Deleveraging Via Asset Sales: Agency Costs, Taxes, and Government Policies

(Job Market Paper)

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Abstract

Do equityholders of a financially distressed firm have an incentive to sell assets and buy back debt to achieve a more sustainable leverage ratio and avoid costly bankruptcy? I develop a dynamic structural model incorporating a dynamic game to determine conditions under which a firm would voluntarily do so. It allows me to assess the impact of debt overhang and asset substitution on the restructuring condition and the holdout problem. I find that as long as the total firm value increases through the debt repurchase, equityholders benefit from it, as well. In a dynamic setting, the debt overhang problem takes the form of too early restructuring. Taxes on cancellation of debt income and government subsidies to debtholders can destroy equityholders' incentives; so does low liquidity in the market for the firm's assets; an asset purchase program fosters them. Finally, via threatening not to tender, debtholders can appropriate a large share of the firm's restructuring gains. However, they cannot stop equityholders from gambling for resurrection which in turn gives equityholders bargaining power to prevent debtholders from holding out.

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1 Introduction

With the recent financial crisis, the question how costly bankruptcy can be avoided, has gained a lot in importance. Any debate on regulatory measures requires first an answer to the question whether firms would *voluntarily* buy back debt in bad economic times to establish a more sustainable leverage ratio and avoid costly bankruptcy. A large part of the discussion on regulatory measures seems to build on the presumption that debt overhang prevents any proactive leverage management by firms altogether. Given that debt repurchases have already been a quite common phenomenon before the crisis, be it in the form of open market repurchases or tender offers, this conclusion is premature. For instance, Mann & Powers (2007) report that between 1997 and 2003 bonds with a total face value of at least 153 billion dollars were tendered. In 2004, the amount of debt repurchased exceeded even 60 billion dollars (Julio, 2007).

Indeed, equity financed debt buy backs most likely fail to increase the value of equity (see Gertner & Scharfstein, 1991; Leland, 1994; Admati et al., 2012). Restructuring will generally increase the total firm value but it is the debtholders that profit from the repurchase in the form of cash for the part of their debt bought back and a lower likelihood of default for the remainder, while equityholders see their initial holdings diluted or cash injections handed over to debtholders[2].

This paper argues that if debt repurchases are financed by asset sales, equityholders can benefit from the restructuring as well[3]. Moreover, whenever debt buybacks financed by asset sales increase the firm value, equityholders will have an incentive to restructure. The debt overhang problem still exists. In a dynamic setting it takes the form of too early restructuring which is the mirror image of the underinvestment problem first noted by Myers (1977).

The model provides a dynamic framework to analyze under which conditions firms have an incentive to delever after being hit by a series of negative economic shocks. I take into account different agency conflicts arising between equity and debtholders. First, debt overhang induces equityholders to make divesture decisions that do not maximize the firm value. Considering the debt overhang problem in isolation, equityholders sell assets and repurchase debt too early compared to what would maximize total firm value. Second, equityholders' limited liability provides an incentive to increase the risk of the firm in bad economic times and gamble for resurrection (Jensen & Meckling, 1976). The asset substitution problem turns out to have the opposite effect on

[2]If renegotiation between equityholders and debtholders is possible, debtholders could cede part of their gain to equityholders in order to incentivize them to carry out a firm value increasing restructuring, which is an application of the Coase Theorem (see Haugen & Senbet, 1978). For bank debt, renegotiation is more likely than for diffusely held public debt. I will address renegotiation in the robustness section 3.3.

[3]Kruse et al. (2009) find that debt tender offers financed by asset sales generate positive announcement returns while equity financed offers do not increase the equity value

the timing of restructuring and, even more surprising, has the potential to overcome the holdout problem present if debtholders have the possibility to coordinate. The theoretical model allows assessing whether different policy measures foster firms' incentives to reestablish a more sustainable leverage ratio or whether they replace equityholders' incentives by taxpayers' money. It also provides conditions for when liquidity issues in the market for the firm's capital goods (fire sales) are likely to prevent voluntary restructuring.

In the model, the capital structure can be dynamically adjusted downward. The firm can sell parts of its productive assets and uses the divesture proceeds to finance a bond buy back; covenants ban the distribution of the proceeds as a special dividends to equityholders. The firm's management, which works in the interest of current equityholders, will endogenously determine when to restructure, default, and at which risk to operate the productive assets. I will look at both methods to buy back debt: tender offers and open market repurchases. When equityholders announce a tender offer, debtholders decide on whether to tender or hold out. When equityholders repurchase debt on the open market, bond prices will reflect debtholders' rational expectations on the secondary market. The interaction between equityholders and debtholders takes place at issuance, restructuring, and default and gives rise to a nonzero-sum stochastic game in continuous time with optimal stopping and instantaneous volatility control. As in Hennessy & Tserlukevich (2008), the solution concept is Markov Perfect Equilibrium.

First, I will confine the analysis to a situation in which debt buybacks are not affected by asset substitution or debtholders collectively holding out. When deciding on whether to sell assets and repurchase debt or not, equityholders trade off the future cashflows foregone because of the capacity reduction against the interest payment decrease and later default both due to the reduction in the notional amount of debt outstanding. If the divesture can be sold at the price of an all equity-financed firm and no taxes were due on the cancellation of debt income, equityholders would always repurchase debt. The reason is that with nonzero bankruptcy costs, the fraction of debt to be repurchased always exceeds the fraction of capacity sacrificed to finance the repurchase. This improves the debt service ratio (cashflow to coupon payments) and makes restructuring profitable for equityholders. Debtholders' gain would be maximized if the buyback took place just prior to default because this allows to reduce the coupon and, as a consequence, defer bankruptcy the most. However, equityholders have an incentive to restructure earlier. For them, repurchasing debt by selling assets is like saving fixed costs which they would like to reduce well in advance before default, even if repurchasing debt earlier means that the coupon reduction is lower.

The benchmark calibration shows that the lion's share of the gains from restructuring goes to debtholders. However, as the equity value is low when the firm considers restructuring, the percentage gains to equityholders are large.

The tax payable on cancellation of debt income can have a large impact on equityholders' incentive to repurchase debt. Firms that would gain little from restructuring may find taxes prohibitively high. Lowering those taxes can be a relatively cheap way for the government to avoid the deadweight losses of bankruptcy. The American Recovery and Reinvestment Act of 2009 did exactly that and has led to a large increase in the amount of debt repurchased (see Levy & Shalev, 2011).

Government subsidies, on the other hand, are an inefficient way to avoid bankruptcy costs. They destroy equityholders' incentives to reduce those welfare losses. In net, government subsidies are costlier than private intervention in this setting.

In order for asset sales to be profitable, a potential liquidity discount in the market for capital goods has to be smaller than bankruptcy costs. The model also shows that fire sales can prevent restructuring in an industry if the firms in the industry are very much alike. When firms differ in their leverage or their exposure to the industry wide shock, voluntary deleveraging financed by asset sales is likely to take place. An asset repurchase program would be an effective government intervention in a situation where firms' incentives to deleverage are lost.

Open market repurchases are very similar to tender offers. Debtholders rationally anticipate that equityholders will buy back debt. At the time of restructuring, debtholders get paid the prevailing market price for a fraction of their holdings. The remaining fraction becomes a claim against the restructured firm. No arbitrage requires that the joint value of both fractions is equal to the market value itself. This can only be the case if the debt value per unit of principal is the same just before and after restructuring which is the same condition that makes debtholders submit their bonds in a tender offer. The only difference is the size of the tax payment. While under the tender offer the whole issue is retired, open market repurchases retire only a fraction of the outstanding principal and, therefore, have a lower taxable income from the cancellation of indebtedness. Thus, in a setting excluding informational aysmmetries or market microstructure issues in the bond market, open market repuchases are the preferred method to buy back debt, as long as the issuer does not require the bondholder's consent to alter material terms of the indenture.

Risk shifting has two opposing effects. First, it decreases the debt reduction potential. The perspective to become a less risky firm if the economic situation improves, provides incentives for debtholders to embrace a coupon reduction when the distance to default shrinks. This offsets the bond price fall when default becomes more likely. As a consequence, letting the firm get closer to default does not accomplish such a large coupon reduction concession as in the case of no risk shifting. Second, higher risk, in general, induces equityholders to hang on to the firm for longer because of the convex nature of their claim. It also induces them to hold on longer to the larger production capacity and restructure later. This patience is rewarded by a larger leverage decrease as more debt can be repurchased leading to a larger reduction in coupon payments. Although the firm's maximal debt reduction potential is lower if the firm can shift its risk, the management will buy back larger amounts of debt because it restructures later.

The limit results are unaffected by the risk shifting problem. Without frictions, firms will always restructure. Without taxes, the same lower bounds for liquidity in the market for capital goods apply. However, taxes are more likely to impede debt repurchases than in the no risk shifting case because of lower debt reduction potential.

If debtholders find a way to coordinate, they can greatly increase their gains from restructuring by turning down the first restructuring offer and holding out until equityholders are just willing to restructure. If the firm cannot shift its risk, then debtholders will always benefit from a holdout. As those gains are large, equityholders have to offer large sums to make them accept the repurchase offer earlier. Since equityholders gains in absolute terms are small they will never afford to pay debtholders their reservation values unless coordination among them is very costly and decreases this value. However, if firms can shift risk, debtholders prefer to restructure earlier than later and will accept the first offer.

1.1 Literature Overview

One of the earlier papers dealing with debt restructuring is Gertner & Scharfstein (1991). They find that equityholders profit from debt tender offers financed by senior debt or cash. Mao & Tserlukevich (2012) look at cash-financed debt repurchases and how they are influenced by taxes and bankruptcy costs. In their model, debtholders do not anticipate the repurchase but once the firm buys its bonds on the market, they understand that they are better off selling. The authors also look at the underinvestment problem and find that a firm should rather use its cash to carry out the investment than to repurchase debt. Julio (2007) looks at the underinvestment problem in particular and finds that repurchasing debt prior to investment can attenuate the problem and also provides empirical evidence for it. Admati et al. (2012) are also interested in whether equityholders have an incentive to restructure and avoid costly bankrutpcy. Using a one period model, they find that if the firm's assets traded at the post restructuring firm value, equityholders would neither expand nor shrink the balance sheet. This result is similar to my finding in section 2.3. However, the dynamic setting of my model reveals that the debt overhang problem is not only limited to whether a firm would buy back debt at all but also affects the timing of the restructuring which, in turn, is influenced by the nature of agency costs involved.

The paper is also related to the debt renegotiation literature. In Mella-Barral & Perraudin (1997), equityholders renegotiate the debt contract ex-post. By threatening to default, they can make debtholders agree to a reduction of coupon payments. Bankruptcy costs are saved entirely but equityholders' ability to engage in ex-post hold-up reduces debt capacity ex-ante. A similar result is derived in Hart & Moore (1994) where equityholders' threat to withdraw value increasing personal skills takes over the role of bankruptcy costs. Renegotiation with all debtholders is very likely to be infeasible with a large number of dispersed bondholders but might be possible with a single or few lenders, such as a bank (see Hackbarth et al., 2007, for a model incorporating both negotiable bank loans and non-negotiable public debt).

While in the previously mentioned models debtholders make concessions in the form of coupon reductions, the model in Christensen et al. (2002) analyzes private workouts that comprise both principal and coupon reductions. Instead of letting the firm go into default, the existing debt is retired and the firm is recapitalized. The firm value increases because bankruptcy costs are saved and the optimal leverage ratio is reestablished. By letting equityholders take a share in the recapitalized firm, debtholders get equityholders agree to the workout. Again, this type of restructuring would require debt to be concentrated. My model, on the other hand, captures restructuring when direct renegotiation between a firm and its debtholders is not feasible.

Bhanot & Mello (2006) consider debt restructurings that are ex-ante specified in bond covenants. They analyze whether debt contracts that prescribe a pre-determined payment to debtholders in case of a downgrade can mitigate the asset substitution problem. The firm commits ex-ante to either inject equity or sell assets in order to retire debt via paying back part of the principal, while my model considers ex-post restructuring at market prices.

2 The Model

A firm operating with capacity K generates an operating income flow of size $Q(K)X_t$, with X_t being a stochastic state variable following a geometric Brownian motion[4]:

$$dX_t = \alpha X_t dt + \sigma X_t dW_t^P \quad (1)$$

W_t^P is a systematic risk factor that commands a market price of risk λ which is assumed to be constant[5]. Under the pricing measure $\mathcal{Q}$, the risk-neutral drift of the stochastic demand factor changes to $\mu = \alpha - \lambda\sigma$ and its $\mathcal{Q}$-dynamics are given by

$$dX_t = \mu X_t dt + \sigma X_t dW_t \quad (2)$$

where W_t is a Brownian motion under the pricing measure $\mathcal{Q}$. One can think of the firm either being a monopolist, then $Q(K)X_t$ would be equal to the inverse demand function times the quantity produced by the firm with capacity K. In this case, X_t represents a stochastic demand shock. Or, the firm can be interpreted as a price taker in a competitive market. Then, $Q(K)$ would represent the production function and X_t the exogenous price of the firm's output.

The firm starts out with an existing capacity K^L which generates operating income of size $Q(K^L)X_t$. For simplicity, I assume that there are no marginal costs of production which makes firm produce at full capacity. As a consequence, the production function for a given capacity is constant ($Q(K) = Q$).

Similar to Carlson et al. (2004), I assume that the firm can switch to different sizes of capacity. In particular, the firm has an option to sell part of its production facilities and downsize its capacity[6] from the *larger* capital stock K^L to the *smaller* $K^S < K^L$. As the firm produces at full capacity, this means that the divesture decreases its operating income from $Q^L X_t$ to smaller $Q^S X_t$. Covenants ban the distribution of asset sales proceeds to equityholders but the money can be used to buy back debt and, thereby, reduce coupon payments, of which the fixed cost character drags down the firm in times of low cash-flows. I employ a similar functional form for the value of the divesture as Lambrecht & Myers (2008) and assume that it depends on the size of the capacity reduction $(Q^L - Q^S)$, which is fixed, the current economic situation, X_t, and fixed costs $\phi_0 < 0$. A parameter $\phi_1 \geq 0$ is used to capture liquidity effects in the market for for capital goods. I allow $\phi > 1$ which

[4]In section 3 I will relax the assumption of a constant volatility and give the management control over how risky it operates the production facilities

[5]A constant market price of risk would be supported, for example, by a CIR economy populated by agents with log utility **?**

[6]A richer model would give the firm free choice over the size of the divesture. Dixit & Pindyck (1998) show that if a firm could increase its capacity in an unlimited way but with decreasing returns to scale then it behaves as if it only considered the next marginal expansion option.

could happen, e.g., if a completely equity financed firm buys the capital goods and levers them up optimally increasing the value of the firm above the present value of the cashflow stream because of the interest tax shield. The price realized will then depend on the bargaining power of the seller and the buyer.

$$P^K(X_t) = \frac{(1-T_C)(Q^L - Q^S)}{r-\mu} X_t \phi_1 + \phi_0 \tag{3}$$

The firm's existing capital structure consists of a single consol bond with coupon C_L. Corporate income is taxed at rate T_C, which yields an after tax income of $(1-T_C)(Q^L X_t - C^L)$. In the absence of stipulations in bond covenants, bankruptcy is declared by equityholders who default upon coupon payments, invoke limited liability, and hand over the firm as a going concern to debtholers. A fraction α of the firm value at bankruptcy is lost due to direct and indirect costs associated with the bankruptcy procedure itself but also with opportunity costs incurred when being financial distressed.

2.1 Tender Offers

Focusing on the most common method, I assume that equityholder offer to pay a fixed, non-negotiable tender offer consideration to all debtholders tendering their bonds. As equityholders face a barrier control problem, the time point when equityholders optimally submit the tender offer will be given by the state variable X_t reaching an optimal threshold. The restructuring threshold will be denoted by X_R. I assume that the firm wants to retire the whole issue[7]. As the asset sale will not suffice to cover the costs of the tender offer, which consist of the tender offer price and the taxes due on the cancellation of indebtedness income, the firm has to issue new debt, denoted by $d(X_R, C^S)$. Let $D^T(X_R)$ be the price offered to debtholders if restructuring takes place at X_R. Then the restructuring condition, which ensures that equityholders don't have to contribute to the restructuring, is given by

$$P^K(X_R) + d(X_R, C^S) = D^T(X_R) + T_{COD}\left(D(X_R, C^L) - D^T(X_R)\right) \tag{4}$$

The tax rate applying to cancellation of debt income (CODI) is denoted by T_{COD} and applies to the repayment value forgiven $D(X_0, C^L) - D^T(X_R)$. The principal is given by $D(X_0, C^L)$ which is the debt value at the issuance date if debt is issued at par[8]. In general, the tax rate on CODI is the

[7] Mann & Powers (2007) find that the median size of the fraction of the issue retired is 94.6%

[8] The income arising from cancellation of debt (CODI = cancelation of debt income) normally corresponds to the repayment value forgiven. Interest payments do not qualify if they would have been tax deductible. With a consol bond, the firm has never to repay the notional amount and the coupon payments do not qualify since they are deductible. I follow Goldstein et al. (2001) and assume that the notional amount is given by the initial value which corresponds to the money raised with the issuance.

same as on corporate income. At the moment, the US government has allowed to defer the COD tax payments for up to 11 years. To allow for active governmet tax policy, I treat T_{COD} as a distinct parameter, not necessarily equal to the corporate tax rate T_C. The tax on CODI reduces the gain that can be made from the debt repurchase and might even make it unprofitable as I will show in 2.3.

The value of the tender offer price, $D^T(X_R)$, will be given by bargaining problem setup. The tender offer price must be such that in induces all debtholders to tender. Suppose a debtholder holds a fraction ϵ of the existing debt. This entitles her to a coupon payment of ϵC^L as long as the firm has not restructured. If she does not tender her bond, the new outstanding coupon will be $\epsilon C^L + C^S$ and $d(X_R, \epsilon C^L + C^S)$ will be the new total debt value. Her position will be worth $\frac{\epsilon C^L}{\epsilon C^L + C^S} d(X_R, \epsilon C^L + C^S)$. She will agree to the tender if

$$\epsilon D^T \geq \frac{\epsilon C^L}{\epsilon C^L + C^S} d(X_R, \epsilon C^L + C^S)$$

To see whether the marginal investor agrees I let ϵ go to zero:

$$\begin{aligned} D^T &\geq \lim_{\epsilon \downarrow 0} \frac{C^L}{\epsilon C^L + C^S} d(X_R, \epsilon C^L + C^S) \\ D^T &\geq C^L \frac{d(X_R, C^S)}{C^S} \end{aligned} \tag{5}$$

If condition (5) holds, then every individual debtholder has the incentive to tender his position. The condition has a very straightforward interpretation. It requires that every bondholder must be offered a price as high as the value that his position would have if he held out and did not tender. $\frac{d(X_R, C^S)}{C^S}$ is the price of the new debt per unit of coupon. The tender offer price per unit of coupon has to be at least as large as the price per unit of coupon debt attains after the repurchase has taken place.

Since equityholders would lose if they offered more than $\frac{C^L}{C^S} d(X_R, C^S)$, this condition will hold with equality in equilibrium. Thus, the tender offer price will be given by

$$D^T(X_R) = \frac{C^L}{C^S} d(X_R, C^S) \tag{6}$$

It also is individually rational for every single bondholder to tender given that no one has tendered yet, if the firm offered D^T to him, because the alternative, not to tender, results in $D^0(X_R, C^L) < D^T$. I will show that if equityholders gain is positive then bondholders gain is positive which yields a sufficient condition for the individual rationality constraint.

The restructuring condition becomes

$$P^K(X_R) + d(X_R, C^S) = \frac{C^L}{C^S} d(X_R, C^S) + T_{COD} \left(D(X_0, C^L) - \frac{C^L}{C^S} d(X_R, C^S) \right) \tag{7}$$

This condition yields the coupon of the new debt $d(X_R, C^S)$ that has to be issued to cover the costs of the tender which consist of paying the tender offer price and the taxes due on cancellation of indebtedness income.

Equityholders have the option to sell part of the firm's production facilities and use the proceeds to reduce leverage. Since they do not care about the total firm value but only about the equity value, they will only do this if deleveraging improves their situation as well. Therefore, they should not have to inject any additional money into the firm for restructuring. All costs associated with the restructuring will be paid from the proceeds of the asset sale. Such costs reduce the amount of money available for buying back debt. Debtholders will anticipate that the equityholders are going to buy back debt at a certain time and will adjust their valuation of debt accordingly. In equilibrium, equityholders strategy will take this into account. I define an equilibrium with tender offers in the following way:

Definition An equilibrium with tender offers is defined as the vector of strategies (X_R, X_D), a vector of prices $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$ and a tender offer price D^T satisfying:

1. X_R is the optimal restructuring threshold given prices are formed by $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$.
2. X_D is the optimal default threshold given the restructuring strategy and prices $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$.
3. Equityholders set the tender offer price D^T optimally such that it induces all debtholders to tender.
4. $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$ are formed rationally, i.e. bond and equityholders rationally anticipate that the firm will restructure at X_R and default at X_D.

To solve the model, I first derive the the value of the firm's securities after it has sold part of its assets and reduced debt. After the divesture the firm operates at the same capacity Q^S and with the same capital structure, C^S, until it defaults. At this stage the firm is called the small firm. Parameters which are specific to this stage will have an S as superscript and its securities will be denoted by small letters. The problem is standard (e.g. see Leland, 1994) and its results are summarized in the following proposition

Proposition 2.1 *The equity value of a firm that generates an EBIT of $Q^S X$ and has a consol bond with coupon C^S outstanding is equal to*

$$e(X) = (1 - T_C)\left(\frac{Q^S X}{r - \mu} - \frac{C^S}{r}\right) - (1 - T_C)\left(\frac{Q^S X_D}{r - \mu} - \frac{C^S}{r}\right)\left(\frac{X}{X_D}\right)^{-\theta} \tag{8}$$

Its debt value is given by

$$d(X) = \frac{C^S}{r} - \left(\frac{C^S}{r} - (1-\alpha)\frac{(1-T_C)Q^S X_D}{r-\mu}\right)\left(\frac{X}{X_D}\right)^{-\theta} \tag{9}$$

where θ is given by

$$\theta = \frac{(r-\mu-\frac{\sigma^2}{2}) + \sqrt{(r-\mu-\frac{\sigma^2}{2})^2 + 2r\sigma^2}}{\sigma^2} \tag{10}$$

The equity value before restructuring (i.e. as long as $X \in (X_R, \infty)$) solves the following Hamilton-Jacoby-Bellman equation:

$$\frac{\sigma^2}{2}X^2E''(X) + \mu X E'(X) + (1-T_C)(Q^L X - C^L) = rE(X) \quad \forall X \in [X_R, \infty) \tag{11}$$

The general solution to this ODE is given by

$$E(X) = (1-T_C)\left(\frac{Q^L X}{r-\mu} - \frac{C^L}{r}\right) + A^{-\theta} \tag{12}$$

The constant A and the optimal restructuring threshold, which still need to be determined, are found from a value matching condition that fixes the equity value at the restructuring boundary and from a smooth pasting condition that serves as an optimality condition.

At the time of restructuring, the firm sells part of its assets and reduces its capacity to Q^S. The divesture allows it to reduce its coupon to $C^S < C^L$. Absence of arbitrage opportunities requires that the equity value function is a continuous function at the restructuring boundary, giving rise to a value matching condition (13). When deciding on the optimal restructuring threshold, the firm trades off the loss from the capacity reduction against the saving of coupon payments which, acting like a fixed cost, drag down the firm in bad economic times. The optimal threshold, X_R is given by the smooth pasting condition (14).

$$\lim_{X \downarrow X_R} E(X) = \lim_{X \downarrow X_R} e(X) \tag{13}$$

$$\lim_{X \downarrow X_R} E'(X) = \lim_{X \downarrow X_R} e'_X(X) + e_C(X)\frac{dC^S}{dX} \tag{14}$$

It is important to note that C^S is itself a function of X^R implicitly determined from the restructuring condition (7). When selecting the optimal repurchase threshold, the firm also takes into account the impact of the restructuring time on its future capital structure via the restructuring condition. This explains, why the smooth pasting condition also involves the partial derivative of $e(X)$ with respect to the coupon C^S.

Thus, the equity value will be given by

$$
\begin{aligned}
E(X) = (1-T_C)&\left[\left(\frac{Q^L X}{r-\mu}-\frac{C^L}{r}\right)-\left(\frac{(Q^L-Q^S)X_R}{r-\mu}-\frac{C^L-C^S}{r}\right)\left(\frac{X}{X_R}\right)^{-\theta}\right] \\
-(1-T_C)&\left[\left(\frac{Q^S X_D}{r-\mu}-\frac{C^S}{r}\right)\left(\frac{X}{X_D}\right)^{-\theta}\right]
\end{aligned}
\tag{15}
$$

The first term in equation (15) is the present value of the income after taxes stream to equity holders. The second term is the value of the restructuring option. The firm gives up capacity but can reduce its interest costs which, acting as a fixed costs, drags down the firm in bad economic times. The third term is the value of the default option.

In order to find the C^S that can be obtained with the asset sale at X^R, I need to derive the debt value of the large firm first. Debt holders are assumed to be rational. In particular, they understand that if the state variable reaches X_R they will be offered a consideration of $D_T(X_R)$ for their bonds tendered. This gives rise to the following value matching condition for debt:

$$\lim_{X\downarrow X_R} D(X,C^L) = \lim_{X\downarrow X_R} D_T(X) \tag{16}$$

$$\lim_{X\downarrow X_R} D(X,C^L) = \lim_{X\downarrow X_R} C^L\frac{d(X_R,C^S(X))}{C^S(X)} \tag{17}$$

where I plugged in the incentive compatible tender offer price to arrive at the second equation.

$$D(X_R,C^L,C^S) = d(X_R,C^S)+P^K(X_R)-T_{COD}\left[D(X_0,C^L,C^S)-D(X_R,C^L,C^S)\right] \tag{18}$$

At the restructuring point, debtholders receive a payment of size $P^K(X_R)$ minus the tax payment due plus new debt with coupon C^S in exchange for their old debt.

From the second equation (16), the debt value of the firm before restructuring is given by

$$D(X,C^L) = \frac{C^L}{r}+\frac{\left(P^K(X_R)-\frac{C^L-C^S}{r}\right)\left(\frac{X}{X_R}\right)^{-\theta}-\left(\frac{C^S}{r}-(1-\alpha)\frac{(1-T_C)Q^S X_D}{r-\mu}\right)\left(\frac{X}{X_D}\right)^{-\theta}}{1-\frac{C^L-C^S}{C^L}T_{COD}\left(1-\left(\frac{X_R}{X_D}\right)^{-\theta}\right)} \tag{19}$$

The first term in the numerator comes from the restructuring option of equityholders. The second term is the value lost in default. The denominator contains the adjustment for the tax payment. If no tax where due, the denominator is one.

That this debt value can be transformed into a much simpler expression can be seen from (17).

$$
\begin{aligned}
D(X) &= \frac{C^L}{r}+\frac{C^L}{C^S}\left((1-\alpha)\frac{(1-T_C)Q^S X_D}{\mu-r}-\frac{C^S}{r}\right)\left(\frac{X}{X_D}\right)^{-\theta} \\
&= \frac{C^L}{C^S}d(X,C^S)
\end{aligned}
\tag{20}
$$

Similar to the equity value, the debt value before restructuring is a function of the coupon payment after restructuring which is implicitly given by the restructuring condition. The debt value before restructuring is just the scaled up value of the debt after restructuring. This is not true anymore in the more general model with asset substitution because at any X_t the large firm could, in general, be run at a different risk than the downsized firm which it is to become.

2.2 Without Frictions, a Firm Would Always Delever

Proposition 2.2 *If restructuring were costless and markets correctly priced the firm's assets then delevering via asset sales is always strictly preferred by equityholders as long as $\alpha \in (0, 1]$.*

Proof See appendix A.1

Proposition 2.3 *If restructuring just prior to default does not improve the debt service ratio, then restructuring is never profitable to equityholders.*

Proof See appendix A.2.

Note, that the improvement of the debt service ratio does not indicate how large the gain to equityholders or debtholders is but just whether restructuring is profitable at all.

The proof of proposition (2.2), reveals why equityholders always have an incentive to repurchase debt under the above mentioned conditions. As long as bankruptcy costs are larger than zero, the fraction of debt to be repurchased is always larger than the fraction of capacity that has to be sold to finance the repurchase. The proof shows that this is always true, under the mentioned conditions, just before the firm declares bankruptcy. The threat of incurring losses in default depresses the price of debt the more, the closer the firm is to bankruptcy. Thus, there always exists a restructuring threshold X_R where equityholders are better off with downsizing than with default!

The proof gives rise to the debt-service ratio as an indicator of whether it is profitable to restructure or not. And proposition 2.3 shows that restructuring is profitable for equityholders, as long as it can improve the debt-service ratio at least at the latest possible restructuring point, namely the default threshold, i.e. $Q^S/C^S(X_D) > Q^L/C^L$. With the increase of the debt-service ratio, equityholders will see their instantaneous dividends ($Q^S X - C^S$) rise. But the advanced ratio will also induce them to default at a later time point ($X_D^0 < X_D$) which immediately raises the distance to default and the value of debt. For debtholders' gain to be positive, a similar (the same) condition has to hold, which implies that it also applies to the firm as a whole.

The farther the firm is away from bankruptcy the larger is the expected value of the coupon payments and the lower the impact of expected bankruptcy costs. With the decreasing impact of bankruptcy costs shrinks the amount of debt that can be repurchased. The amount of debt that can be repurchased for a given asset sale size is not the only

Although the amount of debt that can be repurchased decreases if equityholders restructured earlier, equityholders might prefer a lower debt reduction carried out earlier. A closer look at the gains to equityholders, made in equation (21) reveals the tradeoff. Equityholders trade off losses from capacity reduction with gains coming from two interlinked sources: fixed costs in the form of coupon payments can be reduced and default postponed because of lower coupon payments. Waiting longer with restructuring defers default but at the cost of fixed cost savings. The extreme case being $X_R = X_D$ used in the proof of prop.

Equityholders lose if they wait too long because they carry the burden of a higher coupon for longer. On the other side, if they restructured too early, they would give up production capacity too early and pay too much for the debt reduction. That is the trade-off they face. Note that the optimal restructuring threshold does not maximize the gain to equityholders but the equity value which includes the present value of the gain and, thus, depends on the expected restructuring time. Due to the time preference of equityholders, they will restructure earlier, i.e. the optimal X_R is higher than threshold that would maximize the gain to equityholders.

$$\begin{aligned} G_E^R &= e(X_R, C^S) - E^0(X_R, C^L) \\ &= (1 - T_C)\left[\frac{C^L}{r}\left(1 - \frac{1}{1+\theta}\left(\frac{X_R}{X_D^0}\right)^{-\theta}\right) - \frac{C^S}{r}\left(1 - \frac{1}{1+\theta}\left(\frac{X_R}{X_D}\right)^{-\theta}\right)\right] \\ &\quad - (1 - T_C)\frac{Q^L - Q^S}{r - \mu} X_R \end{aligned} \tag{21}$$

Debtholders take the lion's share of the total gain in the form of lower expected bankruptcy costs. As they have to be compensated for the lower coupon payments in order to be willing to sell their bond holdings (6), they will always gain from restructuring, if it takes place. Note that the restructuring condition implies that the loss given default is independent of whether the firm restructures or not. Only the probability of default changes if debt is repurchased. As figure 1 shows, debtholders profit most if restructuring takes place just prior to default. At this point the coupon can be reduced the most and default happens at the latest possible time point.

$$\begin{aligned} G_D^R =& D(X_R, C^S) - D^0(X_R, C^L) \\ =& \frac{C^L}{C^S} d(X_R, C^S) - D^0(X_R, C^L) \\ =& \frac{C^L}{r} \left((1-\alpha)(1-T_C) \frac{\theta}{1+\theta} - 1 \right) \left[\left(\frac{X_R}{X_D} \right)^{-\theta} - \left(\frac{X_R}{X_D^0} \right)^{-\theta} \right] \end{aligned} \tag{22}$$

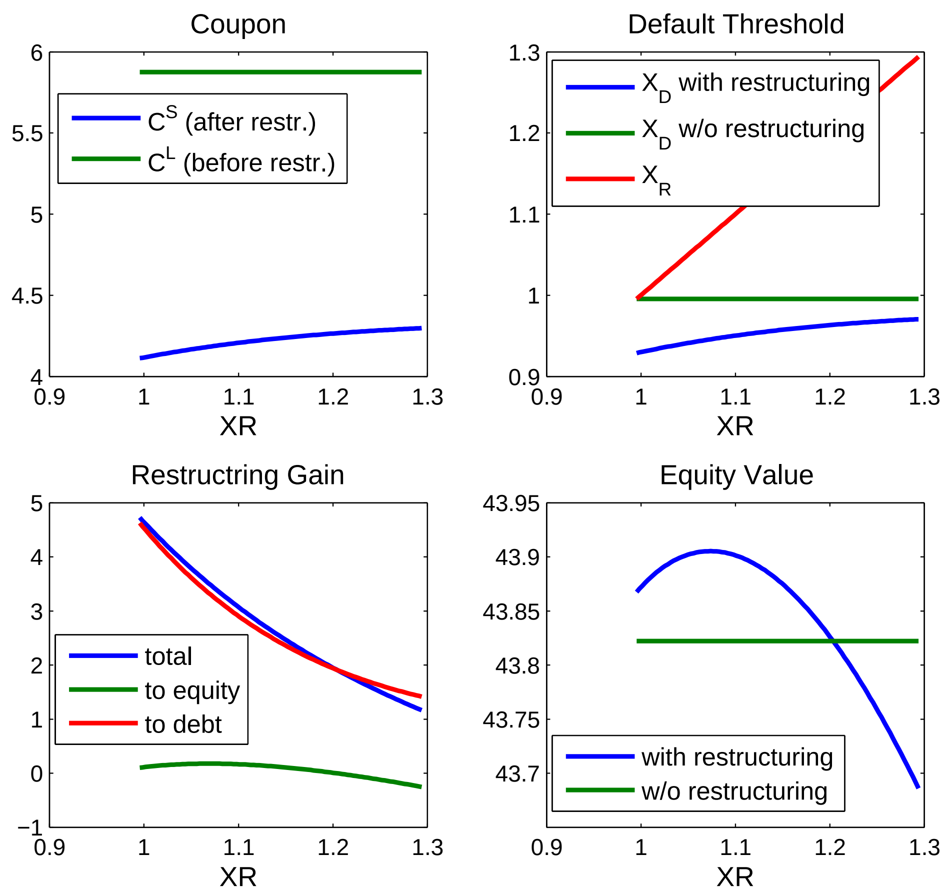
Table 1: Restructuring Gain Analysis using $r = 0.5$ $\mu = 0.01$, $\sigma = 0.3$ $Q^L = 2$, $Q^S = 1.5$, $T_C = 0.3$, $\alpha = 0.3$, $T_{COD} = 0.05$, $\phi = 0.95$, $\phi_0 = 0$, $X_0 = 3$.

	Firm	Equityholders	Debtholders
Asset Sale	-0.50	-0.50	
Tax shield lost later	0.30	0.30	
Tax on CODI	-0.40	-0.40	
Bankruptcy costs saved	3.23	0.70	-2.53
Total	2.62	0.10	2.53
% gain	9%	206%	9%

Table 1 summarizes the sources of the gain from restructuring and its attribution to the different stakeholderes. The gain to equityholders is modest in dollar terms but the equity value is more than twice as high as without restructuring.

Figure 1 depicts the coupon after restructuring. The longer equity holders wait the larger can the coupon reduction be as the debt value falls due to increased default risk. The graph in the lower left corner shows the gains to equity- and debtholders for different restructuring thresholds X_R. To debtholders the gain is highest at the default boundary because they receive the larger coupon for a longer time and the present value of bankruptcy costs to be saved is higher. Equityholders, however, lose if they wait too long because they carry the burden of a higher coupon for longer. On the other side, if they restructured too early, they would give up production capacity too early and pay too much for the debt reduction. That is the trade-off they face. Note that the optimal restructuring threshold does not maximize the gain to equityholders but the equity value which includes the present value of the gain and, thus, depends on the expected restructuring time. Due to the time preference of equityholders, they will restructure earlier, i.e. the optimal X_R is higher than threshold that would maximize the gain to equityholders.

Figure 1: Coupon and default threshold before and after restructuring, restructuring gain, and equity values for different restructuring threshold (X_R) choices using $r = 0.05$, $\mu = 0.01$, $\sigma = 0.3$, $Q^L = 2$, $Q^S = 1.5$, $\alpha = 0.3$, $T_C = 0.3$, $X_0 = 3$. $C^L = 5.88$ is the optimal coupon for the firm with restructuring option.



2.3 When Do Firms Prefer Default to Downsizing?

Proposition 2.2 states that a firm would always strictly prefer reducing leverage via asset sales to going bankrupt as long as $\alpha \in (0, 1]$, if restructuring were costless and markets correctly priced the firm's assets. This proposition I will analyze three different possibilities why firms would prefer default to restructuring: high taxes on cancellation of debt income, government subsidies, liquidity in the market for capital goods.

Proposition 2.2 gives conditions under which equityholders always have an incentive to repurchase debt via asset sales. Together with proposition 2.3, it also indicates conditions under which a firm would rather default. I will analyze three different reasons for why firms would prefer default to restructuring: high taxes on cancellation of debt income (CODI), government subsidies, and liquidity in the market for capital goods.

2.3.1 Prohibitively High Taxes and the American Recovery and Reinvestment Act

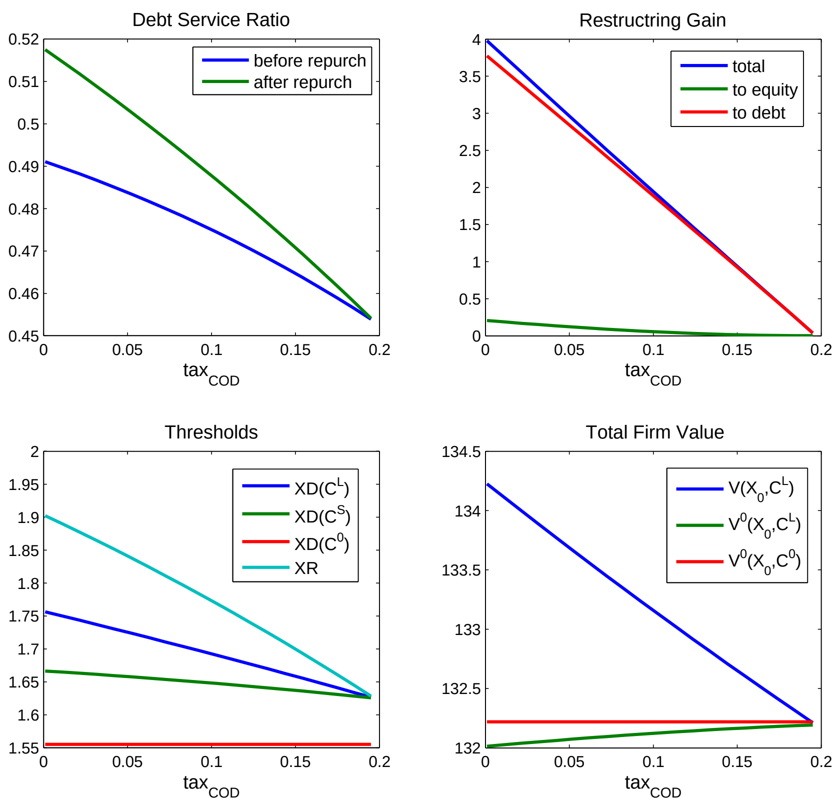
The tax rate applying on cancellation of indebtness income has a considerable impact on a firm's willingness to buy back debt. As the example in figure 2 shows, small changes in the tax rate applying on CODI can considerably shift the restructuring threshold. If the tax rate on the CODI is high, the firm may prefer to go bankrupt before repurchasing any debt. In this example, taxes in the amount of 20% would suffice to eliminate any gains from repurchasing debt. At this level, the debt service ratio cannot be improved any more. A tax rate on CODI as high as the corporate tax rate (30%) would prevent the firm from establishing a more sustainable leverage ratio.

In this respect, the American Recovery and Reinvestment Act, which allowed to defer COD tax costs for up to 11 years, served to improve the capital structure of firms by increasing their incentive to sell assets and buy back debt. Levy & Shalev (2011) provide strong empirical evidence that the introduction of tax rebate has led to a sharp increase in tender offers. .

2.3.2 Government Subsidies and Crowding Out of Equityholders

Government subsidies granted in case of default have the potential to crowd out equityholders in improving the situation of debtholders and the entire firm. A case in point would be deposit insurance or government guarantees, either implicit or explicit, to intervene in case of default and either take over the firm or indemnify the debholders for part or the total of their losses. As the government mitigates debtholders' losses in bankruptcy, debt can becomes too expensive to buy back, especially if taxes on CODI are high. This destroys any gain that equityholders could make and they will never attempt to reduce the debt burden to avoid bankruptcy. The government has

Figure 2: The impact of tax on COD income. T_{COD} varies from 0 to 0.195. The other parameters are $r = 0.05$, $\mu = 0.01$, $\sigma = 0.3$, $Q^L = 2$, $Q^S = 1.5$, $T_C = 0.3$, $\alpha = 0.3$, $\phi_1 = 1$, $X_0 = 3$.



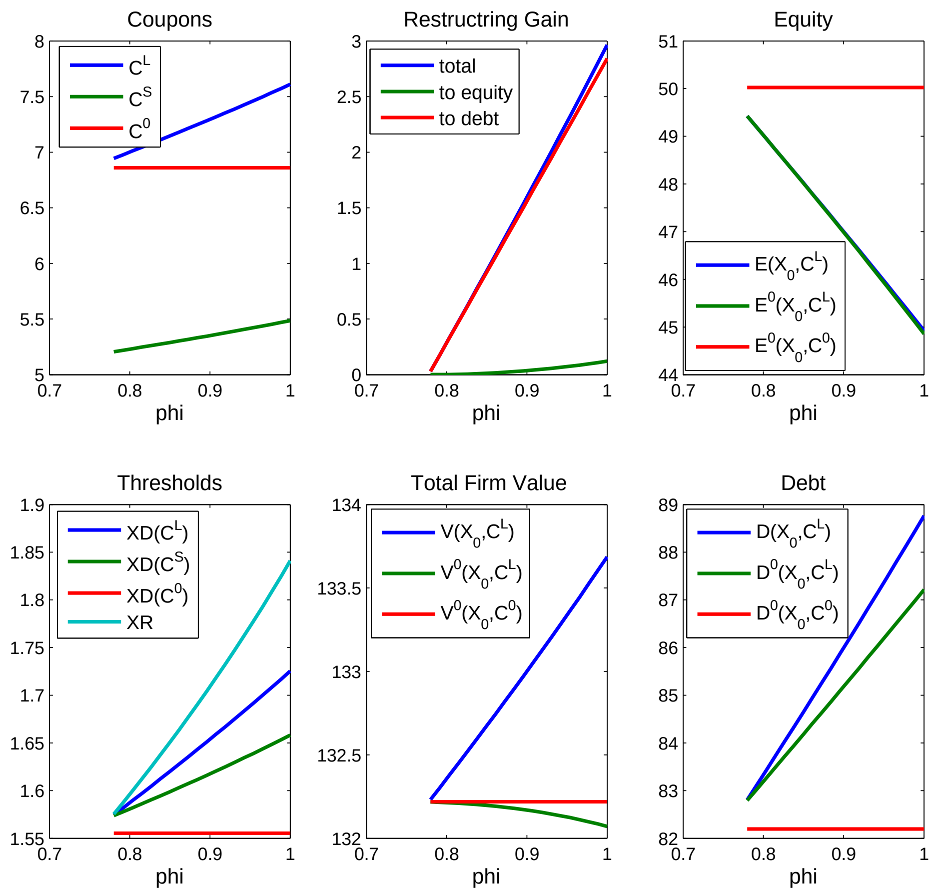
replaced equityholders in saving the firm. The government intervention might be welfare decreasing. Suppose the government takes over the total amount of bankruptcy costs which amount to $\alpha(1-T_C)\frac{Q^L X_D^0}{r-\mu}$ if equityholders do not try to save the firm. However, equityholders' downsizing reduces the deadweight losses in case of bankruptcy to $\alpha(1-T_C)\frac{Q^S X_D}{r-\mu}$, which at the time where the government subsidy would be paid is worth $\left(\frac{X_D^0}{X_D}\right)^{-\theta}$ dollars. Welfare losses are lower under private intervention. Government intervention should be designed in such a way that equityholders first exhaust all their means before the government steps in. This would save costs for the whole society... A more efficient form of government subsidies would be to buy the assets which the firm wants to sell.

2.3.3 Liquidity in the Market for Capital Goods and Fire Sales

Finally, if the market for the firm's assets is not liquid (small ϕ_1), the firm can only retire a small fraction of its debt with the assets to be sold (see figure 3, first panel). If liquidity is very low, the restructuring fails to improve the debt service ratio and any equityholders' incentives to buy back debt are eliminated. Note that a low value of the assets, caused by a bad economic situation, does not impede restructuring. What matters is the difference between the current owner's valuation and that of the potential buyer (captured by ϕ_1). Proposition 2.3 provides a lower bound for market liquidity: ϕ_1 must exceed $(1-\alpha)$, given that no other distortions affect the repurchase. As long as assets are worth more to a new owner than to debtholders in case of default, equityholders will find restructuring profitable.

Although industry equilibrium is not explicitly considered, the model can provide some insights into what happens in case of fire sales. Suppose all firms in the industry are identical and the only source of uncertainty is an industry wide shock. A negative economic shock would hit all firms simultaneously and all would consider downsizing in order to reduce the burden of debt at the same time. If assets could only be sold to other firms in the industry, then the assets would sell at $P^K(X_R) = \frac{v(X_R,C^S)}{Q^S}(Q^L - Q^S)$. $\frac{v(X_R,C^S)}{Q^S}$ can be interpreted at the price per installed capital after restructuring has taken place. This price contains the tax shield and bankruptcy costs of a firm operating with capital of size Q^S and capital structure C^S. At this price, every firm is indifferent between selling the asset and keeping it. Plugging in this price in the restructuring condition reveals, that restructuring does not improve the debt service ratio and equityholders do not have an incentive to repurchase debt in the first place because it is expensive compared to the value of the assets (see proposition 2.3). As a consequence, debt reductions do not take place. A similar result was derived in Admati et al. (2012). They show that equityholders will not repurchase debt if the assets are sold at a price equaling the value which the remaining assets of the firm attain after the debt restructuring.

Figure 3: Comparative Statics: Asset market liquidity varies from 0.7 to 1. The other parameters are $r = 0.5$, $\mu = 0.01$, $\sigma 0.3$, $\lambda = 1.5$, $Q^L = 2$, $Q^S = 1.5$, $\alpha = 0.3$, $T_C = 0.3$, $X_0 = 5$.



However, if firms in the industry are not homogeneous and some firms have suffered less from the economic shock, firms will have different leverage ratios and attach a different value to capital. Firms in greater financial distress could sell their assets to healthier firms which value them more. The price of the capital goods is most likely to exceed $P^K(X_R) = \frac{v(X_R,C^S)}{Q^S}(Q^L - Q^S)$ and restructuring can take place. In addition, demand from firms in other industries might boost the prices of capital goods, as well. Its impact depends on the redeployability of capital goods in the industry.

2.4 Similarities Between Tender Offers and Open Market Repurchases

Instead of caring out a publicly announced tender offer, the firm can simply buy back its own debt at the prevailing market prices. Since the buyback is entirely financed by the asset sale, its revenues will determine how much of the bond can be retired, again taking into account the taxes which fall due on the cancellation of debt income.

$$P^K(X_R, Q^L - Q^S) = \frac{C^L - C^S}{C^L}\left(D(X_R, C^L) + T_C\left[D(X_0, C^L, C^S) - D(X_R, C^L)\right]\right) \tag{23}$$

$\frac{C^L - C^S}{C^L}$ is the percentage of debt the firm can repurchase. Note that the value of the outstanding debt depends on C^S as well as $P(X_R, Q^L, Q^S)$ because debtholders will rationally anticipate the repurchase. If the state variable hits the restructuring boundary, they expect equityholders to buy back a certain fraction of the outstanding debt and realize that after the repurchase they will hold a claim against a firm with lower capacity Q^S and coupon C^S. Thus, the value matching condition for debt becomes:

$$\begin{aligned} D(X_R, C^L, C^S) &= d(X_R, C^S) + \frac{C^L - C^S}{C^L} D(X_R, C^L, C^S) \\ D(X_R, C^L, C^S) &= \frac{C^L}{C^S} d(X_R, C^S) \end{aligned} \tag{24}$$

Open market repurchases are very similar to tender offers. Debtholders rationally anticipate that equityholders will by back debt. At the time of restructuring, debtholders get paid the prevailing market price for a fraction of their holdings. The remaining fraction becomes now a claim against the restructured firm. No arbitrage requires that the joint value of both fractions is equal to the market value itself. This can only be the case if the debt value per unit of principal is the same just before and after restructuring which is the same condition that makes debtholders submit their bonds in a tender offer. The only difference is the size of the tax payment for CODI. The only difference is the tax payment for CODI. As the firm retires only a fraction of its debt, the tax payment is smaller in the case of open market repurchases, which makes them the preferred way of buying back debt if no consent solicitation statement requesting to change material terms of the bond contract is added to the tender offer.

3 Asset Substitution

In the following section, I will relax the assumption that the risk of the firm stays constant. The setup is based on Hennessy & Tserlukevich (2008). The managers of the firm can control the riskiness of how the assets in place are operated. In particular, they can choose an F_t-adapted volatility policy

$$\sigma_t \in \Sigma \equiv [\underline{\sigma}, \overline{\sigma}], \quad \text{where } \ 0 < \underline{\sigma} < \overline{\sigma} < \infty \tag{25}$$

Increasing the riskiness comes at a cost which I assume to reduce the drift of the cashflows produced by the assets in place. One can think of the cost as expenses necessary for establishing and upholding the riskier use of the assets. In particular, I assume that the instantaneous costs are proportional to the increase in variance and amount to $\rho(\sigma_t^2 - \underline{\sigma}^2)$ with $\rho \geq 0$. Risk shifting does not only redistribute the value of the firm from debtholders to equityholders, as in the classical asset substitution problem, but has also valuation consequences for the entire firm value itself, since the risk shifting costs decrease the unlevered asset value if $\rho > 0$. It shrinks the size of the pie to be distributed to the firm's stakeholders. As equityholders can change the size of their slice the might go for a bigger slice from a smaller pie.

If the firm sells parts of its assets, the price that can be obtained in the market will depend on how risky the assets are operated by the potential new owners. If the new owner of the capital goods is an all-equity financed firm, then the assets will be operated as safe as possible. If the assets are brought into a firm that is levered up optimally after the acquisition, then the price of the assets will depend on the risk strategy of the newly levered firm as well as on the gains to be made from the leverage choice, similar as in Fischer et al. (1989), and on the bargaining power between seller and buyer. The buyer, who has the option to optimally adjust his firm's leverage after the acquisition, will operate the assets at a lower risk than the seller, who has sold the assets out of financial distress. For simplicity, I assume that the assets are sold at their unlevered value which leaves the specification of the asset sale price (3) unchanged. Similarly, if debtholders take over the firm in bankruptcy, they will also hold an all-equity financed firm which is run with minimal risk.

Higher equilibrium returns as risk shifting costs - a short digression.

Let α be the P-drift of the cashflow process. The Q-drift is given by $\mu = \alpha - \lambda\sigma$ where $\lambda = \frac{\mu_M^P - r}{\sigma_M}$ is the market price of risk applicable to the firm's (single) risk factor. μ_M^P is the equilibrium instantaneous expected return (P-drift) for this factor. If the firm increases the riskiness of its business, say from σ_1 to $\sigma_2 > \sigma_1$, the exposure vis-a-vis the risk factor increases. As a consequence the Q-drift has to change. I assume here that the firm is small enough to not affect the financial market equilibrium.

$$
\begin{aligned}
\mu_1 &= \alpha - \lambda\sigma_1 = r - (r + \lambda\sigma_1 - \alpha) = r - \delta_1 \\
\mu_2 &= \alpha - \lambda\sigma_2 = r - (\underbrace{r + \lambda\sigma_2}_{\mu_2^P} - \alpha) = r - \delta_2
\end{aligned} \tag{26}
$$

Note that $\mu_i^P = r + \lambda\sigma_i$ is the required expected return of a traded asset with risk exposure σ_i. As $\sigma_2 > \sigma_1$, $\mu_1 > \mu_2$, i.e. the $\mathcal{Q}$-drift of the riskier technology decreases by $\lambda(\sigma_2 - \sigma_1)$. This can be interpreted as a punishment by the capital market for running a riskier business which has valuation consequences in the form of a higher required expected return ($\mu_2 > \mu_1$)! The cost equals the volatility increase (which corresponds to the increase in exposure towards the risk factor) times the market price of risk of the factor. However, my assumption about the cost differs somewhat because it is is proportional to the increase in variance.

3.1 Valuation and Optimal Delevering in the Presence of Risk Shifting

In order to solve the model with a risk shifting option, I will, as in section 2.1, employ backward induction and first solve the problem of a firm having exercised its restructuring option. Using this result, I will then solve the tender offer game and derive the optimal restructuring, default, and risk shifting policies of the firm.

3.1.1 The firm without a divesture option

After having sold the asset and reduced debt the firm operates at the same capacity Q^S and with the same capital structure, C^S, until it defaults. The firm has the possibility to adjust the cashflow risk to different levels $\sigma_t \in \Sigma = [\underline{\sigma}, \overline{\sigma}]$.

As long as the firm has not defaulted ($X \in (X_D, \infty)$), equity fulfills the following Hamilton-Jacobi-Bellman equation:

$$
re(X) = \max_{\sigma \in \Sigma} (1 - T_C)(Q^S X - C^S) + (\mu - \rho(\sigma^2 - \underline{\sigma}^2))Xe'(X) + \frac{\sigma^2}{2} X^2 e''(X) \tag{27}
$$

The optimal volatility policy of the firm is a bang-bang solution:

$$
\sigma^*(X) = \begin{cases} \overline{\sigma}^2 & \text{if} \quad \frac{e''(X)X}{e'(X)} \geq 2\rho \\ \underline{\sigma}^2 & \text{if} \quad \frac{e''(X)X}{e'(X)} < 2\rho \end{cases} \tag{28}
$$

One can interpret the ratio $xe''(X)/e'(X)$ as the firm's risk appetite. If it exceeds twice the marginal cost of increasing volatility, the firm will be run with the highest possible risk; if it is lower than this threshold, then the managers will prefer the low risk strategy.

For the optimal value function $e(X)$ to be smooth enough, the following conditions have to be fulfilled at the threshold where the firm changes its riskiness, denoted by X_S^S:

$$\lim_{X \uparrow X_S^S} e(X) = \lim_{X \downarrow X_S^S} e(X)$$
$$\lim_{X \uparrow X_S^S} e'(X) = \lim_{X \downarrow X_S^S} e'(X)$$

For the transitional boundary X_S^S, the smooth pasting condition above is not an optimality condition. Hennessy & Tserlukevich (2008) show that the optimality condition for X_S^S can be expressed as a super contact condition.

$$\lim_{X \uparrow X_S^S} e''(X) = \lim_{X \downarrow X_S^S} e''(X) \tag{29}$$

Having established $e(X)$ in the continuation region, the optimal default boundary X_D has to be determined. As in the standard cases, the optimal default threshold will be determined by a smooth pasting condition. In addition, to ensure that $e(X)$ is continuous at X_D, it must also satisfy a value matching condition, given by equity's limited liability. Thus the remaining conditions are given by

$$\lim_{X \downarrow X_D} e(X) = 0$$
$$\lim_{X \downarrow X_D} e'(X) = 0$$

The following two propositions will describe the risk taking behavior of a firm with no restructuring option. The first condition states that the firm will always gamble for resurrection if its situation worsens which is very intuitive given the option-like nature of the firm.

Proposition 3.1 *The firm will always be run at maximal risk close to the optimal default threshold.*

Proof See Hennessy & Tserlukevich (2008, p390).

The second proposition states that the firm will also be run at low risk if risk shifting is costly.

Proposition 3.2 *For $\rho > 0$, there exists a region where $\sigma^* = \underline{\sigma}$. For $\rho = 0$, the firm will choose maximal risk.*

Proof Let $\overline{e}(X)$ be the value function for the strategy $\sigma(X) = \overline{\sigma}, \forall X \in (X_D, \infty)$. $\lim_{X \uparrow \infty} \frac{\overline{e}(X)}{X} = (1 - T_C)\frac{Q^S}{r - \mu + \rho(\overline{\sigma} - \underline{\sigma})}$. On the other hand, if $\sigma^*(X) = \underline{\sigma}, \forall X > X_S$, where $X_S \in (X_D, \infty)$ is some switching threshold, then $\lim_{X \uparrow \infty} \frac{e(X)}{X} = (1 - T_C)\frac{Q^S}{r - \mu}$.
Furthermore, since $e''(X) > 0$, $\sigma^*(X) = \overline{\sigma}\ \forall X \in (X_D, \infty)$ if $\rho = 0$.

The value of debt will also be affected by the risk taking behavior of the firm. Higher cashflow risk harms debtholders because it makes default more likely. Arbitrage considerations require that the debt value is sufficiently smooth when the risk of the firm is changed (see Dixit, 1993). Thus, like the equity value, the debt value has to satisfy a value matching and a smooth pasting condition at X_S. Finally, the recovery value in default determines a value matching condition at X_D. As discussed above, debtholders take over an all equity financed firm which they, optimally, immediately run at low risk. The bankruptcy state takes its toll in the form of deadweight losses amounting to α times the value of the unlevered firm.

$$\lim_{X\uparrow X_S} d(X) = \lim_{X\downarrow X_S} d(X)$$
$$\lim_{X\uparrow X_S} d'(X) = \lim_{X\downarrow X_S} d'(X)$$
$$\lim_{X\downarrow X_D} d(X) = (1-\alpha)(1-T_C)\frac{Q^S X_D}{r-\mu}$$

3.1.2 The firm with a divesture option

Similar to the case without risk shifting, equityholders can decide to sell part of the firm's production facilities and use the proceeds to buy back debt. Again, the restructuring condition (7) ensures that equityholders do not have to inject money in order to finance the restructuring. Since equityholders now also have command over how risky the firm is operated, the equilibrium incorporates the optimal risk shifting strategy:

Definition An equilibrium with tender offers is defined as the vector of strategies (X_R, X_S^L, X_S^S, X_D) and a vector of prices $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$ and a tender offer price D^T satisfying:

1. X_R is the optimal restructuring threshold given prices are formed by $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$.
2. X_S^L, X_S^S, X_D are the optimal risk shifting and default threshold given the restructuring strategy and prices $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$.
3. Equityholders set the tender offer price D^T optimally such that it induces all debtholders to tender.
4. $E(\cdot), D(\cdot), e(\cdot), d(\cdot)$ are formed rationally, i.e. bond and equityholders rationally anticipate that the firm will restructure at X_R, default at X_D and shift cashflow risk at X_S^L, X_S^S

The equity value before restructuring solves the following Hamilton-Jacoby-Bellman equation in the continuation region:

$$rE(X) = \max_{\sigma\in\Sigma}(1-T_C)(Q^L X - C^L) + (\mu - \rho(\sigma^2 - \underline{\sigma})XE'(X) + \frac{\sigma^2}{2}X^2E''(X) \ \ \forall X \in [X_R, \infty) \quad (30)$$

Similarly to the case without restructuring option, the optimal volatility strategy is given by a bang-bang solution:

$$\sigma^*(X) = \begin{cases} \overline{\sigma}^2 & \text{if } \frac{E''(X)X}{E'(X)} \geq 2\rho \\ \underline{\sigma}^2 & \text{if } \frac{E''(X)X}{E'(X)} < 2\rho \end{cases} \tag{31}$$

While the management of the firm without restructuring option will always resort to the high risk strategy, at least should the firm find itself in distress, the firm with restructuring option might always stick to the low risk strategy. In particular, if the high risky strategy is very costly (large ρ), the firm might operate its assets cautiously before it sells them. However, just before repurchasing debt the large firm will always be run as risky as the small firm is run just after the restructuring:

While the firm without restructuring option will always resort to the high risk strategy at some point, the firm with restructuring option might prefer never to increase its risk before restructuring. In particular, if the high risky strategy is very costly (large ρ), the firm might operate its assets cautiously before it sells them. In general, though, around the restructuring threshold the large firm will always be run as risky as the small firm it will become after having divested and repurchased debt:

While the firm without restructuring option will at some point be run at high risk, at least close to default, the firm with restructuring option might always stick to the low risk strategy. Contrary to the firm without restructuring option, it is not, in general, the case that the firm will choose the risky strategy. If the high risky strategy is very costly (large ρ), the firm might operate all its production sides cautiously before it sells them. However, it will always be run as risky as the small firm after repurchasing debt:

Proposition 3.3 *If the small firm is run at a high risk around the restructuring threshold, then there exists a right neighborhood of the optimal restructuring threshold where it is optimal for the large firm to follow the high risk strategy too. Formally, if $X_S^S > X_R$ then $X_S^L > X_R$.*

Proof At X_R, $E(X)$ pastes smoothly into $e(X)$, i.e. $E'(X_R) = e(X_R)$. For some neighborhood right to X_R, $E'(X) > e'(X), X > X_R$, otherwise, earlier stopping would have been optimal. Thus, $E''(X_R) > e''(X_R)$. From $\frac{E''(X_R)X_R}{E'(X_R)} > \frac{e''(X_R)X_R}{e'(X_R)} > 2\rho$.

The derivation of the optimal restructuring policy is similar to the case without risk shifting possibility. At X_R, the equity value satisfies the following value matching and smooth pasting conditions:

$$\lim_{X \downarrow X_R} E(X) = \lim_{X \downarrow X_R} e(X)$$
$$\lim_{X \downarrow X_R} E'(X) = \lim_{X \downarrow X_R} e'_X(X) + e_C(X)\frac{dC^S}{dX} \tag{32}$$

To preclude arbitrage with the bonds of the firm, the debt value satisfies the following value matching condition:

$$\lim_{X \downarrow X_R} D(X, C^L) = \lim_{X \downarrow X_R} D_T(X) \tag{33}$$
$$\lim_{X \downarrow X_R} D(X, C^L) = \lim_{X \downarrow X_R} C^L \frac{d(X_R, C^S(X))}{C^S(X)} \tag{34}$$

where the second line is derived by inserting the optimal tender offer price.

As before, the debt value also has to satisfy a value matching and a smooth pasting condition at the risk shifting threshold X_S^L:

$$\lim_{X \uparrow X_S^L} D(X) = \lim_{X \downarrow X_S^L} D(X)$$
$$\lim_{X \uparrow X_S^L} D'(X) = \lim_{X \downarrow X_S^L} D'(X)$$

Note that the solution to the debt value in the case of risk shifting is not simply a scaled up value of the debt value after restructuring as in (20).

3.1.3 The Effect of Risk Shifting on the Restructuring Decision

To illustrate the effect of risk shifting, I contrast a firm with vanishingly small risk shifting potential ($\underline{\sigma} = 0.2, \overline{\sigma} = 0.21$) to a firm that can considerably increase its cashflow risk ($\underline{\sigma} = 0.2, \overline{\sigma} = 0.5$). Figure 4 depicts the coupon after restructuring, the optimal default as well as risk shifting strategies, and the gain to debt and equityholders as a function of the restructuring threshold. The optimal restructuring threshold is 1.78 for the firm with the small risk shifting potential and 1.72 for the firm with the large risk shifting option.

The first striking difference to a firm that is run at the same risk is the lower debt reduction potential. The upper left panel of figure 4 depicts the coupon reduction that can be achieved for different restructuring threshold choices. Equityholders who can control the riskiness of the firm cannot exploit the firm's economic decline at such a scale as equityholders without the ability to change the risk. The coupon after restructuring, C^S, gets even convex in X_R for low values. The underlying reason is that asset substitution lets debtholders embrace a lower coupon if the firm is close to default, because postponing default (and shrinking the region where equityholders gain for resurrection) outweighs the loss due to lower fixed payments. Note that this effect is also present without risk shifting (see Leland, 1994), but only with risk shifting is it substantial enough because equityholders' gambling for resurrection if default becomes more likely makes debt more sensitive to changes in the default threshold. Due to the value increasing effect of the coupon reduction that is the stronger the closer the firm is to default (low X_R), the firm cannot exploit the value reduction of lower X so much and has, in principal, a lower debt reduction potential.

The lower coupon reduction potential eats up part of debtholders' gains. As the coupon does not decrease that much, the default threshold remains higher which enlarges the region where the equity value of the firm before restructuring is sufficiently convex to encourage risk taking. Hence, the fraction of the debt value that arises from the potential to become a safer firm shrinks substantially. This loss outweighs the gain from the decrease in the default threshold. As a consequence, the gain to debtholders even decreases as the firm gets closer to default!

Equityholders who can control the riskiness of the firm face a similar tradeoff as equityholders who cannot. The longer they wait with the repurchase, the more debt they can buy back. But, as the distance-to-default shrinks, the likelihood of a default increases which means that the firm will not profit long from the coupon reduction. For large values of X_R, the increase in the coupon payment savings dominates and the gain rises. As equityholders wait longer, the probability of default increases and reduces their profit from restructuring, generating the concave pattern of the equityholders' gain (see figure 4, lower right panel).

The debt overhang problem is attenuated, as the equityholders' optimal restructuring threshold is closer to the one maximizing the total firm value. However, this comes at the expense of introducing another agency cost through asset substitution which is required in the first place to make the equityholders' optimal restructuring threshold agree more with what is optimal for the total firm value.

Figure 5 summarizes the effect of a larger risk shifting potential, i.e. of a larger difference between $\overline{\sigma}$ and $\underline{\sigma}$. The upper left panel shows that the more the firm can shift its operational risk, the larger is the amount of debt repurchased. Although the debt reduction potential is smaller for the firm with discretion over the riskiness of its cashflows, equityholders make greater use of the repurchase option. The possibility to run the firm with higher risk lets them hold on to it for longer and default later (cf. second panel in figure 5). Note that a greater risk shifting opportunity (higher $\overline{\sigma}$) induces the firm to make greater use of it before restructuring while it will be run saver after having restructured (cf. upper right panel of figure 5).

The gain to debtholders is convex in the risk shifting potential $\overline{\sigma}$. First, it decreases in $\overline{\sigma}$ because the distance to the risk shifting threshold increases, i.e. the firm's chance to become less risky shrinks.

Figure 4: Large vs Small Risk Shifting Option Coupon after restructuring, restructuring gain, and security values for restructuring different threshold choices X_R choices using $r = 0.05$, $\mu = 0.01$, $\underline{\sigma} = 0.2$, $\overline{\sigma} = 0.5$ for large risk shifting option and $\overline{\sigma} = 0.21$ for small risk shifting option, $Q^L = 2$, $Q^S = 1.5$, $\alpha = 0.3$, $T_C = 0.3$, $X_0 = 5$. $C^L = 7.25$ is the optimal coupon for the firm with restructuring option.

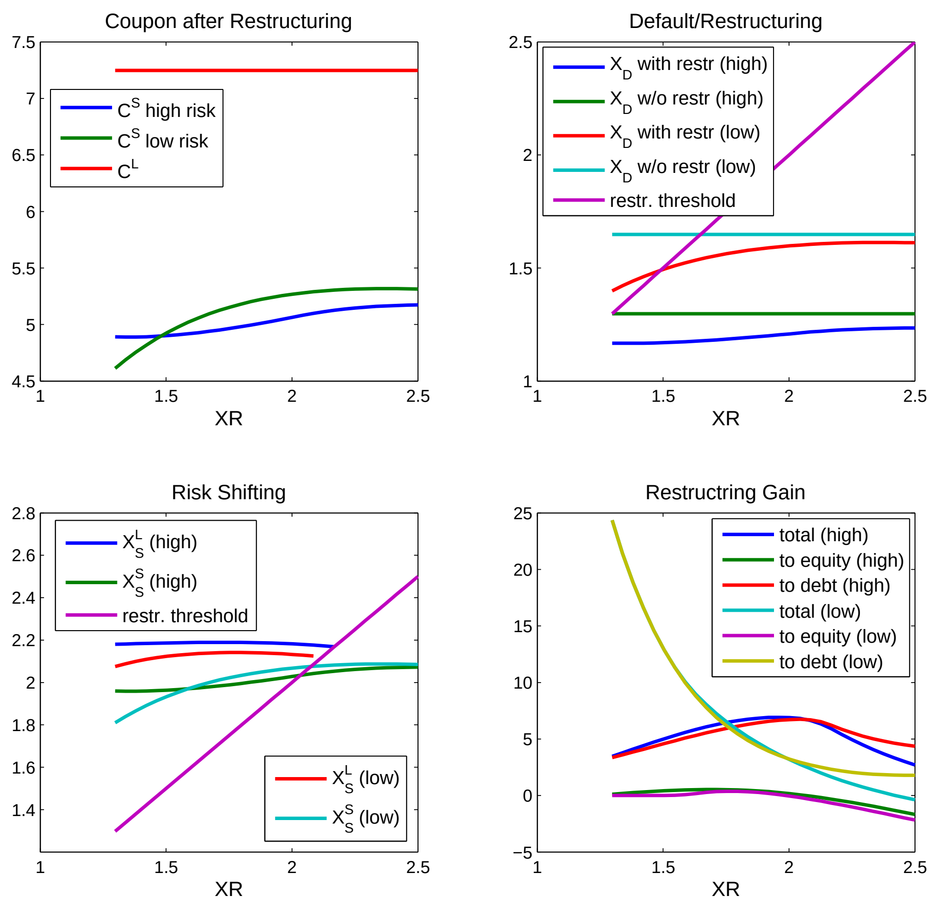
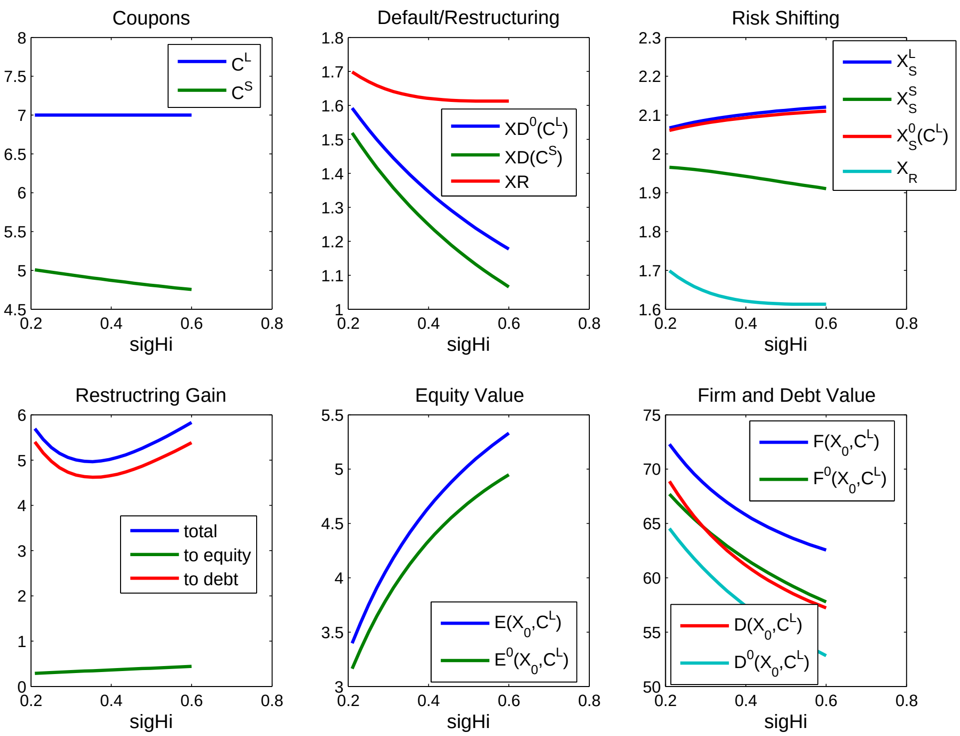


Figure 5: Comparative Statics: The highest possible asset volatility varies from 0.21 to 0.6. The other parameters are $r = 0.05$, $\mu = 0.01$, $\underline{\sigma} = 0.2$, $\lambda = 1.5$, $Q^L = 2$, $Q^S = 1.5$, $\alpha = 0.3$, $T_C = 0.3$, $X_0 = 5$, $C^L = 7$.



This outweighs the gain from becoming saver as the distance-to-default increases. For higher $\overline{\sigma}$ the effect of risk shifting levels off, as the distance to X_S^S shrinks and the bankruptcy costs savings accruing to debtholders dominate.

3.1.4 Risk Shifting and the Choice Between Default and Downsizing?

Proposition 3.4 *In the absence of frictions, risk shifting does not affect the decision of a firm whether to buy back debt or not. It only affects the timing of the debt repurchase.*

Proof In the absence of taxes, the restructuring condition (7) evaluated at $X_R = X_D$ is the same as in the case without risk shifting.

Risk shifting can only take place as long as equityholders have control over the firm. If the firm is close to bankruptcy, the risk shifting decisions of the future owners of the production facilities, namely the buyer of the divesture and the debtholders who take over the remaining firm, matter. In the limit, where $X_R = X_D$, the equityholders risk taking does not impact the restructuring decision any more.

If the firm has to pay taxes on cancellation of debt income, then agency costs caused by the asset substitution problem matter. As these costs decrease the amount of debt that can be repurchased with the asset sale proceeds, a higher risk shifting potential implies that a lower amount of taxes can be born. With the additional loss caused by the asset substitution problem, the tax level preventing an improvement in the debt service ratio will be lower than in the case without risk shifting. As taxes increase, equityholders want to restructure later.

In a similar fashion, a lower level of government subsidies suffices to destroy equityholders' incentives to voluntarily restructure.

The effect of liquidity in the market for capital goods is unchanged by the asset substitution problem, as long as there are no taxes to pay for cancellation of debt income. $1 - \alpha$ is still the lower bound for ϕ_1, i.e. the market price which the assets to be sold have to attain such that repurchasing debt is profitable remains the same.

3.2 The Coordinated Holdout Problem

In section 2.1, I derived the price which equityholders must offer to prevent individual debtholders from holding out. As a single debtholder's reservation value was given by the value of her position if she did not tender this was exactly the price to pay, (6). However, if debtholders could coordinate, there is more to gain. Debtholders know that equityholders will make another offer if they reject the first one, since equityholders are still to profit from restructuring, although at a smaller scale. If debtholders collectively hold out, they can force equityholders to either solicit the tender offer at the time where the gain to debtholders is maximized or, equivalently, pay the corresponding reservation value at an earlier time.

In the example without the risk-shifting option, the combined gain to debtholders is largest if restructuring is timed in such a way that the firm immediately defaults after having delevered. In this setting, debtholders as a group would reject any offer, based on the previously derived incentive compatible consideration (6), where restructuring took place earlier $X_R > X_D$.

Coordination among debtholders is costly. I assume that coordination costs are proportional to the debt value, $qD(X, C^L)$. This form has the appealing feature that the coordination costs are the higher, the further away the firm is from bankruptcy, i.e. the longer debtholders have to hold out. Also, the more dispersed debtholdings are, the harder it will be to coordinate. Proportional costs q will be larger for more diffusely held debt.

Independent of whether debtholders collectively held out or not, they know that they must at least be paid the value of debt after restructuring such that nobody individually hold outs. Let X_R be an arbitrary restructuring point. This minimal offer price is given by

$$\phi(X_R) = \frac{C^L}{C^S(X_R)} d(X_R, C^S(X_R)) \tag{35}$$

This value corresponds to the tender offer price without coordinated hold out. However, here it is not the tender offer consideration because this price will depend on the debtholders reservation value from the collective hold out. The reservation value is given by the value of debt restructured at the threshold that is optimal for debtholders subject to two restrictions. The threshold has to lie in the interval were equityholders are willing to make an offer and the equityholders pursue their optimal equilibrium risk shifting policy. Let X_R^D be the optimal restructuring threshold for debtholders that satisfies the two restrictions and τ_R^D the associated restructuring time. The reservation value of debtholders is given by:

$$\hat{D}(X_t) = E^Q \left[\int_t^{\tau_R^D} e^{-r(s-t)} C_L ds \right] + E^Q \left[e^{-r(\tau_R^D - t)} \phi(X_R^D) \right] \tag{36}$$

Let $\underline{X}_R^E$ be the lowest X where equityholders are still willing to make an offer (their gain to restructuring is just positive). The reservation debt value fulfills the following system of variational inequalities:

$$
\begin{aligned}
\hat{D}(\underline{X}_R^E) &= \phi(\underline{X}_R^E) \\
\hat{D}(X) &\geq \phi(X) \quad \forall X > \underline{X}_R^D \\
C^L + \mathcal{A}^{\sigma^*}\hat{D}(X) - r\hat{D}(X) &\leq 0 \qquad \forall X \geq \underline{X}_R^E \\
\max(\hat{D}(X) - \phi(X), C^L + \mathcal{A}^{\sigma^*}\hat{D}(X)) &= 0 \qquad \forall X \geq \underline{X}_R^E
\end{aligned}
\tag{37}
$$

As $\hat{D}$ is the off-equilibrium path debt value, it is subject to equityholders pursuing their optimal risk strategy $\sigma^*(X)$.

If equityholders do not change the risk profile of the firm, debtholders always have an incentive to hold out.

Proposition 3.5 *If Σ is a singleton or $\rho = 0$, i.e. the firm has a constant risk strategy, then the debtholders would like to restructure as late as possible. Thus, $X_R^D = \underline{X}_R^E$.*

Proof See appendix A.3

Knowing that debtholders will behave that way, equityholders will behave optimally by paying debtholders their reservation value at X_R in order to convince them to accept the first offer. The optimal X_R^* takes this payment into account. Thus, at any restructuring threshold, equityholders have to offer

$$
D^T(X_R) = \max\left(\frac{C^L}{r}d(X_R, C^S(X_R)), (1-q)\hat{D}(X_R, C^L, X_R^D)\right) \tag{38}
$$

Using the benchmark calibration for the firm without risk-shifting option, I find that proportional coordination costs have to be as large as 6% of the current debt value such that restructuring will take place before default. If costs are lower, debtholders reservation value is too large to make restructuring profitable for equityholders.

However, if the firm's management is able to control the riskiness of how the firm is operated, the coordinated hold-out problem might disappear. Risk shifting provides bargaining power to equityholders. Debtholders know that the more they defer restructuring the larger is the equityholders incentive to gamble for resurrection. The increase in gain derived from a later restructuring time is more than offset by the increased riskiness of their position. Proposition 3.6 summarizes this result.

Proposition 3.6 *If the firm can shift the riskiness of the cashflows, debtholders might be willing to immediately accept the offer.*

Debtholders threatening not to tender lets them appropriate a large share of the firm's restructuring gain. However, they cannot stop equityholders from gambling for resurrection which in turn gives equityholders bargaining power to prevent debtholders from holding out.

3.3 Robustness

3.3.1 Nash Bargaining

4 Conclusion

The dynamic, partial equilibrium setting of this model reveals that equityholders have an incentive to sell assets and buy back debt whenever the firm itself profits from the restructuring. The debt overhang problem in a dynamic setting is not that the firm foregoes total firm value increasing restructuring options but that managers will restructure too early which is the mirror image of the underinvestment problem.

Taxes on cancellation of debt income and subsidies to debtholders can make buying back debt too costly for the firm. Government guarantees crowd out equitholders' incentives to restructure and are always less efficient in bearing the deadweight cost of bankruptcy in this setting.

The model also reveals that fire sales are able to prevent restructuring in an industry if the firms in the industry are very much alike. When firms differ in their leverage or their exposure to the industry wide shock, voluntary deleveraging financed by asset sales is likely to take place. An asset repurchase program would be an effective government intervention in a situation where firms' incentives to deleverage are lost.

Debtholders extract the lion's share of the restructuring gains, since every individual debtholder can threaten to hold out. Nevertheless, the percentage gain to equityholders is large because they can considerably postpone default at a time when equity has already lost substantial value. If the management can control the riskiness of its operating income, equityholders can get a larger share of the gains. The possibility to shift the riskiness of the firm is especially valuable to equityholders if the debtholders coordinate and threaten to collectively hold out. While debtholders in total will always profit from holding out in the case of a firm run at the same risk, they will immediately accept the first offer if the management can increase the riskiness.

Debtholders threatening not to tender individually lets them appropriate a large share of the firm's restructuring gains. However, they cannot stop equityholders from gambling for resurrection which in turn gives equityholders bargaining power to prevent debtholders from holding out.

A Proof of Propositions

A.1 Proof of proposition 2.2

I will show that $E(X_0, C^L, C^S) > E^0(X_0, C^L)$ under the above mentioned conditions.

Let τ_R be the time point when the firm buys back its debt and τ_D^0 be the default time if the firm had no asset sale option. The equity values of the firm with the asset sale option and the firm without such an option are given by:

$$
\begin{aligned}
E(X_0, C^L, C^S) =& \mathbb{E}^Q\left[\int_0^{\tau_R} e^{-rt}(1 - T_C\frac{Q^L X_t}{r-\mu} - \frac{C^L}{r}dt\right] + \mathbb{E}^Q\left[e^{-r\tau_R} e(X_R, C^S)\right] \\
E^0(X_0, C^L) =& \mathbb{E}^Q\left[\int_0^{\tau_D^0} e^{-rt}(1 - T_C\frac{Q^L X_t}{r-\mu} - \frac{C^L}{r}dt\right]
\end{aligned}
$$

$E(X_0, C^L, C^S) > E^0(X_0, C^L)$ is equivalent to $e(X_R, C^S) > E^0(X_R, C^L)$, since

$$
\begin{aligned}
E(X_0, C^L, C^S) - E^0(X_0, C^L) =& \mathbb{E}^Q\left[e^{-r\tau_R} e(X_R, C^S)\right] \\
& - \mathbb{E}^Q\left[\int_{\tau_R}^{\tau_D^0 \wedge \tau_R} e^{-rt}(1 - T_C\frac{Q^L X_t}{r-\mu} - \frac{C^L}{r}dt\right] \\
=& \mathbb{E}^Q\left[e^{-r\tau_R}\left(e(X_R, C^S) - E^0(X_R, C^L)\right)\right] \\
>& 0 \\
& \Leftrightarrow \\
e(X_R, C^S) - E^0(X_R, C^L) >& 0
\end{aligned}
$$

I will show that $\exists X_R$ (or equivalently a stopping time defined by $\tau_R = \inf\{t > 0 : X_t = X_R\}$) s.t. $e(X_R, C^S) > E^0(X_R, C^L)$. If the firm used the smallest possible repurchase threshold, namely $X_R = X_D$, the restructuring condition becomes

$$
\begin{aligned}
(1 - T_C)\frac{Q^L - Q^S}{r-\mu} X_D =& \frac{C^L - C^S}{C^S}(1-\alpha)(1 - T_C)\frac{Q^S X_D}{r-\mu} \\
\frac{Q^L}{C^L} <& \frac{Q^S}{C^S}
\end{aligned} \tag{39}
$$

As the default threshold is indirect proportional to Q^i/C^i, it follows from (39) that $X_D^0 > X_D$ and $e(X_D^0, C^S) > E^0(X_D^0, C^L)$.

■

A.2 Proof of proposition 2.3

If restructuring at $X_R = X_D$ does not improve the debt-service ratio, $X_D \geq X_D^0$, since the default thresholds X_D and X_D^0 are indirectly proportional to Q^S/C^S and Q^L/C^L, and we have $Q^S/C^S \leq Q^L/C^L$.

As $\frac{dC(X_R)}{dX_R} > 0$ in a right neighborhood to X_D, any earlier restructuring point $X_R > X_D$ would make the debt-service ratio worse. Since $\frac{Q^S X}{C^S(X)} < \frac{Q^L X}{C^L}$ for all $X_R \in [X_D, X_R]$, $e(X, C^S) < E^0(X, C^S)$ for all $X \in [X_D^0, X_R]$, and, as a consequence $E(X, C^L) < E^0(X, C^L)$ for all $X \in [X, \infty)$. I.e, restructuring is never profitable to equityholders, if it does not improve the debt service ratio at the latest possible restructuring point ($X_R = X_D$).

■

A.3 Proof of proposition 3.5

Let X_R be an arbitrary restructuring threshold satisfying $X_R \in [\underline{X}_R^E, \overline{X}_R^E]$ and τ_R the corresponding restructuring time. Furthermore, assume that equityholders pursue a single risk strategy. The reservation value of debtholders for this arbitrary restructuring threshold is given by:

$$\hat{D}(X_t) = E^Q\left[\int_t^{\tau_R} e^{-r(s-t)} C_L ds\right] + E^Q\left[e^{-r(\tau_R - t)} \frac{C^L}{C^S(X_R)} d(X_R, C^S(X_R))\right] \tag{40}$$

Plugging in for $d(X_R, C^S(X_R))$ yields

$$\hat{D}(X_t) = E^Q\left[\int_t^{\tau_D} e^{-r(s-t)} C_L ds\right] + E^Q\left[e^{-r(\tau_D - t)} (1-\alpha)(1-T_C) \frac{Q^S X_D}{r-\mu} \frac{C^L}{C^S(X_R)})\right] \tag{41}$$

Note that X_R affects X_D and τ_D via $C^S(X_R)$.

$$\hat{D}(X_t) = E^Q\left[\int_t^{\infty} e^{-r(s-t)} C_L ds\right] - E^Q\left[\int_{\tau_D}^{\infty} e^{-r(s-t)} C_L ds\right] + E^Q\left[e^{-r(\tau_D - t)} (1-\alpha)(1-T_C) \frac{Q^S X_D}{r-\mu} \frac{C^L}{C^S(X_R)})\right] \tag{42}$$

$$= \frac{C^L}{r} + E^Q\left[e^{-r(\tau_D - t)} \left((1-\alpha)(1-T_C)\frac{\theta}{1+\theta} - 1\right) \frac{C^L}{r}\right] \tag{43}$$

The second term is smaller than zero. Thus, the larger is τ_D (the smaller X_D), the larger is the reservation value of debt holders.

$$\frac{\partial \hat{D}_t}{\partial X_D} \frac{\partial X_D}{\partial C^S} \frac{\partial C^S}{\partial X_R} < 0 \tag{44}$$

■

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