6.91	5.07	25.50	0.37	25.28
annualized semivolatility	9.49	16.52	13.60	20.32	35.81	21.55	30.73	58.02	30.03	40.09	92.01	39.10
skewness	2.33	0.06	0.62	0.49	-1.29	1.23	0.48	-1.68	2.20	0.39	-1.91	2.25
kurtosis	32.12	10.67	9.48	7.65	13.17	12.50	8.06	17.79	19.72	5.90	11.97	13.60
maximum	11.17	9.77	7.80	12.49	10.37	20.32	19.14	22.21	30.69	19.17	20.80	38.38
minimum	-5.05	-7.12	-6.53	-8.41	-22.93	-9.81	-11.53	-32.50	-20.65	-15.98	-40.42	-15.30
% of positive returns	46.40	44.67	43.92	48.00	50.00	42.02	47.80	51.82	39.90	45.99	51.21	40.76
appraisal ratio	0.15	0.21	-0.05	0.43	0.24	0.04	0.59	0.30	0.02	0.88	0.05	0.28

Notes: See Table 1 for further details.

Table 10: Descriptive statistics for the time-series momentum strategy based on affine alphas, with risk-free rate adjustment

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 5 days			holding period: 10 days			holding period: 22 days		
annualized returns	-4.44	-5.80	1.36	3.06	-22.76	25.82	11.10	-56.52	67.63	105.79	-139.27	245.06
annualized volatility	15.89	24.28	26.60	39.42	49.25	57.96	52.62	69.03	75.95	81.61	98.06	115.52
t-ratio (Newey-West)	-0.02	-0.02	0.00	0.00	-0.02	0.01	0.01	-0.02	0.02	0.05	-0.03	0.05
Panel A: Multifactor Model												
annualized alpha	-5.66	2.64	-8.30	-0.17	-15.69	15.53	9.98	-44.43	54.41	101.79	-135.53	237.32
	(-0.58)	(0.49)	(-1.00)	(-0.01)	(-0.37)	(0.30)	(0.26)	(-0.53)	(0.59)	(1.42)	(-0.91)	(1.40)
market	0.03	0.87	-0.84	0.01	0.77	-0.75	0.08	0.92	-0.83	-0.33	0.72	-1.04
	(1.01)	(15.11)	(-15.15)	(0.13)	(4.56)	(-3.48)	(0.44)	(6.42)	(-3.36)	(-2.21)	(2.40)	(-2.72)
SMB	0.12	-0.09	0.22	-0.24	0.02	-0.26	-0.11	-0.37	0.25	0.17	0.09	0.08
	(1.80)	(-1.94)	(3.77)	(-1.13)	(0.04)	(-0.49)	(-0.35)	(-0.85)	(0.43)	(0.52)	(0.13)	(0.11)
HML	-0.00	0.12	-0.12	0.25	0.27	-0.02	0.18	0.07	0.11	0.27	0.44	-0.17
	(-0.04)	(1.50)	(-1.06)	(0.88)	(0.67)	(-0.04)	(0.49)	(0.12)	(0.18)	(0.64)	(0.60)	(-0.28)
momentum	0.10	-0.05	0.15	0.26	-0.03	0.29	0.15	-0.27	0.42	0.07	0.21	-0.14
	(1.61)	(-1.23)	(3.33)	(2.34)	(-0.23)	(1.72)	(0.78)	(-1.09)	(1.52)	(0.28)	(0.69)	(-0.41)
R^2	0.02	0.81	0.69	0.01	0.16	0.13	0.00	0.14	0.10	0.02	0.04	0.06
Panel B: Returns Distribution												
Sharpe ratio	-0.28	-0.24	0.05	0.08	-0.46	0.45	0.21	-0.82	0.89	1.30	-1.42	2.12
Sortino ratio	-5.63	-4.58	1.11	1.67	-7.32	11.64	4.49	-11.94	27.03	32.95	-21.98	87.22
annualized semivolatility	12.52	20.09	19.58	29.14	49.35	35.21	39.22	75.17	39.71	50.97	100.58	44.60
skewness	-0.81	-0.24	0.26	-0.09	-2.20	1.44	-0.09	-3.13	2.04	0.65	-2.25	2.09
kurtosis	13.94	12.69	11.87	7.25	15.91	14.36	8.23	23.46	16.21	7.22	11.58	10.88
maximum	4.97	10.28	9.53	12.35	11.20	26.67	16.94	15.63	39.19	30.39	11.80	50.69
minimum	-9.08	-9.98	-10.77	-13.19	-24.77	-16.37	-17.98	-40.62	-27.14	-19.57	-41.64	-19.95
% of positive returns	49.50	51.86	46.90	50.62	56.11	45.76	52.45	56.84	44.79	52.48	56.31	47.39
appraisal ratio	-0.26	0.24	-0.45	0.06	-0.21	0.20	0.17	-0.32	0.36	0.83	-0.51	0.79

Notes: See Table 1 for further details.

Table 11: Descriptive statistics for the time-series momentum strategy based on nonparametric appraisal ratios

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 5 days			holding period: 10 days			holding period: 22 days		
annualized returns	-1.37	-4.16	2.79	19.07	14.03	5.04	32.54	13.37	19.17	46.00	-10.59	56.58
annualized volatility	10.90	18.43	18.99	23.15	36.28	35.67	33.08	54.50	53.96	50.11	86.61	84.67
t-ratio (Newey-West)	-0.01	-0.01	0.01	0.04	0.01	0.01	0.05	0.01	0.01	0.05	-0.00	0.02
Panel A: Multifactor Model												
annualized alpha	-2.64	0.34	-2.98	17.71	15.26	2.45	31.15	20.33	10.83	46.79	-7.00	53.79
	(-0.43)	(0.05)	(-0.44)	(1.14)	(0.49)	(0.08)	(1.30)	(0.35)	(0.19)	(1.36)	(-0.06)	(0.48)
market	-0.01	0.55	-0.56	-0.09	0.28	-0.37	-0.06	0.62	-0.68	-0.01	0.75	-0.76
	(-0.46)	(7.96)	(-8.19)	(-1.39)	(2.24)	(-3.48)	(-1.27)	(5.27)	(-6.62)	(-0.06)	(2.44)	(-2.37)
SMB	0.08	0.18	-0.09	0.16	0.08	0.08	0.18	0.42	-0.24	-0.52	-0.29	-0.23
	(1.09)	(2.56)	(-1.04)	(1.08)	(0.21)	(0.22)	(1.23)	(1.41)	(-0.75)	(-1.79)	(-0.42)	(-0.29)
HML	0.18	-0.05	0.23	0.29	-0.13	0.42	-0.01	0.36	-0.36	-0.29	-0.28	-0.01
	(2.19)	(-0.36)	(1.81)	(2.45)	(-0.39)	(1.27)	(-0.04)	(0.64)	(-0.59)	(-0.97)	(-0.63)	(-0.02)
momentum	0.07	0.02	0.06	0.02	0.09	-0.07	0.05	-0.13	0.18	-0.03	0.28	-0.31
	(1.65)	(0.33)	(1.33)	(0.40)	(0.81)	(-0.58)	(0.49)	(-0.62)	(0.93)	(-0.16)	(0.71)	(-0.98)
R^2	0.02	0.48	0.47	0.02	0.03	0.05	0.01	0.11	0.14	0.02	0.04	0.04
Panel B: Returns Distribution												
Sharpe ratio	-0.13	-0.23	0.15	0.82	0.39	0.14	0.98	0.25	0.36	0.92	-0.12	0.67
Sortino ratio	-2.37	-4.25	3.41	18.81	6.76	4.00	20.31	3.84	11.24	18.30	-1.82	26.07
annualized semivolatility	9.18	15.55	13.01	16.09	32.98	20.00	25.44	55.30	27.07	39.89	92.26	34.45
skewness	-1.41	-0.50	0.55	-0.12	-1.57	1.57	-0.39	-2.22	2.45	-0.86	-2.57	2.55
kurtosis	26.56	9.51	11.84	7.43	13.24	15.87	7.55	23.80	21.96	13.66	15.60	15.83
maximum	4.89	6.67	7.53	7.47	10.34	21.46	10.58	23.45	30.20	16.68	16.54	41.08
minimum	-7.74	-7.18	-7.74	-9.69	-20.17	-10.11	-12.28	-33.28	-20.98	-28.19	-40.10	-15.58
% of positive returns	47.52	50.12	42.93	48.63	53.87	42.27	50.94	55.08	39.52	50.70	54.78	40.00
appraisal ratio	-0.13	0.05	-0.17	0.68	0.29	0.05	0.75	0.19	0.12	0.76	-0.02	0.25

Notes: See Table 1 for further details.

Table 12: Descriptive statistics for the time-series momentum strategy based on affine appraisal ratios

	long-short	long	short	long-short	long	short	long-short	long	short	long-short	long	short
	holding period: 1 day			holding period: 5 days			holding period: 10 days			holding period: 22 days		
annualized returns	5.91	-2.99	8.90	5.25	-33.31	38.56	8.73	-67.78	76.51	40.68	-146.40	187.07
annualized volatility	13.98	23.54	25.37	34.25	48.05	54.39	48.86	68.54	72.85	73.14	96.55	109.47
t-ratio (Newey-West)	0.03	-0.01	0.02	0.01	-0.02	0.02	0.01	-0.03	0.03	0.02	-0.03	0.04
Panel A: Multifactor Model												
annualized alpha	5.06	5.27	-0.22	2.57	-26.26	28.83	7.19	-56.21	63.39	37.53	-142.09	179.62
	(0.62)	(1.15)	(-0.03)	(0.14)	(-0.63)	(0.60)	(0.20)	(-0.67)	(0.70)	(0.62)	(-0.97)	(1.08)
market	0.02	0.86	-0.84	0.03	0.74	-0.71	0.03	0.89	-0.86	-0.22	0.78	-1.00
	(0.52)	(15.56)	(-16.35)	(0.23)	(4.33)	(-3.15)	(0.14)	(6.24)	(-3.25)	(-1.86)	(2.64)	(-3.11)
SMB	0.10	-0.09	0.19	-0.24	0.02	-0.26	-0.19	-0.35	0.15	0.14	0.22	-0.08
	(1.21)	(-1.55)	(2.89)	(-1.30)	(0.05)	(-0.51)	(-0.51)	(-0.83)	(0.26)	(0.47)	(0.32)	(-0.13)
HML	-0.06	0.09	-0.15	0.03	0.27	-0.24	0.30	0.18	0.12	0.21	0.52	-0.31
	(-0.78)	(1.09)	(-1.32)	(0.15)	(0.66)	(-0.51)	(0.74)	(0.30)	(0.20)	(0.52)	(0.69)	(-0.49)
momentum	0.07	-0.04	0.11	0.23	-0.05	0.28	0.14	-0.25	0.40	0.09	0.21	-0.12
	(1.10)	(-1.23)	(2.52)	(2.40)	(-0.35)	(1.79)	(0.72)	(-1.00)	(1.41)	(0.35)	(0.66)	(-0.34)
R^2	0.02	0.82	0.75	0.02	0.16	0.15	0.00	0.14	0.11	0.01	0.05	0.06
Panel B: Returns Distribution												
Sharpe ratio	0.42	-0.13	0.35	0.15	-0.69	0.71	0.18	-0.99	1.05	0.56	-1.52	1.71
Sortino ratio	9.69	-2.32	7.98	3.26	-10.63	19.72	3.62	-13.82	34.60	13.52	-22.10	70.27
annualized semivolatility	9.67	20.44	17.71	25.57	49.77	31.03	38.24	77.85	35.10	47.75	105.15	42.26
skewness	0.51	-0.37	0.64	-0.10	-2.46	1.90	-0.34	-3.46	2.38	0.59	-2.59	2.16
kurtosis	10.65	13.57	13.70	7.90	17.87	17.56	10.54	26.50	18.67	8.85	13.25	11.29
maximum	5.95	10.96	10.86	11.32	11.42	25.97	17.10	15.55	38.84	29.28	11.80	49.39
minimum	-5.11	-10.01	-10.46	-11.11	-24.67	-15.53	-20.64	-42.17	-27.42	-20.69	-42.28	-20.04
% of positive returns	50.25	54.71	46.90	51.25	55.74	46.38	51.32	57.97	44.67	49.55	56.94	45.10
appraisal ratio	0.40	0.59	0.06	0.14	-0.35	0.36	0.14	-0.40	0.42	0.37	-0.54	0.61

Notes: See Table 1 for further details.