Measuring Counterparty Credit Risk from Reinsurance

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Abstract

Direct insurers are exposed to credit risk when entering into reinsurance contracts. This particular form of credit risk, which is sometimes referred to as security risk, is discussed and a methodology to determine the credit loss distribution for the direct insurer's reinsurance portfolio is proposed. The methodology extends the CreditRisk$^+$ methodology in order to account for reinsurance specific aspects. In particular, the element of potentiality in the credit exposure is modeled and the twofold character of diversification for reinsurance portfolios is analised.

Contents

1 Introduction

Reinsurance is insurance for insurance companies. A more precise definition is given in [4] (translation to English taken from [2]):

> "Reinsurance is the transfer of part of the hazards or risks that a direct insurer assumes by way of insurance contract or legal provision on behalf of an insured, to a second insurance carrier, the reinsurer, who has no direct contractual relationship with the insured."

In particular, the fact that the insured has no direct contractual relationship with the reinsurer has the consequence that the risk that the reinsurer fails to honour its financial obligations is carried by the direct insurer. In this paper we present an approach to quantifying this particular form of credit risk, a risk component to which most direct insurers are exposed to and is also referred to as *security risk*.
The approach we present is based on the well-known CreditRisk$^+$ methodology introduced by Credit Suisse Financial Products [1] which is widely used by financial institutions to quantify the credit risk of e.g. loan portfolios. The objective of our model is to determine the credit loss distribution for the entire reinsurance portfolio of a direct insurer.
An important difference between a loan portfolio and a reinsurance portfolio is that in a reinsurance portfolio there is an additional diversification dimension: not only are different risks reinsured by different reinsurers but additionally a single risk can be reinsured by several reinsurers, each one only taking a share of the risk. In Figure 1 these two diversification dimensions are shown: they correspond to passing from Portfolio 1 to Portfolio 2 by using different reinsurers for different risks and to passing from Portfolio 2 to Portfolio 3 by choosing several reinsurers for a single risk. In a loan portfolio only the first form of diversification occurs, i.e. diversification over counterparties.
Before presenting the details of our model we will give a very brief overview on the main concepts and the terminology in reinsurance which are relevant for credit risk.
A key element in security risk is the potentiality of the credit exposure: as long as the insured risks do not materialize the extent of the credit exposure towards the reinsurer is not known. This is analogous to the situation encountered in credit risk from derivatives. This aspect is briefly discussed in Section 2. Our distinction between current (known) exposure and (unknown) potential exposure is explained in Section 3.
For the practical applicability of financial risk models the availability and simplicity of the input data is an important issue. This is why we detail the input data which is required by the model in Section 4. In the same section we also discuss the simplifying assumptions made in order to obtain a tractable, yet pertinent model.
Finally, the mathematical algorithm to compute the credit loss distribution for the direct insurer's reinsurance portfolio is discussed in Section 5 and some test portfolios are discussed in Section **??**.

2 Credit risk from reinsurance: the main aspects

2.1 Reinsurance in a nutshell

In this section we give a very brief overview on the main aspects of reinsurance. An excellent, more comprehensive, and detailed introduction is given in [2]. Reinsurance contracts are usually either conceived to cover a large individual risk (e.g. reinsurance of liability for a specific product) or an entire portfolio of (similar) risks (e.g. reinsurance of a portfolio of life policies, or a number of ships). The first case is referred to as *facultative reinsurance* and the second case is called *treaty reinsurance*.

Besides distinguishing the above aspects, reinsurance contracts are designed to enable the reinsurer to participate in the risk of the direct insurer in various ways. No need to say that a multitude of reinsurance contract types has evolved in the industry with details that can be rather complicated. Here we just discuss the main distinction:

- *Proportional reinsurance*: Here the reinsurer and the primary insurer share both the premiums paid by the client as well as the claims paid, i.e. the reinsurer accepts a percentage of both the benefits and the costs entailed by the primary insurance contract. Forms of proportional reinsurance are *quota share* and *surplus treaty*.
- *Non proportional reinsurance*: The reinsurer covers the portion of a claim exceeding a certain amount, e.g. the part of a claim above 100 Mio. Usually the reinsurer also limits the maximum claim he is prepared to cover, e.g. the part of a claim above 100 Mio but up to a maximal payment of 100 Mio. This is an example of an *excess of loss reinsurance*. The direct insurer has a self retention of 100 Mio and the reinsurer covers the excess above 100 Mio up to a maximum payment of 100 Mio.

Roughly speaking proportional reinsurance involves a participation by the reinsurer which increases linearly with respect to the insured loss suffered by the insured party, whereas in non proportional reinsurance contracts the dependence between the insured loss and the payment due by the reinsurer is nonlinear. Clearly the particular reinsurance structure has a significant impact on the credit exposure and must be taken into account in any attempt to model security credit risk.

2.2 Potential exposure

It is evident from the very nature of a reinsurance contract that the future payments an insurer will receive from his reinsurer are not known today [1] since they depend on the future losses suffered by the insured party. Therefore the credit exposure that the insurer holds towards the reinsurer has a decisive element of potentiality and requires a probabilistic treatment if it is to be quantified.

2.3 Analogy to credit risk from derivatives

The element of potentiality in counterparty exposures in reinsurance has a strong analogy with the credit risk incurred when holding derivatives positions: due to the potentially very high value changes of the positions, counterparty exposures can rapidly reach preoccupying dimensions, similarly in the aftermath of a big insured catastrophe an insurer has to rely on the often very conspicuous payments from the reinsurer.

Yet there is also an important difference between reinsurance credit exposure and derivatives: derivatives prices are jointly driven by a common set of underlying asset prices and the movement of the underlying factors happens in a highly correlated way. Thus holding many different derivatives contracts (with the same counterparty) will not necessarily diversify credit risk away. Insurers and reinsurers, however can diversify their risk portfolios by choosing from a

[1] What is of course known by the direct insurer are receivables from past claims which are due by the reinsurer.

large variety of independent risks. Consequently, as a first approximation, it is presumably not too misleading to assume that the various (reinsured) risks are independent[2], an erroneous and dangerous assumption in derivatives credit risk.

3 Definition of counterparty credit exposure towards a reinsurer

In this section we will clarify what we mean by credit exposure resulting from the reinsurance contracts held with a reinsurer. As already discussed we will distinguish between *current exposure* and *potential exposure*. The (total) exposure is then given by the sum of these two components. In Section 4 we will cast the considerations of this section in mathematical language. Particular attention will be paid to the discussion of the assumptions made in the model.

3.1 Current exposure

The current exposure is given by all known receivables due by the reinsurer. This receivables are the consequence of known past losses incurred by the insured. The current exposure is thus determined by considering known past events. In principle the information can be extracted from the accounting figures produced by the direct insurer. The direct insurer's accounting of his relationship with the reinsurer is rather tricky and involves some technical jargon (consult e.g. [3] for more details). Nevertheless we will try to briefly discuss the accounting positions the insurer needs to consider for determining the current credit exposure towards a reinsurer:

- *Receivables for paid claims*: These are receivables due by the reinsurer resulting from claims that the direct insurer has settled with the insured party. The loss event will be completely settled for the direct insurer when he will have received the outstanding payment from the participating reinsurer.
- *Receivables for reported unpaid claims*: the direct insurer has received the claim from the insured party but has not made his payment yet. The direct insurer knows that he will in turn receive a payment by the reinsurer, which in the direct insurers accounting results as a "receivable for reported unpaid claims".
- *Receivables for claims incurred but not reported*: the direct insurer knows that the insured party has suffered a loss for which it has insurance cover. The insured party has not reported the exact claim amount but an estimate can be made by the insurer. This estimate enters the insurers accounting under "receivables for claims incurred but not reported" or briefly "IBNRs".
- *Prepaid reinsurance premiums*: this is the portion of the premium for the remaining duration of the reinsurance contract that the direct insurer has ceded to the reinsurer to obtain the reinsurance protection over the policy period. If a reinsurer defaults then the remaining reinsurance cover has to be replaced. The credit risk is this replacement value.
- *Offsetting positions*: The credit exposure arising from the above positions is reduced by deposits, letters of credit, guarantees, etc. from the reinsurer.

[2]Hurricanes in Florida and earthquakes in California happen independently of each other, turmoil in different stock markets very often does not.

3.2 Potential exposure

By potential exposure we understand the payments that could become due by the reinsurer within the considered future time period as a consequence of the reinsured risks. In particular, we think of rare extreme events that could trigger large payments by the reinsurer. Thus we will model the potential exposure as a random variable where we will be forced to assign probabilities for the reinsured risks to materialize. Furthermore the impact of the adverse events on the reinsurers obligations has to be assessed by inspecting the modalities of all the reinsurance contracts involved.

4 A model based on CreditRisk$^+$

This section presents an approach to integrating the relevant aspects of reinsurance credit risk into a mathematical model with the objective to obtain a consistent quantification of this risk over the entire reinsurance portfolio held by the direct insurer. The main aspects which ought to be reflected by the model are:

- The credit quality of the various reinsurers expressed e.g. in terms of their credit ratings and associated default probabilities over the considered time period.
- The different "amount" of risk taken by each reinsurer and the probabilities for the reinsured risks to materialize.
- Diversification or concentration effects with respect to the distribution of the credit exposure among the reinsurers in the portfolio.
- Diversification or concentration effects with respect to the number and size of independent risks each reinsurer has assumed within the direct insurer's portfolio.

Before we proceed to present the model in Subsection 4.2 we discuss the data that will be used as input to the model.

4.1 Input data and main model assumptions

In this section we discuss the data used in the model and we discuss the main assumptions that need to be made to obtain a tractable model. The data used for determining current and potential exposures is given in Table 1 and 2, respectively.

Reinsurer	Rating	Current exposure
Safe Re	AAA	100'000'000
Reliable Re	A	20'000'000
Medium Re	BBB	2'000'000
⋮	⋮	⋮

Table 1: Data used to model current exposure.

Contract	Reinsurer	Rating	Potential exposure	Probability of a large claim
Contract 1	Greatest Re	AA	2'000'000	0.005
"	Safe Re	A	5'000'000	"
"	Medium Re	BBB	10'000'000	"
"	Hyper Re	AAA	50'000'000	"
"	Super Re	B	50'000'000	"
Contract 2	Reliable Re	A	15'000'000	0.01
"	New Re	AA	10'000'000	"
"	Old Re	BB	5'000'000	"
"	Double Re	AA	1'000'000	"
Contract 3	Safe Re	A	100'000'000	0.05
⋮	⋮	⋮	⋮	⋮

Table 2: Data used to model potential exposure.

We note that the data for current exposure are completely analogous to the data used to model credit losses resulting from holding a loan portfolio. The situation is different when modeling potential exposure: here we are forced to consider each reinsurance contract individually. The reason is that in general the various contracts cover different and independent risks. As a consequence for a given counterparty the (potential) exposure contributions arising from each contract cannot simply be added!

The following important simplifying assumptions will be made:

- Counterparty defaults are independent, i.e. information on the default of a counterparty does not alter the default probability of any other counterparty.
- Absence of clash effects, i.e. it is assumed that the various reinsurance contracts cover independent risks.
- The payments triggered by the materialization of the reinsured risks do not alter the default probability of the involved reinsurers, i.e. the default events are independent of the reinsured loss events.

The next section explains how the above information can be incorporated into a probabilistic model.

4.2 The random variable modeling credit losses

In this section we introduce a random variable L which models the credit loss after a certain time horizon. Here we will always consider a one year time horizon. For the following we fix a sample space Ω and a probability $P : 2^{\Omega} \to [0,1]$. The one year credit loss L of the direct insurer can be expressed as the sum of exposure contributions over all counterparties $j = 1, \ldots, n$ and all reinsurance contracts $k = 1, \ldots, m$ in his reinsurance portfolio..

$$L = \sum_{j=1}^{n} D_j \left[E_j + \sum_{k=1}^{m} C_k W_{j,k} \right] \quad , \tag{1}$$

where D_j , C_k , $W_{j,k}$, and E_j have the following interpretation:

- For $j \in \{1, \ldots, n\}$ we write D_j for the binomial random variable indicating the default of counterparty j , i.e. we set

  $$D_j : \Omega \to \{0, 1\} \text{ and } P(D_j = 1) = d_j \ , \ P(D_j = 0) = 1 - d_j \ ,$$

  where d_j is the one-year default probability of the counterparty j .
- For $k \in \{1, \ldots, m\}$ we denote the binomial random variable indicating that the potential exposure from contract k has materialized by C_k , i.e. we set

  $$C_k : \Omega \to \{0, 1\} \text{ and } P(C_k = 1) = p_k \ , \ P(C_k = 0) = 1 - p_k \ ,$$

  where p_k is the probability that the potential exposure from contract k becomes effective.
- $W_{j,k}$ is a nonnegative number describing the payment due by the reinsurer j should a large claim materialize in the contract k, i.e. $W_{j,k}$ is the payment due by the reinsurer j in case of the event $C_k = 1$.
- The current exposure towards reinsurer $j \in \{1, \ldots, n\}$ is denoted by the nonnegative number E_j .

The main assumptions already stated in the previous subsection can now be specified more precisely within the framework of this model.

Assumptions 4.1 *For the remainder of this paper the three following simplifying independence assumptions will be made:*

- *The random variables D_j , $j \in \{1, \ldots, n\}$ are independent. This models the independence of the default events.*
- *The random variables C_k , $k \in \{1, \ldots, m\}$ are independent. This models the independence of the insured risks.*
- *For $j \in \{1, \ldots, n\}$ and $k \in \{1, \ldots, m\}$ the random variables D_j and C_k are independent. This models the independence of default events and loss events.*

The main task now consists in computing the probability distribution of L, i.e. we are interested in the probabilities

$$P(L = X) \ , \ X \geq 0$$

that a credit loss of any given size $X \geq 0$ occurs. The next subsection explains the various computational steps in determining the loss distribution of L .

5 Iterative computation of the credit loss distribution

The loss distribution of L can be obtained in three steps:

- Step 1: For each $j \in \{1, \ldots, n\}$ the distribution of

$$E_j + \sum_{k=1}^{m} C_k W_{j,k}$$

  is determined by applying the CreditRisk$^+$ methodology.

- Step 2: The distributions of the random variables

$$L_j := D_j \left[E_j + \sum_{k=1}^{m} C_k W_{j,k} \right] \ , \ j = 1, \ldots, n$$

  are determined from the distributions obtained in the first step by using the fact that D_j is a binomial random variable.

- Step 3: Finally, the distribution of $L = \sum_{j=1}^{n} L_j$ is determined by successive convolution of the probability distributions of the independent variables L_j , $j = 1, \ldots, n$ determined in Step 2.

We proceed to discuss these three steps in more detail:

5.1 The first step

In this first step we are facing the problem of computing the probability distribution of

$$E_j + \sum_{k=1}^{m} C_k W_{j,k}$$

for each $j \in \{1, \ldots, n\}$. Since E_j is just a constant we only need to consider the random variable

$$\sum_{k=1}^{m} C_k W_{j,k} \, .$$

This random variable is exactly the starting point of the CreditRisk$^+$ methodology applied to a portfolio of m loans of size $W_{j,k}$ held by m different counterparties with default probabilities p_k , $k \in \{1, \ldots, m\}$.
As is shown in [1] [3]

the probability distribution can be approximated with satisfactory precision by an iterative computation: for that purpose the m exposures $W_{j,k}$ are assigned to exposure bands, say

$$B_1, \ldots, B_{N(j)} \ ,$$

[3] For simplicity we only consider the simplest case of vanishing default rate volatility, i.e. we assume the default probabilities to be constant rather than stochastic over the considered time period. The introduction of systematic risk by introducing a default rate volatility is discussed in [1]. It is straightforward to adapt our method to the more general situation.

according to their size. We can write the total exposure $E_j + \sum_{k=1}^m W_{j,k}$ in the following way

$$E_j + \sum_{k=1}^{m} W_{j,k} \;=\; U_j\, e_j \;+\; U_j \sum_{i=1}^{N^{(j)}} m_i^{(k)}\, n_i^{(k)} \;+\; \delta \;,$$

where U_j is the exposure unit and e_j , $m_i^{(j)}$ and $n_i^{(j)}$ are integers denoting the following quantities

- e_j current exposure towards counterparty j expressed in exposure units U_j rounded to the nearest integer.
- $m_i^{(j)}$ is the average exposure (rounded to the nearest multiple of the unit U_j) of the counterparties in band i expressed in exposure units.
- $n_i^{(j)}$ number of counterparties in band i .

The number δ is the error made by rounding the current exposure and the average exposure in each band i to the nearest integer multiple of the exposure unit U_j . The rounding error δ can be kept small by choosing a sufficiently small exposure unit U_j .

5.1.1 Computation of the probability distribution of $\sum_{k=1}^m C_k W_{j,k}$

The application of the CreditRisk$^+$ method leads to the following result: the probability distribution of

$$F_j \equiv \sum_{k=1}^{m} C_k W_{j,k}$$

can be determined by the following iteration starting with

$$P(F_j = 0) = e^{-\sum_{i=1}^{N^{(j)}} \lambda_i}$$

and iteration step

$$P(F_j = l\, U_j) \approx \frac{1}{l} \sum_{i=1}^{N^{(j)}} m_i^{(j)} \lambda_i\, P\left(\, F_j = (l - m_i^{(j)}) U_j \,\right) \;,$$

where

$$l = 1, 2, \ldots, M_j \equiv \sum_{i=1}^{N^{(j)}} n_i^{(j)} m_i^{(j)}$$

and λ_i is given by the sum over the probabilities p_k of the exposures $W_{j,k}$ assigned to band i , i.e.

$$\lambda_i \equiv \sum_{\{\, l\, ;\, W_{j,l} \,\in\, \text{band } i \,\}} p_l \;\; .$$

In the above iteration we set

$$P\left(\, F_j = (l - m_i^{(j)}) U_j \,\right) \equiv 0$$

whenever $l - m_i^{(j)} < 0$. Furthermore note that

$$M_j = \sum_{i=1}^{N^{(j)}} n_i^{(j)} m_i^{(j)}$$

is the total number of exposure units U_j exposed to the counterparty j. Therefore it is the greatest value that the random variable F_j can attain. This explains why the above iteration ends at $l = M_j$.

The probability distribution of $E_j + F_j$ is then obtained by "shifting" the probability distribution of F_j, i.e.

$$P(E_j + Fj = l\,U_j) = P\left(F_j = (l - e_j)\,U_j\right)\ ,$$

where

$$l = 0, 1, \ldots, e_j + M_j\ ,$$

and where we set

$$P\left(F_j = (l - e_j)\,U_j\right) \equiv\ 0$$

whenever $l - e_j < 0$, i.e. for $l = 0, \ldots, e_j - 1$.

5.2 The second step

The second step consists in determining the probability distribution of

$$L_j = D_j[E_j + F_j]$$

for $j = 1, \ldots, n$ by making use of the probability distributions of $E_j + F_j$ already determined in the first step. This can now easily be done, in fact

$$(2)\ P(L_j = l\,U_j) = \begin{cases} d_j\,P(E_j + F_j = 0)\ +\ (1 - d_j)\displaystyle\sum_{s=0}^{e_j+M_j} P(E_j + F_j = s\,U_j) & \text{for } l = 0\ , \\ d_j\,P(E_j + F_j = l\,U_j) & \text{for } l \neq 0\ , \end{cases}$$

where

$$l = 0, 1, \ldots, e_j + M_j\ .$$

Observe that the above formula can be simplified since

$$\sum_{s=0}^{e_j+M_j} P(E_j + F_j = s\,U_j) = 1\ .$$

In order to understand how the above formula is obtained, consider first the probability $P(L_j = 0)$. In Figure 2 we illustrate the observation that $L_j = 0$ can be zero for two reasons:

- $D_j = 1$ and $P(E_j + F_j = 0)$ with a probability of

  $$d_j\,P(E_j + F_j = 0)$$

  for this to happen.

- $D_j = 0$ and as consequence any value of $E_j + F_j$ leads to $L_j = 0$. The probability for this to happen is

  $$(1 - d_j)\sum_{s=0}^{e_j+M_j} P(E_j + F_j = s\,U_j)\,.$$

Thus adding the above probabilities yields the expression for $P(L_j = 0)$ given in Formula (2). The idea to find Formula (2) for the case $l \neq 0$ is obviously that $P(L_j \neq 0)$ can only occur if $D_j = 1$, which happens with probability d_j.

5.3 The third step

The final step is to determine the probability distribution of $L = \sum_{j=1}^{n} Lj$, i.e. to aggregate the probability distributions determined in Step 2. Note that by the Assumptions 4.1 the random variables L_j , $j = 1, \ldots, n$ are independent. This aggregation can therefore be performed by applying the following simple result on discrete random variables.

Lemma 5.1 *Let X and Y be independent random variables with finitely many different values in $\mathbb{N}$ (i.e. X and Y are so-called simple random variables) on a sample space Ω. Then the probability distribution of the random variable $X + Y$ can be determined from the probability distributions of X and Y by*

$$P(X + Y = l) = \sum_{\{(r,s)\,;\,r+s=l\}} P(X = r)\, P(Y = s) \quad , \tag{3}$$

where $l = 0, 1, \ldots, \max(X + Y)$.

The above operation is sometimes called *convolution* of the probability distributions of X and Y . Clearly the probability distribution of the sum of more than two random variables is obtained by successive pairwise convolution of the probability distributions.
For the numerical computation of the sum in the convolution (3) it is possible to use the Fast Fourier Transform (FFT) to get a more efficient evaluation of the convolution. For portfolios of moderate size the sum can also be evaluated explicitly.

References

[1] CreditRisk$^+$. A Credit Risk Management Framework. Credit Suisse Financial Products, 1997. Group, September 28, 1998.

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[3] Proportional reinsurance accounting. Accounting and statistical procedures. Swiss Re Report. Zurich 1999.

[4] M. Grossmann, Rückversicherung-Eine Einführung. Ed. and publ. by Institut für Versicherungswirtschaft, University of St. Gall, 3rd ed., 1990.

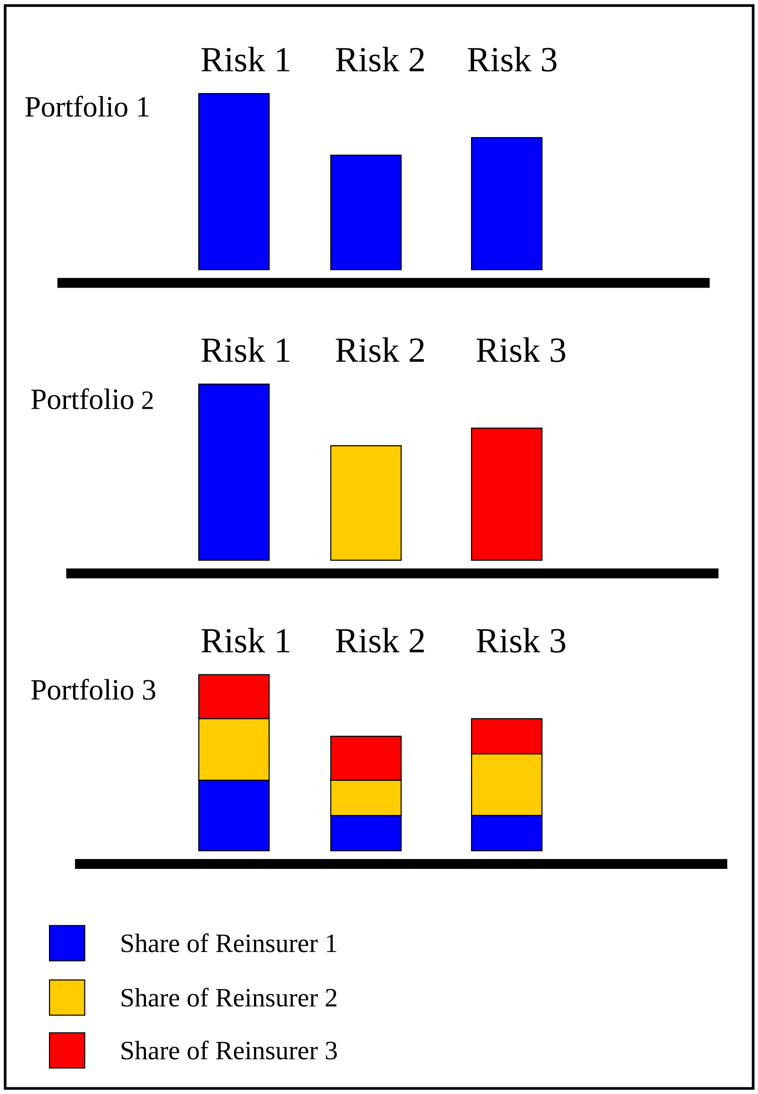

Figure 1: The two dimensions of diversification in reinsurance: diversification over counterparties by passing from reinsurance Portfolio 1 to 2 and diversification in a single contract by replacing Portfolio 2 by Portfolio 3.

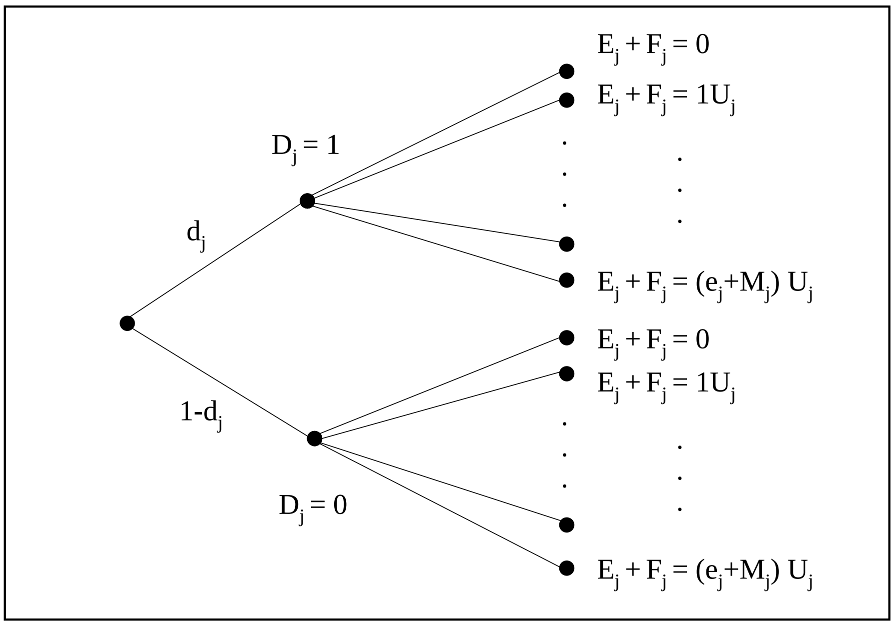

Figure 2: Illustration of the idea leading to Formula (2) for the case $l = 0$.