Stochastic Capital Depreciation and the Comovement of Hours and Productivity

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Abstract

In this article, we demonstrate that a small degree of stochastic variation in the depreciation rate of capital can greatly reduce the comovement between hours worked and labor productivity in a neoclassical growth model. The depreciation rate is modeled as a Markov process to place a strict upper bound and ensure that variation and not the level of the rate is driving the result. Markov switching implies nonlinear decision rules in the dynamic stochastic general equilibrium model (DSGE). Our contribution to DSGE solution methodologies is to apply Judd's (1998) projection method to nonlinear decision rules. This approach allows for nonlinear decision rules in a richer set of models with many more state variables than can be solved with grid-based approximations. The results presented here suggest that Markov switching parameters offer a powerful extension to DSGE models.

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1. Introduction

Numerous dynamic stochastic general equilibrium (DSGE) macroeconomic models now allow for variation in the depreciation rate of capital. One approach treats the depreciation rate as an endogenous variable such that the choice to use capital intensively or to spend little on maintenance and repair results in high depreciation [Greenwood, Hercowitz and Huffman (1988); Burnside, Eichenbaum and Rebelo (1996); King and Rebelo (2000) for the former; McGrattan and Schmitz (1999), Collard and Kollintzas (2000) and Licandro and Puch (2000) for the latter]. Procyclical variation in capital utilization amplifies the effect of a technology shock on output. Thus, the variance of technology shocks can be lower in a model with variable capital utilization, with fewer implied occurrences of technical regress. When the depreciation rate is a function of maintenance and repair, the assumption is that each unit of capital is matched with labor input that is geared toward either production or capital maintenance and repair. In this case, technology shocks have an income and a substitution effect on the rate of depreciation. The substitution effect is positive because a positive productivity shock causes goods production to be relatively more efficient than maintenance. The income effect is negative because a positive productivity shock reduces the labor input needed to produce a given quantity of consumer goods, freeing labor for alternative activities including maintenance. In both of these scenarios, variation in the depreciation rate is a means and not an end. Endogenous depreciation equates margins at less than full capital utilization or introduces a role for large, countercyclical expenditures on maintenance and repair. In this way, endogeous depreciation serves to amplify and add to the persistence of the effects of technology shocks on output. But, fully endogenous depreciation does not allow for random changes in the depreciation rate as an independent source of economic fluctuations.

Alternatively, the depreciation rate can be stochastic in DSGE models, putting depreciation shocks on a par with technology shocks as fundamental driving forces behind macroeconomic fluctuations. At the same time, one interpretation of exogenous variation in the depreciation rate is that it serves as a shorthand approach to complicated endogenous scrappage decisions that are hard to work into a DSGE model. For example, a high rate of obsolescence of energy-intensive capital in the face of the 1970s oil price shocks might very well have been an endogenous response to a particular type of shock. Rather than add a lot of structure, a modeler might choose to treat such an episode as a surprise and temporary exogenous increase in the depreciation rate. Moreover, one might truly believe that variation in the depreciation rate is mostly stochastic and not a function of choice variables. Ambler and Paquet (1994) introduce stochastic depreciation as a persistent autoregressive process much like technology. Such depreciation shocks help the model match an important feature of U.S. labor markets that elude real business cycle (RBC) models: hours and productivity have only a small positive correlation, rather than the large positive correlation implied by the standard RBC model that relies solely on technology shocks.

The reason why Ambler and Paquet's (1994) autoregressive depreciation rate leads to a low hours-productivity correlation is somewhat ambiguous, however. A standard RBC model with a constant rate of depreciation can imply a low hours-productivity correlation if the depreciation rate is sufficiently high, as shown in Figure 1. Nevertheless, a quarterly rate of depreciation above 3 percent is generally considered implausibly high, so the standard RBC model cannot generate a low positive correlation between hours and productivity at empirically plausible parameter values. Ambler and Paquet's (1994) autoregressive process generates annual depreciation rates that fluctuate between roughly 4 and 14 percent. Thus, especially in light of

Figure 1, Ambler and Paquet's autoregressive process does not firmly establish that stochastic fluctuations limited to low, empirically plausible rates of depreciation can account for a low hours-productivity correlation.

This article seeks to demonstrate that stochastic fluctuation of the depreciation rate within a narrow band at a moderate level is by itself able to generate a low hours-productivity correlation, without the occurrence of high rates of depreciation. In this way, we reinforce Ambler and Paquet's point that stochastic depreciation induces a substantial reduction in the comovement of hours and labor productivity. The intuition is that a random increase in the depreciation rate causes hours worked and labor productivity to respond in opposite directions. When the depreciation rate is in the high state, the capital-output ratio begins to fall, so labor hours are substituted for capital. Consequently, hours and labor productivity move in opposite directions. This source of negative correlation between hours and labor productivity can counteract the positive correlation implied by technology shocks to result in a correlation between hours and productivity in order to match the data.

To limit the range of fluctuation and the highest level of the depreciation rate, we model stochastic depreciation rates as the outcome of a two-state Markov switching process. Provided that the high depreciation state lies below the region where the level of the depreciation rate affects the hours-productivity correlation, we can be sure that any effect of stochastic depreciation on the hours-productivity correlation is coming from random changes in the depreciation rate, as opposed to realizations of a very high depreciation rate. In fact, our results show that the high and low depreciation states do not have to be very far apart to generate low hours-productivity correlations at empirically plausible rates of depreciation.

This article also makes an important contribution to the methodologies used in DSGE models to implement nonlinear decision rules. Markov switching parameters imply nonlinear decision rules, although one approach followed by Gong (1995) was to calculate linear decision rules for each Markov state and then weight the rules by their probabilities. Andolfatto and Gomme (2001) introduce a Markov switching money growth rate and use a grid-based approximation to the nonlinear decision rule functions. As we discuss in Section 4, however, the grid-based approach can only be applied to relatively simple models with a very small number of state variables—three or less. Instead, we apply Judd's (1998) projection method of polynomial approximations to the nonlinear decision rules. The projection remains feasible for DSGE models with ten state variables or more. Without such methods, research involving Markov-switching parameters or other forms of nonlinear decision rules would be very limited in terms of dimensionality.

The article is organized as follows. Section 2 motivates our use of a Markov switching process for a time-varying depreciation rate. Various uncertainties concerning the persistence of depreciation are highlighted. Section 3 presents the baseline RBC model. The same section discusses calibration strategy. The estimation procedure with nonlinear decision rules is outlined in Section 4. Section 5 presents the main results in the form of sensitivity analysis and impulse responses. Section 6 concludes.

2. Difficulties in Calibrating Time-Varying Depreciation

Empirical evidence on time-varying depreciation rates is scant for the U.S. economy. The accounting methods used for the construction of national accounts data typically assume constant depreciation rates. Hence, evidence on time-varying depreciation often comes from sectoral data that are not necessarily representative of the economy as a whole.

In one of the few empirical studies on estimating time-varying depreciation rates, Abadir and Talmain (2001) use annual observations on real net investment and on real gross investment to calculate the macroeconomic depreciation rate from their implied capital stock measure.[1] They find that depreciation rates differ substantially across countries for the period 1970 to 1996. The estimates for the United States suggest the following characteristics: annual depreciation rates exhibit strong persistence, they fluctuate in a relatively narrow (trendless) band and they do not appear to be strongly procyclical. This suggests that capital is often destroyed or scrapped through factors that are independent of capital utilization. Innovation that leads to obsolescence and scrappage is not always procyclical. In the service sector, which is heavily reliant on computers, the technology cycle of microprocessors—popularly known as Moore's law—is independent of the business cycle. Capital is also destroyed through natural disasters and other random events.

Our strategy for modeling time-varying depreciation is to assume that depreciation is driven by a Markov process that is independent of the technology shock.[2] The rationale for this is to allow varying levels of persistence

[1]Their data driven procedure rests on the assumption that at one point in the sample $\delta_{t+1} = \delta t$, which allows them to derive an estimate for the implied capital stock. Through a series of identities they are then able to obtain an estimate for the depreciation rate.

[2]In a similar setup with independent shocks, Bernanke, Gertler, and Gilchrist (1999)

for the different depreciation states that are assumed to be independent from the technology shock. In our setup, we rely on the empirical work of Abadir and Talmain (2001) and assume that depreciation rates are weakly correlated with the business cycle. We experiment with conditional means for annual depreciation that are all strictly lower than 10 percent and consider various degrees of persistence in the Markov process.

3. Model Structure and Calibration

The model is a standard DSGE model with indivisible labor and no artificial frictions aside from a shopping-time motive for holding money. The model economy is populated by a large number of infinitely lived agents whose expected utility is defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t (ln(c_t) + \theta ln(1 - h_t) + \theta \frac{L_t}{\hat{L}} [ln(1 - \hat{L} - h_t) - ln(1 - h_t)]), \qquad (1)$$

where β is the time discount factor; c_t is private consumption; θ is a positive scalar that determines the relative disutility of non-leisure activities; h_t is shopping time, L_t is expected work time under a Rogerson (1988) em-

and Carlstrom and Fuerst (2000) consider the influence of adjustment and agency costs by considering a time-varying component on investments in the capital stock equation.

ployment lottery; $\hat{L}$ is the indivisible time spent at work for those working [Hansen (1985)].[3]

Aggregate output, Y_t, is assumed to depend on the total amount of capital, K_t, and on total hours of work, N_t, with labor-augmenting technological progress at the gross rate λ

$$Y_t = e^{z_t} K_t^{\alpha} (\lambda^t N_t)^{1-\alpha}. \tag{2}$$

The shopping-time motive for holding money is really not distinct from a cash-in-advance constraint. Many cash-in-advance models have a cash good and a credit good. If one assumes that buyers have imperfect advance knowledge of which sellers require cash and which offer credit, then holding more money results in less time lost from mistaking a cash-only seller for a credit-offering seller. The shopping time technology specifies the amount of time that must be spent shopping within period t as a function of consumption c_t relative to the amount of real money balances, m_t. The shopping time technology follows the specification given in King and Wolman (1996):

$$h_t = \kappa \frac{m_t}{P_t c_t} + \xi^{1/\nu} (\frac{\nu}{1-\nu}) (\frac{m_t}{P_t c_t})^{\frac{\nu-1}{\nu}}, \tag{3}$$

[3] We follow the general practice, where lower case letters are used to denote individual choices and upper case letters denote economy-wide per-capita quantities.

where κ defines the finite satiation level of real cash balances, $m = \xi c \kappa^{-\nu}$ when the nominal interest rate is zero. A time constraint restricts leisure, shopping time, and work to sum to one:

$$l_t + h_t + n_t = 1. \tag{4}$$

The technology shock, z_t, is assumed to follow an AR(1) process with the following law of motion:

$$z_t = \rho z_{t-1} + \epsilon_t, \; \epsilon_t \sim N(0, \sigma_\epsilon^2). \tag{5}$$

The random variable, ϵ_t, is drawn from a normal distribution with mean zero and standard deviation σ_ϵ.

The capital stock evolves according to

$$k_{t+1} = (1 - \delta_{S_t}) k_t + i_t, \tag{6}$$

where i_t is the chosen level of investment and δ_{S_t} is the rate of depreciation of capital, which is assumed to follow a two-state Markov process that is independent of z_t.[4] Markov switching generates uncertainty for a single period before agents realize in which state they are in. The installation of

[4]With our setup, equation (6) is unable to reduce the probability of technical regress as discussed in Burnside, Eichenbaum, and Rebelo (1996) and King and Rebelo (2000).

capital takes one period, making the capital stock predetermined at time t, but there are otherwise no installation or adjustment costs. The aggregate resource constraint of the economy is given by

$$y_t = c_t + i_t. \tag{7}$$

We close the model with an interest rate rule that is a function of actual inflation and an inflation target:

$$\Delta r_{t+1} = 1.2[\pi_t - \pi^*] + \eta_t. \tag{8}$$

Here π is inflation, π^* is the inflation target and η_t is an *i.i.d.* error term. Deviations from the inflation target are corrected with a feedback component set to 1.2.

Calibration for the Baseline Model

The model is calibrated to the parameter values listed in Table 1. The rate of time preference and the Cobb-Douglas production function coefficients are standard. The values for indivisible labor are taken from Li (1999). The parameters in the shopping-time technology ($\nu = 0.75$ and $\zeta = 0.04875$) are taken from King and Wolman (1996). The autoregressive coefficient for the technology process, z, is set to 0.95. The standard deviation of the technology shocks is set at $\sigma_\rho = 0.0045$ to match the variance of output in the data when depreciation switches as follows. In the extreme case. the depreciation rate switches between 0.015 in the low state and 0.025 in the high state. These values imply an annual depreciation rate between 6 and 10 percent. The low state value is consistent with Stokey and Rebelo (1995) and the high state value with King and Rebelo (2000). Much of our analysis works with a quarterly depreciation rate set to 0.21, which is consistent with values used by Gilchrist and Williams (2000). The persistence for the two states is defined by $p_\delta = 0.90$ and $q_\delta = 0.85$.

4. Numerical Implementation

Two features of the dynamic general equilibrium described above make use of traditional solution techniques problematic. Since Markov switching is present in the model's parameters, there is no model steady state to serve as a center of approximation. Even in the absence of the shocks to technology or monetary policy, switches in other model parameters will prevent the economy from approaching a steady state value. In addition, the values of these switching parameters in different states are of crucial importance in determining the decisions of agents. Thus the switching cannot simply be "turned-off" to provide a deterministic steady state. As a secondary problem, since the agent does not have full information as to the state of monetary policy, the agent's decisions have to depend on his beliefs of the current stance of monetary policy. This necessitates the introduction of the agent's beliefs as state variables in the agent's optimization problem. While this alone doesn't necessarily preclude traditional methods, these states have very nonlinear transitions through time.

In light of these problems, we use a solution technique first discussed by Judd (1998); dubbed the projection method. The idea is to approximate the agent's decision rules by polynomials that "nearly" solve the agent's optimization problem in a way made formal below. Using polynomials allows us to represent the approximate rules in a very compact form. While the number of coefficients in a high-degree polynomial in many variables can be somewhat large, this number is nowhere near the number of values that must be kept track when using grid-based methods. For an example, a second-degree polynomial in three variables requires 10 coefficients for its specification. A grid-based approximation to a function in three variables using five grid points in each dimension would require specifying 5^3 or 125 parameters. Grid-based approximations to a function of a slightly larger number of variables - we use 8 variables below - are impossible for even sparse grids.

To apply the projection method, we first express the agent's optimization problem in the following form:

$$\underset{u_t}{Max}\, E_0 \left[\sum_{t=0}^{\infty} \beta^t r(x_t, u_t, D_t) \right]$$

subject to the constraints

$$x_{t+1} = g(x_t, u_t, D_t, \varepsilon_{t+1})$$

with x_0 a given. In this formulation, we assume that x_t is an $n \times 1$ vector of state variables, known by the agent at time t. u_t is an $m \times 1$ vector of the agent's decision variables. The $d \times 1$ vector D_t is a vector of variables referred to as economy-wide variables. These are variables that the agent assumes are unaffected by his decisions and which lack fixed transition equations. They will be determined by a set of equilibrium conditions. Finally, ε_{t+1} is an $e \times 1$ vector of random variables.

State variables can be further sub-divided into two groups although we don't want to complicate the notation by using different symbols for them. One group will consist of variables that are purely exogenous to the agent. Their transitions will depend only on factors outside of the agent's control, namely themselves and economy-wide variables. These variables could represent things like the technology level and the stance of monetary policy. We further assume that those state variables that are under the control of the agent do not have transitions dependent on their current values. For such a variable, x_t^i, controlled by the agent, x_{t+1}^i depends only on u_t, D_t, and possibly ε_{t+1}. These variables are things specific to the agent such as his savings and money holdings. Our assumption as to transitions for these variables are not really restrictive in that we have yet to come across a dynamic general equilibrium that couldn't be put in a form that satisfies this assumption. The primary reason for this restriction is to enable us to write the agent's first order conditions as described below without having to deal with a value function as well.

For what follows, we denote differentiation by a decision variable by a Greek subscript, ψ. Differentiation by a state variable will be denoted by a Roman subscript. With this convention then, and the assumption on transitions described above, we can express the first-order conditions of the agent's optimization problem as:

$$r_\psi(x_t, u_t, D_t) + \beta E_t \left[\sum_{i \in C} r_i(x_{t+1}, u_{t+1}, D_{t+1}) g_\psi^i(x_t, u_t, D_t, \varepsilon_{t+1}) \,| x_t \right] = 0$$

for $\psi = 1, \ldots, m$. The sum of products of derivatives of the return function and the transitions functions takes over those variables that are under the agent's direct control, hence the shorthand notation, $i \in C$. The equilibrium we seek can actually be characterized as a set of functions, $u_t \equiv u(x_t)$ and $D_t \equiv D(x_t)$, that determine the agent's decisions and the values of the economy-wide variables as functions of today's states. The agent's first order conditions can be looked at as m functional equations in these unknown functions. To complete the determination of these unknown functions, we need d more functional equations. These will be equilibrium conditions for the economy and can take a wide range of forms. The usual case is for them to take a simple form such as $e(x_t, u(x_t), D(x_t)) \equiv 0$.

In the concrete case we discuss above, we can express the vectors above as follows:

$$x_t = (z_t, b_{p,t}, b_{z,t}, \ln(R_t), \ln(P_{t-1}), \mu_{t-1}, \delta_t, k_t)$$

where μ_t is the money growth rate, defined as M_t/M_{t-1}. Note that the technology level, beliefs about the current state of the Markov switching parameters, prices, nominal interest rates, and money growth rates are variables out of the agent's control. We can define a decision vector as:

$$u_t = (k_{t+1}, m_{t+1}, l_t)$$

where m_{t+1} represents per capita money balances. Finally, the only variable left needed to determine the agent's decisions at time t is the money growth rate needed for the bank to hit its interest rate target, thus we can define D_t simply as μ_t. From these variables, we can calculate the agent's consumption and define $r(x_t, u_t, D_t)$ as the agent's utility $U(c_t, l_t)$. We have three first order conditions derived from the three decision variables. Since we must have per capita money balances m_{t+1} identically equal to 1 in equilibrium, however, we can interpret these three functional equations as implicitly determining k_{t+1}, l_t, and μ_t.

It is, of course, impossible to derive an analytic expression for these unknown functions, so we will approximate them with polynomials. These polynomials approximations will take the general form:

$$u(x_t) \approx \sum_{d_1+d_2+\cdots+d_n \leq D} c_{(d_1,d_2,\ldots,d_n)} \varphi_{(d_1,d_2,\ldots,d_n)}(x_t)$$

where D is the upper bound on the degree of the polynomial. The φ functions are polynomials that take the form:

$$\varphi_{(d_1,d_2,\ldots,d_n)}(x_t) = T^{d_1}(x_t^1)T^{d_2}(x_t^2)\cdots T^{d_n}(x_t^n)$$

where $T^{d_i}(x)$ is a polynomial in x with degree d_i. $T^{d_i}(x)$ could, technically, be simply x^{d_i}, but these polynomials are notorious for their extremely poor approximation properties. Consequently, we will use suitably scaled and translated Tchebychev polynomials for the $T^{d_i}(x)$ functions. These not only have excellent approximation properties, but are also easy to evaluate with the intrinsic functions that come with most standard software packages. Usually defined on the interval $[-1, 1]$ as $T^n(x) = \cos(n \arccos(x))$, they can be easily evaluated by any package that has an intrinsic cosine and arccosine function.

To derive these approximations, we must find conditions that we can use to determine their coefficients. Note that in general we will have $m + d$ functional equations that implicitly determine both $u(x_t)$ and $D(x_t)$. To vastly simply the discussion below, we'll refer to them as:

$$R_\psi(x_t, u(x_t), D(x_t)) = 0$$

for $\psi = 1, \ldots, (m + d)$. We need to choose coefficients for the polynomial approximations so that the approximations "nearly" solve the set of functional equations above. There are numerous ways to do this, many described in detail in Judd (1998). We feel the most natural way to choose these coefficients is to try and choose them so that:

$$\int R_\alpha(x, u(x), D(x))\varphi_{(d_1,\ldots,d_n)}(x)dx = 0$$

where these integrals are taken over some pre-determined region of space thought to capture most of the dynamic behavior of the economy and the $\varphi(x)$ functions have been translated to center on this region. This approach is appealing since it transparently gives one equation for each unknown coefficient in each functional approximation. It is also eminently reasonable in that, if for some reason, the equations $R_\alpha(x, u(x), D(x)) = 0$ were satisfied exactly by polynomial u and D, these conditions will determine their coefficients exactly. The reader can refer to Judd (1998) for a more in depth discussion of the selection of this particular set of equations, and a good explanation for his calling this a projection method. Furthermore, in practice there is no need to try to evaluate these integrals to any great degree of accuracy. As long as the R_α functions are zero in some average sense over a region, we should have reasonable approximations to the agent's decision rules over this region. Consequently rather that trying to evaluate these integrals to a high degree of precision, we simply use a pseudo-random Monte Carlo method to calculate them. Thus derivation of the coefficients in our polynomial approximations has been reduced to the solution of a set of nonlinear equations in these coefficients. While evaluation of the equations themselves can be slow, we have had no trouble solving them with a standard method (Broyden's method) for nonlinear equations.

5. Empirical Results

This section presents empirical results in the form of sensitivity analysis and impulse responses. The sensitivity analysis highlights several findings surrounding the influence of depreciation on the hours-productivity correlation. In particular, we emphasize that it is the time-varying nature of depreciation, not its level, that is important for generating the sought after correlation. Thereafter impulse responses are presented to explain the hours-productivity correlation.

Sensitivity Analysis

We begin our analysis by reviewing the main properties from the case where depreciation is held constant. Figure 1 plots the hours-productivity correlation for constant (quarterly) depreciation rates, δ, between 0.01 and 0.04. The correlation is 0.86 for δ ranging between 0.01 and 0.027. Extreme values of 0.037 or greater are able to generate negative correlations. A depreciation rate of 0.035 is in line with the empirical correlation; such a rate seems unrealistically high, however.

The low hours-productivity correlation arising from high depreciation rates is explained by two features. The first is the increasing use of labor over capital. The second, in terms of economic fluctuations, the variance of hours falls considerably faster than that of output. In other words, the variance of capital increases as depreciation increases. This implies that the fluctuations in productivity must increase, which leads to a lower hours-productivity correlation.

Next, we consider the influence of switching between states with a low and a high depreciation rate. The correlation results are given in Figure 2 for three different processes defined by $\delta + / - \mu$, where $\delta = (0.017, 0.02,$ and $0.022)$ and $\mu = (0, 0.0005, ..., 0.005)$. The transition matrix for the two state process is set to $p_1 = 0.9$ and $p_2 = 0.85$. The results find that relatively small deviations from the conditional mean between $+/-0.003$ and $+/-0.0035$ generate correlations that are consistent with the empirical data. It is important to note that this result stems from the switching and not from high depreciation rates.

Of further interest is to consider the influence of persistence in the states for δ on the correlation levels. Three cases are considered: high-high, low-low and high-low persistence. High-high persistence is defined as $p_1 = 0.9$ and $p_2 = 0.85$ in the transition matrix. High-low persistence takes on the values $p_1 = 0.9$ and $p_2 = 0.75$ and low-low persistence is defined as $p_1 = 0.75$ and $p_2 = 0.75$.

The results, summarized in Figure 3 for $\delta = 0.02$, find that if the degree of persistence is high in both states, then the depreciation rates do not need to deviate much from each other in order to match the empirical correlations. The opposite result is true when the transition matrix reflects low-low persistence. In this case, considerable switching between the high and low states occurs, but the distance between the mean depreciation rates in the low and high states have to lie further apart in order to obtain the desired hours-productivity correlation. The more realistic case of a highly persistent (low mean) state together with a less persistent (high mean) state lies in between the two extreme cases.

Table 2 presents correlations and standard deviations of our model based data with actual data for the period 1958:1 to 1999:4. The first column gives information of the model with only the technology shock. This model has difficulty matching the investment ratio and the hours-productivity correlation. The introduction of switching in the depreciation rate is unable to resolve the problem with the investment ratio, but offers a substantial improvement for the hours-productivity correlation. It falls from 0.86 to 0.22.

Impulse Responses

The precise meaning of impulse responses to a switch in a Markov process requires explanation. In particular, we have to define the 'shock' behind an impulse response. We run parallel simulations of the DSGE model to have 'switch' and 'no switch' scenarios. Both simulations share in common technology shocks and a depreciation rate that randomly follows the Markov switching process until 25 periods before the end of the sample of length T. At that point, the first simulated series puts depreciation into the low state for the next six periods; the second series puts depreciation into the low state for the next period, the high state for the next four periods, and then the low state again in the sixth period. A switch in the depreciation rate becomes known in the same period and the endogenous variables reflect this new imformation, such that the variables should respond in the impulse before the capital stock is affected. After the specified return to the low state in the sixth period, the two series again share a common set of realizations of the Markov switching process for depreciation. The four period duration of the high-depreciation state roughly reflects the half life of a spell in that state according to its transition probability, $q = 0.85$ ($0.85^4 = 0.51$). The difference between the paths of variables with and without the four period sojourn to the high-depreciation state serves as a measure of the impulse response to a switch to the high-depreciation state. The reported impulse response is the average response from 200 simulations of the model.

Figure 4 plots the impulse responses of output, capital, hours and productivity to a switch to the high-depreciation state that lasts four periods. In the low state the depreciation rate δ is set to 0.018 with transition probability 0.9, whereas in the high state $\delta = 0.022$ with transition probability 0.85.

The shock to capital generates the typical non-humped shaped dynamics common among RBC models (see Gilchrist and Williams, 2000). More important for us is the observation that depreciation switching tends to push the hours-productivity correlation downward or even make it negative, depending on the degree of switching. With enough variation in the depreciation rate, the negative effect on the hours-productivity correlation can overcome the tendency of technology shocks to induce a positive correlation.

The depreciation shock causes a negative level shift in output. Hours fall initially because the capital stock is relatively high at first in relation to the lower level of output. As the capital stock falls, however, labor hours must be substituted for capital to maintain steady output. When the depreciation rate switches back to the low state, output returns immediately to its initial level. To achieve this level of output, hours must jump above their initial level because the capital-output ratio is lower than it was initially. Gradually hours decline as the rebuilding of the capital stock allows for substitution of capital for labor. Throughout this process, the capital-output ratio moves in the opposite direction from hours. Thus, labor productivity moves in the opposite direction from hours. If depreciation switching is of sufficient magnitude and frequency, the negative correlation it imparts between hours and productivity can overcome the positive correlation implied by technology shocks, as shown in Figure 2.

6. Summary and Conclusions

In a dynamic stochastic general equilibrium (DSGE) model, we investigate whether a stochastic depreciation rate for capital can account for a low correlation between hours worked and labor productivity when depreciation is only allowed to fluctuate within a narrow range. In the somewhat uninteresting case, as we illustrate, an exceptionally high level of the depreciation rate, rather than its fluctuations, can reduce the hours/productivity correlation. Of greater interest is to is limit the range of fluctuations around an acceptable rate of depreciation. To do is, we use a two-state Markov process. Such stochastic depreciation switching can capture sudden shifts that occur, for example, when energy-intensive forms of capital are rendered obsolete from an energy shock.

Markov switching in capital depreciation makes the decision rules nonlinear. Our contribution to DSGE solution methodologies is to apply Judd's (1998) projection method to nonlinear decision rules. This approach allows for nonlinear decision rules in a richer set of models with many more state variables than can be solved with grid-based approximations.

Our calibration strategy for the time-varying depreciation rates relies on the empirical work of Abadir and Talmain (2001). They find that depreciation rates for the United States are highly persistent and fluctuate within a narrow band. These properties can be easily incorporated in a Markov-switching process.

With this setup, several results emerge from a simple RBC model augmented with Markov-switching depreciation. The level of depreciation is relatively unimportant for the hours-productivity correlations when depreciation is subject to Markov switching. With relatively small fluctuations in depreciation and switches to the high depreciation state that have a half-life of one year, the model replicates the low 0.24 hours-productivity correlation found in the data, whereas the model without depreciation switching delivers the standard result that the hours-productivity correlation exceeds 0.85. As the impulse response function shows, a depreciation shock induces a correlation between hours and labor productivity that is about minus one. An appropriate mix of stochastic shocks between technology and depreciation can result in the small positive correlation observed in the data. Such dramatic results suggest that Markov switching is a powerful extension to DSGE models, and the methods we introduce can allow for several Markov switching parameters simultaneously.

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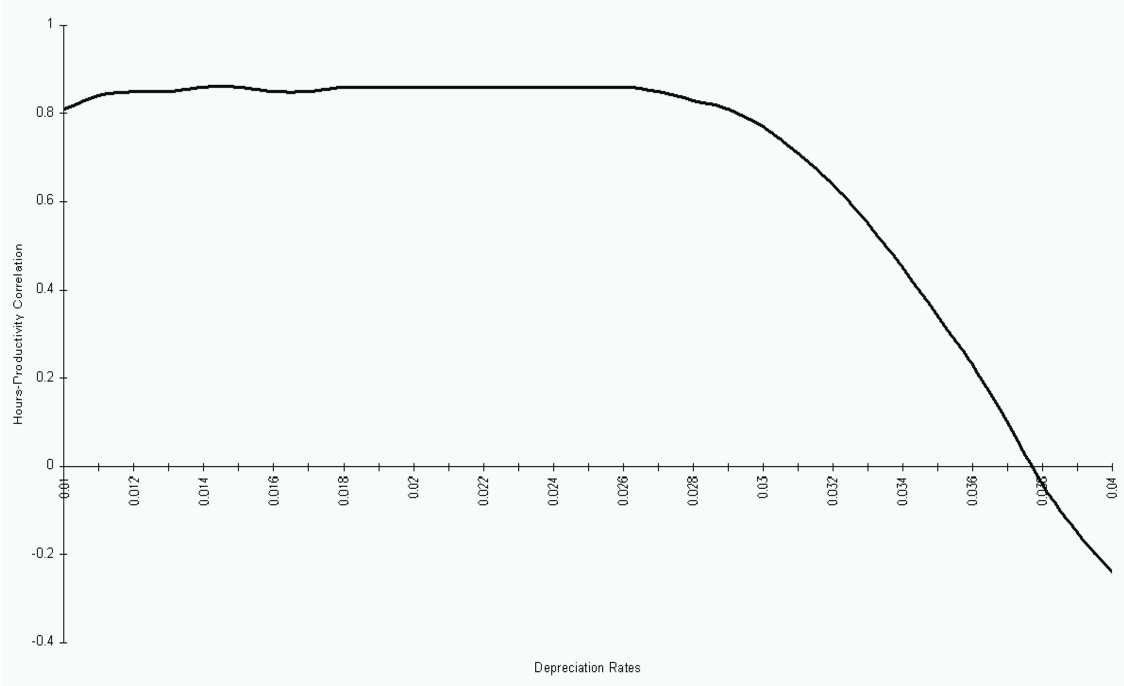
Figure 1: Hours-Productivity Correlation with Constant Depreciation Rates



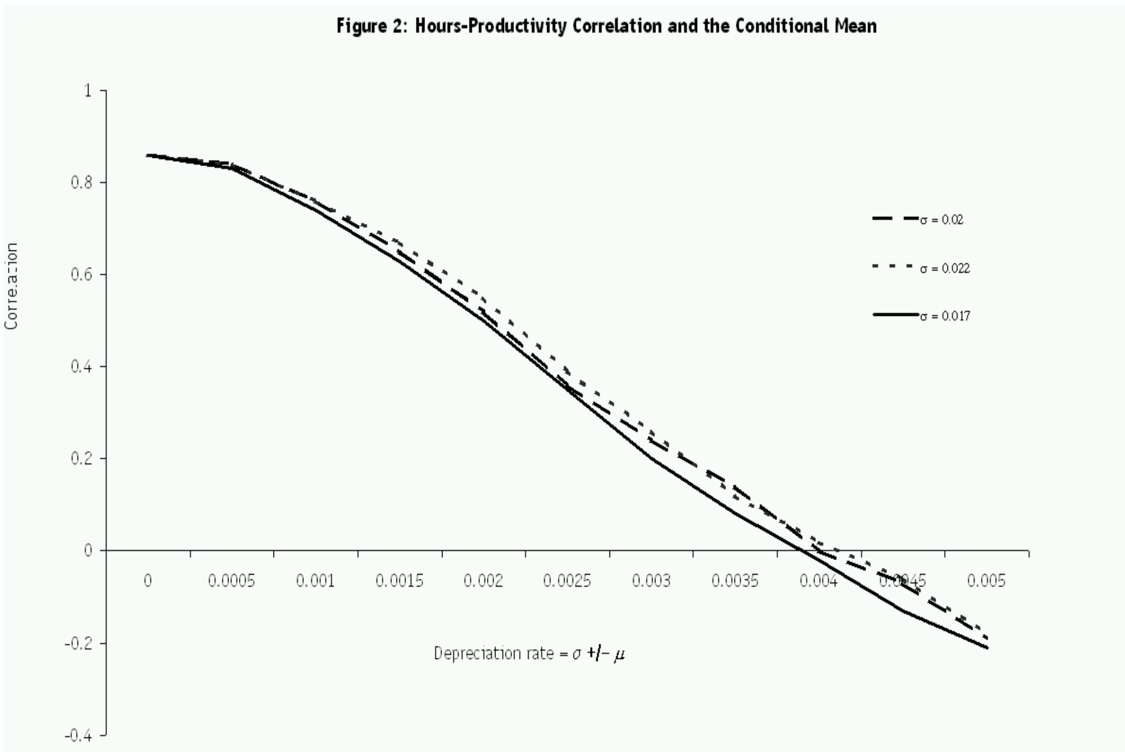


Figure 2: Hours-Productivity Correlation and the Conditional Mean
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
Correlation
0
0.0005
0.001
0.0015
0.002
0.0025
0.003
0.0035
0.004
0.0045
0.005
Depreciation rate = σ +/- μ
σ = 0.02
σ = 0.022
σ = 0.017

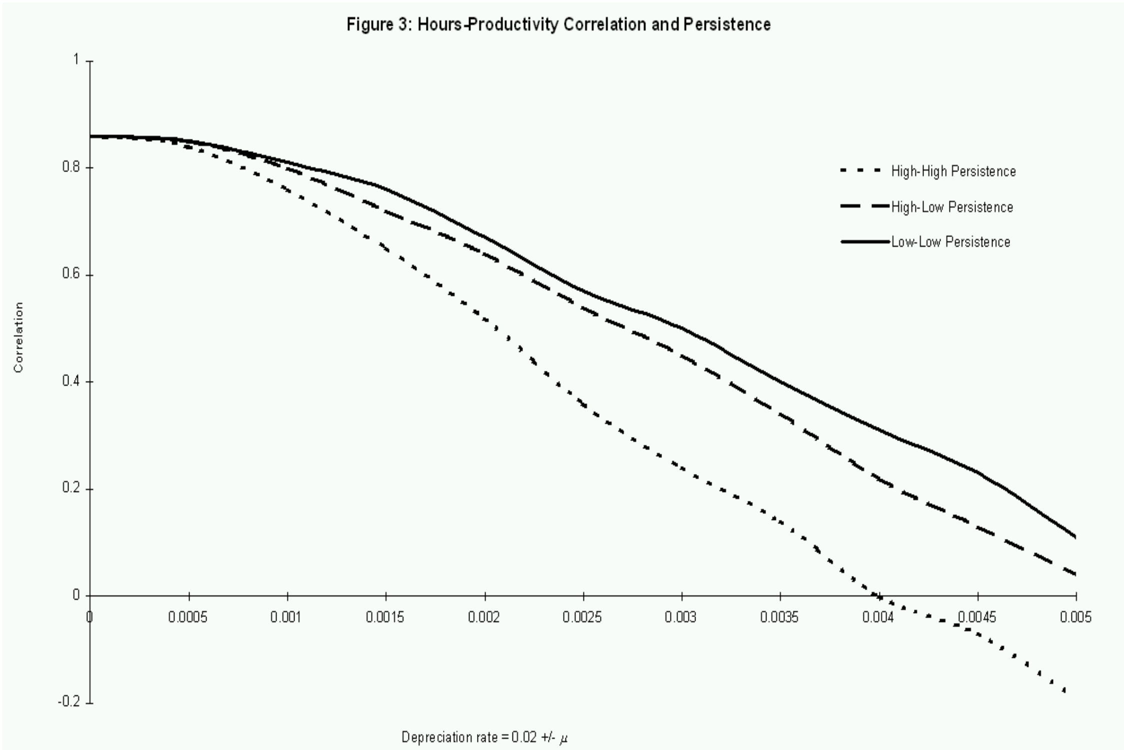


Figure 3: Hours-Productivity Correlation and Persistence
Correlation
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.0005
0.001
0.0015
0.002
0.0025
0.003
0.0035
0.004
0.0045
0.005
High-High Persistence
High-Low Persistence
Low-Low Persistence
Depreciation rate = 0.02 +/- μ

Figure 4: Impulse responses to a switch to the high-depreciation state that lasts four periods

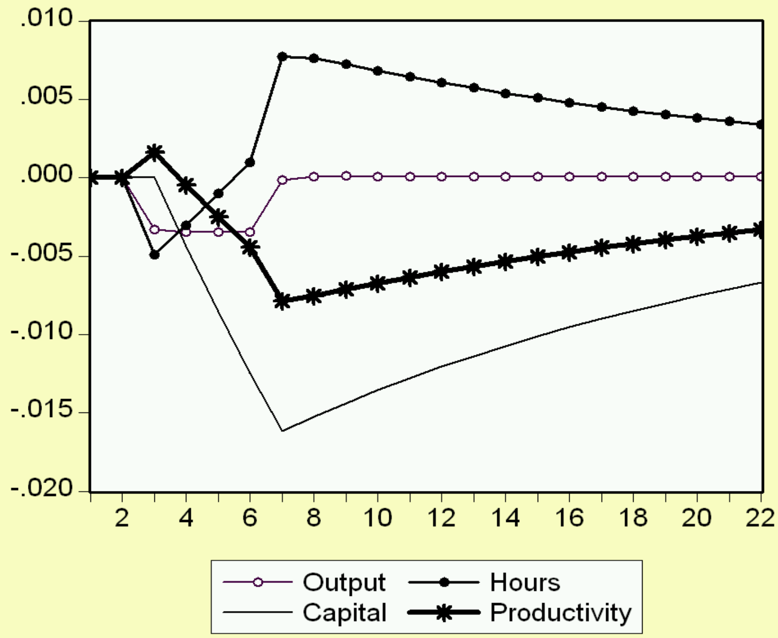


Table 1: Calibration of Markov Switching Economy

Rate of time preference	β	0.9917
Production function coefficient	α	0.33
Leisure coefficient	θ	2.9474
Labor productivity growth	λ	1.00373
Indivisible labor	$\hat{L}$	0.25
Shopping time tech.	ν	0.75
Shopping time tech.	ζ	0.04875
Depreciation rates	δ	0.015, 0.025
Depreciation transition probability	p_δ, q_δ	0.90 0.85
AR technology	ρ	0.95
std. deviation of technology error	σ_ρ	0.0045

Table 2: Business Cycle Statistics for the Switching Model

	Tech. shock only	with Time-Varying Dep.	Actual Data
σ_y	1.07	1.08	1.24
σ_c/σ_y	0.36	0.45	0.52
σ_i/σ_y	3.10	3.37	2.50
σ_n/σ_y	0.69	0.79	0.77
$\sigma_{y/n}/\sigma_y$	0.35	0.46	0.46
$\sigma_n/\sigma_{y/n}$	2.00	1.72	1.64
corr(c,y)	0.94	0.68	0.84
corr(i,y)	0.94	0.95	0.92
corr(n,y)	0.99	0.89	0.88
corr(y/n,y)	0.94	0.63	0.66
corr(y/n,n)	0.86	0.22	0.25