Liquidity and Credit Risk*

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Abstract

We develop a simple binomial model of liquidity and credit risk in which a bondholder has the option to time the sale of his security, given a distribution of potential buyers, bids and liquidity shocks. We examine as a benchmark the case without default and find that our model predicts a decreasing term structure of liquidity premia, in accordance with the empirical findings of Amihud & Mendelson (1991). Then, we study the default risky case and show that credit risk influences liquidity spreads in a non-trivial way. We find that liquidity spreads are an increasing function of the volatility of the firm's assets and leverage - the key determinants of credit risk. Furthermore we show that bondholders are more likely to sell their holdings voluntarily when bond maturity is distant and when default becomes more probable. Finally, in a sample of US corporate bonds, we find support for the time to maturity effect and the positive correlation between credit and liquidity risks.

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1 Introduction

Credit risk and liquidity risk have been put forward as two of the main justifications for the existence of yield spreads above benchmark Treasury notes or bonds (see Fisher (1959)). While a rapidly growing body of literature has focused on credit risk[1] since Merton (1974), liquidity has remained a relatively unexplored topic, in particular for defaultable securities. Meanwhile, concern about market liquidity issues has become increasingly marked since the autumn of 1998.[2] The purpose of this paper is to develop a simple structural model of liquidity and credit risk in an attempt to better understand the interaction between these two sources of risk and their relative contributions to the yield spreads on corporate bonds.

Throughout the paper, we define liquidity as the ability to sell a security promptly and at a price close to its value in frictionless markets. We thus think of an illiquid market as one in which a sizeable discount may have to be incurred for immediacy.

Structural credit risk models along the lines of Merton (1974) are often criticized on the following two counts. First, it is argued that the levels of yield spreads generated by the models are too low to be consistent with observed spreads.[3] This may indeed be indicative of an inadequate modeling approach. However it should be noted that such casual empiricism fails to take into account that these models only price credit risk. If prices on corporate bonds reflect compensation for other sources of risk such as illiquidity then one should expect to find that these models overprice bonds.

A second criticism often brought up is that the levels of credit spreads obtained with most structural models are negligible for very short maturities and that this is inconsistent with empirical evidence.[4] Our model implies non trivial liquidity premia for short maturities and can thus help to reconcile structural models with this stylized fact

In recent empirical work on credit spreads, Anderson & Sundaresan (2000) show that although extensions of the Black & Cox (1976) model are often able to capture large parts of the variation in spread indices, during some sub-samples they fail to do so. This suggests that another factor exogenous to their model is at play. Duffee (1999) estimates a three factor[5] model of bond prices on corporate debt data. His findings suggest that the factors are not sufficient to fully capture bond yield spreads. We posit that a natural candidate for an additional factor should be one that captures the illiquidity of corporate debt markets.

We model credit risk in a modified Merton (1974) framework. Although interest rate risk is an important determinant of corporate bond prices we abstract from it in order to allow any interaction between credit and liquidity risk to be analyzed in isolation. Structural models with stochastic interest rates have been proposed for example by Kim et al. (1993), Shimko et al. (1993), Nielsen et al. (1993) and Wang (1999).

We introduce two distinct sources of liquidity risk. First, when the firm is solvent, the bearer of a bond is subjected to random liquidity shocks. Such shocks can for example reflect cash constraints or a need to rebalance a portfolio because the specific bond is no longer appropriate for hedging or diversification purposes. Thus, with a given probability he may have to sell his bond immediately. The price he would have to sell at is assumed to be a random fraction of the price in a perfectly liquid market. The distribution of this fraction is modeled as a function of the number of traders active in the market for a particular bond. As a first step, we let this distribution be time and state independent. Later in the paper we provide an extension where the bond dealers' behavior is an endogenous function of the bond's credit risk and remaining time to maturity.

The supply side of the market is an endogenous function of the state of the firm and the probability of liquidity shocks. When there is no liquidity shock, the bondholder still has the option to sell if the price he can obtain is high enough. Although if a bondholder could hold the bond until maturity he would avoid accepting a discount altogether, he will sell if the price is better than the expected value of waiting and exposing himself to the risk of being forced to sell at a less favorable price.

The second important assumption we make is that heterogeneity in liquidity is maintained in the market for distressed debt. Bonds that were relatively illiquid before default remain less liquid than other distressed bonds. This is supported by empirical evidence. Wagner (1996) studies the market for distressed debt and finds that medium and small defaulted issues outperform the returns on larger (and hence more liquid) issues. The relative size of issues is likely to be invariant to whether a firm is solvent or in default (at least prior to the issuance of restructured assets). Hence we would expect the liquidity of an issue to be positively related to the volume outstanding, before and after financial distress.

Our model predicts that liquidity spreads are decreasing functions of time to maturity. This is consistent with empirical evidence on markets for Treasury securities. Amihud & Mendelson (1991) examine the yield differentials between US Treasury notes and bills - securities with differing liquidity and find that term structures of liquidity premia indeed have this particular shape. Kempf & Uhrig (1997) study liquidity effects on German government bonds and their findings support the conclusions of Amihud & Mendelson (1991) as to the shape of the term structure of liquidity premia. The reason that our model implies a decreasing term structure of liquidity spreads is that there is an upper bound on the dollar losses that can follow from liquidity shocks and that this translates into decreasing spreads. The convexity derives from a bondholder's optimal selling strategy - in trying to minimize losses from liquidity shocks he will lower the cap on marketability costs below the level which would prevail if he was restricted from trading.

In addition to the shape of the term structure, our model implies that the level of liquidity premia is correlated to the probability of financial distress. The premia will be higher when default is likely as bondholders will have less time to look for attractive offers and may have to liquidate their positions on unfavorable terms. This correlation is reinforced when we extend the bidding behavior of bond dealers to depend on the risk profile of the bond - as the riskiness of the bond increases near default, larger discounts may be required to induce a trade.

Furthermore, we show that the optimal trading behavior of the bondholder is a function of time to maturity, firm risk and leverage. The bondholder's behavior is summarized by the discount that he is willing to sell his security at. When time to maturity is short, there is a relatively small chance that he will be forced to sell his bond at an unfavorable price. He will thus be unwilling to accept a large discount relative to the frictionless price. However when the bond maturity is distant he will be willing to sell at a relatively lower price. This effect is consistent with the stylized fact that the market for recent vintages is more active than that for seasoned ones. Trading should be more active the riskier the issue. When the firm is near default the probability of a sudden shortening of a bond's effective maturity may be substantial and the willingness of bondholders to sell will increase.

Reduced form models of credit risk such as Duffie & Singleton (1999) or Lando (1998) are typically able to include liquidity as a component of their total spreads. However in these models, only aggregate spreads can be derived from actual data and one cannot distinguish a situation with a high liquidity premium and little default risk from one of a very liquid but risky bond. Separating liquidity from credit spreads is not only of academic interest, it is also crucial to hedging and portfolio management. In order to set up effective strategies for these purposes it is not sufficient to merely know how much compensation one receives or pays for different risks, but also to which risks one is exposed and to what extent.

The Longstaff (1995) model lies close to ours in spirit. He measures the value of liquidity for a security as the value of the option to sell it at the most favorable price for a given time window. Although our results are not directly comparable because the author derives upper bounds for liquidity discounts for a given sales-restriction period, his definition of liquidity comes close to our own.

To date, Tychon & Vannetelbosch (1997) is, to our knowledge, the only paper which explicitly models the liquidity of corporate bonds endogenously. They use a strategic bargaining setup in which transactions take place because investors have different views about bankruptcy costs. Although some of their predictions are similar to ours, their definition of liquidity risk differs significantly. Notably, as their liquidity premia are linked to the heterogeneity of investors' perceptions about the costliness of financial distress, their model predicts that liquidity spreads in Treasury debt markets should be zero.

Two of our model's empirical implications - the positive correlation between default and liquidity risk and the negative slope of the term structure of liquidity spreads - are tested on a panel of approximately 500 US corporate bonds. Controlling for credit risk by a set of commonly used variables, we examine the impact of two proxies for liquidity risk: a measure of liquidity risk in Treasury markets and a measure of bond age. A comparison of parameter estimates across subsamples constructed along credit ratings and bond maturities reveals support for our model's predictions.

The structure of the paper is the following. Section 2 presents the model and describes the default generating mechanism and the sources of illiquidity. Section 3 analyzes liquidity spreads in the default-free case and reports the general shape of the term structure of liquidity spreads in this context. Sections 4 and 5 describe the default risky case and its implications for bond trading respectively. In section 6 we extend our analysis to a more realistic model of dealer behavior. Section 7 reports on our empirical tests on the model's predictions and section 8 concludes.

2 The Model

We assume for simplicity that bondholders are risk-neutral. We thus avoid the delicate issue of determining a suitable equivalent martingale measure for an illiquid and therefore incomplete market.[6] Although risk aversion might affect the quantitative outputs of our model they are unlikely to change the qualitative results.

The uncertainty relating to the firm value $\{v_t\}_{t=0}^{t=T}$ is modeled using a standard multiplicative binomial formulation[7] in which σ denotes the volatility of the firm's assets and Δt is the time interval between two nodes[8]. The risk-free interest rate r is assumed to be constant. Following Merton (1974), we assume that the firm is financed by a single issue of discount debt with maturity T and promised principal repayment P. We consider the value of the perfectly liquid bond $B_L(t)$ to be the benchmark against which we will compare the value of an illiquid discount bond $B_I(t)$. The price of a "liquid" security is given by the price that would obtain under the assumptions made in a Merton (1974)-like setting.

At maturity the holder receives the principal repayment when the firm is solvent or a fraction of the value of the firm when it is in default.

$$B_L(T) = P\, I_{v_T \geq P} + \max(v_T - K, 0)\, I_{v_T < P}.,$$

where I_A is the indicator function taking the value 1 if event A is true and 0 otherwise and K represents the costs of financial distress.[9] These are taken to include both direct costs (legal fees etc.) and indirect costs arising from suboptimal operating decisions (due to e.g. over or underinvestment incentives in financial distress), lost business relationships, etc.

We assume that at maturity the firm is liquidated and proceeds are distributed to the respective claimants. There is hence no market liquidity problem at this date and the illiquid bond price is

$$B_I(T) = B_L(T).$$

Following Black & Cox (1976) (and in contrast to Merton (1974)), we assume that financial distress can be triggered prior to the maturity of a bond when the value of the firm's assets reaches a lower boundary L. We have specified L as a fraction $g \leq 1$ of the present value of the debt's principal :

$$L = g\, \exp(-rT) P.$$

We thus impose an upper bound on the firm's leverage in terms of its "quasi-debt ratio".[10] When a realization of firm value v_t becomes known we observe if the firm is solvent. If not, the bonds are worth:

$$\begin{aligned} B_L(t) &= \max(L-K,0), \\ B_I(t) &= \max(L-K-\kappa,0), \end{aligned}$$

where $\kappa > 0$ is an additional cost reflecting the illiquidity of distressed debt. The intuition for this parameter is that the market for distressed bonds is at least as illiquid as that for otherwise identical but non-defaulted bonds. This cost implies a higher expected return for previously illiquid debt in the distressed debt market, as supported by the empirical work of Wagner (1996). Suppose we set $\kappa = 0$; we would then have the counter-intuitive result that investors holding illiquid debt would be better off if the issuer defaulted because of the better liquidity of the market for distressed securities.[11]

An equivalent but less convenient approach to modeling this idea would be to apply our model of liquidity for non-distressed bonds to the default states. A sufficiently high κ would then correspond to a lower number of active traders in the market for defaulted debt.

In our model, the following events occur given that the firm remains solvent, i.e. $v_t > L$ (Figure 1 summarizes the sequence of events). First, the bondholder finds out whether he is forced to sell his bond due to a liquidity shock. Such shocks may occur as a result of unexpected cash shortages, the need to rebalance a portfolio in order to maintain a hedging or diversification strategy, or to meet capital requirements. The bondholder could for example be an insurance company which faces a sudden jump in claims because of an earthquake. The probability of being forced to sell during a particular period Δt is θ.

Given that he is forced to sell, the discount that he faces is modeled as follows. The price offered by any one particular trader is assumed to be a random fraction $\widetilde{\delta}_t$ of the perfectly liquid price B_L. We assume this fraction to be uniformly distributed on $[0,1]$. He may however obtain several offers and will retain the best one. Given that he goes to market, he will obtain N offers, where N is assumed to be Poisson with parameter γ

$$N \sim Po(\gamma),$$

so that γ is the expected number of offers.[12]. One may also think of γ as the number of active traders in the market for a particular type of bond. This number may differ for institutional reasons. For example, banks may be less active in the market for highly rated debt as a result of the way that capital requirements are structured.[13] The choice of distribution and support for the individual discounts is admittedly simplistic but we will nevertheless retain it for illustrative purposes.[14] The expected best fraction of the liquid price that he will be offered will thus be[15]

$$\bar{\delta} \equiv E\left[\tilde{\delta}_t\right] = \sum_{n=0}^{\infty} e^{-\gamma} \frac{\gamma^n}{n!} \cdot \frac{n}{n+1}.$$

Note that as γ tends to infinity, $\bar{\delta}$ tends to 1 as an ever greater number of market makers compete for the same security and the price converges to a purely liquid price. Our model then coincides with a discretized version of Black and Cox (1976). We do not consider the degenerate case where $\gamma = 0$ and no-one trades the bond.

The motivation for the randomness of $\widetilde{\delta}_t$, i.e. the implicit assumption that different prices for the same security can be realized at any one time is the same as for the occurrence of liquidity shocks. Some agents trade for hedging or cash flow reasons and may thus accept to buy at a higher (or sell at a lower) price than other traders.[16] The model of dealer behavior we propose in section 6 will make this assumed heterogeneity more explicit.

This setup is consistent with the structure of the US corporate bond market, an OTC market dominated by a limited number of dealers. Information asymmetries can therefore occur and result in several prices being quoted in a given market at the same time[17].

FIGURE 1

We assume that the distributions of liquidity shock arrivals and discounts are independent of firm value uncertainty.[18] This could apply to our above example of an insurer facing a sudden jump in claims. The expected value of the bond given a forced sale is thus

$$E_t\left[\tilde{\delta}_t B_L(t)\middle|\,\text{forced sale}\right] = B_L(t)\,E\left[\tilde{\delta}_t\right] = B_L(t)\,\bar{\delta},$$

where $E_t[.]$ denotes conditional expectation with respect to information available at date t after the possible realization of a liquidity shock but before the arrival of bids from bond dealers.[19] If he is not forced to sell (with probability $(1-\theta)$) he still has the option to sell his bond should the best offer made to him be acceptable. If he decides to sell he will receive a payment of

$$\tilde{\delta}_t B_L(t)\,,$$

and if he decides not to sell, the holding value is

$$e^{-r\Delta t}E_t\left[B_I(t+1)\right].$$

Hence, just prior to t (at $t-$, where the value of the firm is known but the potential liquidity shock and distribution of offers are not) the expected value of the illiquid bond if the firm is solvent is

$$E_{t-}\left[B_I(t)\right] = E_{t-}\left[\theta\bar{\delta}_t B_L(t) + (1-\theta)\max\left(\tilde{\delta}_t B_L(t)\,, e^{-r\Delta t}E_t\left[B_I(t+1)\right]\right)\right]. \tag{1}$$

We denote by δ_t^* the reservation price fraction above which the bondholder will decide to sell at time t and below which he will keep his position until the next period unless he faces a liquidity shock. This allows us to rewrite

$$E_{t-}\left[\max\left(\tilde{\delta}_t B_L\left(t\right), e^{-r\Delta t}E_t\left[B_I\left(t+1\right)\right]\right)\right],$$

as[20]:

$$\begin{aligned} & E_{t-}\left[\widetilde{\delta}_t B_L\left(t\right) I_{\widetilde{\delta}_t>\delta_t^*} + e^{-r\Delta t}E_t\left[B_I\left(t+1\right)\right] I_{\widetilde{\delta}_t\leq\delta_t^*}\right] \\ = \quad & B_L\left(t\right) E_{t-}\left[\widetilde{\delta}_t I_{\widetilde{\delta}_t>\delta_t^*}\right] + P\left[\widetilde{\delta}_t\leq\delta_t^*\right] e^{-r\Delta t}E_t\left[B_I\left(t+1\right)\right], \end{aligned}$$

The critical value for the offered price fraction $\tilde{\delta}_t$, above which the bondholder will decide to sell is

$$\delta_t^* = \frac{e^{-r\Delta t}E_t\left[B_I\left(t+1\right)\right]}{B_L\left(t\right)}.$$

In order to compute bond prices, we use backward induction: we begin at the maturity of the bonds where values are known and move recursively through the tree until we reach the initial date.

So far, we have taken the prevailing market conditions to be fixed. The average number of traders willing to trade a given bond has been assumed constant. This does not allow for adverse shocks to the economy as a whole, accompanied by worsening credit-market conditions. This assumption could be relaxed as follows. We could assume that normal market conditions are represented by an average number of traders γ^N and that with probability θ^C there is a deterioration in the willingness of traders to deal with corporate bonds to the extent that the Poisson parameter for the expected number of offers drops to $\gamma^C < \gamma^N$. [21] We do not present this extension as it does not alter the qualitative results of the model.

Now that we have discussed our assumptions and the resulting modeling framework, we proceed to present our numerical results. We begin with the case of credit risk-free debt and then introduce the case of corporate defaultable debt.

3 Results in the Default Risk-free Case

We study the case of a default risk-free bond first. This serves a dual purpose. First, we define a benchmark case necessary to analyze the interaction of credit and liquidity risk. Without knowing what the term structure of liquidity spreads looks like in the credit risk-free case, it would be hard to understand the full impact of default risk. Second, we allow the implications of the model to be related to empirical results which are available for the Treasury markets.

For all simulations in the following sections, we define a set of parameters which will be used unless stated otherwise. This is to facilitate comparisons between the various graphs and tables. This base case has the following parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock. In the next section (default risky case), we will supplement the base case with values of bankruptcy costs and of the default parameters.

FIGURE 2

Figure 2 plots two specifications of the liquidity spread. The dashed line, which is almost straight, is the liquidity spread when the bondholder cannot sell his position before maturity unless he is forced to do so by a liquidity shock. The solid line is the liquidity spread when we lift the constraint of no early sale. It is then a decreasing and convex function of time to maturity. This illustrates that it is the ability to sell voluntarily prior to maturity which yields the shape of the liquidity curve. This particular shape is consistent with empirical evidence of Amihud & Mendelson (1991) and Kempf & Uhrig (1997) for the US and German government debt markets respectively.

Amihud & Mendelson (1991) study the markets for US Treasury notes and bills of equal maturity, instruments with identical interest rate risk exposure and remaining payoffs. The difference between these instruments lies in the liquidity of their secondary market. They find that the yield spreads on these securities differ on average by about 42 basis points and that the differential is a convex and decreasing function of time to maturity.

Kempf & Uhrig (1997) test for the existence of a liquidity spread in longer government bonds in the German market.[22] The authors find a statistically significant average spread of 17 basis points. This level and that found by Amihud & Mendelson (1991) can readily be replicated by our model for reasonable parameter values.

Note that a decreasing function for *yield spreads* does not mean that liquidity has a smaller impact on *prices* for long bonds. On the contrary, as will be shown in the next section, percentage price differences increase in maturity. Rather it is the presence of an upper bound on dollar losses due to illiquidity that generates decreasing spreads.

We now turn to the case of defaultable debt.

4 Results in the Default-risky Case

In this section we add credit risk into our framework in order to study the interaction between liquidity risk and the possibility of financial distress.

We first present the shape of liquidity spread curves in the two cases when voluntary sales before maturity are precluded or allowed. We then analyze the respective share of yield spreads explained by liquidity and credit risk and show that liquidity premia can explain a significant proportion of spreads in our model. Bounds are then obtained for price discounts (the difference between the price of a liquid and an otherwise identical illiquid bond) and some comparative statics of the liquidity spreads are provided in subsection 4.4.

4.1 The Term Structure of Liquidity Spreads

FIGURE 3

Let us first review some notation. Recall that θ denotes the probability of a liquidity shock in a given time interval Δt. The parameter γ can be thought of as the number of traders active in the market for a particular corporate bond. Leverage is measured by the quasi-debt ratio q which relates the risk-free present value of debt commitments to the current asset value, while κ is a reduced form measure for the cost of illiquidity in the market for distressed debt.

As before, the base case for simulations will be $\Delta t = 1/12, r = 7\%$, $\gamma = 7$, $\theta = 0.427\%$. We now add parameters specific to default risk: the quasi-debt ratio $q = 0.8$, the volatility of assets is $\sigma = 0.16$, bankruptcy costs are $K = 30$ and the supplementary costs for illiquid securities are $\kappa = 20$. Finally, the default boundary is $L = g\ \exp(-rT)P$, where $g = 1$.

We note that spreads are still decreasing in debt maturity so that the qualitative shape of the liquidity term structures does not change with the introduction of credit risk into the analysis (Figure 3).[23] This is in line with the results of Longstaff (1994) who studied the Japanese market and found similar patterns for credit risky bonds issued by the Japan Finance Corporation of Municipal Enterprise, those of the Tokyo Metropolitan Government and debentures of the Industrial Bank of Japan.[24]

The decreasing term structure of liquidity spreads can help to explain a stylized fact for high grade corporate bond spreads. Structural models cannot reproduce the flat shape of term structures of spreads above Treasury benchmarks for high grade bonds, as reported by Duffee (1999). These models produce increasing term structures of credit spreads for low risk securities. However, if we add the liquidity component, it is clearly possible to reconcile a structural model with a flat term structure of spreads, the increase in credit spreads being offset by the decrease in liquidity spreads.[25]

4.2 The Components of Yield Spreads

FIGURE 4

Marketability premia are an important part of the total yield spread for short maturities, less so for longer bonds (see Figures 4 and 5, obtained using the base case parameters[26]). This follows from our model of credit risk which predicts that default is unlikely for very short maturities as well as from our model of marketability risk which predicts higher liquidity spreads at the short end of the term structure. This qualitative result is consistent with the results of Longstaff (1994) who finds a similar split for Japanese data.

FIGURE 5

4.3 Bounds on price discounts due to illiquidity

In order to understand the predicted shape of the term structure it will be useful to gain some intuition for the behavior of price discounts. Consider Figure 6 which plots these discounts in the case when no early sales are allowed, those in the unrestricted case and finally, their upper and lower bounds.

Price discounts cannot be greater than the immediate payment of the additional bankruptcy discount κ if the bond is subject to default risk. In the default risk-free case, it cannot exceed the expected discount irrespective of traders quotes $1-\overline{\delta}$. The price discount is thus bounded above by $max\left(\kappa, 1-\overline{\delta}\right)$

The discount is bounded below by $1-\delta^*$, otherwise the bondholder would be willing to immediately sell off his position. Within these bounds, the discount increases and is concave, reflecting the higher probability of an adverse liquidity shock. Naturally, the probability of a liquidity shock during the life of a very long bond tends to 1 and if no early sale is allowed, the spread converges to its upper asymptote. However this is not the case when we can sell during the life of the bond because the probability of a voluntary sale also increases with time to maturity and thus offsets some of the impact of the increasing likelihood of a shock. In fact, it converges to the lower bound, because for longer maturities the probability that a bondholder sells in anticipation of a liquidity shock approaches one.[27] The convergence to the lower asymptote due to voluntary sales enforces a smaller cap on the dollar losses arising from illiquidity. The presence of an upper bound on price discounts by itself yields a downward sloping term structure of liquidity spreads. The possibility of preemptive trades by the bondholder to ward off future liquidity shocks further lowers the long end of this term structure and by doing so, tends to give it a convex shape.

FIGURE 6

4.4 Comparative Statics

Our main finding so far is the decreasing shape of the term structure of liquidity spreads. We now proceed to study in greater detail how the compensation for liquidity risk depends on other model parameters.

The figures below show how three of the most important parameters of our model impact on liquidity spreads for different maturities. We first consider θ which is the probability of facing a liquidity shock. Figure 7 reports the term structures of liquidity spreads for various values of θ corresponding respectively to an annual probability of at least one liquidity shock of 5% $(\theta = 0.427\%)$, 10% $(\theta = 0.874\%)$ and 20% $(\theta = 1.842\%)$. As expected, the greater this probability, the greater the liquidity spread because the more likely the bondholder is to be forced to sell at an unfavorable price.

Figure 8 plots liquidity spreads for various levels of a firm's asset volatility. It is clear from this plot that liquidity spreads tend to increase with credit risk. This has important implications for empirical work on corporate debt valuation models. These results show that credit and liquidity risk cannot be treated independently. One cannot simply add a liquidity premium above credit spreads. Spreads above benchmarks will be wider for speculative-grade debt not only to compensate for credit risk but also because default risk impacts on liquidity spreads.

However, credit risk is not only a matter of default probability (mainly influenced by leverage and volatility), but is also determined by the loss realized in default. The last figure (Figure 9) plots the relationship between the additional loss incurred by illiquid securities upon default and the liquidity premium. Again a positive relationship is found.

FIGURES 7,8,9

5 Bondholder Trading Behavior

In this section we analyze the willingness of a bondholder to sell his security at a discount to the price that would prevail in a perfectly liquid market. When financial distress approaches, the bondholder realizes that there is an increased likelihood that his option to seek out favorable offers prior to maturity will become worthless. Hence he will be willing to sell at a lower price. Again this would suggest that trading volume increases as default becomes more likely. An empirical study on high-yield debt by Schulman et al. (1993) supports the idea that anticipation of default results in greater trading activity.

FIGURE 10

Figure 10 illustrates in a binomial tree the percentage price discounts[28] that are acceptable to the bondholder as a function of time to maturity and proximity of default. We see that the closer we are to maturity, the lower the acceptable discount because the bondholder only has a short time to wait before the bond is redeemed and the likelihood of a forced sale at an unfavorable price is smaller. On the other hand, when the issuer of the bonds approaches financial distress, the bondholder will be more willing to sell his security at a greater discount to avoid the risk of facing reorganization costs.

Table 1 focuses more specifically on the impact of default risk on liquidity discounts. Default risk is measured by leverage (quasi-debt ratio: $P \exp(-rT)/v_0$) and the volatility of assets : σ. The discount the bondholder is ready to sell his bond at clearly increases in both leverage and volatility and can reach considerable levels. These results are robust to the choice of the other model parameters $(\gamma, \theta, K, \kappa, r)$.

TABLE 1

6 Dealer Trading Behavior

Until now we have assumed that the demand side of the market is unaffected by events that impact on bond value. In this section we develop a simple inventory model of dealer behavior in order to gauge the impact on term structures of liquidity spreads of a more realistic specification of the demand for corporate bonds. We show that it is possible to extend our framework to allow for offer distributions which are dependent on the value of the firm (riskiness of the bond). For example, bond dealers might widen their bid-ask spreads as the credit quality of a bond declines and their inventory becomes more risky.

Suppose a risk averse dealer's preferences can be described by an exponential utility function over his terminal wealth. We will normalize his initial wealth W_0 to unity. The fraction of his wealth initially invested in the risky bond is denoted as x. The remainder of his wealth is assumed to be held as cash. In case the dealer does not trade, his terminal wealth (after one period) is:

$$W_{NT} = x\,(1+R) + (1-x)\,(1+r)$$

where r is the riskless rate and R is the risky return on his current bond position.

If the dealer purchases an extra unit of the corporate bond at the price δB_L, his terminal wealth (wealth with trade W_T) can be written as

$$W_T = (x + B_L)\,(1+R) + (1 - x - \delta B_L)\,(1+r)\,.$$

This equation shows that the dealer pays δB_L for a bond whose purely liquid value is B_L. Given that the dealer is not affected by the same liquidity shocks as the original bond holder, the value of the bond for the dealer is B_L. However, the fact that the dealer can buy for δB_L a security whose value for him is B_L does not imply that he will necessarily purchase it. If he does, the riskiness of his portfolio (share of his wealth invested in risky asset) will increase and his utility may decrease due to risk aversion.

W_T can be rewritten as

$$\begin{aligned} W_T &= W_{NT} + F\left(1 + R - (1+r)\,\delta\right) \\ &\equiv W_{NT} + Q \end{aligned}$$

where F is the fraction of the "true" value of the bond position under consideration relative to the dealer's total initial wealth W_0, i.e. $F = \frac{B_L}{W_0}$ (recall $W_0 = 1$). Note that we assume that the dealer finances a purchase of the bond at price δB_L by borrowing at the risk free rate.

We now wish to solve for the discount δ that the dealer will charge for buying the bond. The market maker will only be willing to buy the extra unit of the security at a price such that his expected utility is at worst unchanged.

$$E\left[U\left(W_{NT}\right)\right] = E\left[U\left(W_T\right)\right] \tag{2}$$

We have assumed that the dealer has negative exponential utility over terminal wealth

$$U(w) = -\exp\{-bw\}$$

We show in the appendix that the solution to (2) is given by

$$\delta(x) = \frac{1+\bar{R}}{1+r} - \frac{1}{bF(1+r)} \ln\left(\frac{1 + \frac{1}{2}b^2 var(R)(x+F)^2}{1 + \frac{1}{2}b^2 x^2 var(R)}\right), \tag{3}$$

where $\bar{R}$ is the expected value of R and $var(R)$ its variance.

It is clear from this expression that for a given risk level, the required discount $(1-\delta)$ is smaller the greater the expected return granted as compensation $\bar{R}$. Conversely, for a given level of expected return, the liquidity discount is greater the more a trade contributes to the risk of the overall portfolio. Figure 11 depicts the relationship between the fraction δ the dealer is willing to pay for the illiquid bond and the level of bond risk and return.

FIGURE 11

6.1 Bond pricing with state dependent dealer behavior

In order to incorporate this simple model into our bond pricing framework, we need to make assumptions about the arrival of dealers. We will then be able to compute expected discounts conditional on a voluntary sale and in the case of a forced sale. These discounts and the probability of a voluntary sale are the three necessary inputs for our bond pricing formula (equation 1).

We assume that when the bondholder goes to market he is faced with uncertainty relative to the type of dealer he can trade with. We will further assume that the heterogeneity among dealers relates to their initial portfolio bond holdings. For simplicity, we let this initial fraction $\widetilde{x}$ be uniformly distributed over $[0, 1]$.

Given the expressions for the discounts and the probability of a voluntary sale derived in the appendix, we can solve for illiquid bond prices and liquidity spreads as before using a recursion of equation (1). Figure 12 reports the liquidity spreads for varying levels of firm risk σ.

The results obtained with this simple model of dealer behavior are consistent with those reported in previous sections with exogeneous demand. We still find a decreasing term structure of liquidity spreads in the new setup and the correlation between credit (as measured by σ) and liquidity risk remains positive.

FIGURE 12

7 Testing the Model's Predictions

In this section we ask whether corporate bond data support two of our model's predictions. First, we investigate whether liquidity spreads and credit spreads are positively related in the data. Second, we wish to test whether the term structure of liquidity spreads is decreasing. Full structural estimation of our model lies beyond the scope of this paper. Rather, we test its implications by regressing bond yield spreads on two sets of variables, one to control for credit risk, the other to proxy for liquidity risk. We then compare parameter estimates across subsamples defined along credit ratings and bond maturities.

We estimate the following panel regression with fixed effects for the bond spread y_{it} of issue i at time t

$$\begin{aligned} y_{it} &= c_i + \beta_1 VIX_t + \beta_2 SPRET_t + \beta_3 M2M1_t \\ &\quad + \beta_4 SLOPE_t + \beta_5 r_{it} + \beta_6 DEFPREM_t \\ &\quad + \beta_7 OTR_{it} + \beta_8 TLIQ_t + \beta_9 y_{it-1} + \varepsilon_{it} \end{aligned} \tag{4}$$

We use the White (1980) heteroskedasticity consistent covariance matrix estimator for the coefficient covariances. The variables will be discussed in more detail below.

The panel consists of 522 zero-coupon bond issues, yielding a total of 35198 monthly price observations. The data spans the period 1986 to 1996. Zero-coupon bonds are particularly well suited for a study of liquidity because they are not biased by coupon effects.[29]

TABLE 2

Spreads are calculated as the difference between the risky bond yield and the risk-free rate obtained by the Nelson & Siegel (1987) procedure. A more detailed description of the construction of spreads is provided in appendix.

7.1 The data

We will now describe the data used in running the regression in (4). We include six variables in order to capture variations in the bond yield spreads that we do not attribute to liquidity risk. We include measures of stock market return and volatility, an indicator for the credit cycle, two term structure variables and a metric for the aggregate default risk in the economy. We then add a proxy for the liquidity of each individual issue and one for that of the fixed income markets as a whole.

1. Stock market volatility (VIX)

   Asset volatility and leverage are the two most important determinants of default risk in a structural model of credit risk. Asset volatility is however not directly observable for most firms and it is typically proxied in empirical work by the volatility of the stock when the issuing firm has publicly traded equity. In this paper, we will be focusing on an aggregate measure of asset volatility across firms and will use the volatility of the stock market as a proxy for asset volatility. We have chosen to include implied volatility rather than historical volatility[30] because implied volatility is a forward looking measure (the traders' expectation of volatility). The measure of implied volatility we use is the Chicago Board Options Exchange VIX index which is a weighted average of the implied volatilities of eight options with 30 days to maturity. We expect the volatility to enter the regression with a positive sign, since a greater volatility implies a greater risk of default and should be reflected in higher spreads.

2. The stock market return ($SPRET$)

   A primary determinant of the probability of financial distress is the leverage of a firm. The book values of debt outstanding for firms are likely to be substantially less volatile than the market value of the firms' equity. Hence, we would expect that, on average, a positive stock market return is associated with an increase in leverage and decrease in the likelihood for financial distress for a firm. We use the monthly S&P 500 return as provided by CRSP and expect it to be negatively related to the levels of bond yield spreads.

3. The credit cycle ($M2M1$)

   This series is constructed as the difference between the two monetary aggregates M2-M1 published by the Federal Reserve. It measures the amount of lending made by banks and is commonly used by practitioners to capture some of the cyclicality of the lending activity. We have no strong view about the expected sign for this variable. On the one hand, one could argue that it should have a negative link with spreads, as large bank lending (high $M2M1$) should coincide with cheap borrowing on the markets (tight spreads). On the other hand, a widening of spreads could induce corporates to borrow more from banks rather than tap the market.

4. The slope of the yield curve ($SLOPE$)

   We measure the slope of the yield curve by computing the difference between the yields on the 10 year and 2 year benchmark bonds given by Datastream. We interpret this term structure variable as an indicator of overall economic conditions. If recessions are associated with a downward sloping and expansions with an upward sloping yield curve, then we would expect a negative relationship between this proxy and bond yield spreads.

5. The risk-free rate (r_{it})

   Corporate yields can be broken down into a risk-free rate component and a spread. How these two components interact has been a matter of debate in the literature. Do spreads increase when the risk-free rate rises or do they decrease? Recent evidence in Duffee (1999) has shown that one could expect a negative sign for the risk-free rate at least for investment grade bonds, i.e. that spreads tend to fall when Treasury yields rise. Morris et al. (1998) make a distinction between a negative short term impact and a positive long term impact of changes in risk-free rates on corporate spreads. We include the yield on a hypothetical credit risk free discount bond with the same maturity. Our panel is a mixture of high quality and speculative grade bonds and some include embedded call options which have been shown to lessen the negative impact of the risk-free rate on spreads. We prudently expect it to carry a negative sign.

6. Aggregate default premium ($DEFPREM$)
   We include as a regressor the difference between Moody's Baa and Aaa rated bond yield indices. We take this variable as an additional proxy for the probability of financial distress in the economy and consequently expect it to be positively related to individual issue credit spreads.

7. Treasury market liquidity ($TLIQ$)
   It has been documented (see Amihud & Mendelson (1991)) that Treasury securities with identical cash flows and varying marketability can trade at different spreads. We include as an indicator the yield differential between the previous long bond and the most recently issued 30 year benchmark bond. We intend this proxy to capture the liquidity in the Treasury bond market. After the Russian default in 1998, the US long bond (30 year benchmark) was trading at a 35 basis point premium versus the second longest bond with just a few months less to maturity. (see Poole (1998))

8. On-the-run bonds (OTR)
   A stylized fact about bonds is that they are more liquid immediately after issuance and rapidly lose their marketability as a larger share of the issues becomes locked into portfolios.[31] Therefore, we use as a proxy for individual issue liquidity a dummy which indicates if the bond was issued in the last 3 months. We expect that the parameter for this variable will be negative to reflect the fact that older and less liquid issues trade at higher spreads than recently issued bonds.

Figure 13 provides a graphical representation of the evolution of these variables with the exception of the OTR dummy over the period spanned by our sample. We now turn to a discussion of our regression results.

7.2 Results

The two hypotheses we wish to test are i) whether liquidity spreads are higher for more credit-risky securities and ii) whether the term structure of liquidity spreads is decreasing. We do this by comparing parameter estimates for our liquidity proxies in subsamples defined by credit ratings and maturities.

Table 2 reports summary statistics for the variables used in our analysis. The average yield spread in the sample, which comprises 35198 observations for 522 bonds, is 40 basis points within a broad range. The low mean spread is consistent with the high average credit rating of issues in our sample. The maturities of the bonds range between one and almost thirty years and are quite evenly distributed. [32] The average age of the bonds is about 5 years, ranging from a couple of weeks to 16 years.

We present the regression results in table 3.[33] For the regression with all yield spread observations, we note that the non-liquidity related variables all enter signs consistent with our expectations. Stock market volatility is significantly positively associated with the level of yield spreads. Structural models of credit risk derive high volatilities from high levels of leverage and thus this relation is consistent with higher levels of distress probabilities leading to higher bond spreads. However, the sign is reversed for bonds with high credit rating and long maturities, although the economic significance diminishes considerably.

The same intuition is valid for the negative and significant relationship between S&P 500 returns and yield spreads. A positive return is likely to be associated with a decreased leverage and thus also default probability and spread. This finding is consistent across all subsamples except for long maturity bonds, when the parameter estimates are close to zero and insignificant.

The credit cycle indicator is insignificant except for the case of long maturity bonds and its impact on yield spreads is not very large. The slope of the term structure is consistently negatively related to the levels of yield spreads. This is consistent with the explanation that positive slopes are associated to good overall economic conditions.

The level of the risk free interest rate is always negatively related to the spread levels. It is strongly significant except for one particular specification with long term bonds. The relationship is more marked for firms with low credit rating or bonds with short maturities.

Not surprisingly, the aggregate market default premium as measured by the spread between Moody's Baa and Aaa yield indices is clearly positively related to the level of individual bond spreads. Again, the impact is larger for short term bonds and issues with lower credit rating.

The signs of the OTR dummy and $TLIQ$ are consistent with our interpretation that they proxy for liquidity. On average, a recently issued bond in the full sample can expect to trade at around 14 basis points less than if it were more seasoned. A greater illiquidity premium in Treasury markets translates into higher yield spreads in the corporate bond market. This effect is weaker though, a 10 basis point increase in $TLIQ$, yields only a 1.2 basis point increase in the risky bond yield spread. In all ten regressions the parameter estimates are statistically significant.

In order to see if our two predictions are borne out by the data, we consider two subsamples: one based on credit rating, the other on bond maturity. We will first discuss whether we find evidence of a positive correlation between credit and liquidity spreads.

We find that the parameter estimate for the OTR dummy is approximately three times larger in the subsample of bonds with low credit rating. This suggests that bonds that are off-the-run and credit risky will have to reward their holders with an additional yield which can be three times higher in basis points than the corresponding extra yield for high credit quality bonds. Similarly, the impact of $TLIQ$ is almost four times larger in the low rating sample. This is consistent with a decrease in liquidity in Treasury markets that propagates into corporate debt markets with a markedly stronger impact on the speculative grade segment. Both of these findings are supportive of the positive relationship between credit and liquidity risk implied by our model.

Turning to the shape of the term structure of liquidity spreads, we find that the impact of the OTR dummy differs across bond maturities. The benefit to being on-the-run for short term bonds is about 50% higher than for long bonds - suggesting that the liquidity component of yield spreads diminishes with maturity. The difference between the parameter estimates for TLIQ is not statistically significant and does hence not add further support to our hypothesis.

Overall, we believe that the empirical results reported in this section are supportive of two empirical implications derived from our model of credit and liquidity risk.

8 Concluding Remarks

We have developed a simple model to illustrate the impact of liquidity risk on the yield spreads of corporate bonds. Despite its simplicity, the model has a number of interesting features. Our main qualitative finding is that the level of liquidity spreads should be positively correlated with credit risk and that they should be decreasing functions of time to maturity.

Another finding is that for reasonable parameter inputs the model is able to generate non-negligible yield spreads even for short maturities. This addresses a common criticism of structural bond pricing models and helps to reconcile them with empirical evidence.

Our results are consistent with previous empirical research not only as far as the shape of the term structure (decreasing and convex) is concerned but also in terms of the levels. The relative shares of spreads explained by credit risk and liquidity risk in our model are comparable with earlier empirical findings. In addition, our model is aligned with a number of stylized facts about debt securities. For example, it provides some justification for the greater liquidity of young issues and the higher frequency of trades nearer default reported in the literature.

Finally, we find that US corporate bond data support the prediction of our model that liquidity spreads should be positively correlated with the likelihood of default and that spreads should decrease with time to maturity.

Appendix A : Expected Discounts

The expected best fraction of the liquid price that the seller will be offered is

$$E\left[\tilde{\delta}_t\right] = \sum_{n=0}^{\infty} P\left(N=n\right) \int_0^1 \delta f^n\left(\delta\right) d\delta,$$

where the density $f^n\left(x\right)$ is the probability that x is the best price fraction obtained given n offers. Given only one offer, for a uniform distribution the probability of getting a fraction of less than x is

$$F\left(x\right) = x,$$

where F is the cumulative distribution. The probability of getting none higher than x with n independent offers is thus

$$\left(F\left(x\right)\right)^n = x^n,$$

and so the desired density function f^n is

$$f^n\left(x\right) = \frac{\partial\left(F\left(x\right)\right)^n}{\partial x} = nx^{n-1},$$

and thus, given that the number of offers is Poisson with parameter γ,we get

$$\begin{aligned} E\left[\tilde{\delta}_t\right] &= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!} \cdot \int_0^1 n\delta^n d\delta, \\ &= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!} \cdot \frac{n}{n+1}. \end{aligned}$$

Now, in order to compute the value of an illiquid bond at a given node in the binomial tree we need to solve

$$E_{t-}\left[\tilde{\delta}_t \cdot I_{\tilde{\delta}_t > \delta_t^*}\right]$$

and

$$P\left(\tilde{\delta}_t > \delta_t^*\right) = E\left[I_{\tilde{\delta}_t > \delta_t^*}\right]$$

Recall that

$$P\left(N = n\right) = e^{-\gamma}\frac{\gamma^n}{n!},$$

and that conditional on n offers our assumption of uniformly distributed offers yields the following density for the price fraction δ offered

$$f^n\left(\delta\right) = n\delta^{n-1}.$$

Then it follows that

$$\begin{aligned}
E\left[I_{\tilde{\delta}_t > \delta_t^*}\right] &= \sum_{n=0}^{\infty} P\left(N = n\right) E\left[I_{\tilde{\delta}_t > \delta_t^*} | N = n\right] \\
&= \sum_{n=0}^{\infty} P\left(N = n\right) \int_{\delta_t^*}^{1} n\delta^{n-1} d\delta \\
&= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!}\left(1 - \left(\delta_t^*\right)^n\right).
\end{aligned}$$

$$\begin{aligned}
E_{t-}\left[\tilde{\delta}_t \cdot I_{\tilde{\delta}_t > \delta_t^*}\right] &= \sum_{n=0}^{\infty} P\left(N = n\right) E\left[\tilde{\delta}_t \cdot I_{\tilde{\delta}_t > \delta_t^*} | N = n\right] \\
&= \sum_{n=0}^{\infty} P\left(N = n\right) \int_{\delta_t^*}^{1} n\delta^{n} d\delta \\
&= \sum_{n=0}^{\infty} e^{-\gamma}\frac{\gamma^n}{n!}\frac{n}{n+1}\left(1 - \left(\delta_t^*\right)^{n+1}\right).
\end{aligned}$$

Appendix B : Spread Constructions

Spreads are defined as the difference between the yield on a corporate bond and the yield on a U.S. Treasury bond with same maturity. We use zero-coupon bonds only so that our spread calculations are not biased by coupon effects. Given that there does not exist a U.S. Treasury bond for all maturities, we have chosen to construct a whole term structure of risk-free rates from existing bond prices for each month end from January 1986 to December 1996 (132 months).

We use the Nelson & Siegel (1987) algorithm to obtain a smooth yield curve from zero coupon bonds. This procedure is a four parameter yield-curve calibration method whose flexible specification allows us to replicate most term structures shapes usually observed on the market. Formally, the yield at time t on a bond with maturity T is given by

$$R(t,T) = \beta_0 + (\beta_1 + \beta_2)\frac{1 - \exp(-T/\beta_3)}{T/\beta_3} - \beta_2 \exp(-T/\beta_3)$$

Using risk-free zero-coupon bonds (mainly strips) to derive the benchmark curves enables us to obtain a nearly perfect fit of observed riskless rates by maximum likelihood (we use the CML tool in Gauss). However we find that the Nelson-Siegel procedure is over-parametrized for zero-coupon bonds and leads to wide differences in the parameter estimates in spite of only mild variations in their initial values. We thus impose a restriction on the first parameter which is the only one with a clear economic interpretation. More precisely, the first parameter represents the yield of a perpetual risk-free bond $R(t,\infty)$. We approximate it by the 30 year U.S. Treasury rate and thus obtain a consistent and robust set of optimal parameters. The constraint also turns out to yield positive forward rates for all maturities and all observation periods, thereby avoiding one of the main criticisms of the algorithm. For each month, we exclude risky bonds whose maturity falls outside the range spanned by the risk-free bonds to avoid the imprecisions of the interpolation procedure outside this range.

Appendix C: Dealer behavior

In this appendix, we first derive the δ reported in section 6. We then calculate the probability of a voluntary sale in the inventory model, as well as the expected discount in case of a forced sale or a voluntary sale.

Required discount

As a starting point we require that the utility of the bond dealer be equal in the case of a purchase and the no trade case. This will be the maximum price the dealer will be willing to pay to purchase the bond from the bond holder. It is also the price that should prevail if the dealer market is competitive. We thus write:

$$E\left[U\left(W_{NT}\right)\right] = E\left[U\left(W_{T}\right)\right].$$

A second order Taylor expansion around the expected terminal wealth yields:

$$U\left(\bar{W}_{NT}\right) + \frac{1}{2}U''\left(\bar{W}_{NT}\right)E\left[\left(W_{NT} - \bar{W}_{NT}\right)^2\right] = U\left(\bar{W}_{T}\right) + \frac{1}{2}U''\left(\bar{W}_{T}\right)E\left[\left(W_{T} - \bar{W}_{T}\right)^2\right]$$

Given our assumption of negative exponential utility, we can write

$$\begin{aligned} U'(w) &= b\exp\{-bw\} = -bU(w) \\ U''(w) &= -b^2\exp\{-bw\} = b^2U(w) \\ var\left(W_{NT}\right) &\equiv E\left[\left(W_{NT} - \bar{W}_{NT}\right)^2\right] \end{aligned}$$

and therefore:

$$\begin{aligned} U\left(\bar{W}_{NT}\right)\left(1 + \frac{1}{2}b^2 var\left(W_{NT}\right)\right) &= U\left(\bar{W}_{T}\right)\left(1 + \frac{1}{2}b^2 var\left(W_{T}\right)\right) \\ -\exp\left\{-b\bar{W}_{NT}\right\}\left(1 + \frac{1}{2}b^2 var\left(W_{NT}\right)\right) &= -\exp\left\{-b\left(\bar{W}_{NT} + \bar{Q}\right)\right\}\left(1 + \frac{1}{2}b^2 var\left(W_{T}\right)\right) \end{aligned}$$

where $\bar{Q} = E[Q] = E[F(1 + R - (1 + r)\delta)]$.

Solving for $\bar{Q}$, we find

$$\begin{aligned}\bar{Q} &= \frac{1}{b}\ln\left(\frac{1+\frac{1}{2}b^2 var(W_T)}{1+\frac{1}{2}b^2 var(W_{NT})}\right) \\ &= F\left(1+\bar{R}\right) - (1+r)\delta\end{aligned}$$

and

$$\delta = \frac{1+\bar{R} - \frac{1}{bF}\ln\left(\frac{1+\frac{1}{2}b^2 var(W_T)}{1+\frac{1}{2}b^2 var(W_{NT})}\right)}{(1+r)}$$

which yields equation (3), after substitution for W_T and W_{NT}.

Expected discounts

We have assumed that the fraction of the dealer's wealth initially invested in the risky bond follows a uniform distribution with support $[0, 1]$. We now want to compute expected discounts conditional on a voluntary sale and in the case of a forced sale, as well as the probability of a voluntary sale. This will facilitate the numerical work in the binomial tree. Let us first introduce some notations. From the calculations above, we know that $\delta(x)$ can be written as:

$$\delta(x) = A - B\ln\left(\frac{1+C(x+F)^2}{1+Cx^2}\right),$$

with $A = \frac{1+\bar{R}}{1+r}$, $B = \frac{1}{bF(1+r)}$ and $C = \frac{1}{2}b^2 var(R)$.

Expected discount (forced sale)

In case of a forced sale, the expected discount is:

$$
\begin{aligned}
E\left[\delta_t\left(x\right)\right] &= \int_0^1 \delta_t\left(x\right) dx \\
&= \left[Ax\right]_0^1 - B\left[\mathcal{I}\right]_0^1,
\end{aligned}
$$

with

$$
\mathcal{I} = \int \ln\left(\frac{1+C\left(x+F\right)^2}{1+Cx^2}\right) dx.
$$

Integrating, $\mathcal{I}$ can be shown to be equal to

$$
\begin{aligned}
\mathcal{I} &= \left(x+F\right)\ln\left(1+Cx^2+2CxF+CF^2\right) - x\ln\left(1+Cx^2\right) \\
&\quad + \frac{2}{\sqrt{C}}\left(\arctan\sqrt{C}\left(x+F\right) - \arctan\sqrt{C}x\right),
\end{aligned}
$$

and finally:

$$
\begin{aligned}
E\left[\delta\left(x\right)\right] &= A - BF\ln\left(\frac{1+C+2CF+CF^2}{1+CF^2}\right) \\
&\quad + B\ln\left(\frac{1+C+2CF+CF^2}{1+C}\right) \\
&\quad + \frac{2B}{\sqrt{C}}\arctan\left(\frac{-C^{\frac{3}{2}}F\left(1+F\right)}{CF^2+1+C\left(1+F\right)}\right).
\end{aligned}
$$

Expected discount (voluntary sale)

In order to compute $E\left[\delta_t\left(x\right)\cdot I_{\delta_t>\delta_t^*}\right]$ all we need to do is to redefine the boundaries of the undefined integral above.

$$
\begin{aligned}
E\left[\delta_t\left(x\right)\cdot I_{\delta_t>\delta_t^*}\right] &= \left[Ax\right]_{\delta_t^*}^1 - B\left[\mathcal{I}\right]_{\delta_t^*}^1 \\
&= A\left[1-\delta^*\right] - B\mathcal{I}_{\delta_t^*}^1,
\end{aligned}
$$

with the function $\mathcal{I}$ defined as above.

Probability of a voluntary sale

In order to compute

$$P\left(\delta > \delta_t^*\right),$$

note that $\delta(x)$ is monotonically decreasing in x. Now define $\widehat{x}$ such that $\delta(\widehat{x}) = \delta_t^*$. Then

$$P\left(\delta > \delta_t^*\right) = P\left(x < \widehat{x}\right),$$

and we must have:

$$\delta_t^* = A - B \ln\left(\frac{1 + C\left(\widehat{x} + F\right)^2}{1 + C\widehat{x}^2}\right),$$

which yields the quadratic equation:

$$C\left(1 - \exp\left\{\frac{A - \delta_t^*}{B}\right\}\right)\widehat{x}^2 + 2CF\widehat{x} + 1 + CF^2 - \exp\left\{\frac{A - \delta_t^*}{B}\right\} = 0.$$

Solving for $\widehat{x}$, we find:

$$\widehat{x} = \frac{-F}{1 - \exp\left\{\frac{A-\delta_t^*}{B}\right\}} + \frac{\sqrt{4C^2F^2 - 4C\left(1 - \exp\left\{\frac{A-\delta_t^*}{B}\right\}\right)\left(1 + CF^2 - \exp\left\{\frac{A-\delta_t^*}{B}\right\}\right)}}{2C\left(1 - \exp\left\{\frac{A-\delta_t^*}{B}\right\}\right)}.$$

Notes

[1]See for example Black & Cox (1976), Kim et al. (1993), Shimko et al. (1993), Nielsen et al. (1993), Longstaff & Schwartz (1995), Anderson & Sundaresan (1996), Jarrow & Turnbull (1995), Duffie & Singleton (1999), Lando (1998).

[2]Indeed, the BIS Committee on the Global Financial System underlined the need to understand the sudden deterioration in liquidity during the 1997-1998 global market turmoil. See Bank for International Settlements (1999).

[3]Such an argument can be found for example in Jones et al. (1984), Longstaff & Schwartz (1995) and Mella-Barral & Perraudin (1997).

[4]This argument is one of the motivations for the article by Duffie & Lando (2000).

[5]Two of the factors capture term structure risk and the other credit risk.

[6]Few papers have addressed the problem of default risk in incomplete markets. Lotz (1997) studies local risk minimization (equivalent to the problem of pricing under the Minimal Martingale Measure) in a reduced form framework of credit risk. Moraux & Villa (1999) also work under this measure but derive prices in the Merton (1974) model when the assumption of asset tradeability is lifted. We believe that the issue of an appropriate martingale measure is important for practical purposes but that dealing with it within the context of our current framework would only serve to obscure the fundamental forces at work.

[7]Thus the probability of an up move in a given time interval Δt is given by

$$p = \frac{e^{r\Delta t} - d}{u - d},$$

where

$$u = \frac{1}{d} = e^{\sigma\sqrt{\Delta t}}.$$

[8]Note that the choice of Δt is not irrelevant to our results. Δt can be seen as the potential trading frequency of the bondholder or the time necessary to receive a new set of offers. If we were to let $\Delta t \longrightarrow 0$, the liquidity spread would vanish because in any finite interval, the bond holder would receive an infinity of offers and would get an offer with zero discount with probability one.

[9]Note that in contrast to Tychon & Vannetelbosch (1997), we do not assume that bankruptcy costs are investor specific.

[10]The quasi-debt ratio was used as a leverage measure by Merton (1974) and defined as the present value of its debt obligations at the risk free rate in relation to the firm's value:

$$q = \frac{e^{-rT}P}{v_0}.$$

[11]This result is technically analogous to the effect observed in the credit risk literature for models of the Black & Cox (1976) type. In some circumstances bondholders would be better off in default because they might recover funds faster.

[12]The assumption of arrival of trades or offers as Poisson processes has been used in a different context in the microstructure literature (e.g. Easley et al. (1996)).

[13]Under the current guidelines (under revision), capital requirements do not discriminate across bond ratings. Hence banks are at a competitive disadvantage in the investment-grade corporate debt market.

[14]Given that our model is numerical, it can in principle accommodate any distribution.

[15]We have gathered the details of the calculations in the appendix.

[16]Our assumption is similar to the concept of liquidity traders in market

microstructure.

[17]See for example Schultz (1998) and Chakravarty & Sarkar (1999).

[18]We relax the independence of $\widetilde{\delta}_t$ and v_t in section 6.

[19]The distribution of offers is assumed constant through time so that $E_t\left[\tilde{\delta}_t\right] = E\left[\tilde{\delta}_t\right] = \bar{\delta}$.

[20]Details of the calculations of $P\left[\tilde{\delta}_t > \delta_t^*\right]$ and $E_{t-}\left[\tilde{\delta}_t I_{\tilde{\delta}_t > \delta_t^*}\right]$ can be found in the appendix.

[21]This extension would not affect the structure of our bond pricing formula (**??**) but would change the quantitative output. The probability of obtaining a given number of offers would be

$$P\left(N = n\right) = \left(\left(1 - \theta^C\right) e^{-\gamma^N} \frac{\left(\gamma^N\right)^n}{n!} + \theta^C e^{-\gamma^C} \frac{\left(\gamma^C\right)^n}{n!}\right)$$

and this would directly affect the expected fraction that a bondholder could expect to obtain both in the event of a forced sale and a preemptive trade.

[22]They compare the yields of a sample of large issues (issue size is used as a proxy for liquidity) against those of smaller issues over the period 1992-94. All 143 bonds in their study were issued between 1982 and 1994 with initially 10 years to maturity and had a residual maturity between 0.5 and 10 years.

[23]This result is robust to changes in the parameters. In some extreme cases, there is a hump that peaks at very short maturties but then most of the curve still retains the described shape.

[24]The author also finds jumps in spreads around six to seven years to maturity but these are due to specificities of the Japanese market.

[25]The lack of smoothness of the curves is due to the presence of a barrier in the tree.

[26] $\Delta t = 1/12, r = 7\%,\ \gamma = 7,\ \theta = 0.427\%,\ q = 0.8,\ \sigma = 0.16,\ K = 30,\ \kappa = 20,$ $g = 1.$

[27] The reason that there is no discounting effect follows from our specification of quotes in illiquid markets as fractions of their value in perfectly liquid markets.

[28] defined as $((1 - \delta_t^*) \times 100)$.

[29] For a review of these effects, see Sundaresan (1997) chapter 5.

[30] We have also carried out the regressions on 30-day and 90-day historical S&P500 volatility and the results were very similar.

[31] See for example chapter 10 in Fabozzi & Fabozzi (1995).

[32] We have excluded bonds with less than one year to maturity because of the extreme sensitivities of short bond spreads to small changes in price.

[33] Table 4 provides a correlation matrix for the independent variables. For variables that are cross-section specific, the table reports the mean of correlations computed for each cross-sectional unit.

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Table 1 : The effect of default risk on the bondholder reservation liquidity discount.

The table reports the acceptable ($t = 0$) percentage discount (δ_0^*) as a function of leverage and firm risk. The debt maturity is set to 10 years. Other parameters are set according to the base case scenario: $\delta t = \frac{1}{12}$, $r = 7\%$ *p.a.*, $\gamma = 7$, $\theta = 0.874\%$ (corresponding to a 10% liquidity shock probability on a yearly basis), $q = 0.60$, $K = 10$, $\kappa = 10$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$.

		Firm risk (σ)				
		0.10	0.15	0.20	0.25	0.30
	0.20	3.72	3.72	3.72	3.72	3.72
Quasi	0.40	3.72	3.72	3.76	3.95	4.02
debt	0.60	3.72	3.80	4.38	4.64	7.47
ratio	0.80	3.77	5.14	5.95	17.62	19.49

Table 2: Descriptive statistics of bond issues

	y_{it}	Maturity	Age	Credit rating
Mean	40	12.4	5.3	1.3
Median	31	11.9	5.2	1.0
Maximum	1,017	29.7	16.3	8.0
Minimum	1	1.0	0.0	1.0
Std. Dev.	41.28	7.02	3.02	0.74
Skewness	8.02	0.28	0.64	3.24
Kurtosis	117.65	2.10	3.33	16.05

Table 3: Differential impact of liquidity proxies in rating and maturity subsamples.

The results are based on the following panel regression during the period between January 1986 to December 1996.

$$\begin{aligned} y_{it} &= c_i + \beta_1 VIX_t + \beta SPRET_t + \beta_3 SLOPE + \beta_4 r_{it} + \beta_5 DEFPREM_t \\ &\quad + \beta_6 OTR_{it} + \beta_7 TLIQ_t + \beta_8 y_{t-1} + \varepsilon_{it} \end{aligned}$$

where VIX denotes the implied volatility index, $SPRET$ the monthly S&P 500 return, $SLOPE$ the difference between the 10 and 2 year Treasury yields, r_{it}, the Treasury rate corresponding to the maturity of the particular bond, $DEFPREM$ the difference between Moody's BBB and AAA rated corporate bond yield indices, OTR a dummy which indicates whether a given bond is on-the-run, assumed to mean less than 3 months of age. $TLIQ$ denotes the difference in yield between the most recently issued 10 year Treasury bond and the yield on the next most recent. Due to the presence of serial correlation in the time series for individual bond spreads we include the lagged spread as a regressor. The coefficient for this variable is close to 0.85 for all regressions and highly significant. The adjusted R^2 for the regressions is slightly higher than 90% in all cases. The regressions are run with a White heteroskedasticity consistent covariance matrix for the residuals. The first line reports the coefficient estimates and the row below the t-statistics.

	VIX	$S\&P$ return	M2M1	Slope	r_{it}	Moody's BBB-AAA	OTR	T-liq
All	0.081	-0.057	-0.004	-0.019	-3.981	0.247		
	2.895	-3.410	-1.261	-5.061	-14.093	15.816		
	0.077	-0.063	-0.003	-0.015	-4.079	0.249	-13.851	
	2.746	-3.835	-0.958	-4.084	-14.279	15.790	-5.625	
	0.077	-0.050	-0.002	-0.018	-3.760	0.255		0.119
	2.721	-3.040	-0.683	-4.707	-13.348	16.295		4.623
Low	0.331	-0.188	0.010	-0.055	-12.405	0.570	-32.999	
rating	4.057	-2.702	0.811	-2.541	-11.885	10.829	-2.886	
	0.340	-0.145	0.014	-0.052	-11.821	0.577		0.216
	4.153	-2.027	1.079	-2.376	-10.866	10.930		2.180
High	-0.063	-0.048	-0.001	-0.004	-1.910	0.137	-11.867	
rating	-4.318	-4.644	-0.513	-1.491	-11.814	17.628	-15.694	
	-0.061	-0.039	-0.002	-0.008	-1.701	0.140		0.058
	-4.154	-3.798	-0.701	-2.703	-10.363	17.713		3.657
Short	0.205	-0.112	0.010	-0.019	-6.126	0.311	-19.498	
maturity	5.792	-4.087	1.874	-2.446	-16.030	15.303	-7.144	
	0.209	-0.092	0.011	-0.022	-5.799	0.311		0.106
	5.898	-3.307	1.957	-2.807	-14.724	15.239		2.552
Long	-0.095	-0.006	-0.007	-0.023	-0.735	0.191	-10.829	
maturity	-4.807	-0.440	-3.083	-6.429	-3.109	19.063	-10.682	
	-0.100	0.000	-0.006	-0.025	-0.429	0.201		0.119
	-5.023	0.035	-2.660	-7.101	-1.795	19.725		5.700

Table 4: Correlation matrix of independent variables

	VIX	$SPRET$	$M2M1$	$SLOPE$	$DEFPREM$	$TLIQ$	OTR	r_{it}
VIX	100%	-37%	-38%	-16%	56%	-11%	6%	-26%
$SPRET$	-37%	100%	0%	-9%	6%	-4%	-11%	-2%
$M2M1$	-38%	0%	100%	13%	-70%	-40%	-42%	-72%
$SLOPE$	-16%	-9%	13%	100%	-9%	7%	24%	-59%
$DEFPREM$	56%	6%	-70%	-9%	100%	17%	70%	6%
$TLIQ$	-11%	-4%	-40%	7%	17%	100%	-7%	-21%
OTR	6%	-11%	-42%	24%	70%	-7%	100%	29%
r_{it}	-26%	-2%	-72%	-59%	6%	-21%	29%	100%

Binomial Setting

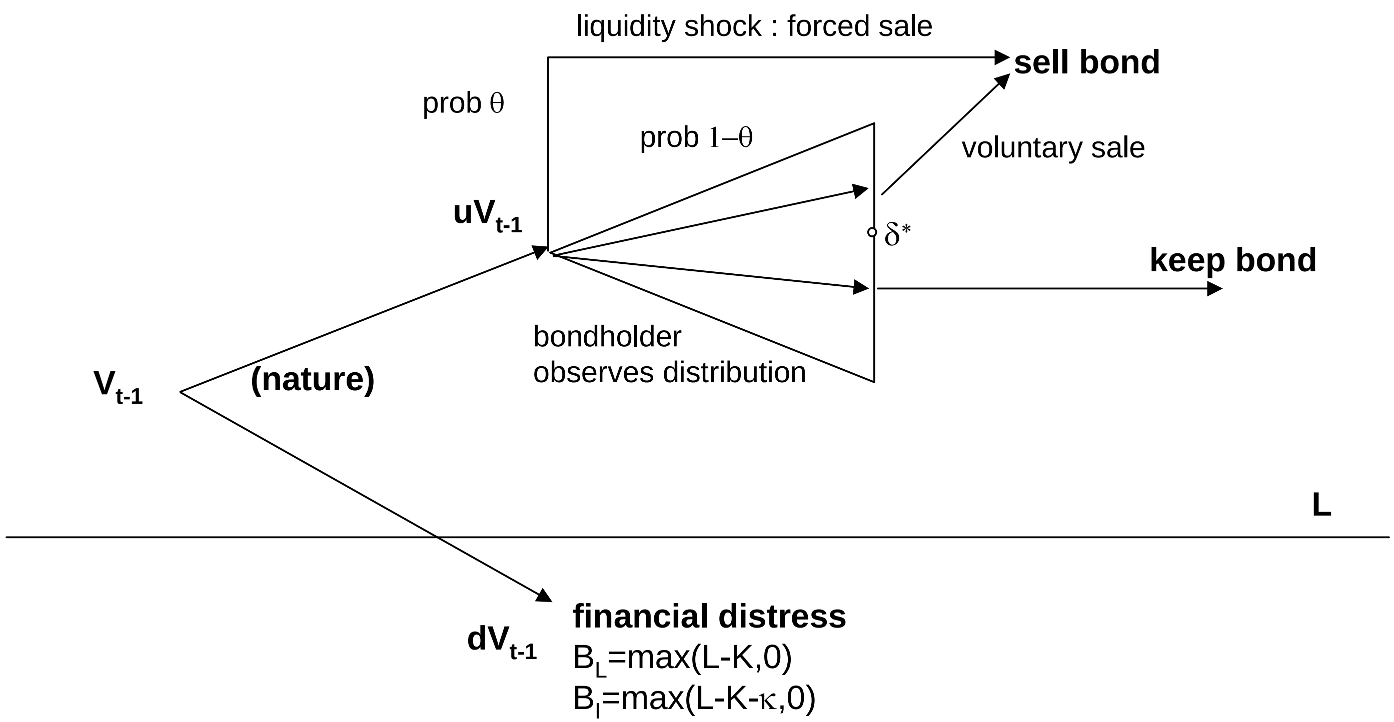

Figure 1: Description of the sequence of events for a given time period in the binomial tree for the firm's asset value. First, the value of the firm is realized. Then a liquidity shock may materialize. Otherwise the bondholder decides whether or not to sell voluntarily. This game is repeated until we reach the maturity date of the bond.

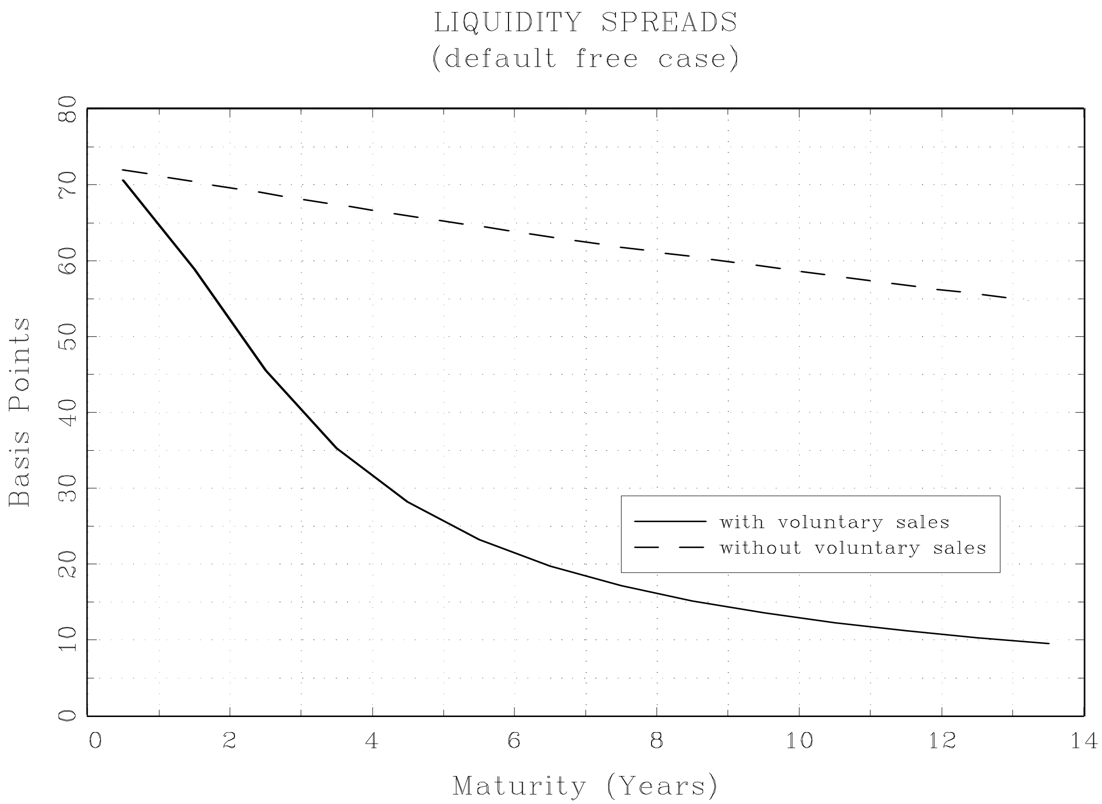

Figure 2: Liquidity spreads with and without voluntary sales in the case without default risk. Parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock.

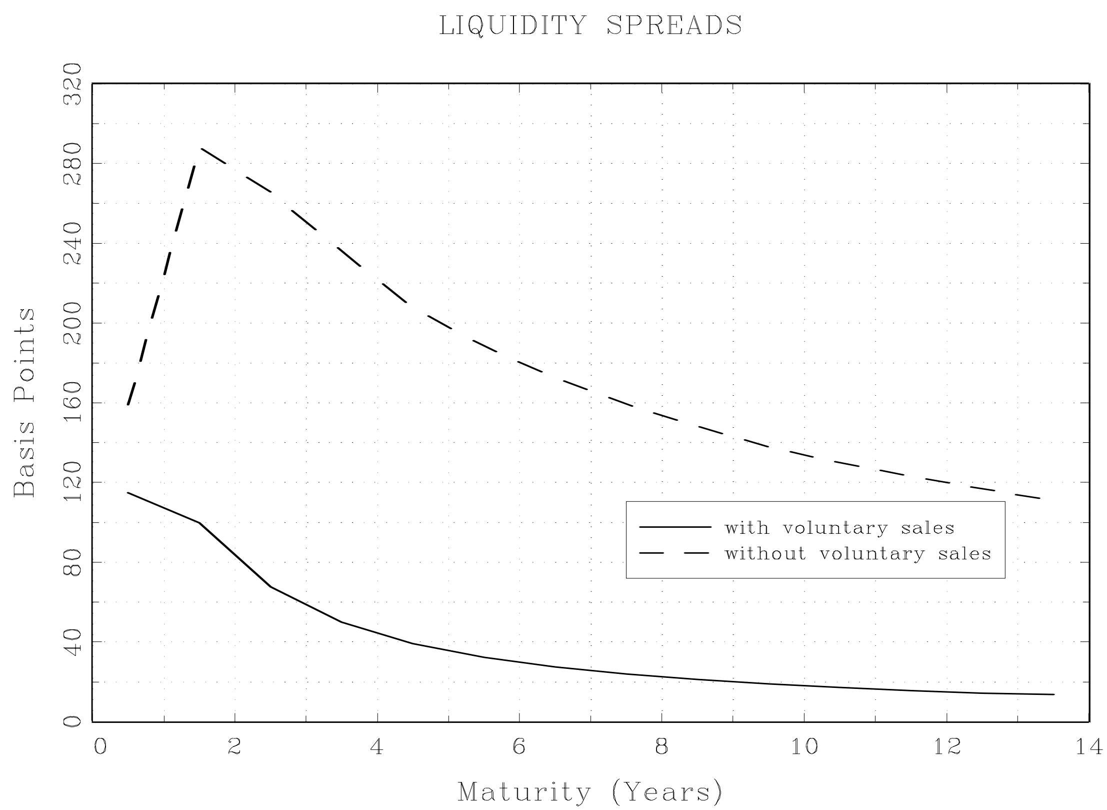

Figure 3: Liquidity spreads with and without voluntary sales in the default-risky case. Parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock and the quasi-debt ratio is $q = 0.8$. The default risk related parameters are the asset volatility ($\sigma = 0.16$), the bankruptcy costs $K = 30$ and $\kappa = 20$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$.

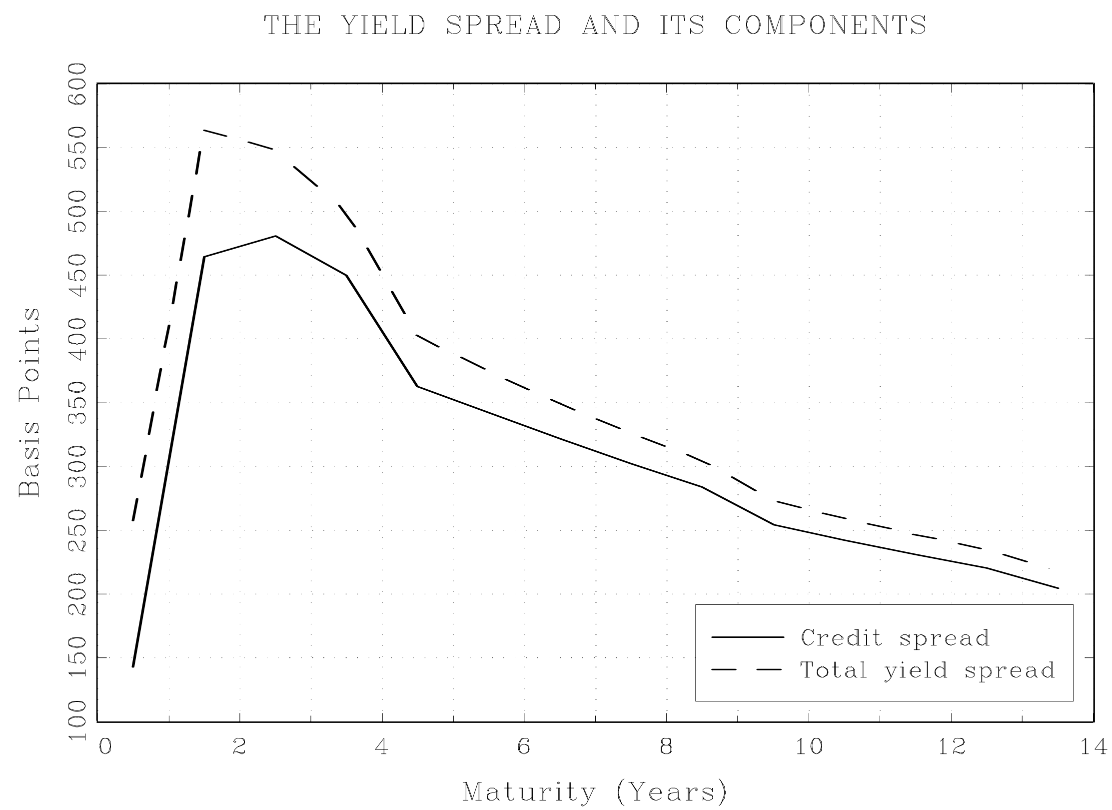

Figure 4: Credit spread and total yield spread for the base case scenario. Parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock and the quasi-debt ratio is $q = 0.8$. The default risk related parameters are the asset volatility ($\sigma = 0.16$), the bankruptcy costs $K = 30$ and $\kappa = 20$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$.

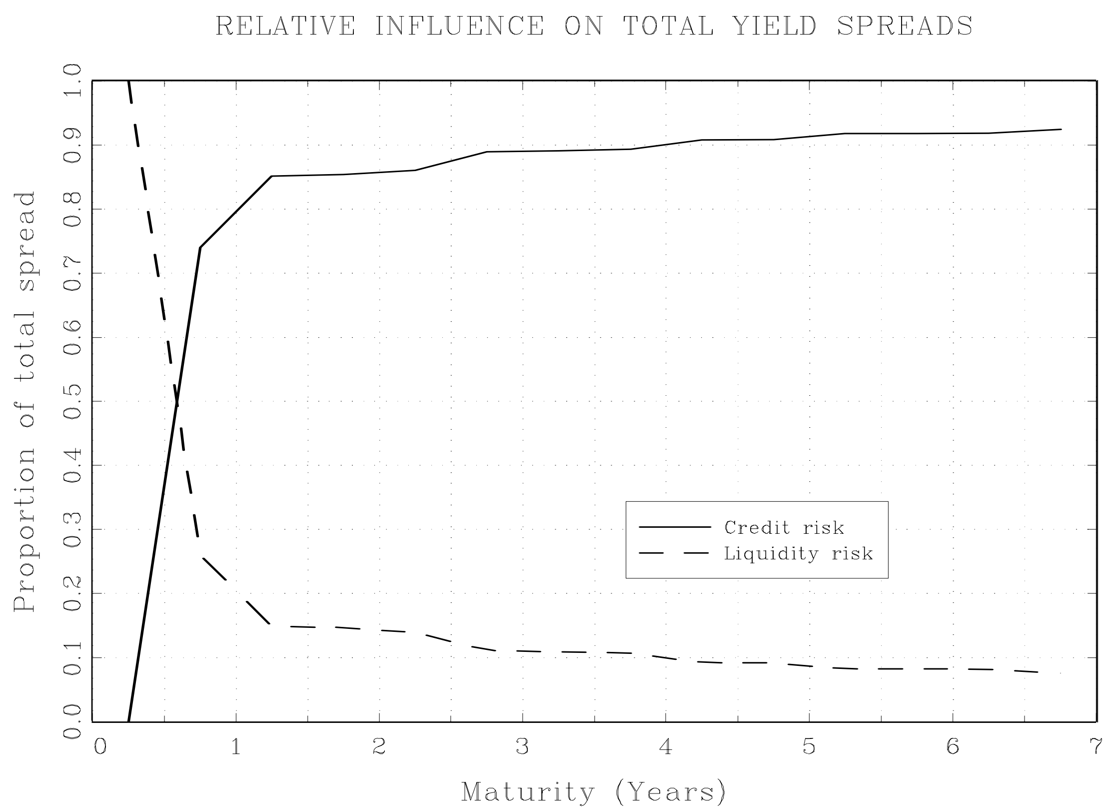

Figure 5: The percentage proportion of liquidity and credit risk compensation in the total yield spread. Parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock and the quasi-debt ratio is $q = 0.8$. The default risk related parameters are the asset volatility ($\sigma = 0.16$), the bankruptcy costs $K = 30$ and $\kappa = 20$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$.

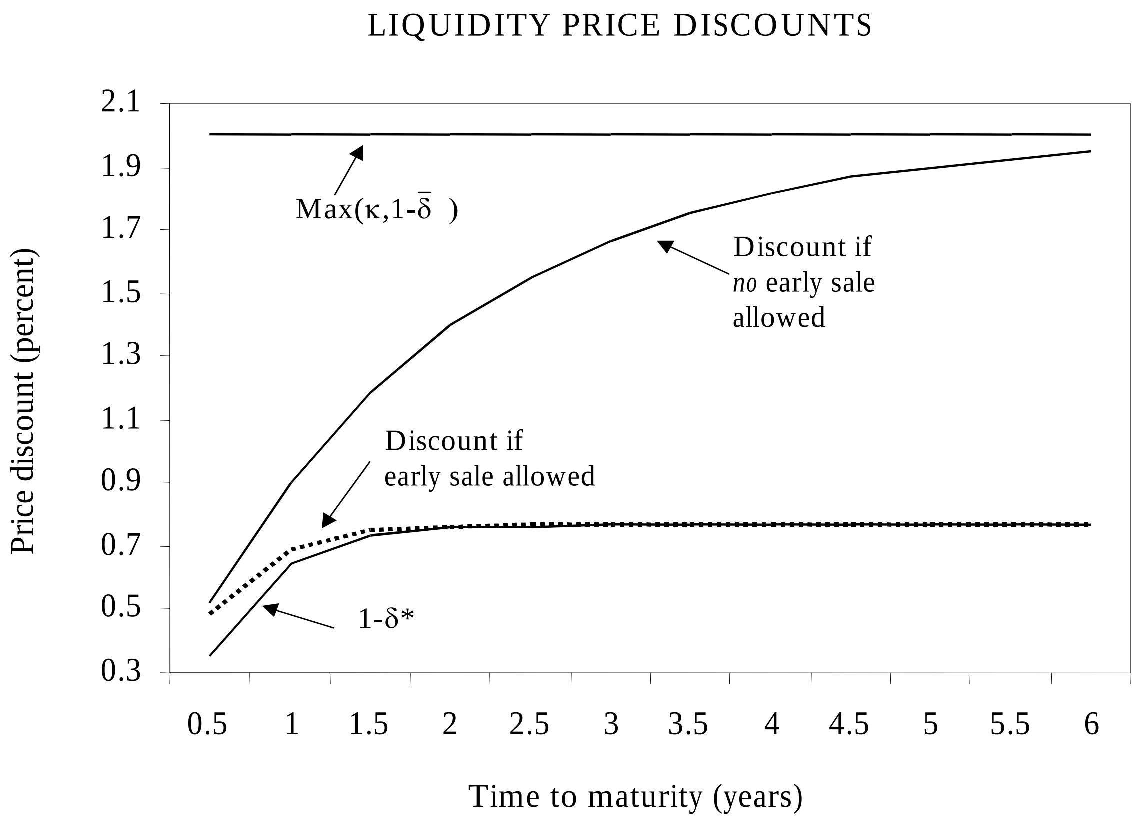

Figure 6: The price discount and its upper and lower bounds as functions of time to maturity. The upper bound is corresponds to the worst case when the bondholder is immediately enters into financial distress or is forced to sell. When there are no voluntary sales, the liquidity discount will eventually approach this upper bound as the probability of a forced sale approaches one. However, as this happens the likelihood of a voluntary sale increases, until the discount eventually approaches the lower bound corresponding to the reservation discount of the bondholder.

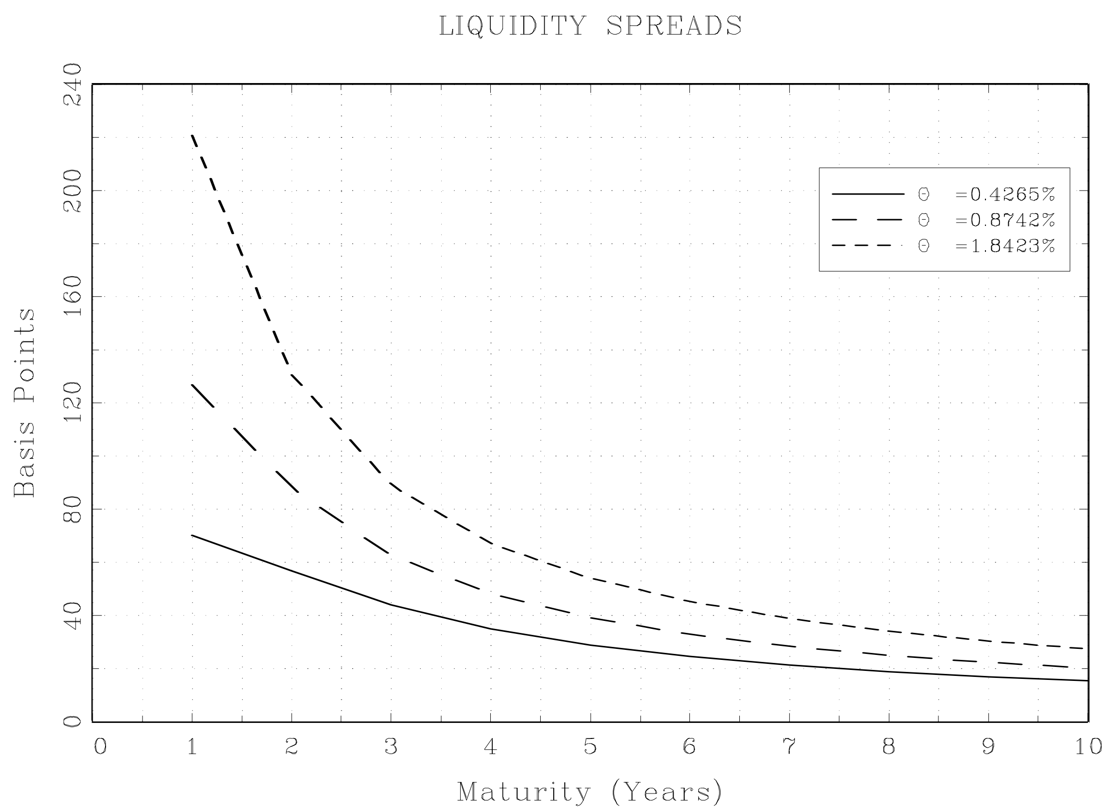

Figure 7: Liquidity spreads for varying liquidity shock probabilities ($\theta's$ corresponding to 5%, 10% and 20% yearly probabilities respectively). Other parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$. The default risk related parameters are the asset volatility ($\sigma = 0.16$), the bankruptcy costs $K = 30$ and $\kappa = 20$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$.

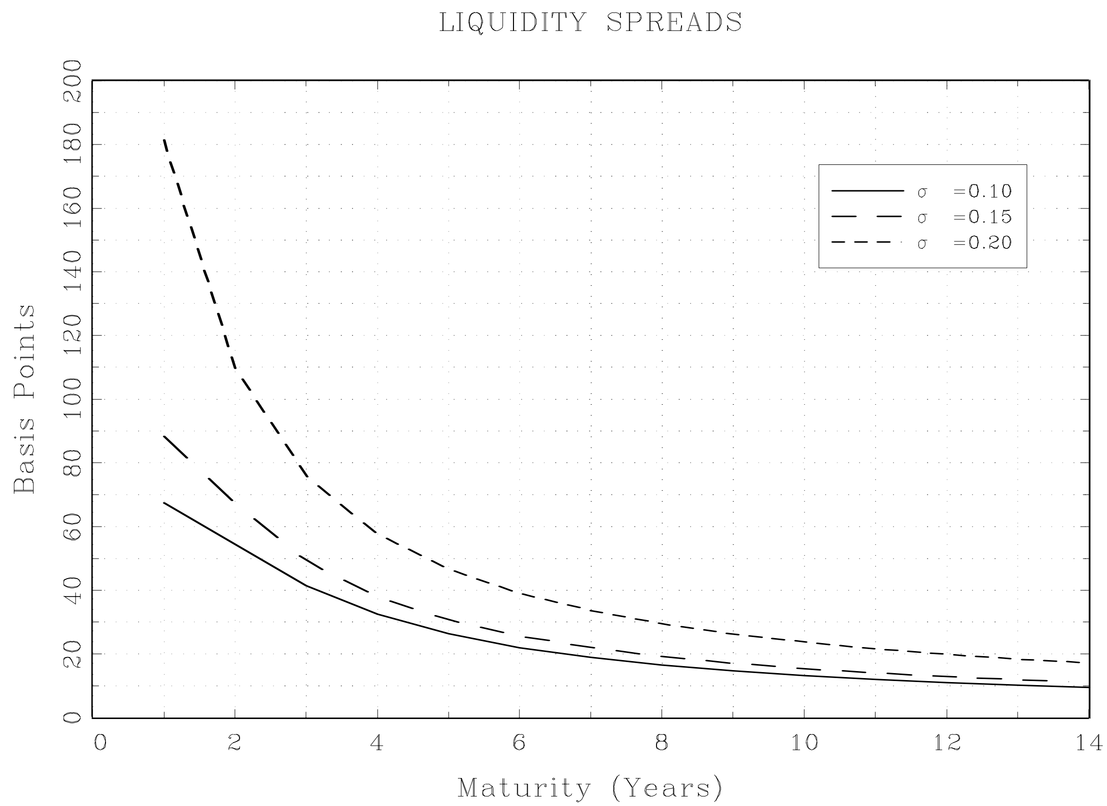

Figure 8: Liquidity spreads for varying levels of asset volatility. Other parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock and the quasi-debt ratio is $q = 0.8$. The bankruptcy costs are $K = 30$ and the cost of illiquidit in the market for distressed debt is $\kappa = 20$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$.

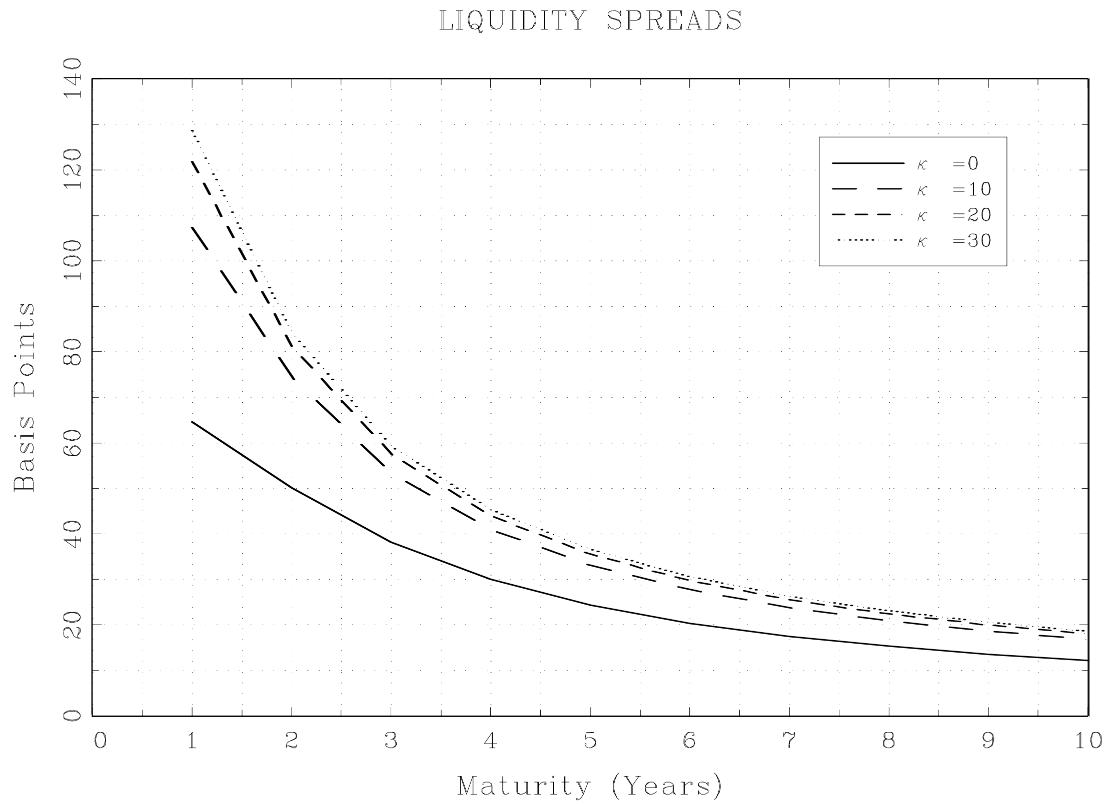

Figure 9: Liquidity spreads for varying levels of κ, the cost of illiquidity in the market for distressed debt. Other parameters : the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock and the quasi-debt ratio is $q = 0.8$. The default risk related parameters are the asset volatility ($\sigma = 0.16$), the bankruptcy costs $K = 30$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$.

$t=0$	1	2	3	4	5	6	7	8	9	T
									0	
								0.71		
							1.37		0	
						1.95		0.71		
					2.44		1.37		0	
				2.84		1.95		0.71		
			3.17		2.44		1.37		0	
		3.48		2.9		1.95		0.71		
	3.94		3.5		2.71		1.37		0	
5.32		4.96		4.37		3.31		0.71		
	13.3		12.57		11.81		10.51		0	
		def.		def.		def.		def.		
			def.		def.		def.		def.	
				def.		def.		def.		
					def.		def.		def.	
						def.		def.		
							def.		def.	
								def.		
									def.	

Figure 10: The maximum percentage discounts $(1 - \delta_t^* \times 100)$ that the bondholder will be willing to sell the bond at as a function of firm value and time to maturity. The time step is $\Delta t = 1$ and the time to maturity $T = 10$. The risk-free interest rate is $r = 7\%$, the average number of market-makers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock and the quasi-debt ratio is $q = 0.8$. The default risk related parameters are the asset volatility ($\sigma = 0.16$), the bankruptcy costs $K = 30$. The default threshold is set in proportion to the quasi debt ratio $L = exp(-rT)P$.

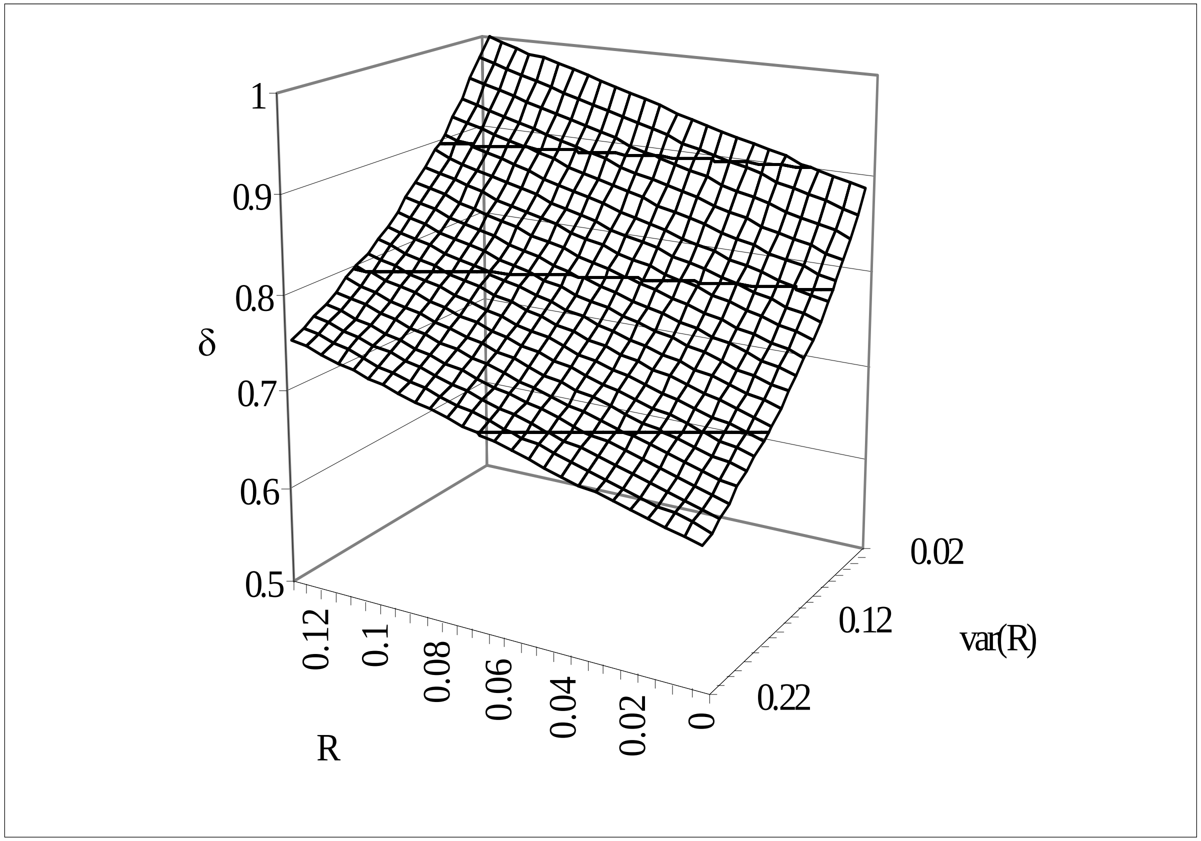

Figure 11: The price of immediacy: the discount required by the dealer in the inventory model. The risk aversion parameter of the dealer is $b = 3$, the risk free interest rate is 7%, the size of the bond position is the same size as the dealers' initial wealth and the dealer's initial cash position is 50% of his portfolio holdings.

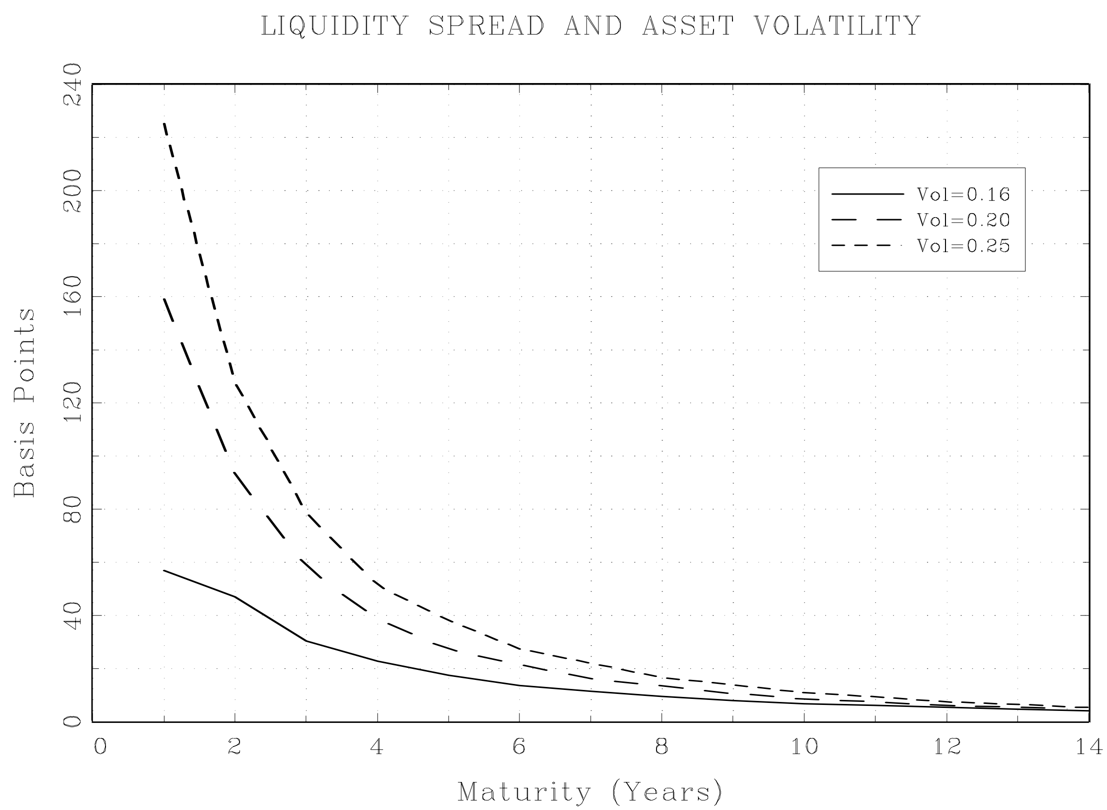

Figure 12: Liquidity spreads with a risk averse dealer for varying levels of σ. Other parameters: the time step is $\Delta t = 1/12$ (one month), the risk-free interest rate is $r = 7\%$, the average number of dealers active for the security is $\gamma = 7$, the probability of a liquidity shock is $\theta = 0.427\%$ which corresponds to a yearly probability of 5% of having at least one shock and the quasi-debt ratio is $q = 0.8$. The bankruptcy costs are $K = 10$ and the cost of illiquidity in the distressed debt market is $\kappa = 10$. The default threshold is set in proportion to the quasi debt ratio $L = \exp(-rT)P$. The relative risk aversion of the dealers is $b = 5$.

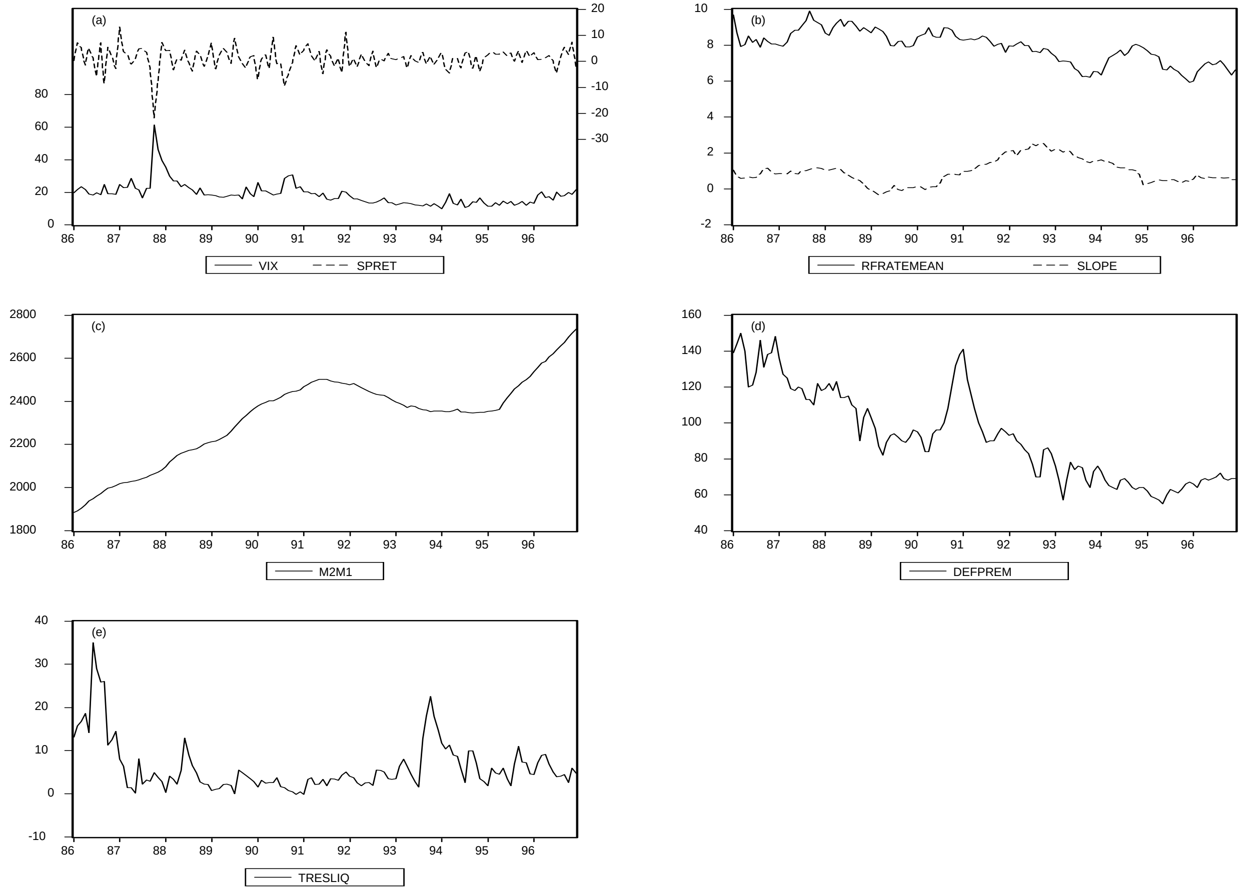

Figure 13: Plots of spread regressors. Panel (a) depicts the stock market volatility proxied by the VIX (left hand scale) and the return on the S&P 500 index ($SPRET$ - right hand scale). Panel (b) represents the slope of the term structure ($SLOPE$; yield differential between 10 and 2 year benchmark Treasury bonds) and the mean issue specific yield to maturity, both in percentage points. Panel (c) plots our credit cycle indicator and panel (d) plots the spread between Moody's BBB and AAA corporate bond yield indices. Finally panel (e) is a plot of the difference in yield between the on-the-run and next most recently issued 30 year Treasury bonds.