Subjective measures of risk aversion and portfolio choice

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Abstract

The paper investigates risk attitudes among different types of individuals. We use several different measures of risk atttitudes, including questions on choices between uncertain income streams suggested by Barsky et al. (1997) and a number of ad hoc measures. As in Barsky et al. (1997) and Arrondel (2000), we first analyse individual variation in the risk aversion measures and explain them by background characteristics (both "objective" characteristics and other subjective measures of risk preference). Next we incorporate the measured risk attitudes into a household porftolio allocation model, which explains portfolio shares, while accounting for incomplete portfolios. Our results show that the Barsky et al. (1998) measure has little explanatory power, whereas measures based on a factor analysis of ad hoc measures do a considerably better job. We provide a discussion of the reasons for this finding.

1 Introduction

The theory of the so-called "Modern Portfolio Theory" as pioneered by Markowitz (1952, 1959) stresses the idea that portfolio diversification leads to a risk reduction. The concept of an optimal portfolio allocation, already introduced by Keynes, Hicks and Kaldor in their theories of money, was further developed by Tobin (1958). Agents would diversify their savings between a risk-free asset (money) and a single portfolio of risky assets: different combinations of money and that unique portfolio would arise from different attitudes towards risk.

The standard model of lifetime consumption and portfolio choice of Samuelson (1969) and Merton (1969, 1971) predicts that without transaction costs agents should invest their savings in all asset classes and that the allocation across assets will vary across individuals according to their preferences, wealth and investment horizon.

*We thank Miles Kimball for suggesting the question format used in this paper.

The portfolio theory developed by Markowitz is at the basis of "tactical asset allocation", the systematic short horizon allocation of investment portfolios across asset classes such as bonds, stocks and cash. Brennan, Schwartz and Lagnado (1997) studied this portfolio problem in the presence of time variation in expected returns on the asset classes. They found that the optimal portfolio proportions of an investor with such a short horizon are significantly different from the ones of an investor with a longer horizon.

In conventional financial planning advice, young investors are usually encouraged to take more risk than older investors, and similarly, old investors are typically encouraged to hold more bonds, relative to stocks, than younger investors. The rationale of this advice is that young people face a longer investment horizon than old people, so that they either have "time to ride out the ups and downs of the market" (Campbell and Viceira, 2001) or can adjust their labor supply in response to uncertain investment returns (Bodie, Merton and Samuelson, 1992). This advice is economically valid as long as the investor's human wealth is relatively uncorrelated with stock returns (Jagannathan and Kockerlakota, 1996). The implicit idea is then that the length of one's investment horizon affects the riskiness of one's portfolio, so that short horizons usually lead to rather conservative portfolio strategies (Brennan, Schwartz and Lagnado, 1997; Gollier and Zeckhauser, 1999; Gollier, 2002).

It has nevertheless been argued that the distinction between short and long investment horizon may not make sense under certain conditions on return distributions and utility functions. More precisely, when investors' relative risk aversion does not depend systematically on their wealth and when investors have only financial wealth, people should behave myopically and their portfolio should meet the best short-term characteristics (Campbell and Viceira, 2000).

In most cases, observed portfolio choices are not consistent with standard asset allocation models. As a consequence, several studies have focused on empirical failures of portfolio optimization theory. The greatest failure is given by the fact that the majority of individuals do not hold fully diversified portfolios, even though the percentage of households holding risky assets has increased in the last decade (Haliassos and Hassapis, 2000). The observed heterogeneity in portfolio choices may be partly explained by nonfinancial income and costs of stock market participation (Vissing-Jorgensen, 1999).

The sub-optimal degree of international diversification known as "home asset bias" has been analyzed, among others, by French and Poterba (1990, 1991), Tesar and Werner (1992, 1994, 1995), Cooper and Kaplanis (1994), Glassman and Riddick (2001). Possible reasons of the over-investment in domestic assets have been identified in different transaction costs between countries, additional sources of risk for foreign investing and explicit omission of assets from the investor's opportunity set. A more fundamental piece of evidence against the rational model of portfolio allocation is provided by Benartzi and Thaler (2001) who find that the allocation of investors is heavily dependent upon the choices offered to them. Roughly speaking, if they are offered n choices they tend to allocate $\frac{1}{n}$ of their investment to each of the choices offered independent of the risk characteristics of the investment opportunities.

Recent papers (Heaton and Lucas, 2000; Faig and Shum, 2002) have pointed out that certain individuals, specifically young investors and entrepreneurs, hold a larger than expected share of safe financial assets in their portfolios, in order to diversify the risk of their business and personal illiquid projects.

The effect of labor income risk and liquidity contraints on portfolio choice is the topic of a large body of literature (Heaton and Lucas, 1997; Koo, 1998; Viceira, 2001; Campbell, Cocco, Gomes and Maenhout, 2002), whose main claim is that investors facing borrowing contraints are more vulnerable to the risk of their financial portfolios and thus should avoid risky assets. Tepla (2000) has characterized optimal intertemporal portfolio policies for investors with CRRA utility facing a borrowing contraint with or without shortsale restrictions.

In several ways the rational model of choice on which the modern portfolio theory is based appears to be unable to explain several empirical findings. The connection between theory and empirical evidence is however often somewhat tenuous, because too many intervening factors may explain why theoretical predictions are not borne out by data. For this reason some authors have turned to more direct, subjective evidence on preferencess to reduce the distance between theory and empirical facts. A prominent example is the paper by Barsky et al. (1997) who elicit several pieces of subjective information to improve our understanding of intertemporal choice and portfolio allocation. We follow the same approach by considering several measures of risk attitude. One measure is based on hypothetical choices between uncertain income streams in a household survey. The methodology is closely related to Barsky et al. (1997) and Arrondel (2000). The Barsky et al. measure has a nice direct interpretation if individuals have CRRA preferences. We will find however, that the measure also has theoretical and empirical problems. Hence we also consider alternative measures of risk attitude. We relate the different measured risk attitudes to observed portfolio choice of households. To deal with incomplete portfolios, we set up a simple rationing model that can endogenously generate corner solutions in the portfolio allocation. Thus, we formulate and estimate a complete system of portfolio demand equations incorporating subjective measures of risk aversion. The model is closely related to rational portfolio theory and seems to do a reasonable job in describing differences in allocation across individuals who differ in socio-economic characteristics, wealth, and risk attitudes.

The paper is organized as follows. In the next section we describe the data we use in the analysis. In particular we present descriptive statistics on the various risk attitude measures and on the portfolio composition of households. Section 3 discusses some results from the literature regarding the classical theory of portfolio choice. Based on these results we formulate in Section 4 present a simple static asset allocation model with rationing. The rationing emerges as the result of corner solutions (i.e. the existence of incomplete portfolios). We then derive an econometric model with switching regimes, where each regime is characterized by a particular asset ownership pattern. The model has two components: the first component determines the regime, and the second component describes portfolio shares of assets conditional on the regime. Section 5 presents empirical results. We find that the risk tolerance measure of Barsky et al. (1997) does a poor job in explaining choices between risky and less risky assets. Other, simpler, risk attitudes measure do have a more significant effect on the choice of risky assets. It thus appears that these simple attitude measures are a better measure of risk tolerance.

2 Data on risk aversion and precaution and how they were collected

The data used in this paper have been collected from the households in the CentERpanel. The CentERpanel is representative of the Dutch population, comprising some 2000 households in the Netherlands. The members of those households answer a questionnaire at their home computers every week. These computers may either be their own computer or a PC provided by CentERdata, the agency running the panel[1]. In the weekends of August 7-10 and August 14-17 of 1998 a questionnaire was fielded with a large number of subjective questions on hypothetical choices. The questionnaire was repeated in the weekends of November 20-23 and November 27-30 of 1998 for those panel members who had not responded yet. For this paper we exploit the section involving choices over uncertain lifetime incomes. We merge these data with data from the CentER Savings Survey (CSS). The CSS collects data on assets, liabilities, demographics, work, housing, mortgages, health and income, and many subjective variables (e.g. expectations, savings motives) from annual interviews with participants in the CentERpanel. Typically the questions for the CSS are asked in May of each year, during a number of consecutive weekends.

2.1 Choices of uncertain lifetime income

Our analysis focuses on a number of questions involving risky choices over lifetime incomes. This methodology, taken from Barsky et al. (1997) (BJKS, from now on) and also adopted by Arrondel (2000), allows us to rank individuals with respect to their risk aversion without having to assume a particular functional form for the utility function.

In the BJKS experiment, questions are posed to all respondents, consisting of individuals aged over 50. Arrondel (2000) asked the questions to a representative sample of French households. In our case, the questions are asked only to people who

[1] The description refers to the time of the survey. Nowadays, CentERdata does not provide a PC any longer but a set-top box.

have a job *and* who are the main breadwinner in a household (i.e. the person in the household who brings in the largest amount of money).

The structure of the questions is depicted in Figure 1. In the first round, respondents are asked the following question:

Imagine your doctor recommends that you move because of allergies. You follow his advice, and it turns out you have to choose between two possible jobs. Both jobs are about equally demanding (for example, both jobs involve the same number of working hours per week), but the income in one job is much more certain than the income in the other job.

The first job guarantees your current income for the rest of your life. In addition, we assume that income other members of your household may have, will also remain unchanged. In this situation, you know for sure that during the remainder of your life, the net income of your household will be equal to Dfl. Y .

The second job is possibly better paying, but the income is also less certain. In this job, there is a 50% chance that you will earn so much that the income of your household will be doubled for the rest of your life, that is, be equal to Dfl. Y x*2.*

There is, however, an equally big chance (50%) that you will earn substantially less in the second job. In the latter case, the net monthly income of your household will for the rest of your life be equal to Dfl. Y x*07.*

Which job would you take?

1 the job with the guaranteed fixed household income of Dfl. Y

2 the job that involves a 50% chance that the income of your household will for the rest of your life be equal to Dfl. Y x*2, but also involves a 50% chance that the income of your household will for the rest of your life be equal to* Y x*07.*

Various quantities in the question vary per respondent, exploiting the computerized nature of the interviews. The quantity Y is the respondent's selfreported after tax household income. Yx2 is twice the household income; Yx07 is household income times .7, etc. This is in contrast to the experiments by BJKS and Arrondel (2000), in which the incomes were the same for all individuals. Obviously, the question involves a choice between a certain and an uncertain outcome: the former is given by the actual income the respondent receives (Y), the latter is a 50-50 gamble over a good outcome (Yx2) and a bad outcome (Y x*07*).

In the second round each individual is asked a similar question. If she has chosen the certain outcome (Y) in the first round, she now faces another gamble where the risky outcome is more attractive. The 50-50 gamble now involves Yx2 and Yx08 (0.8 times income). If she has chosen the risky prospect in the first round, she is now asked to choose between her income for sure and a less attractive gamble, i.e. 50% chance of Yx2 and 50% chance of Yx05.

Similarly, in the third round the gamble becomes more attactive for those respondents who have once again chosen a certain income stream in the second round (the

50-50 gamble now involves Yx2 and Yx09), and less attractive for those respondents who preferred the risky choice (the 50-50 gamble now involves Yx2 and Yx025)

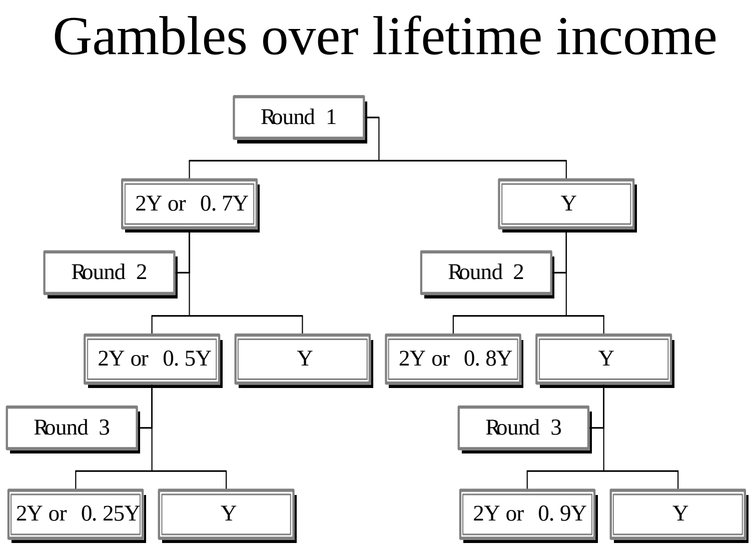

Figure 1: Choices of uncertain lifetime income

The answers to the questions allow us to identify six groups ranked from most risk averse to least risk averse (or equivalently from least risk tolerant to most risk tolerant; we will generally denote the variable defined by the six classes as "risk tolerance"). Both the BJKS study and Arrondel's involve only two rounds of questions rather than three as ours. For comparison we temporarily combine the two most extreme groups into one. Thus we have four categories of individuals, from I to IV, where the I-group is the union of the 1 and the 2 groups and the IV-group is the union of the 5 and 6 groups. We can then compare the risk tolerance across the three studies. Table 1 gives the results. To facilitate a comparison with the BJKS study we split our sample in two age groups: 50 and younger and over 50[2].

An unfortunate aspect of the sample selection (respondents being employed and being the main breadwinner) is that it severely limits the number of observations. This clearly reduces the possibility of obtaining statistically significant results. Keeping this in mind, a comparison between France and The Netherlands on the basis of the complete age range suggests that there is a greater spread of risk aversion in The Netherlands than in France. The Dutch respondents are more heavily represented in the two extreme categories (almost 53% of the Dutch belong to the least risk averse group compared to 43% of the French, whereas 12% of the Dutch belong to the most risk averse group compared to 6% for the French). Summing the percentages of the

[2]The BJKS sample consists of respondents over 50.

first two groups and the percentages of the last two groups respectively, suggests that the Dutch are less risk averse than the French (only 69.5% of the Dutch belong to the first two groups compared to 82.5% of the French, whereas 30.5% of the Dutch belong to the last two groups compared to 17.5% of the French).

Considering the subsamples of respondents over 50, it appears that the Dutch have similar risk preferences to the Americans, although the Americans may be slightly more risk tolerant than the Dutch. Compared to the Dutch and the Americans, the French appear to be much more risk averse.

Table 1: Risk Tolerance in USA, France and The Netherlands

	Total sample		Respondents over 50		
Group	**France**	**Neth.**	**USA**	**France**	**Neth.**
I	43.1	52.8	64.6	48.6	66.3
II	39.4	16.7	11.6	36.8	13.5
III	11.2	18.1	10.9	8.7	9.0
IV	6.3	12.4	12.8	5.9	11.2
Total	100	100	100	100	100
Obs.	2954	657	11707	?	178

Turning to a closer analysis of the Dutch data, we once again distinguish six classes of risk tolerance. Table 2 presents the distribution of respondents by risk category and gender. Notice that there is a very uneven distribution of males and females in the sample. This is the result of the fact that only employed main breadwinners have been selected. The vast majoriy of the respondents fall in the most risk averse categories. Although the table might suggest that females are more risk averse than males, the differences in the percentages across gender are minor and indeed a χ^2 test of the null that risk aversion is the same for males and females does not lead to rejection.

Table 2: Risk Tolerance by gender (percentages)

	% by gender		Total sample	
Group	Males	Females	Frequency	Percentage
1	25.9	26.7	160	25.7
2	26.6	27.8	167	26.7
3	15.7	23.3	110	17.6
4	18.9	13.3	111	17.8
5	6.5	3.3	36	5.8
6	6.4	5.6	39	6.4
Total	100	100	100	100
Obs.	567	90	657	
$\chi^2(5) = 5.45$, $p = .36$				

Table 3 suggests that better educated individuals are generally less risk averse. The first two groups comprise 60% of low educated people and 50.4% of highly educated people. Similarly, the last two groups comprise 7% of low educated respondents and 13.4% of highly educated respondents. The differences in risk tolerance between the three levels of education are statistically significant.

Table 3: Risk Tolerance in the Netherlands by levels of education (percentages)

	Education		
Group	Low education	Middle Education	High Education
1	39.6	20.3	21.1
2	21.4	27.8	29.3
3	15.7	16.5	18.9
4	16.4	20.8	17.3
5	1.9	8.0	6.8
6	5.0	6.6	6.8
Total	100	100	100
Obs.	159	212	266
$\chi^2(10) = 27.8$, $p = .002$			

Table 4 compares differences in risk tolerance between employees and self-employed. Although the table suggests that the self-employed are substantially more risk tolerant than employees, the small number of observations of self-employed respondents leads to statistically insignificant differences.

Table 4: Risk Tolerance of employees and self-employed (percentages)

	Employment type	
Group	Employees	Self-employed
1	26.4	20
2	26.8	25.7
3	16.9	14.3
4	18.0	20.0
5	6.1	5.7
6	5.8	14.3
Total	100	100
Obs.	622	35
$\chi^2(5) = 4.58$, $p = .47$		

2.2 Risk attitude measures based on principal components

The CSS-questionnaire contains six direct questions about investment strategies. These are reproduced below. Respondents can express their agreement or disagreement with these statements on a seven point scale (1 means complete disagreement and 7 means complete agreement.

SPAAR1
I find it more important to invest safely and to get a guaranteed return than to take risks in order to possibly get a higher return.
SPAAR2
Investing in stocks is something I don't do, since it is too risky.
SPAAR3
If I believe an investment will carry a profit, I am willing to borrow money for it.
SPAAR4
I want to be sure my investments are safe
SPAAR5
I am increasingly convinced that I need to take more financial risks if I want to improve my financial position.
SPAAR6
I am willing to run the risk of loosing money if there is also a chance that I will make money.

The CSS also contains 13 questions about savings motives. Below we reproduce three of them that are related to precautionary motives and uncertainty. Answers can be given on a 1 to 7 scale, where 1 means "very unimportant" and 7 means "very important".

SPAARM03
To have some savings in case of unforeseen expenses due to illness or an accident.

SPAARM12
As a reserve for unforeseen events
SPAARM13
To have enough money in the bank, so that I can be sure to be able to meet my financial obligations.

Applying principal components analysis to these nine indicators of risk aversion and precaution, we find that three underlying factors can explain most of the variance in the answers (see Table 5). After applying varimax rotation we find the factor loadings presented in Table 5. The largest factor loadings in each column are given in bold face. We note that the factors which we have callled "riskat1" and "riskat3" mainly explain the six "spaar" variables, whereas "riskat2" mainly explains the "spaarm" variables. In view of the wordings of the nine questions, we interpret riskat1 and riskat3 as measures of risk aversion, whereas riskat2 is mainly a measure of prudence. Thus we would expect riskat2 to play a role in savings decisions whereas riskat1 and riskat3 should affect portfolio choice.

Table 5: Rotated factor loadings (varimax rotation)

Variables	**riskat1**	**riskat2**	**riskat3**	**uniqueness**
spaarm03	-0.038	**0.796**	-0.017	0.365
spaarm12	0.163	**0.838**	0.023	0.270
spaarm13	0.154	**0.785**	0.036	0.360
spaar1	**0.838**	0.111	0.058	0.282
spaar2	**0.632**	-0.007	0.370	0.464
spaar3	0.006	0.030	**0.724**	0.475
spaar4	**0.817**	0.151	0.046	0.307
spaar5	-0.015	0.037	**0.785**	0.382
spaar6	0.346	-0.027	**0.758**	0.304

Table 6 presents a number of descriptive statistics of the three risk attitude measures by several demographic and socio-economic characteristics. The p-values refer to one way analyses of variance of each of the risk attitude measures on the characteristics considered. The risk attitude measures are normalized such that they have mean zero and variance one for the complete sample. The variance varies slightly across subgroups but usually very little. Table 6 shows that all three risk attitude measures are significantly less for males than for females, implying that males are less risk averse and have a less strong precautionary motive. Education only has a significant effect for riskat1, with a pattern that is hard to interpret. Despite the small number of self-employed in our sample we do find significant less risk aversion among the self-employed than among employees for riskat2 and riskat3. For riskat1, the difference is not significant.

Table 6: Risk attitude variables by background characteristics(means)

Characteristic	**riskat1**	**riskat2**	**riskat3**	**Nobs**
Male	-.071	-.092	-.126	529
Female	.123	.159	.219	304
p-value	*.007*	*.0005*	*.0000*	
Low education	.032	.087	.060	259
Middle education	.075	-.007	-.048	255
High education	-.119	-.066	-.026	283
p-value	*.06*	*.21*	*.422*	
Employees	-.098	.039	-.092	407
Self-employed	.259	-.356	-.575	21
p-value	*.13*	*.07*	*.03*	
Whole sample	-1.85e-09	9.98e-10	4.42e-10	833

2.3 Direct questions on precaution and risk aversion

The third type of subjective measures are the answers to the following two questions, which were included in the same module of the CentERpanel as the BJKS risk aversion measure. The questions read as follows:

Would you rather describe yourself as a carefree person, or rather as a careful person?
When there is possible danger, do you take many precautions?

Responses in both cases could be given on a seven point scale. We will refer to the first variable as "careful" and to the second variable as "precaution". Table 7 presents a number of descriptive statistics of the two risk attitude measures by several demographic and socio-economic characteristics. The table contains the mean scores on the seven point scales for both variables. The p-values refer to χ^2-tests of the null that the distribution of answers over the seven possible categories is the same across different values of a characteristic[3]. We observe that females are significantly more "careful" than males. Both careful and precaution appear negatively related with education.

[3]So the χ^2-tests are similar as for tables 2, 3 and 4. To save space we do not present the full distribution of answers by characteristic.

Table 7: *Risk attitude variables by background characteristics(means)*

Characteristic	**careful**	**precaution**	**Nobs**
Male	5.001	4.843	977
Female	5.110	4.896	734
p-value	*.04*	*.44*	
Low education	5.096	4.948	560
Middle education	5.048	4.877	531
High education	5.003	4.764	523
p-value	*.001*	*.04*	
Employees	4.916	4.711	786
Self-employed	4.914	4.851	47
p-value	*.90*	*.31*	
Whole sample	5.047	4.866	1711

2.4 How are the subjective measures related?

Before turning to the analysis of the relation between the various measures of risk tolerance and portfolio composition we present analyses of the interrelation between the various measures. First of all, comparing the outcomes presented in Tables 2 through 7 we notice the following patterns. Splitting the sample by gender, risk tolerance does not reveal significant differences, but riskat1, riskat2, riskat3, and careful all indicate that males are more risk tolerant and less precautious than females. Distinguishing respondents by education shows that risk tolerance, riskat1, careful, and precaution all suggest that people with higher education are less risk averse and less precautious. Riskat2 and riskat3 do not differ significantly by education. Finally, only riskat2 and riskat3 differ significantly (at the 7% and 3% level respectively) between self-employed and employees. The lack of significant differences for the other variables does not necessarily mean that employees and self-employed do not differ, but may rather be a reflection of the small number of self-employed in the sample.

Table 8 presents results of regressing each of the measures on a number of background variables and the other measures. For the explanation of risk tolerance, careful and precaution the method of analysis is ordered probit. For riskat1 through riskat3 we use regression. The ordinal variables risk tolerance careful and precaution are simply coded from 1 to 7 when used as explanatory variables. Replacing the simple coding by dummies does not alter the outcomes appreciably. Since the variables do not all come from the same interview and because not all questions are asked to all respondents, the number of observations is fairly small, reflecting the partial overlap in observations. Of course, the regressions should be viewed as purely descriptive. They present a way of showing the nature of the interrelations between the six measures. Although we briefly discuss the effects of background variables, like age, income, wealth, gender, education, and being selfemployed, one should keep in mind that the effects of these variables are all conditional on all the other risk attitude measures in the regressions.

Age is only significant for risk tolerance and riskat1. Figure 2 draws the quadratic age functions implied by the estimates for all six measures. The figure shows that both risk tolerance and riskat1 imply that risk aversion increases with age. The variables income and total wealth are both coded in the form of an inverse hyperbolic sine[4]. Income has a significant effect on risk tolerance and riskat1. The signs of the effects imply in both cases decreasing risk aversion with income. Wealth is (marginally) significant in four out of six regressions. The estimation results for risk tolerance suggest increasing relative risk aversion with increasing wealth, which would be consistent with a constant absolute risk aversion utility function, for instance. The results for riskat3 suggest decreasing risk aversion with increasing wealth, but since we do not know how riskat3 would be related exactly to the parameters of a known utility function, the result is hard to interpret. The two precautionary measures (precaution and riskat2) increase with wealth. Gender is never significant and education and being self-employed are only significant for risk tolerance. One should recall once again, that these effects are to be interpreted as being conditional on the other risk attitude measures.

Turning to the interrelationships between the risk attitude measures, we observe that precaution and riskat2 appear to be significantly related, as one would expect from the interpretation of riskat2. Somewhat less expected, careful, riskat3, and risk tolerance are also significantly related to precaution.

[4]The inverse hyperbolix sine of x is $h(x) = \ln[1 + \sqrt{x^2 + 1}]$

Table 8: Interrelationships between the various measures (ordered probit and ols estimates)

Expl. variables	Risk tol.	Careful	Precau.	Riskat1	Riskat2	Riskat3
Age	.046	.100	-.052	.051	-.090	.026
Age squared	-.0008	-.001	.0005	-.0004	.001	-.0003
p-value age eff.	*.002*	*.133*	*.423*	*.050*	*.186*	*.864*
Income	.533	-.321	.206	-.503	-.110	.004
t-value	*2.79*	*1.67*	*1.09*	*2.91*	*.65*	*.02*
wealth	-.070	.125	-.052	.036	.065	-.075
t-value	*1.90*	*3.37*	*1.39*	*1.05*	*1.94*	*2.27*
Gender	-.047	-.072	-.168	-.072	.379	.204
t-value	*.26*	*.40*	*.93*	*.44*	*2.34*	*1.28*
Self-empl.	.624	-.008	.186	.238	-..497	-.246
t-value	*2.12*	*.03*	*.62*	*.87*	*1.85*	*.93*
Middle educ.	.435	-.131	.164	.067	-.174	.108
High educ.	.331	-.125	.175	.042	-.278	.071
p-value ed eff.	*.023*	*.676*	*.492*	*.901*	*.149*	*.745*
Careful	-.058	-	.531	.189	.055	-.044
t-value	*1.00*	-	*9.72*	*3.63*	*1.09*	*.87*
Precaution	-.152	.518	-	-.013	.102	.103
t-value	*2.69*	*9.62*	-	*.26*	*2.02*	*2.07*
Risk tolerance	-	-.049	-.127	-.113	.058	-.067
t-value	-	*1.09*	*2.85*	*2.77*	*1.44*	*1.71*
Riskat1	-.172	.233	-.009	-	-	-
t-value	*2.90*	*3.90*	*.15*	-	-	-
Riskat2	.083	.086	.134	-	-	-
t-value	*1.33*	*1.40*	*2.20*	-	-	-
Riskat3	-.120	-.021	.130	-	-	-
t-value	*1.94*	*.33*	*2.10*	-	-	-
Number of obs.	342	342	342	342	342	342
(Pseudo) R^2	0.066	.156	.128	.158	.076	.060

2.5 Assets and liabilities

The CSS collects extensive information on assets and liabilities. Respondents are asked for ownership and quantity of different categories of assets, both real and financial, and of liabilities and mortgages. Table 9 reports data on financial assets at the household level. We group assets in categories that are somewhat homogeneous with respect to their risk profile. We have indicated the category an asset belongs to by *NR* for non-risky assets, by *R* for risky assets and by *O* for "other" assets. Non-risky assets are checking accounts, savings accounts, deposits, and insurances. Risky assets are defined as the sum of growth and mutual funds, options, stocks and

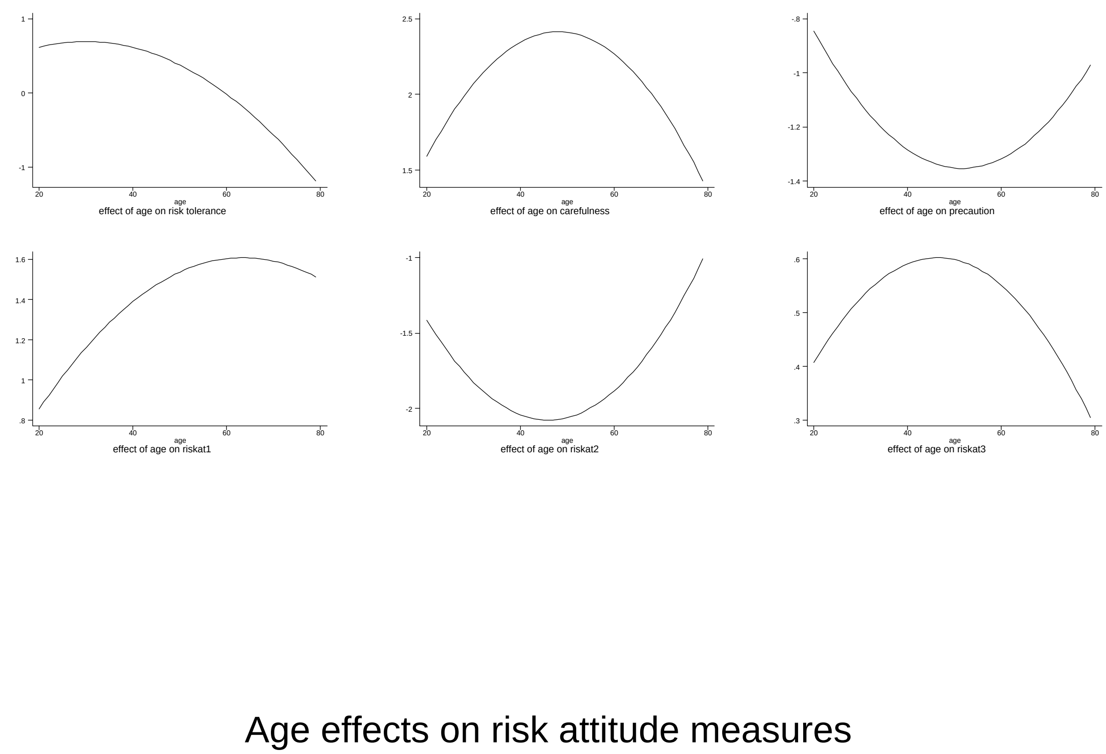

Figure 1: Age functions for the six risk attitude measures

business equity. Other assets are the sum of real estate, mortgage, bonds, money lent out and financial debt. The table presents relative frequencies of ownership, mean values and shares of each asset category in the total portfolio. Notice that the data not only refer to the respondents who have been posed the questions about uncertain life time incomes.

Checking accounts, savings accounts, deposits, and insurances are held by 96.3% of the sample. This safe asset makes up close to 50% of average financial wealth in the sample. Business equity is another sizeable component of financial wealth, although it is only held by 7.1% of the sample. Generally, the households in the sample use very little credit: Financial debts amount to less than 10% of total financial assets (and hence financial wealth is more than 90% of total financial assets). Only 4.5% of the sample holds bonds and/or mortgage bonds, whereas 16.5% hold stocks; 22.6% of the sample holds growth or mutual funds. Of course the group of stock holders overlaps with the owners of mutual funds or growth funds. 30.4% of the sample households have stocks and/or growth and mutual funds. Clearly, for most households real estate (usually the primary residence) dominates the portfolio. Financial assets are only 27% of total assets and financial wealth is only 33% of total wealth.

At the aggregate level, both risky and non-risky assets are basically 50% of average financial wealth, but the percentage of people owning risky assets (34.5) is far less than half of that of people owning safe assets (96.3). Other assets are widely spread over the sample (82.8%) and they are 175.9% of financial assets.

Table 9: Households' Assets and liabilities in 1998

Assets	Type	Obs.	% own	Mean	% fin.w.	% tot.w.
Checking, dep., ins., etc.	*NR*	1427	96.3	45630	48.7	13.2
Growth, mutual funds	*R*	1427	22.6	11603	12.3	3.3
Bonds	*O*	1427	4.5	2187	2.3	.6
Stocks	*R*	1427	16.5	15683	16.7	4.5
Options	*R*	1427	1.1	115	.1	.03
Money lent out	*O*	1427	8.3	1943	2.1	.6
Business equity	*O*	1427	7.1	16591	17.7	4.8
Total financial assets		1427	-	93753	100	27.0
Real estate	*O*	1422	68.0	252650	-	72.9
Total assets		1422	-	346693	-	100
Financial debt	*O*	1427	-	8976	9.6	2.6
Mortgage	*O*	1422	55.8	82952	-	23.9
Financial wealth		1427	-	84776	90.4	24.4
Non-risky assets	*NR*	1427	96.3	45630	48.7	13.2
Risky assets	*R*	1427	34.5	43992	46.9	12.7
Other assets	*O*	1422	82.8	346456	175.9	47.6
Total wealth		1422	-	254803	-	73.5

NR: non-risky; *R*: risky; *O*: other assets

3 Some theory

To motivate our empirical model, it is useful to summarize some concepts and results from the literature. In the exposition below we mainly follow the excellent new book by Gollier (2001).

3.1 Comparative risk aversion

Agent 1 is more risk averse than agent 2 if

$$E[u_2(w_0 + \tilde{x}) \leq u_2(w_0) \Rightarrow E[u_1(w_0 + \tilde{x}) \leq u_1(w_0)] \tag{1}$$

where u_2 and u_1 are utility functions, w_0 is initial wealth and $\tilde{x}$ is a risky asset with zero expected return. This is equivalent with $A_1(w_0) \geq A_2(w_0)$, where $A_i(z) \equiv -\frac{u''(z)}{u'(z)}$, the coefficient of absolute risk aversion. Of course, if the coefficient of absolute risk aversion is larger for individual 1, then this is also true of the coefficient of relative risk aversion: $R_i(z) = z.A_i(z)$.

3.2 HARA (harmonic absolute risk aversion) utility functions

$$u(z) = \zeta(\eta + \frac{z}{\gamma})^{1-\gamma} \tag{2}$$

Absolute risk tolerance (the inverse of absolute risk aversion) for this utility function is equal to

$$T(z) = \frac{1}{A(z)} = -\frac{u''(z)}{u'(z)} = \eta + \frac{z}{\gamma} \tag{3}$$

Thus, absolute risk tolerance (inverse absolute risk aversion) is linear in wealth, which explains the name of this class of utility functions. Notice that the coefficient of relative risk aversion then equals

$$R(z) = \frac{z}{\eta + \frac{z}{\gamma}} \tag{4}$$

and the degree of absolute prudence:

$$P(z) = -\frac{u'''(z)}{u''(z)} = \frac{\gamma+1}{\gamma}(\eta + \frac{z}{\gamma})^{-1} \tag{5}$$

The degree of relative prudence is $zP(z)$.

Notice that if $\eta = 0$, the utility function reduces to

$$u(z) = \zeta(\frac{z}{\gamma})^{1-\gamma} \tag{6}$$

which is the CRRA utility function with coefficient of relative risk aversion γ (cf. (4)). Similarly, if $\gamma \to \infty$, it can be shown that the utility function reduces to:

$$u(z) = -\frac{\exp(-Az)}{A} \tag{7}$$

where A is the coefficient of absolute risk aversion ($A = \frac{1}{\eta}$). Finally, for $\gamma = -1$, we obtain a quadratic utility function.

3.3 Risk aversion and portfolio choice

For CARA preferences (see (7)) the share of wealth to invest in a risky asset is

$$\frac{\alpha^*}{w_0} = \frac{\mu}{\sigma^2}\frac{1}{w_0 A} = \frac{\mu}{\sigma^2}\frac{1}{R(w_0)} \tag{8}$$

where μ and σ^2 are the mean and variance of the distribution of the excess return of the risky asset and α^* is the amount invested in the risky asset. For non-CARA preferences formula (8) is approximate.

HARA preferences (see (2)): For this case no explicit solution is available, but a numerical solution can be found in a rather simple way. Let a be the solution of the equation

$$E\boldsymbol{\ae}(1+\frac{a\boldsymbol{\ae}}{\gamma})^{-\gamma}=0$$

then the general solution for α^* is equal to

$$\alpha^*=a(\eta+\frac{w_0}{\gamma})=aT(w_0) \tag{9}$$

using (3). So we see that the **amount** invested in the risky asset is directly proportional to the degree of absolute risk tolerance. The **share** of total wealth invested in the risky asset is then inversely proportional to the degree of relative risk aversion. This is qualititatively similar to the result for CARA-preferences.

Next we consider the case of a vector of risky assets. We will restrict ourselves to CARA-preferences. To motivate the econometric model to be used in the sequel, we provide the derivation of the optimal portfolio for this case. Let μ be the $(k-1)$-vector of mean excess returns and Σ the variance covariance matrix of the excess returns. Let W be begin of period wealth, r is the riskfree interest rate. The $(k-1)$-vector α denotes the quantities invested in the respective risky assets, with stochastic returns given by the vector $\boldsymbol{\ae}_0$. Let ι be a $(k-1)$-vector of ones. Then $\iota'\alpha$ is the amount of money invested in the risky assets and $W-\iota'\alpha$ is the amount invested in the riskfree asset (No non-negativity restrictions are imposed). Consumption z is equal to the value of the assets at the end of the period. Thus consumption is:

$$z=(W-\iota'\alpha)(1+r)+\alpha'(\iota+\boldsymbol{\ae}_0)=W(1+r)+\alpha'(\boldsymbol{\ae}_0-r)\equiv w_0+\alpha'\boldsymbol{\ae} \tag{10}$$

where $w_0=W(1+r)$ and $\boldsymbol{\ae}=\boldsymbol{\ae}_o-r.\iota$ .We assume $\boldsymbol{\ae}$ to be normally distributed, so that $\boldsymbol{\ae}\sim N(\mu,\Sigma)$. The consumer wants to maximize the expectation of end of period utility subject to (10) by choosing α optimally. Inserting (10) in (7), neglecting the multiplicative constant A, and taking expectations yields

$$\begin{aligned}
V(\alpha) &= -(2\pi)^{-n/2}\left|\Sigma\right|^{-1/2}\int\exp(-A(w_0+\alpha'x))\exp(-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu))dx \\
&= \exp(-Aw_0-A\alpha'\mu+\frac{1}{2}A^2\alpha'\Sigma\alpha).(2\pi)^{-n/2}\left|\Sigma\right|^{-1/2} \\
&\quad .\int\exp[-\frac{1}{2}(x-\mu+A\Sigma\alpha)'\Sigma^{-1}(x-\mu+A\Sigma\alpha)]dx \\
&= -\exp(-Aw_0-A\alpha'\mu+\frac{1}{2}A^2\alpha'\Sigma\alpha)
\end{aligned} \tag{11}$$

Maximizing (11) with respect to α yields:

$$\alpha^*=\frac{1}{A}\Sigma^{-1}\mu \tag{12}$$

We can also write this in terms of portfolio shares. In that case (8) generalizes to:

$$w = \Sigma^{-1}\mu . \frac{1}{R(w_0)} \tag{13}$$

4 An econometric model of portfolio choice

Our interest will be in ownership and portfolio shares of a number of asset categories that vary in riskiness. We want to allow for other factors determining portfolio composition than just the distribution of excess returns. To introduce these other factors in a utility consistent way, we replace (11) by

$$V^*(\alpha) = -\exp(-Aw_0 - A\alpha'\mu - A^2 w_0 \alpha'\Sigma z + \frac{1}{2}A^2\alpha'\Sigma\alpha) \tag{14}$$

where z is a vector of taste shifters:

$$z = \Lambda x + \varepsilon \tag{15}$$

where x is a vector of individual (or household) characteristics, Λ is a parameter matrix, and ϵ an i.i.d. error term. We will interpret ϵ as representing unobservable variations in taste across individuals.

Maximizing (14) with respect to the quantity vector α yields the following expression for the vector of risky asset shares:

$$\tilde{w} = z + \frac{1}{R}\Sigma^{-1}\mu \equiv z + \Gamma\mu^* \tag{16}$$

where $\Gamma = \Sigma^{-1}$ and $\mu^* = \frac{1}{R}\mu$.

Notice that no sign restrictions are imposed on the elements of $\tilde{w}$. If we impose the condition that assets have to be non-negative -the empirically relevant case- the maximization of (14) has to take place subject to the condition $\alpha \geq 0$. Given that Γ is positive definite, necessary and sufficient conditions for a maximum are then:

$$\begin{aligned} \kappa &\geq 0 \\ w &\geq 0 \\ \kappa' w &= 0 \\ w &= z + \frac{1}{R}\Sigma^{-1}(\mu + \kappa) = z + \Gamma(\mu^* + \lambda) \end{aligned} \tag{17}$$

where $\lambda = \kappa/R$, and κ is a vector of Lagrange multipliers. The share of the riskless asset in the portfolio is equal to $1 - \iota_{k-1}' w$. Since the share of the riskless asset follows directly from the shares of the risky assets through adding up, we restrict our attention to the shares of the risky assets.

To characterize the Kuhn-Tucker coditions (17) it is convenient to define "virtual prices" $\tilde{p} \equiv \mu^* + \lambda$. It follows from the Kuhn-Tucker conditions that the virtual prices are equal to the corresponding elements of μ^* if the corresponding budget share is not equal to zero. To calculate virtual prices for the assets whose share equals zero, we introduce some notation. Let S' (k_1 x $(k-1)$) and D' (k_2 x $(k-1)$) be selection matrices with $k_1 + k_2 = k - 1$, i.e. $\begin{pmatrix} S' \\ D' \end{pmatrix}$ is a permutation of I_{k-1}, the $(k-1)$ x $(k-1)$ identity matrix. The matrix S' selects the elements of w which are zero and D' selects the elements of w which are non-zero. Some useful properties of S and D are:

$$S'S = I_{k_1} \quad D'D = I_{k_2} \quad SS' + DD' = I_{k-1} \quad D'S = 0 \tag{18}$$

Given that S' selects the elements of w that are zero, there holds $S'w = 0$, and similarly $D'\lambda = 0$.

The share equations in (17) can then be written as

$$\begin{pmatrix} 0 \\ D'w \end{pmatrix} = \begin{pmatrix} S' \\ D' \end{pmatrix} z + \begin{pmatrix} S' \\ D' \end{pmatrix} \Gamma[SS' + DD']\tilde{p} \tag{19a}$$

The top-half of (19a) gives as a solution for the virtual prices:

$$S'\tilde{p} = -(S'\Gamma S)^{-1}\{S'z + (S'\Gamma D)D'\mu^*\} \tag{20}$$

using the fact that $D'\tilde{p} = D'\mu^*$. Substituting this in the bottom half of (19a) yields for the portfolio shares of the non-zero assets:

$$\begin{aligned} D'w &= D'z + \Pi S'z + (D'\Gamma D)D'\mu^* + \Pi(S'\Gamma D)D'\mu^* \\ &\equiv D'z + \Pi S'z + \Psi D'\mu^* \end{aligned} \tag{21}$$

where $\Pi \equiv -D'\Gamma S(S'\Gamma S)^{-1}$, which is a $(k_2 \text{x} k_1)$-matrix. The $(k_2 \text{x} k_2)$ matrix Ψ is defined as $\Psi \equiv (D'\Gamma D) + \Pi(S'\Gamma D)$.

For later purposes, it is useful to rewrite this equation somewhat. Recall the definition of $\tilde{w}$ (cf. (16)). We will sometimes refer to $\tilde{w}$ as "latent" portfolio shares, to indicate that they are generally not all observed. Instead w is observed. Using the fact that

$$\begin{aligned} \Psi D' - D'\Gamma &= D'\Gamma DD' - D'\Gamma S(S'\Gamma S)^{-1}S'\Gamma DD' - D'\Gamma \\ &= D'\Gamma(I_{k-1} - SS' - S(S'\Gamma S)^{-1}S'\Gamma DD' - I_{k-1}) \\ &= -D'\Gamma\left(S(S'\Gamma S)^{-1}S'\Gamma SS' + S(S'\Gamma S)^{-1}S'\Gamma(I_{k-1} - SS')\right) \\ &= -D'\Gamma S(S'\Gamma S)^{-1}S'\Gamma = \Pi S'\Gamma \end{aligned} \tag{22}$$

we can then write (21) as

$$\begin{aligned} D'w - D'\tilde{w} &= D'z + \Pi S'z + \Psi D'\mu^* - D'z - D'\Gamma\mu^* \\ &= \Pi S'z + (\Psi D' - D'\Gamma)\mu^* \\ &= \Pi S'z + \Pi S'\Gamma\mu^* = \Pi S'\tilde{w} \end{aligned} \tag{23}$$

So we observe that the non-zero portfolio shares are equal to their latent counterparts plus a linear combination of the latent budget shares corresponding to the zero assets. Defining $\Delta' \equiv D' + \Pi S'$, this can also be written as $D'w = \Delta'\tilde{w}$.

Also note that the non-zero Lagrange multipliers are found as

$$
\begin{aligned}
S'\lambda &= S'\tilde{\mu} - S'\mu^* = -(S'\Gamma S)^{-1}\{S'z + (S'\Gamma D)D'\tilde{\mu}\} - S'\mu^* \\
&= -(S'\Gamma S)^{-1}\{S'z + S'\Gamma DD'\mu^* + S'\Gamma SS'\mu^*\} \\
&= -(S'\Gamma S)^{-1}\{S'z + S'\Gamma[DD' + SS']\mu^*\} \\
&= -(S'\Gamma S)^{-1}S'\tilde{w} \qquad (24)
\end{aligned}
$$

The econometric model of portfolio shares can now be written as follows:

$$
\begin{aligned}
\tilde{w}_i &= z_i + \Gamma\mu_i^* = \bar{z}_i + \Gamma\mu_i^* + \epsilon_i \\
&\left\{ \begin{array}{c} D_i'w_i = \Delta_i'\tilde{w}_i \\ S_i'\lambda_i = -(S_i'\Gamma S_i)^{-1}S_i'\tilde{w}_i \end{array} \right\} \quad \textbf{iff} \quad \left\{ \begin{array}{c} \Delta_i'\tilde{w}_i \geq 0 \quad \textbf{and} \\ (S_i'\Gamma S_i)^{-1}S_i'\tilde{w}_i \leq 0 \end{array} \right\} \qquad (25)
\end{aligned}
$$

where a subscript i has been added to index observations and $\bar{z}_i$ is the systematic part of z_i, i.e. $z_i = \bar{z}_i + \epsilon_i$. The selection matrices D_i' and S_i' vary by observation. The Kuhn-Tucker conditions guarantee that for each realization of the latent shares $\tilde{w}_i$ there is only one unique combination of D_i' and S_i' such that the inequality conditions (25) are satisfied.

4.1 Identification

Using (25) we observe that the vectors z are identified up to a scaling constant from the simple probit equations explaining ownership patterns. Furthermore, we note that the elements of μ^* vary proportionately, so that *given* z we obtain $k-1$ pieces of information on Γ from the probits based on (25). To fully identify all parameters we need to consider the equations for the non-zero shares (21). The number of free elements in Π is equal to $(k_2 \mathrm{x} k_1)$. The number of elements in Ψ is equal to $(k_2 \mathrm{x} k_2)$, but since all elements of μ^* are proportional to each other, we can only identify k_2 elements. Thus for a given pattern of non-zero asset shares, and given z, we have $k_2 + k_2.k_1 = k_2(k_1+1) = k_2(k-k_2)$ pieces of information that can be identified from the rationed equations. To determine the total number of pieces of information on Γ that can be identified from the rationing equations, we have to account for all possible patterns of missing asssets. We find that the number of restrictions imposed on Γ is equal to:

$$
R(k) \equiv \sum_{k_2=1}^{k-1} \binom{k-1}{k_2} (k-k_2)k_2 \qquad (26)
$$

Since Γ is symmetric, the number of free elements in Γ is equal to $k(k-1)/2$. In addition we need $(k-1)$ scaling constants to identify z, but on the other hand the probits provide $k-1$ pieces of information on Γ, so these cancel out. In total we thus need $k(k-1)/2$ pieces of information. Table 10 presents the number of free elements in Γ and the numbef of restrictions $R(k)$ for different values of k. For $k \geq 2$, the parameters in the model are identified, at least by the simple counting rule we have applied here.

Table 10: Number of assets and restrictions on Γ

k	**Free elements of** Γ	R(k)
2	1	1
3	3	6
4	6	24
5	10	80
6	15	240
7	21	672
8	28	1792
9	36	4608
10	45	11520

4.2 The Likelihood

The likelihood is based on (25). We consider two cases. The first case is where all asset shares are non-zero. In this case the observed shares are equal to the latent shares and the likelihood contribution is the joint density of the asset shares as implied by the first equation in (25). The second case is where one or more of the asset shares are zero. For observed values D_i', $D_i' w_i$, and S_i' the likelihood contribution of this observation is:

$$\begin{aligned} g(D_i'w \quad & | \quad \Delta_i'\varepsilon_i \geq 0,\, (S_i'\Gamma S_i)^{-1}S_i'\varepsilon_i \leq 0). \Pr(\Delta_i'\varepsilon_i \geq 0,\, (S_i'\Gamma S_i)^{-1}S_i'\varepsilon_i \leq 0) \\ & = \quad \Pr(\Delta_i'\varepsilon_i \geq 0,\, (S_i'\Gamma S_i)^{-1}S_i'\varepsilon_i \leq 0 \mid D_i'w_i).h(D_i'w_i) \\ & = \quad \Pr((S_i'\Gamma S_i)^{-1}S_i'\varepsilon_i \leq 0 \mid D_i'w).h(D_i'w_i) \end{aligned} \tag{27}$$

using the fact that $D_i'w_i = \Delta_i'\varepsilon_i$ and with obvious definitions for the conditional density g and the marginal density h of $D_i'w_i$. To evaluate the likelihood contribution, we need to find the marginal distribution of $D_i'w$ and the conditional distribution of $(S_i'\Gamma S_i)^{-1}S_i'\varepsilon_i$ given $D_i'w_i$. We assume normality of the error vector ϵ_i throughout, with $\epsilon_i \sim N(0,\Omega)$. Given this normality assumption these are straightforward exercises.

We first consider the joint distribution of $(S_i'\Gamma S_i)^{-1}S_i'\varepsilon_i$ and $D_i'w_i = \Delta_i'\varepsilon_i$. We immediately have that $(S_i'\Gamma S_i)^{-1}S_i'w_i$ and $D_i'w_i$ are jointly normal with variance-covariance matrix equal to

$$\begin{pmatrix} (S_i'\Gamma S_i)^{-1}S_i'\Omega S_i(S_i'\Gamma S_i)^{-1} & (S_i'\Gamma S_i)^{-1}S_i'\Omega\Delta_i \\ \Delta_i'\Omega S_i(S_i'\Gamma S_i)^{-1} & \Delta_i'\Omega\Delta_i \end{pmatrix} \tag{28}$$

The means of the marginal distributions of $(S_i'\Gamma S_i)^{-1}S_i'\mathbf{e}_i$ and $D_i'w_i$ are equal to:

$$\mathrm{E}[(S_i'\Gamma S_i)^{-1}S_i'\mathbf{e}_i] = (S_i'\Gamma S_i)^{-1}S_i'[\overline{z}_i + \Gamma\mu_i^*] \tag{29}$$

and

$$\mathrm{E}[D_i'w_i] = \Delta_i'[\overline{z}_i + \Gamma\mu_i^*] \tag{30}$$

The conditional variance-covariance matrix of $(S_i'\Gamma S_i)^{-1}S_i'w_i$ given $D_i'w_i$ is given by

$$(S_i'\Gamma S_i)^{-1}[S_i'\Omega S_i - S_i'\Omega\Delta_i(\Delta_i'\Omega\Delta_i)^{-1}\Delta_i'\Omega S_i](S_i'\Gamma S_i)^{-1} \tag{31}$$

and the conditional mean of $(S_i'\Gamma S_i)^{-1}S_i'w_i$ given $D_i'w_i$ is given by

$$(S_i'\Gamma S_i)^{-1}S_i'[\overline{z}_i + \Gamma\mu_i^*] + (S_i'\Gamma S_i)^{-1}S_i'\Omega\Delta_i(\Delta_i'\Omega\Delta_i)^{-1}[D_i'w_i - \Delta_i'[\overline{z}_i + \Gamma\mu_i^*]] \tag{32}$$

Appendix A provides the details of the likelihood for the case $k = 3$, which will be considered in our empirical work.

4.3 Results for the full model

Table 11 presents estimation results for several versions of the model described above. We have restricted observations to households with positive wealth only to avoid problems with the definitions of portfolio shares. Given the other selections (the need to have complete observations on all explanatory variables and the portfolio shares), the restriction to households with positive wealth involves a loss of about 12% of the observations. Three versions of the model are presented in the table. The first and second version use a combination of risk attitude variables to parameterize risk aversion. For the first version, we specify risk tolerance as

$$\frac{1}{R} = \frac{1}{1 + \exp[\lambda.\mathrm{riskat1} + (1-\lambda).\mathrm{riskat2}]} \tag{33}$$

where the parameter λ can be estimated jointly with the other parameters in the model. For the second version, we use (33) but with riskat1 and riskat3 replaced by careful and precaution. In the third version the term $[\lambda.\mathrm{riskat1}+(1-\lambda).\mathrm{riskat2}]$ is replaced by the variable risk.The number of observations varies per version, reflecting sample selections and skipping patterns in the questionnaires, as discussed before. In all versions the estimate of γ_{12} (the off-diagonal element of Γ) had to be bounded from below to maintain positive definiteness of Γ. Somewhat arbitrarily we have restricted the quantity $\gamma_{12}/(\gamma_{11}\gamma_{22})$ to be greater than -0.9.

We observe that γ_{11} and γ_{22} (the diagonal elements of Γ) are only signficantly different from zero in the first version where we use riskat1 and riskat3 as indicators of risk aversion. The version with careful and precaution has also been estimated on exactly the same sample as was used for riskat1 and riskat3. The likelihood in that case turned out to be equal to -337.07, which is to be compared to -317.27

for the version with riskat1 and riskat3. The version with risk as the risk aversion parameter appears to provide the poorest fit, even taking into account the lower number of observations. For example, if we multiply the log-likelihood for this version by (675/437) we obtain -352.1, which is quite a bit lower than the value of -317.27 for the version with riskat1 and riskat3. The estimates of the effect of the other explanatory variables are qualitatively similar across the three versions. Income has a positive effect on the portfolio share of asset 2 (asset R in Table 9) and a negative effect on the portfolio share of asset 3 (asset O in Table 9). The effects of wealth are the opposite of those for income. Age and age squared are always jointly significant, where education is never significant. The parameters of the estimated age functions imply that the share of the first risky asset will rise after age 40, whereas the share of the second risky asset will fall monotonically. Gender does not exert a statistically significant influence on portfolio choice.The parameter λ is estimated around .5, so that both variables making up the risk tolerance variable are having an almost equal influence.

The variables riskat1 and riskat3 are linear combinations of the underlying responses to the subjective questions listed in Section 2.2 above. In principle therefore, one can also use these responses directly in the definition of the risk tolerance measure analogous to (33). Table 12 provides the estimates of the weights of each of these variables in the risk tolerance measure. The other parameters of this version of the model have been suppressed for reasons of space. They are similar to the results reported in Table 11 for riskat1 and riskat3. We observe from Table 12 that the subjective variables are dominated by "spaar2". Recall that this is the response to the question "Investing in stocks is something I don't do, since it is too risky". The fact that the word "stocks" is mentioned explicitly in this question may explain why this variable dominates all others in explaining portfolio choice.

Finally, we note that by inverting Γ we obtain an estimate of the variance covariance matrix of excess returns Σ, as perceived by households. We find $\Sigma = \begin{pmatrix} 1.43 & \\ .90 & 17.4 \end{pmatrix}$, where the off-diagonal element is the correlation rather than the covariance between the excess returns. Note that the scale of Σ is arbitrary at this point, as the scale of the risk tolerance measure (33) is arbitrary. We do note that the variance covariance matrix implies that the second risky asset has a higher perceived variance in returns than the first one.

Table 11: Estimation results for the full model (total assets)

	Riskat1/3		**Careful/Prec.**		**Risk**	
Parameter/variable	**Est.**	**t-val.**	**Est.**	**t-val.**	**Est.**	**t-val.**
Risky Asset 1						
log-income	.087	2.25	.093	2.76	.021	.35
gender	.031	1.15	-.007	.29	.106	1.74
middle education	.015	.45	.042	1.41	-.009	.17
higher education	.051	1.64	.055	1.86	.066	1.33
log-wealth	-.059	3.74	-.085	5.75	-.065	2.68
age	-.020	2.69	-.006	1.00	-.010	.92
age squared	.0002	3.68	.0001	2.22	.0002	1.54
constant	-.015	.04	.159	.50	.547	.93
Risky Asset 2						
log-income	-.115	2.88	-.119	3.79	-.071	1.41
gender	-.026	.94	-.013	.59	-.085	1.59
middle education	.004	.12	-.008	.27	.010	.23
higher education	-.038	1.13	-.046	1.62	-.042	.98
log-wealth	.218	17.2	.241	22.7	.212	13.8
age	.008	1.02	.001	.23	.012	1.15
age squared	-.0001	2.01	-.0001	1.52	-.0002	1.69
constant	-.874	2.47	-.999	3.55	-1.28	2.71
ω_1	.248	20.1	.301	24.8	.288	15.3
ω_2	.330	29.4	.356	31.5	.322	21.2
ρ	-.78	28.6	-.861	56.7	-.840	27.7
γ_{11}	1.15	3.69	1.54	1.22	.107	.43
γ_{22}	.102	1.99	.119	1.04	.003	.41
λ	.487	.084	.441	1.16	-	-
Number of obs.	675		1149		437	
log-likelihood	-317.27		-642.30		-227.98	

Table 12: The estimates for the separate risk attitude variables

Variable	**Estimate**	**t-value**
spaar1	-.064	.90
spaar2	.710	5.42
spaar3	-.036	.57
spaar4	.047	.56
spaar5	.093	1.34
spaar6	.209	1.77
spaarm03	-.022	.24
spaarm12	-.027	.27
spaarm13	.089	1.17
Joint significance	$\chi^2(8)$=320.6	p=.000

5 Concluding remarks

We have explored the explanatory power of a number of different subjective measures of risk aversion for the explanation of portfolio choice. The results appear to be mixed. The variable risk, which has the firmest grounding in economic theory, appears to have very little explanatory power. There are a few different possible explanations for this. First of all, the question is quite complicated and many respondents may have a hard time understanding the exact meaning of the question. Secondly, the question conditions on a respondent's current situation. So for instance an individual with a risky portfolio may be induced to choose a save income stream, since she is already exposed to considerable risk. Conversely, an individual with a very safe portfolio can afford to choose a riskier income path. In both cases the observed relationship between the measured risk tolerance and portfolio choice is attenuated.

The variables riskat1 and riskat3 extracted from a factor analysis of ad hoc measures appear to be doing considerably better in terms of explanatory power. Yet, closer analysis reveals that most of the explanatory power of these variables comes from one question, which asks directly for the subjective evaluation of the riskiness of investing in stocks. This raises the possibility that this measure at least partly reflects an ex post rationalization of non-stock ownership.

The modelling of incomplete portfolios by explicit use of Kuhn-Tucker conditions is fairly straightforward and estimation of the resulting model yields well determined estimates of a number of key parameters. However, we had to restrict parameters in order to make sure that the underlying utility function is well-behaved. This suggests that more research into an adequate specification is needed.

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A The likelihood for $k = 3$

For $k = 3$ there are four possible ownership patterns for the two risky assets: (1) $w_1 = 0$, $w_2 = 0$; (2) $w_1 \neq 0$, $w_2 = 0$; (3) $w_1 = 0$, $w_2 \neq 0$; (4) $w_1 \neq 0$, $w_2 \neq 0$. We will discuss these consecutively. For later use we introduce some scalar notation:

$$\begin{aligned} \Gamma &\equiv \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}, \quad \Omega \equiv \begin{pmatrix} \omega_1^2 & \rho\omega_1\omega_2 \\ \rho\omega_1\omega_2 & \omega_2^2 \end{pmatrix}, \quad w_i \equiv \begin{pmatrix} w_{i1} \\ w_{i2} \end{pmatrix} \\ \overline{z}_i &\equiv \begin{pmatrix} \overline{z}_{i1} \\ \overline{z}_{i2} \end{pmatrix}, \quad \mu_i^* \equiv \begin{pmatrix} \mu_{i1}^* \\ \mu_{i2}^* \end{pmatrix} \end{aligned} \tag{34}$$

For the various normal distributions and densities the following notation is adopted. $B\Phi\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}; \upsilon, \Sigma\right)$ represents the joint probability that two normally distributed random variables, with mean vector υ and variance-covariance matrix Σ, are less than or equal to x_1 and x_2, respectively. $B\phi\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}; \upsilon, \Sigma\right)$ is the value of the corresponding density at the point $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. $\Phi[x; \mu, \sigma]$ is the probability that a normally distributed variable, with mean μ and variance σ^2, is less than or equal to x. $\phi[x; \mu, \sigma]$ is the value of the corresponding density at the point x.

A.1 $w_1 = 0$, $w_2 = 0$

For this case the likelihood contribution is the bivariate probability that both Lagrange multipliers are non-negative, or in other words that $\Gamma^{-1}\epsilon_i \leq 0$. Thus the likelihood contribution for this case is:

$$\mathcal{L}_{1i} = B\Phi\left[(0); \left(\Gamma^{-1}[\bar{z}_i + \Gamma\mu_i^*]\right), \Gamma^{-1}\Omega(\Gamma^{-1})'\right] \tag{35}$$

The matrix Γ is assumed symmetric, so the transposition sign is superfluous, strictly speaking. Define $D \equiv |\Gamma| = \gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}$. Then we can write

$$\Gamma^{-1}[\bar{z}_i + \Gamma\mu_i^*] = \frac{1}{D}\begin{pmatrix} \gamma_{22}\bar{z}_1 - \gamma_{12}\bar{z}_1 + \mu_1^* \\ \gamma_{11}\bar{z}_2 - \gamma_{21}\bar{z}_1 + \mu_2^* \end{pmatrix} \tag{36}$$

$$\Gamma^{-1}\Omega(\Gamma^{-1})' \equiv \frac{1}{D^2}\begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12} & \Theta_{22} \end{pmatrix} \tag{37}$$

with

$$\begin{aligned} \Theta_{11} &= (\gamma_{22})^2\omega_1^2 - 2\gamma_{12}\gamma_{22}\rho\omega_1\omega_2 + (\gamma_{12})^2\omega_2^2 \\ \Theta_{12} &= -\gamma_{22}\gamma_{21}\omega_1^2 + (\gamma_{11}\gamma_{22} + \gamma_{12}\gamma_{21})\rho\omega_1\omega_2 - \gamma_{11}\gamma_{12}\omega_2^2 \\ \Theta_{22} &= (\gamma_{11})^2\omega_2^2 - 2\gamma_{11}\gamma_{21}\rho\omega_1\omega_2 + (\gamma_{21})^2\omega_1^2 \end{aligned} \tag{38}$$

A.2 $w_1 \neq 0$, $w_2 = 0$

For this case we have $S_i' = \begin{pmatrix} 0 & 1 \end{pmatrix}$, $D_i' = \begin{pmatrix} 1 & 0 \end{pmatrix}$, $S_i'\Gamma S_i = \gamma_{22}$. Thus, $(S_i'\Gamma S_i)^{-1}w_i = \frac{w_{i2}}{\gamma_{22}}$. Hence, $(S_i'\Gamma S_i)^{-1}w_i \leq 0$ is equivalent with $w_{i2} \leq 0$. Furthermore we have $\Delta_i' = D_i'[I - \Gamma S_i(S_i'\Gamma S_i)^{-1}S_i'] = \begin{pmatrix} 1 & -\frac{\gamma_{12}}{\gamma_{22}} \end{pmatrix}$, $S_i'\Omega\Delta_i = -\frac{\gamma_{12}}{\gamma_{22}}.\omega_2^2 + \rho\omega_1\omega_2$, and $\Delta_i'\Omega\Delta_i = \omega_1^2 - 2\rho\frac{\gamma_{12}}{\gamma_{22}}\omega_1\omega_2 + \left(\frac{\gamma_{12}}{\gamma_{22}}\right)^2\omega_2^2$. Hence, the marginal density of w_{i1} is normal with variance

$$\sigma_1^2 \equiv \Delta_i'\Omega\Delta_i = \omega_1^2 - 2\rho\frac{\gamma_{12}}{\gamma_{22}}\omega_1\omega_2 + \left(\frac{\gamma_{12}}{\gamma_{22}}\right)^2\omega_2^2 \tag{39}$$

and mean

$$\chi_{i1} \equiv \Delta_i' \overline{z}_i + \Delta_i' \Gamma \mu_i^* = -\frac{\gamma_{12}}{\gamma_{22}} \overline{z}_{i2} + \overline{z}_{i1} + (\gamma_{11} - \frac{\gamma_{21}\gamma_{12}}{\gamma_{22}})\mu_{i1}^* \tag{40}$$

The conditional variance of the latent budget share $\tilde{w}_{i2}$ given w_{i1} (cf. (31), but without pre- and postmultiplication by $(S_i' \Gamma S_i)^{-1}$) becomes

$$\eta_2^2 \equiv S_i' \Omega S_i - S_i' \Omega \Delta_i (\Delta_i' \Omega \Delta_i)^{-1} \Delta_i' \Omega S_i = \omega_2^2 - \frac{\left(-\frac{\gamma_{12}}{\gamma_{22}}.\omega_2^2 + \rho \omega_1 \omega_2\right)^2}{\omega_1^2 - 2\rho \frac{\gamma_{12}}{\gamma_{22}} \omega_1 \omega_2 + \left(\frac{\gamma_{12}}{\gamma_{22}}\right)^2 \omega_2^2} \tag{41}$$

Similarly we have $S_i' \Omega \Delta_i (\Delta_i' \Omega \Delta_i)^{-1} = \frac{-\frac{\gamma_{12}}{\gamma_{22}}.\omega_2^2 + \rho \omega_1 \omega_2}{\omega_1^2 - 2\rho \frac{\gamma_{12}}{\gamma_{22}} \omega_1 \omega_2 + \left(\frac{\gamma_{12}}{\gamma_{22}}\right)^2 \omega_2^2}$. Thus it follows from (32) (omitting premultiplication by $(S_i' \Gamma S_i)^{-1}$) that the conditional mean of w_{i2} given w_{i1} is given by

$$\begin{aligned} \nu_{i2} &\equiv S_i' \overline{z}_i + S_i' \Gamma \mu_i^* + S_i' \Omega \Delta_i (\Delta_i' \Omega \Delta_i)^{-1} [D_i' w_i - \Delta_i' \overline{z}_i - \Psi_i S_i' \mu_i^*] \\ &= \overline{z}_{i2} + \gamma_{21} \mu_{i1}^* + \gamma_{22} \mu_{i2}^* + \\ &\quad \frac{-\frac{\gamma_{12}}{\gamma_{22}}.\omega_2^2 + \rho \omega_1 \omega_2}{\omega_1^2 - 2\rho \frac{\gamma_{12}}{\gamma_{22}} \omega_1 \omega_2 + \left(\frac{\gamma_{12}}{\gamma_{22}}\right)^2 \omega_2^2} \{w_{i1} - \chi_{i1}\} \end{aligned} \tag{42}$$

For the likelihood contribution of this case we obtain:

$$\mathcal{L}_{2i} \equiv \Phi[0; \nu_{i2}, \eta_2].\varphi[w_{i1}; \chi_{i1}, \sigma_1] \tag{43}$$

A.3 $w_1 = 0$, $w_2 \neq 0$

For this case we have $S_i' = \begin{pmatrix} 1 & 0 \end{pmatrix}$, $D_i' = \begin{pmatrix} 0 & 1 \end{pmatrix}$. Similar to previous case, the condition that the Lagrange multiplier for the binding constraint is non-negative is equivalent with $w_{i1} \leq 0$. Furthermore, $\Delta_i' = D_i' [I - \Gamma S_i (S_i' \Gamma S_i)^{-1} S_i'] = \begin{pmatrix} -\frac{\gamma_{21}}{\gamma_{11}} & 1 \end{pmatrix}$, $S_i' \Omega \Delta_i = -\frac{\gamma_{21}}{\gamma_{11}}.\omega_1^2 + \rho \omega_1 \omega_2$, $\Delta_i' \Omega \Delta_i = \omega_2^2 - 2\rho \frac{\gamma_{21}}{\gamma_{11}} \omega_1 \omega_2 + \left(\frac{\gamma_{21}}{\gamma_{11}}\right)^2 \omega_1^2$. Hence the marginal density of w_{i2} is normal with variance

$$\sigma_2^2 \equiv \Delta_i' \Omega \Delta_i = \omega_2^2 - 2\rho \frac{\gamma_{21}}{\gamma_{11}} \omega_1 \omega_2 + \left(\frac{\gamma_{21}}{\gamma_{11}}\right)^2 \omega_1^2 \tag{44}$$

and mean

$$\chi_{i2} \equiv \Delta_i' \overline{z}_i + \Delta_i' \Gamma \mu_i^* = -\frac{\gamma_{21}}{\gamma_{11}} \overline{z}_{i1} + \overline{z}_{i2} + (\gamma_{22} - \frac{\gamma_{21}\gamma_{12}}{\gamma_{11}})\mu_{i1}^* \tag{45}$$

.The conditional variance of latent budget share w_{i1} given w_{i2} as given in (31) (but without pre- and postmultiplication by $(S_i'\Gamma S_i)^{-1}$) becomes

$$\eta_1^2 \equiv S_i'\Omega S_i - S_i'\Omega\Delta_i(\Delta_i'\Omega\Delta_i)^{-1}\Delta_i'\Omega S_i = \omega_1^2 - \frac{\left(-\frac{\gamma_{21}}{\gamma_{11}}.\omega_1^2 + \rho\omega_1\omega_2\right)^2}{\omega_2^2 - 2\rho\frac{\gamma_{21}}{\gamma_{11}}\omega_1\omega_2 + \left(\frac{\gamma_{21}}{\gamma_{11}}\right)^2\omega_1^2} \tag{46}$$

Similarly we have $S_i'\Omega\Delta_i(\Delta_i'\Omega\Delta_i)^{-1} = \frac{-\frac{\gamma_{21}}{\gamma_{11}}.\omega_1^2 + \rho\omega_1\omega_2}{\omega_2^2 - 2\rho\frac{\gamma_{21}}{\gamma_{11}}\omega_1\omega_2 + \left(\frac{\gamma_{21}}{\gamma_{11}}\right)^2\omega_1^2}$. Thus it follows from (32) (omitting premultiplication by $(S_i'\Gamma S_i)^{-1}$) that the conditional mean of w_{i1} given w_{i2} is given by

$$\begin{aligned}\nu_{i1} &\equiv S_i'\overline{z}_i + S_i'\Gamma\mu_i^* + S_i'\Omega\Delta_i(\Delta_i'\Omega\Delta_i)^{-1}[D_i'w_i - \chi_{i2}] \\ &= \overline{z}_{i1} + \gamma_{11}\mu_{i1}^* + \gamma_{12}\mu_{i2}^* + \\ &\quad \frac{-\frac{\gamma_{21}}{\gamma_{11}}.\omega_1^2 + \rho\omega_1\omega_2}{\omega_2^2 - 2\rho\frac{\gamma_{21}}{\gamma_{11}}\omega_1\omega_2 + \left(\frac{\gamma_{21}}{\gamma_{11}}\right)^2\omega_1^2}\{w_{i2} - \chi_{i2}\}\end{aligned} \tag{47}$$

For the likelihood contribution of this case we obtain:

$$\mathcal{L}_{3i} \equiv \Phi[0; \nu_{i1}, \eta_1].\varphi[w_{i2}; \chi_{i2}, \sigma_2] \tag{48}$$

A.4 $w_1 \neq 0$, $w_2 \neq 0$

This case is straightforward. The likelihood contribution is simply the bivariate density of w_1 and w_2 as generated by (25). Thus the likelihood contribution for this case is:

$$\mathcal{L}_{4i} = B\phi\left[\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}; \begin{pmatrix} \overline{z}_{i1} + \gamma_{11}\mu_{i1}^* + \gamma_{12}\mu_{i2}^* \\ \overline{z}_{i2} + \gamma_{21}\mu_{i1}^* + \gamma_{22}\mu_{i2}^* \end{pmatrix}, \Omega\right] \tag{49}$$