Core and 'Crust': Consumer Prices and the Term Structure of Interest Rates *

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Abstract

We propose a no-arbitrage model that jointly explains the dynamics of consumer prices as well as the nominal and real term structures of risk-free rates. In our framework, distinct core, food, and energy price series combine into a measure of total inflation to price nominal Treasuries. This approach captures different frequencies in inflation fluctuations: Shocks to core are more persistent and less volatile than shocks to food and, especially, energy (the 'crust'). We find that a common structure of latent factors determines and predicts the term structure of yields and inflation. The model outperforms popular benchmarks and is at par with the Survey of Professional Forecasters in forecasting inflation. Real rates implied by our model uncover the presence of a time-varying component in TIPS yields that we attribute to disruptions in the inflation-indexed bond market. Finally, we find a pronounced declining pattern in the inflation risk premium that illustrates the changing nature of inflation risk in nominal Treasuries.

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1 Introduction

A general view in the empirical macro-finance literature is that financial variables do little to help forecast consumer prices. In particular, most empirical studies find that there is limited or no marginal information content in the nominal interest rate term structure for future inflation (Stock and Watson (2003)). The challenge to reconcile yield curve dynamics with inflation has become even harder during the recent financial crisis due to the wild fluctuations in consumer prices, largely driven by short-lived shocks to food and, especially, energy prices (Figure 1). There is hardly any trace of these fluctuations in the term structure of interest rates. Core price indices, which exclude the volatile food and energy components, have been more stable. Nonetheless, attempts to forecast core inflation using Treasury yields data have also had limited success.

We propose a dynamic term structure model (DTSM) that fits inflation and nominal yields data well, both in and out of sample. We price both the real and nominal Treasury yield curves using no-arbitrage restrictions. In the tradition of the affine DTSM literature (e.g., Duffie and Kan (1996), Piazzesi (2010), Duffie, Pan, and Singleton (2000)), we assume that the real spot rate is a linear combination of latent and observable macroeconomic factors. The macroeconomic factors are the three main determinants of consumer prices growth: core, food, and energy inflation. We model them jointly with the latent factors in a vector autoregression (VAR). Nominal and real bond prices are linked by a price deflator that grows at the total inflation rate, given by the weighted average of the individual core, food, and energy measures.

This framework easily accommodates the properties of the different inflation components. Shocks to core inflation are much more persistent and less volatile compared to shocks to food and, especially, energy inflation (the 'crust' in the total consumer price index). The model fits these features by allowing for different degrees of persistence and volatility of the shocks to each of the three inflation measures, and for contemporaneous and lagged dependence among the factors. The three individual components combine into a single measure of total inflation that we then use to price the nominal yield curve.

When we estimate the model on a panel of nominal Treasury yields and the three inflation series, we find a considerable improvement in the fit compared to DTSM specifications that rely on a single inflation factor (either total or core). In particular, we see a significant improvement in the out-of-sample performance of the model when forecasting inflation. This is most evident in core consumer price index (CPI) forecasts, which we find to systematically outperform the forecasts of various univariate time series models, including the ARMA(1,1) benchmark favored by Ang, Bekaert, and Wei (2007) and Stock and Watson (1999). Our model does well on total CPI too, often improving on the ARMA and other benchmarks. Remarkably, it is at par with the Survey of Professional Forecasters (SPF) on total inflation and it outperforms the University of Michigan survey forecasts. Finally, total inflation forecasts from our preferred no-arbitrage DTSM are more precise than forecasts from unconstrained VAR models estimated on interest rate and inflation data, including specifications that use core, food, and energy inflation series.

These results underscore the advantages of modeling the dynamics of the individual inflation components. A DTSM that prices bonds out of a single measure of inflation delivers forecasts for the specific proxy of inflation used for estimation (e.g., total, core, or a principal component of several price series). In contrast, jointly modeling the three inflation factors (core, food, and energy) produces forecasts for total inflation as well as each of its components. Moreover, this approach proves to be more robust to the extreme fluctuations observed in some price indices. In particular, the estimation finds shocks to energy inflation to be short-lived and to have limited impact on the yield curve and long-run inflation expectations.

Our inflation forecasts not only reflect information from past price realizations, but also from yield curve dynamics. In fact, we find that the latent factors explain a large fraction of the variation in both nominal yields and core inflation. In particular, we allow the latent factors to shape the conditional mean of core inflation, and model estimation supports such dependence. When we decompose the variance of the forecasting error for core inflation, we find that the latent factors explain approximately 60% of it at the five-year horizon. This fraction remains sizeable even at the short one-year horizon (>18%), and it increases even further when we perform an unconditional variance decomposition.

A related analysis shows that the latent factors are the main drivers in bond yields' variation and crowd out inflation variables in explaining the term structure of interest rates. This is consistent with the model of Joslin, Priebsch, and Singleton (2010), who impose restrictions on the model coefficients such that the loadings of the yields (or their linear combinations) on macroeconomic variables are zero. In contrast, we do not impose such conditions a priori. We estimate an unconstrained model and find factor loadings on the inflation series that are nearly zero. We then demonstrate using simulated yields and inflation series that our model replicates the empirical linkage between yields and inflation data extremely well.

The model produces estimates for the real term structure of interest rates. We find a real spot rate pattern that is tightly linked to the history of monetary policy intervention. Longer maturity real yields show a much smoother behavior. At all maturities, real rates exhibit a declining pattern since the 1980s.

While we do not use data on Treasury Inflation Protected Securities (TIPS), we compare our real rates estimates to TIPS yields during the sub-sample for which those data are available. In the early years of TIPS trading, TIPS rates are systematically higher than model-implied real rates, with a spread of approximately 150bps at the ten-year maturity in the first quarter of 1999. The spread progressively shrinks to near zero by 2004. This evidence points to the presence of a time-varying liquidity premium in the TIPS market as documented by D'Amico, Kim, and Wei (2010), Fleckenstein, Longstaff, and Lustig (2010), Haubrich, Pennacchi, and Ritchken (2009), and Pflueger and Viceira (2012). More interestingly, the TIPS-real-rate spread widens again during the financial crisis, with a peak immediately after the collapse of Lehman Brothers. This is related to disruptions in the TIPS market, where liquidity dried up in fall 2008 and remained scarce for several months.[1] In contrast, long-term real rates implied by our model remain smooth; only the real spot rate shows a moderate increase in fall 2008 due to heightened short-term deflationary expectations. We obtain these results by estimating our model *solely* on nominal yields and inflation data, without relying on survey- or market-based measures of real rates and expected inflation.

Similar to real rates, the model-implied inflation risk premium is high in the 1980s and declines over time, as in Ang, Bekaert, and Wei (2008) but at odds with Haubrich, Pennacchi, and Ritchken (2009). Most notably, the premium shrinks to zero shortly after 2005, a period during which long-term yields are low in spite of prolonged restrictive monetary policy. Greenspan (2005) refers to this development as a 'conundrum'; our model associates it with low expected future inflation and a reduction in the inflation risk premium. This mechanism is at play again towards the end of our sample period, when the inflation risk premium turns even negative. These results suggest that Treasuries carry significant inflation risk in the 1980s, while they behave close to inflation hedges in recent times, providing insurance against recessions in which deflation risk is high.[2]

The model provides a natural setting to study the pass-through effect of shocks in energy prices on core inflation and the yield curve. We find that energy shocks have had a limited impact on core inflation through the early 2000s. The effect was stronger in the 1980s and declining ever since. A similar pattern applies to conditional and unconditional correlations in shocks to energy and core inflation, except for a moderate increase in these measures in recent years. Not surprisingly, bond yields are largely unaffected by energy shocks.

Finally, we perform a number of robustness checks and explore some technical issues. First, we perform maximum-likelihood estimation using different methods to extract the latent factors (inverting them from a subset of the yields as in Chen and Scott (1993), or estimating them via the Kalman filter). Second, we explore model estimation on different data sets of yields (CRSP zero-coupon rates with maturity up to five years vs. constant-maturity Treasury yields with maturity up to 20 years) and inflation (CPI vs. personal consumption expenditures, or PCE, data). Third, we perform estimation directly on the yields, or on their principal components (as in, e.g., Adrian and Moench (2010), Hamilton and Wu (2011), and Joslin, Singleton, and Zhu (2011)). Fourth, we explore estimation over different sample periods (a long sample going back to 1962Q1 vs. the post-1984 period).

Related Literature Ang, Bekaert, and Wei (2007, 2008) estimate nominal and real term structures for U.S. Treasury rates with no-arbitrage models that include latent factors and one inflation factor (measured by either total or core realized inflation). The authors consider

[1]For instance, a panel of inflation risk professionals convened in New York to discuss developments in the market of inflation-linked products (Risk Magazine, 2009). The panel noted that the TIPS market was disrupted to a point that trading took place only 'by appointment'.

[2]This supports the evidence of Campbell, Sunderam, and Viceira (2011), who find the covariance between stock and bond returns to be positive in the 1980s and negative in the 2000, a change that alters bond risk premia and the shape of the Treasury yield curves. We reach similar conclusions without relying on stock market and TIPS data.

specifications with and without regime switches in the inflation dynamics. They find that term structure information does not generally lead to better inflation forecasts and often leads to inferior forecasts compared to those produced by models that use only aggregate activity measures. Their evidence confirms the results in Stock and Watson (1999), and extends them to a wide array of specifications that combine inflation, real activity, and yield dynamics. The relatively poor forecasting performance of term structure models applies to simple regression specifications, iterated long-horizon VAR forecasts, no-arbitrage affine models, and non-linear no-arbitrage models. They conclude that while inflation is very important for explaining the dynamics of the term structure (e.g., Ang, Bekaert, and Wei, 2008), yield curve information is less important for forecasting future inflation. Yet, the yield curve should reflect market participants' expectations of future consumer price dynamics. We propose a DTSM model that is successful at extracting such information and produces more accurate inflation forecasts.

Several studies incorporate market expectations in fitting real and nominal term structures of interest rates. For instance, Adrian and Wu (2010), Campbell, Sunderam, and Viceira (2011), Christensen, Lopez, and Rudebusch (2010), D'Amico, Kim, and Wei (2010), and Grishchenko and Huang (2010) combine nominal off-the-run yields constructed in Gürkaynak, Sack, and Wright (2007) with TIPS zero-coupon rates from Gürkaynak, Sack, and Wright (2010). Chen, Liu, and Cheng (2010) use raw U.S. TIPS data, while Barr and Campbell (1997) and Hördahl and Tristani (2010) focus on European index-linked bonds. Kim and Wright (2005) and Pennacchi (1991) rely on survey forecasts, while Haubrich, Pennacchi, and Ritchken (2009) introduce inflation swap rates to help identify real rates and expected inflation. In these studies, estimation typically forces the model to match survey- and market-based measures of real rates and expected inflation (TIPS data, survey inflation forecasts, or inflation swaps) up to a measurement error. Hence, model-implied real rates and inflation forecasts inherit the properties of these inputs by construction. In contrast, we propose a model that relies *entirely* on nominal U.S. Treasury and inflation data to jointly estimate real rates, expected inflation for total, core, food, and energy price indices, and the inflation risk premium. Remarkably, our inflation forecasts are in line with the SPF forecasts and outperform the University of Michigan survey; nominal yields forecasts improve upon the SPF. Our estimates for real rates and the inflation risk premium are also in line with related market-based measures.

A vast related literature explores the relation between *nominal* interest rates and the macroeconomy. Early works directly relate current bond yields to past yields and macroeconomic variables using a vector auto-regression approach (e.g., Estrella and Mishkin (1997), and Evans and Marshall (1998, 2007)). This literature has successfully established an empirical linkage between shocks to macroeconomic variables and changes in yields. More recently, several studies have explored similar questions using no-arbitrage dynamic term structure models (e.g., Ang and Piazzesi (2003), Ang, Piazzesi, and Wei (2006), Diebold, Rudebusch, and Aruoba (2006), Duffee (2006), Hördahl, Tristani, and Vestin (2006), Moench (2008), Diebold, Piazzesi, and Rudebusch (2005), Piazzesi (2005), Rudebusch and Wu (2008)). Other contributions have extended these models to include market expectation in the form of survey forecasts (e.g., Chernov and Mueller (2008), Chun (2010), and Kim and Orphanides (2005)).

Recent work explores the role of no-arbitrage and dynamic restrictions in canonical Gaussian affine term structure models (e.g., Joslin, Singleton, and Zhu (2011), Duffee (2011b), and Joslin, Le, Singleton (2011)). These studies question whether no-arbitrage restrictions affect out-of-sample forecasts of yields and macroeconomic factors relative to the forecasts produced by an unconstrained factor model. In our framework, no-arbitrage restrictions allow us to identify the inflation risk premium and therefore to compute real rates, which are both an important part of our analysis. Moreover, our model departs from the canonical Gaussian DTSM class. First, we impose additional restrictions on the physical factor dynamics (Calvet, Fisher, and Wu (2010)) as well as on the interactions between latent and inflation factors. Second, we fix some of the risk premia coefficients at zero. Further, similar to Duffee (2010) we estimate the model under the constraint that conditional maximum Sharpe ratios stay close to their empirical realizations.[3] We confirm that with these restrictions our preferred DTSM outperforms unconstrained VAR models estimated on interest rate and inflation data, including specifications that use core, food, and energy inflation series.

Several scholars study the link between bond risk premia and the macroeconomy (e.g., Cieslak and Povala (2010), Cochrane and Piazzesi (2005), Duffee (2011a), Joslin, Priebsch, and Singleton (2010)). This literature focuses on the predictability of bond returns. We concentrate on no-arbitrage models of the nominal and real term structures, and explore their implications for expected inflation and the inflation risk premium.

The rest of the paper proceeds as follows. Section 2 presents the model. We discuss data and the estimation method in Section 3. Section 4 reports on the time-series and no-arbitrage restrictions favored by the model specification analysis. The empirical results are in Section 5, while Section 6 concludes the paper.

2 The Model

We assume that K_1 latent factors $L_t = \left[\ell_t^1, ..., \ell_t^{K_1}\right]$ and K_2 inflation factors $\Pi_t = \left[\pi_t^1, ..., \pi_t^{K_2}\right]$ describe the time t state of the economy. Collecting the state variables in a vector $F_t = [L_t, \Pi_t]'$, we define the state dynamics via a Gaussian vector auto-regression (VAR) system with p lags,

$$F_t = \phi_0 + \phi_1 F_{t-1} + ... + \phi_p F_{t-p} + \Sigma u_t \,, \tag{1}$$

[3]Joslin, Singleton, and Zhu (2011) conclude that improvements in the conditional forecasts of the pricing factors in Gaussian dynamic term structure models are due to the combined structure of no-arbitrage and $\mathbb{P}$-distribution restrictions. Duffee (2011b) and Joslin, Le, Singleton (2011) reach similar conclusions. We discuss restrictions on factor dynamics, model Sharpe ratios, and risk premia in more detail in Sections 3, 4, and 5.

where ϕ_0 is a $(K_1+K_2)\times 1$ vector of constants and ϕ_i, $i=1,...,p$, are $(K_1+K_2)\times(K_1+K_2)$ matrices with the autoregressive coefficients. The $(K_1+K_2)\times 1$ vector of independent and identically distributed (i.i.d.) shocks Σu_t has Gaussian distribution $N(0,V)$, with $V=\Sigma\Sigma'$.

We stack the contemporaneous unobservable factors, $X_t^u = L_t = \left[\ell_t^1,...,\ell_t^{K_1}\right]$, together with the contemporaneous and lagged observable inflation factors, $X_t^o=[\Pi_t,...,\Pi_{t-(p-1)}]$, in a $K\times 1$ vector $X_t=[X_t^u,X_t^o]'$, where $K=K_1+K_2\times p$. With this notation, we introduce the VAR dynamics in first-order compact form,

$$X_t=\Phi_0+\Phi X_{t-1}+\Omega\varepsilon_t\,, \tag{2}$$

where $\varepsilon_t=[u_t',0,...,0]'$, and the $K\times K$ matrix Ω contains the matrix Σ and blocks of zeros that correspond to the elements of the lagged inflation factors.

2.1 Real Bond Prices

The one-period short real rate, r_t^*, is an affine function of the state vector X_t,

$$r_t^*=\delta_0+\delta_1'X_t\,. \tag{3}$$

The coefficient δ_1 has dimensions $K\times 1$ and is subject to the identifying restrictions, $\delta_1^{\ell^1},...,\delta_1^{\ell^{K_1}}=1$ (e.g., Dai and Singleton (2000)). Moreover, we impose the constraint that the short rate depends only on contemporaneous factor values. That is, we fix the elements of the δ_1 coefficient corresponding to lagged inflation variables at zero, $\delta_1 = \left[\left(\delta_1^{\ell^1},...,\delta_1^{\ell^{K_1}}\right),\left(\delta_1^{\pi^1},...,\delta_1^{\pi^{K_2}}\right),0,...,0\right]'$.

We follow Ang, Bekaert, and Wei (2007, 2008) and specify the real pricing kernel m_{t+1}^* as

$$m_{t+1}^*=\exp\left(-r_t^*-\frac{1}{2}\lambda_t'\lambda_t-\lambda_t'\varepsilon_{t+1}\right)\,, \tag{4}$$

where the market price of risk λ_t is affine in the state vector X_t,

$$\lambda_t=\lambda_0+\lambda_1X_t\,, \tag{5}$$

for a $K\times 1$ vector λ_0 and the $K\times K$ matrix λ_1. Combining equations (3)-(4), we obtain

$$m_{t+1}^*=\exp\left[-\frac{1}{2}\lambda_t'\lambda_t-\delta_0-\delta_1'X_t-\lambda_t'\varepsilon_{t+1}\right]. \tag{6}$$

Given the pricing kernel m_{t+1}^*, the time t price of a real zero-coupon bond with $(n+1)$ periods to maturity is the present expected value of the time $(t+1)$ price of an n-period bond:

$$p_t^{*n+1}=E_t\left[m_{t+1}^*p_{t+1}^{*n}\right]\,. \tag{7}$$

Since the model is affine, equation (7) has solution

$$p_t^{*n}=\exp\left(\bar{A}_n^*+\bar{B}_n^{*\prime}X_t\right)\,, \tag{8}$$

where the coefficients $\bar{A}_n^*$ and $\bar{B}_n^*$ solve the ordinary difference equations (ODEs):

$$\begin{aligned} \bar{A}_{n+1}^* &= -\delta_0 + \bar{A}_n^* + \bar{B}_n^{*\prime}\left(\Phi_0 - \Omega\lambda_0\right) + \frac{1}{2}\bar{B}_n^{*\prime}\Omega\Omega'\bar{B}_n^* \\ \bar{B}_{n+1}^{*\prime} &= -\delta_1' + \bar{B}_n^{*\prime}\left(\Phi - \Omega\lambda_1\right) . \end{aligned} \tag{9}$$

The real short rate equation (3) yields the initial conditions $\bar{A}_1^* = -\delta_0$ and $\bar{B}_1^{*\prime} = -\delta_1'$ for the ODEs (9). Thus, the real yield on an n-period zero-coupon bond is

$$y_t^{*n} = -\frac{\log\left(p_t^{*n}\right)}{n} = A_n^* + B_n^{*\prime} X_t \,, \tag{10}$$

where $A_n^* = -\frac{\bar{A}_n^*}{n}$ and $B_n^* = -\frac{\bar{B}_n^*}{n}$.

2.2 Nominal Bond Prices

If we define Q_t to be the price deflator, then the time t price of a nominal $(n+1)$-period zero-coupon bond, p_t^{n+1}, is given by

$$p_t^{n+1} = p_t^{*n+1} Q_t = E_t\left[m_{t+1}^* \frac{Q_t}{Q_{t+1}} p_{t+1}^{*n} Q_{t+1}\right] = E_t\left[m_{t+1} p_{t+1}^n\right] \,, \tag{11}$$

where, as in Ang, Bekaert, and Wei (2007, 2008), we have defined the nominal pricing kernel m_{t+1} to be

$$m_{t+1} = m_{t+1}^* \frac{Q_t}{Q_{t+1}} = m_{t+1}^* \exp(-\pi_{t+1}) = \exp\left(-r_t^* - \pi_{t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right) . \tag{12}$$

We assume that the inflation rate $\pi_t \equiv \log(Q_t/Q_{t-1})$ at which investors deflate nominal asset prices is a weighted sum of the inflation factors in Π_t, $\pi_t = \sum_{j=1}^{K_2} \omega^j \pi_t^j$, where $0 \leq \omega^j \leq 1$. The Ang, Bekaert, and Wei (2007, 2008) model without regime switches is a special case of this setting, in which the factor Π_t contains a single measure of inflation (either total or core inflation). We obtain this case by fixing the weight associated with a specific inflation factor at one, and setting all other weights at zero.

Considering the state dynamics in equation (2), we define $\Phi_0^\pi = \sum_{j=1}^{K_2} \omega^j \Phi_0^{\pi^j}$, where $\Phi_0^{\pi^j}$ is the element of the vector Φ_0 that corresponds to the inflation factor π^j, $j = 1, \ldots, K_2$. Similarly, consider the $1 \times K$ vectors $\Phi^\pi = \sum_{j=1}^{K_2} \omega^j \Phi^{\pi^j}$ and $\Omega^\pi = \sum_{j=1}^{K_2} \omega^j \Omega^{\pi^j}$, where Φ^{π^j} and Ω^{π^j} are the rows of the Φ and Ω matrices that correspond to the inflation factor π^j. Then, Appendix A shows that nominal bond prices are an affine function of the state vector X:

$$p_t^n = \exp\left(\bar{A}_n + \bar{B}_n' X_t\right) \,, \tag{13}$$

where the coefficients $\bar{A}_n$ and $\bar{B}_n'$ solve the ODEs:

$$\begin{aligned} \bar{A}_{n+1} &= -\delta_0 + \bar{A}_n + \bar{B}_n'\left(\Phi_0 - \Omega\lambda_0\right) - \Phi_0^\pi + \frac{1}{2}\bar{B}_n'\Omega\Omega'\bar{B}_n + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime} + \Omega^\pi\lambda_0 - \bar{B}_n'\Omega\Omega^{\pi\prime} \\ \bar{B}_{n+1}' &= -\delta_1' - \Phi^\pi + \bar{B}_n'\left(\Phi - \Omega\lambda_1\right) + \Omega^\pi\lambda_1 \,, \end{aligned} \tag{14}$$

with initial conditions $\bar{A}_1 = -\delta_0 - \Phi_0^\pi + \Omega^\pi \lambda_0 + \frac{1}{2}\Omega^\pi \Omega^{\pi\prime}$ and $\bar{B}_1' = -\delta_1' - \Phi^\pi + \Omega^\pi \lambda_1$. Thus, the yield on a nominal n-period zero-coupon bond is affine in the state vector,

$$y_t^n = -\frac{\log(p_t^n)}{n} = A_n + B_n' X_t \,, \tag{15}$$

where $A_n = -\frac{\bar{A}_n}{n}$ and $B_n = -\frac{\bar{B}_n}{n}$.

2.3 Benchmark Models

The literature has proposed a wide array of models to forecast inflation (e.g., Stock and Watson (1999, 2003, and 2007)). Of these, the ARMA(1,1) and the random walk have proven particularly resilient in predicting consumer price dynamics over different sample periods. Thus, we consider both of these univariate models for comparison with our term structure specifications. As in Atkeson and Ohanian (2001), the random walk (RW) forecast for an inflation series at any future horizon is the average of the realizations during the past four quarters. The ARMA(1,1) model for an inflation series π^i is

$$\pi_t^i = \mu + \rho\, \pi_{t-1}^i + \varepsilon_t + \theta\, \varepsilon_{t-1} \,. \tag{16}$$

As in Faust and Wright (2012), we also consider various models that combine distinct core, food, and energy series, but leave out interest rates data. First, we construct forecasts for total inflation as a weighted sum of the ARMA(1,1) forecasts of each component, $E_t[\pi_{t+n,n}^{tot}] = \omega^c E_t[\pi_{t+n,n}^c] + \omega^f E_t[\pi_{t+n,n}^f] + \omega^e E_t[\pi_{t+n,n}^e]$, where $\pi_{t+n,n}^i$ denotes inflation realized from t to $t+n$. We term such forecast ARMA$_W$. Second, we use an unconstrained VAR estimated on core, food, and energy inflation data to forecast the three inflation components. Such forecasts recombine into a measure of total expected inflation, as in the ARMA$_W$ case. We label this model VAR$_3$.

Ang, Bekeart, and Wei (2007) argue that inflation surveys outperform other popular forecasting methods (see also Faust and Wright (2009)). Surveys are conducted for a limited number of price series. Whenever available, we include them as additional benchmarks, as described in Section 3 below.

Recent work explores the role of no-arbitrage and dynamic restrictions in canonical Gaussian affine term structure models (e.g., Joslin, Singleton, and Zhu (2011), Duffee (2011b), and Joslin, Le, Singleton (2011)). These studies question whether no-arbitrage restrictions affect out-of-sample forecasts of yields and macroeconomic factors relative to the forecasts produced by an unconstrained factor model. Therefore, as a final benchmark we also consider an unconstrained VAR estimated on interest rates, core, food, and energy inflation data. To obtain $E_t[\pi_{t+n,n}^{tot}]$ forecasts, we weigh the forecasts for the individual inflation components, as in the ARMA$_W$ case.

3 Data and Estimation

We jointly use nominal U.S. Treasury yields and inflation data for model estimation. We consider two sample periods, both ending in December 2011. The first starts in January

1985; it excludes the Fed's monetary experiment of the early 1980s and it is therefore less likely to include different regimes in inflation and interest rates. The second sample period is much longer and begins in January 1962, i.e., the first date from which all data series described below become available. We consider two data sets of Treasury yields:

1. The first data set comprises quarterly observations on zero-coupon yields with maturities of 1, 4, 12, and 20 quarters. The bond yields (4, 12, and 20 quarters maturities) are from the Center for Research in Security Prices (CRSP) Fama-Bliss Discount Bonds file, while the 1-quarter rate is from the CRSP Risk-Free Rates File. All bond yields are continuously compounded.

2. The second data set extends the maturity of available yields up to 30 years; it consists of daily constant-maturity par yields computed by the U.S. Treasury and distributed by the Board of Governors in the H.15 data release. Prior to analysis, we interpolate the par yields into zero-coupon yields using a smoothed spline interpolation, as described in Section A.1 of the Online Appendix.[4] On each day, we construct the term structure of zero-coupon rates from all available yield maturities. However, for model estimation we only use yields with maturities of 1, 3, 5, 10, and 20 years. We then aggregate the daily series at the quarterly frequency. The 1-quarter par yield in the H.15 release becomes available from September 1, 1981. Thus, to allow estimation over a long sample period, we combine the interpolated zero-coupon yield series with maturities from 1 to 20 years with the 1-quarter rate from the Fama CRSP Treasury Bill files. When estimating the model with data post 1984, we confirm that using our interpolation of the 1-quarter zero-coupon rate from the H.15 constant-maturity par yields gives similar results.

Moreover, we focus on two widely used measures of inflation:

1. We collect monthly data on four Consumer Price Indices (CPI) constructed by the Bureau of Labor Statistics (BLS): (1) the total CPI for all Urban Consumers (all items CPI-U); (2) the core CPI (all items less food and energy); (3) the food CPI; and (4) the energy CPI.

2. We also repeat the analysis with Personal Consumption Expenditure (PCE) data released by the Bureau of Economic Analysis (BEA). Similar to the CPI series, we consider total, core, food, and energy PCE indices.

Section A.5 in the Online Appendix describes the main constituents of the core, food, and energy indices and explains the differences between the CPI and PCE series.

All price series are seasonally adjusted. We compute quarterly price indices by averaging over the monthly observations. Growth rates are quarter over quarter logarithmic differences in the index levels. Appendix B explains how we measure the weights ω^c, ω^f, and ω^e associated with the core, food, and energy components.

[4]We confirm that our estimation results are unchanged when we compute zero-coupon rates using a linear term structure interpolation (similar to the unsmoothed Fama-Bliss method).

Table 1 contains CPI and PCE summary statistics for the long (Panel A) and short sample periods (Panel B). The CPI- and PCE-weighted series are the total inflation series computed from their core, food, and energy components using the relative importance weights. Summary statistics for CPI- and PCE-weighted series are nearly identical to those computed for the total CPI and PCE inflation series released by the BLS and the BEA. Moreover, we find that the correlation between CPI and CPI-weighted total inflation series is 99.73% in the post 1984 sample period, while it is 99.59% in the long sample period. For PCE data, the correlation is higher than 99.9% in both sample periods. This evidence shows that our measure of total inflation constructed as a weighted average of the various components is a close proxy to the actual inflation series computed from the total CPI index.

Table 1 also illustrates the difference in persistence across inflation series. For both sample periods, the first-order auto-correlation for CPI-core inflation exceeds 0.83; higher-order auto-correlations remain high. The CPI-food series is much less persistent, with a first-order auto-correlation of 0.48 and 0.63 in the two sample periods, and declining at longer lags. In contrast, the shocks to CPI-energy series are short lived, with a first-order auto-correlations of 0.30 and 0.21 in the two periods. Shocks die away quickly, resulting in second- and third-order correlations that are close to zero or even negative. Consequently, total CPI inflation is less persistent than core inflation, especially since 1984 when shocks to both food and energy prices have become less persistent (Stock and Watson (2007)). This is also evident from Figure 1, which plots the four inflation series over the full sample period. PCE inflation shares similar properties with the CPI series.

For both CPI and PCE series, the core component has a predominant weight in the total inflation index. The average relative importance of CPI core is 0.74 in the long sample period, compared to 0.77 since 1984. The average weights for the PCE-core series are slightly higher and remain stable across sample periods (0.82 and 0.86, respectively). The food CPI component has average weights of 0.18 and 0.15 in the two samples, while the average energy weight is 0.08. In the PCE series, the relative importance weights of food and energy are slightly lower. The weights show limited time variation, with a standard deviation that is very small and nearly zero after 1984. Related, the auto-correlation for these series is high at all lags. In our analysis, we consider three alternative approaches in using the weights series to compute a proxy for total inflation. First, we fix the weights at their average value over the sample period. Second, we fix them at the value observed at the end of the sample. Third, we allow the weights to vary over time.[5] Since there is little time variation in the weights, the three approaches yield similar results. In what follows, we report findings based on the third approach, which allows us to use the most current information at the time we price the bonds in the sample. Hence, for consistency we add a subscript t to denote the time-t value of a relative importance weight.

[5]The bond pricing formula derived in Section 2.2 still holds when weights are time varying, under the assumption that over the life span of the bond the weights remain equal to the value observed at the time we compute their prices. This is a reasonable approximation since there is little time variation in the weights series.

We also collect two sets of survey forecasts of inflation and nominal U.S. Treasury yields that we use to assess the performance of our models:

1. The Michigan survey forecasts based on the Survey of Consumers conducted by the University of Michigan's Survey Research Center. We use the median inflation forecast, which is available since January 1978.

2. The median forecasts from the Survey of Professional Forecasters (SPF) for total CPI inflation; the three-month Treasury bill rate; and the 10-year Treasury bond rate. These series are available since the third quarter of 1981 (CPI inflation and the three-month rate) and the first quarter of 1992 (the 10-year rate). We do not use CPI core, PCE total and core forecasts since they become available only recently, in the first quarter of 2007.

We estimate the benchmark ARMA models by maximum likelihood and the VAR models by ordinary least squares (OLS). As for the term structure models, we use two alternative methods:

1. We apply the Kalman filter to estimate the model via maximum likelihood. The observable variables are the inflation factors Π_t and linear combinations of the nominal bond yields. Our preferred approach is to include the first four principal components extracted from the panel of yields (e.g., Adrian and Moench (2010), Duffee (2011b), Hamilton and Wu (2011), Joslin, Singleton, and Zhu (2011)). As a robustness check, we also estimate the model directly on the cross section of the yields. In either case, we assume i.i.d. zero-mean Gaussian errors on the yields' principal components (or the individual yields), while the inflation factors are measured without error.

2. In the empirical term structure literature, it is common to obtain a measure of the latent state vector from a subset of the bonds in the sample and proceed with maximum likelihood estimation (e.g., Chen and Scott (1993) and many others since them). This method requires arbitrary assumptions on what bonds are priced without error. Nonetheless, we also explore this approach for comparison with previous studies.

Duffee (2010) argues that conditional maximum Sharpe ratios implied by fully flexible four-factor and five-factor Gaussian term structure models are astronomically high. To solve this problem, he estimates the model coefficients with the constraint that the sample mean of the filtered conditional maximum Sharpe ratios does not exceed an upper bound. Similar to Duffee, during estimation we penalize the likelihood function when model parameters produce conditional maximum Sharpe ratios that deviate from empirical realizations. The penalty takes the form of a gamma density for the model-implied conditional maximum Sharpe ratio, computed as a function of the model coefficients. We fix the mean of the gamma distribution at 0.25, a value that Duffee finds to be in line with the Sharpe ratios of U.S. Treasury returns, and its standard deviation at 0.025. In the model estimation, we maximize the sum of the logarithmic likelihood function and its penalty.[6]

4 Model Specifications and Fit

During estimation, we impose time-series and cross-sectional restrictions via constraints on the Φ and Ω matrices in the physical factor dynamics (2) and on λ_0 and λ_1 in the market price of risk, equation (5). Here we outline our baseline case as well as some alternative specifications and special cases.

4.1 Baseline Case: the DTSM$_{3,3}$ Model

In our preferred specification, there are $K_1 \geq 3$ latent factors with a 'recursive' structure similar to Calvet, Fisher, and Wu (2010). In this setting the factors are correlated, with the k^{th} latent factor mean-reverting to the lagged realization of the $(k-1)^{th}$ factor:

$$\ell_t^k = \left(1 - \phi_1^{\ell^k,\ell^k}\right) \ell_{t-1}^{k-1} + \phi_1^{\ell^k,\ell^k} \ell_{t-1}^k + \sigma_{\ell^k} u_t^{\ell^k}\,, \tag{17}$$

where the shocks $u_t^{\ell^k}$, $k = 1, \ldots, K_1$, are uncorrelated. Moreover, as in Calvet, Fisher, and Wu (2010), we impose a non-linear decay structure on the auto-regressive coefficients, $\phi_1^{\ell^k,\ell^k} = \exp\{-\beta_k\}$, $\beta_k = \beta_1\, b^{k-1}$, with $\beta_1 > 0$, $b > 1$ and $k = 1, \ldots, K_1$. This parsimonious representation naturally ranks the latent factors in order of persistence and therefore avoids issues related to possible factors rotations (e.g., Collin-Dufresne, Goldstein, and Jones (2008), Dai and Singleton (2000), Hamilton and Wu (2010), Joslin, Priebsch, and Singleton (2010)).[7]

In the baseline model, the vector of inflation factors contains $K_2 = 3$ components that are measures of core, food, and energy inflation, $\Pi_t = [\pi_t^c,\, \pi_t^f,\, \pi_t^e]$. Market participants deflate nominal asset prices in equation (12) at the total inflation rate, computed as the weighted sum of the three inflation series. That is, $\pi_t = \pi_t^{tot} = \omega_t^c \pi_t^c + \omega_t^f \pi_t^f + \omega_t^e \pi_t^e$, where ω_t^c, ω_t^f, and ω_t^e represent the relative importance of core, food, and energy prices in the total price index.

We allow the lagged latent factors, ℓ_{t-1}^1,..., $\ell_{t-1}^{K_1}$, to have a direct impact on inflation and we assume that the inflation series follow an AR(p) processes with $p \leq 4$. Further, core and food inflation can respond to lagged realizations of energy inflation, π_{t-i}^e, $i = 1, \ldots, 4$. This

[6]This is similar to an approach commonly used in the empirical macro literature for the estimation of state space models via Bayesian methods, e.g., An and Schorfheide (2007).

[7]While there are common elements with Calvet, Fisher, and Wu (2010) term structure model, there are also significant differences. First, our vector of state variables includes inflation series in addition to latent factors. Second, we price both the nominal and real term structures. Third, we allow the real spot rate to depend on all latent factors as well as the inflation variables. This is in contrast to their assumption that the nominal spot rate equals the least persistent latent factor.

gives the following conditional mean dynamics:

$$\begin{aligned} E_{t-1}\left[\pi_t^c\right] &= \phi_0^{\pi^c} + \sum_{k=1}^{K_1} \phi_1^{\pi^c,\ell^k} \ell_{t-1}^k + \sum_{i=1}^{4} \phi_i^{\pi^c,\pi^c} \pi_{t-i}^c + \sum_{i=1}^{4} \phi_i^{\pi^c,\pi^e} \pi_{t-i}^e \\ E_{t-1}\left[\pi_t^f\right] &= \phi_0^{\pi^f} + \sum_{k=1}^{K_1} \phi_1^{\pi^f,\ell^k} \ell_{t-1}^k + \sum_{i=1}^{4} \phi_i^{\pi^f,\pi^f} \pi_{t-i}^f + \sum_{i=1}^{4} \phi_i^{\pi^f,\pi^e} \pi_{t-i}^e \\ E_{t-1}\left[\pi_t^e\right] &= \phi_0^{\pi^e} + \sum_{k=1}^{K_1} \phi_1^{\pi^e,\ell^k} \ell_{t-1}^k + \sum_{i=1}^{4} \phi_i^{\pi^e,\pi^e} \pi_{t-i}^e . \end{aligned} \quad (18)$$

The covariances between shocks to the three inflation series, $\left(\sigma_{\pi^c,\pi^f}, \sigma_{\pi^c,\pi^e}, \sigma_{\pi^e,\pi^f}\right)$, in the matrix V are non-zero, while shocks to the inflation variables are orthogonal to shocks to the latent factors.

We consider market prices of risk in which the elements of the λ_1 matrix in equation (5) are zero, except for those in the first row that load on the latent factors and core inflation. This is the specification used by Duffee (2011a) and is motivated by the findings of Cochrane and Piazzesi (2008), who argue that market prices of risk are earned only in compensation for exposure to shocks in the 'level' factor. Moreover, similar to Duffee (2011a), we allow the elements in λ_0 associated with the latent factors to be non-zero.

In our baseline case, we estimate the DTSM$_{K_1,3}$ with $K_1 = 3$ by maximum likelihood via the Kalman filter. We fit the model on the first four principal components of nominal zero-coupon yields with maturities up to 10 years and inflation series that start in 1985 and end in 2011. In this and all other models discussed below, we fix ϕ_0^π at a value such that the unconditional mean of the inflation process matches the sample mean of the realized inflation series. We check in unreported results that treating ϕ_0^π as a free parameter produces a similar fit. Further, we explore additional restrictions on the state dynamics by using standard specification tests (information criteria and coefficient t-ratios). This analysis favors an AR(1) model for all inflation series, i.e., $\phi_i^{\pi^c,\pi^c} = \phi_i^{\pi^f,\pi^f} = \phi_i^{\pi^e,\pi^e} = 0$ for $i > 1$. We also explore the dependence of core and food on energy inflation. We do not find dependence of current food inflation on lagged energy realizations, i.e., $\phi_i^{\pi^f,\pi^e} = 0$, $i = 1, \ldots, 4$, in equation (18). In contrast, the link between energy and core changes across sample periods. Realizations of energy inflation with one quarterly lag have an impact on current core inflation. The effect is positive and significant in sample periods that start in the early 1960s and end on or after 1985. However, the magnitude of the coefficient declines steadily as the end date of the sample increases, as we document in more detail in Section 5.5. This result indicates the presence of limited pass-through of energy shocks on core inflation that has gradually declined since the 1980s. Realizations of energy inflation with lags higher than one quarter do not have a significant impact on core inflation, i.e., $\phi_i^{\pi^c,\pi^e} = 0$, $i = 2, \ldots, 4$.[8]

With these restrictions, the DTSM$_{3,3}$ model does a very good job at explaining the term structure of interest rates. The three latent factors successfully span the various frequencies

[8]We also consider models in which current core inflation depends on the average of the past four quarterly energy realizations. Our specification tests reject this restriction.

in the yields' fluctuations, which produces a tight fit for the entire yield curve (the root mean squared errors range from 4.2 to 7.1 basis points across yields' maturities). Moreover, a combination of the first lag in core inflation along with lagged realizations of the first and third latent factors explain core inflation fluctuations well. The first latent factor is highly persistent, delivering a distinct tent shape to the conditional mean of the core process. The second and third ones accommodate shorter-lived fluctuations in prices and interest rates.

4.2 Alternative Specifications

The DTSM$_{3,2}$ Model In this second specification, the vector of inflation factors contains both total and core inflation, $\Pi_t = [\pi_t^{tot},\ \pi_t^c]$, and market participants deflate nominal asset prices in equation (12) at the total inflation rate, $\pi_t = \pi_t^{tot}$. That is, π_t is the weighted sum of π_t^{tot} and π_t^c with weights $\omega^{tot} = 1$ and $\omega^c = 0$.

We assume that the conditional mean of core inflation π_t^c follows an AR(1) process and is driven by a combination of the latent factor $\ell_t^1, ..., \ell_t^{K_1}$. Similarly, total inflation, π_t^{tot}, mean-reverts to core inflation, π_t^c, and a linear combination of the same latent factors. In particular, we model the conditional means of core and total inflation as:

$$
\begin{aligned}
E_{t-1}\left[\pi_t^{tot}\right] &= \phi_0^{\pi^{tot}} + \sum_{k=1}^{K_1} \phi_1^{\pi^{tot},\ell^k} \ell_{t-1}^k + \left(1 - \phi_1^{\pi^{tot},\pi^{tot}}\right) \pi_{t-1}^c + \phi_1^{\pi^{tot},\pi^{tot}} \pi_{t-1}^{tot} \\
E_{t-1}\left[\pi_t^c\right] &= \phi_0^{\pi^c} + \sum_{k=1}^{K_1} \phi_1^{\pi^c,\ell^k} \ell_{t-1}^k + \phi_1^{\pi^c,\pi^c} \pi_{t-1}^c\,. \qquad (19)
\end{aligned}
$$

The variance matrix V allows for non-zero cross-correlations among shocks that hit the two inflation processes. We find that allowing for correlation between ℓ_3 and core inflation improves the model fit, while shocks to inflation and the other latent factors are orthogonal.

Similar to the DTSM$_{3,3}$ case, we estimate this model by maximum likelihood via the Kalman filter on the same sample of yields as well as total and core inflation data. During estimation, we explore the following two constraints and find them to be favored by model specification tests. First, we set $\phi_1^{\pi^c,\ell^1} = \left(1 - \phi_1^{\pi^c,\pi^c}\right)$ and, second, we assume that the AR(1) coefficients of core and total inflation follow a non-linear decay structure. In particular, we set $\phi_1^{\pi^c,\pi^c} = \exp\{-\beta_{core}\}$, where $\beta_{core} = \beta_1 b_\pi$ and $b_\pi > 1$. Similarly, for total inflation we have $\phi_1^{\pi^{tot},\pi^{tot}} = \exp\{-\beta_{tot}\}$, where $\beta_{tot} = \beta_{core} b_\pi = \beta_1 b_\pi^2$. This specification resembles the recursive structure adopted for the latent factors in the $K_1 \geq 3$ case, with the additional restriction that the first latent factor determines the central tendency of core inflation. In turn, total inflation reverts back to the more persistent core inflation series. With these restrictions, fitting the conditional mean of core and total inflation requires the estimation of a single new coefficient, b_π, as β_1 is the same coefficient that determines the speed of mean reversion of the first latent factor ℓ^1, $\phi_1^{\ell^1,\ell^1} = \exp\{-\beta_1\}$ in equation (17).

Other Cases For comparison with the previous literature, we also explore models that use univariate measures of inflation (either core or total). We focus on the DTSM$_{2,1}$ without regime switches of Ang, Bekaert, and Wei (2007, 2008).[9] To illustrate the benefits of separately modeling the core from the crust in consumer prices, we extend that model to include the three inflation components (instead of core or total inflation alone) while keeping all other parts unchanged. We label this specification $\mathrm{DTSM}_{2,3}$. We estimate the $\mathrm{DTSM}_{2,1}$ and $\mathrm{DTSM}_{2,3}$ cases using the methods described in Section 3; the results are in the Online Appendix.

5 Empirical Results

Here we report the main empirical findings. First, we use the events of the recent financial crisis to contrast the implications of our baseline $\mathrm{DTSM}_{3,3}$ to those of other models. Second, we discuss the out-of-sample performance of the models in forecasting inflation. Third, we examine the implications of our preferred model for the term structure of real rates and model risk premia. Fourth, we explore the determinants of interest rates and inflation in our baseline DTSM. Fifth, we assess the pass-through effect of energy shocks on core inflation. Sixth, we briefly discuss nominal yields forecasts.

5.1 Dynamic Term Structure Models and the U.S. Financial Crisis

The U.S. financial crisis took a dramatic turn in fall 2008 after the bankruptcy of Lehman Brothers. The total CPI index decreased by 1% in October and 1.7% in November 2008. Energy prices were the main determinant of this decline, with the CPI energy index falling by 8.6% and 17% in the months of October and November. This extreme drop continued the downward pattern in energy prices observed since the previous summer, resulting in a 32.4% total fall from their July 2008 peak. In contrast, core CPI prices declined 0.1% in October, and remained flat in November. These fluctuations in consumer prices produce the extreme 9.41% drop in total inflation that we report in Table 2 for the fourth quarter of 2008, expressed in percent per annum. These extreme events provide a useful framework to develop intuition for the working of different model specifications.

We fit several flavors of our term structure models using data through the end of 2008 and forecast inflation for year 2009.[10] First, we focus on the $\mathrm{DTSM}_{2,1}$, which we estimate on CRSP zero-coupon rates and either total or core CPI as a measure of the inflation factor. This model forecasts total and core CPI inflation to be -5.60% and -0.18% in 2009, respectively (Table 2). Both values are far from the subsequent realizations observed in 2009 (1.47% and 1.73%, respectively). That is, when estimated with total CPI inflation, the model extrapolates the -9.41% inflation rate realized in the fourth quarter of 2008, and predicts strong deflation in 2009. Fitting the model with the less volatile core inflation series produces

[9]Ang, Bekaert, and Wei (2007) show that accounting for regime shifts in inflation and latent factors produces only moderate improvements to the out-of-sample model performance.

[10]Table 2 reports results computed with the most recent CPI data releases, which includes small revisions since fall 2008. Model estimation with real time data as of the beginning of 2009 has given similar results.

a much less dramatic deflation scenario.

This analysis underscores several advantages of modeling the dynamics of the individual inflation components. With the $\mathrm{DTSM}_{2,1}$ we are forced to choose whether bonds are priced out of total or core inflation. Either choice produces forecasts for one series but not for the other. In contrast, jointly modeling three inflation factors, CPI core, food and energy, yields forecasts for total inflation as well as each of its components. Moreover, this approach proves to be more robust to the extreme energy price fluctuations observed during this period. For instance, the $\mathrm{DTSM}_{2,3}$ produces much higher total CPI forecasts for 2009, 0.17% compared to -5.60% for the $\mathrm{DTSM}_{2,1}$ when estimated on the same panel of CRSP yields. This is because the model finds shocks to energy inflation to be short lived. It expects energy prices to decline moderately in 2009, with only a limited pass-though effect on total CPI inflation.

Finally, we estimate our baseline $\mathrm{DTSM}_{3,3}$ on a sample of zero-coupon rates with maturity up to 10 years that starts in the first quarter of 1985. In this case, the model downplays the effect of energy shocks even more when forecasting total and core CPI inflation. The 2009 forecasts are 1.49% and 1.20%, respectively. These forecasts are close to the 2009 observations: The last column of Table 2 shows realized total and core CPI rates of 1.47% and 1.73%. Energy prices show a rebound (a 1.14% projected increase in 2009), while food prices are expected to grow at 3.29%. Both series are much less persistent and more volatile than core CPI. This is consistent with a higher forecast error, as seen from the last column of Table 2.

The bottom row of Table 2 shows model-implied estimates of the five-year real rate as of the end of the sample period, and contrasts them to two popular market-based estimates of real rates, (1) the average five-year zero-coupon rate on TIPS over the fourth quarter of 2008;[11] and (2) the difference between the five-year zero-coupon nominal yield in the fourth quarter of 2008 and trailing 2008 inflation.

Real rates estimates from the $\mathrm{DTSM}_{2,1}$ are as high as 2.92%, in line with the stark deflation outlook predicted by this model. The 2.73% TIPS rate could also indicate a high deflation risk. However, many market participants noticed that the TIPS market was greatly disrupted by the poor liquidity conditions prevalent in financial markets in fall 2008 and deemed TIPS rates to be an unreliable measure of inflationary expectations.[12] Thus, the second market-based real rate estimate of 0.55% in the last column of Table 2 is arguably a more accurate forecast than the TIPS yield. This value is very close to the real rates estimated by our $\mathrm{DTSM}_{2,3}$ and $\mathrm{DTSM}_{3,3}$ with separate core, food, and energy inflation factors (0.42% and 0.68%, respectively).

[11] The data are from the Federal Reserve Board; their staff compute daily TIPS zero-coupon rates using the approach of Gürkaynak, Sack, and Wright (2010).

[12] For instance, on November 9, 2009 Paul Krugman writes in his New York Times blog, The Conscience of a Liberal: "The yield on TIPS shot up after Lehman fell; ordinary bond yields plunged over the same period. Was this a collapse in expected inflation? Not really, or at any rate not mostly: TIPS are less liquid than regular 10-year bonds, so in the rush for liquidity they became very underpriced for a while. Correspondingly, as markets calmed down there was a fall in TIPS yields and a rise in ordinary bond yields; this probably didn't have much to do with changing inflation expectations."

5.2 Inflation Forecasts

We repeatedly estimate the $\text{DTSM}_{K_1,3}$ using quarterly yields data over the period beginning in 1985Q1 and ending on date t, where t ranges from 1998Q4 through 2010Q4.[13] For each set of coefficients obtained with data up to and including quarter t, we forecast core, food, and energy inflation at quarter $t+j$, $j = 1, \ldots, 4$. As in Ang, Bekaert, and Wei (2007), for each series i we sum the four quarterly forecasts to estimate inflation realization over the next year, $\pi^i_{t+4,4} = \pi^i_{t+1} + \pi^i_{t+2} + \pi^i_{t+3} + \pi^i_{t+4}$, where $\pi^i_{t+j} = \log(Q_{t+j}/Q_{t+j-1})$. Moreover, we use the weights ω^c_t, ω^f_t, and ω^e_t to compute a forecast for the realization of total inflation during the next year, $E_t[\pi^{tot}_{t+4,4}] \equiv \omega^c_t E_t[\pi^c_{t+4,4}] + \omega^f_t E_t[\pi^f_{t+4,4}] + \omega^e_t E_t[\pi^e_{t+4,4}]$. We assess the forecast error against realized inflation based on the root mean squared error criterion (RMSE),

$$\text{RMSE} = \sqrt{E[(E_t(\pi^i_{t+4,4}) - \pi^i_{t+4,4})^2]} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(E_t(\pi^i_{t+4,4}) - \pi^i_{t+4,4})^2}\,, \tag{20}$$

for each inflation series i. In particular, for total inflation we compare the forecast $E_t[\pi^{tot}_{t+4,4}]$ with the actual realization, $\pi^{tot}_{t+4,4}$, not with the weighted proxy $\omega^c_t\pi^c_{t+4,4} + \omega^f_t\pi^f_{t+4,4} + \omega^e_t\pi^e_{t+4,4}$. Table 3 reports RMSEs for the $\text{DTSM}_{K_1,3}$ cases with $K_1 = 3$ and 4, estimated on CMT yields with maturity up to 10 years or their first four principal components. Panel A focuses on CPI inflation measures, while Panel B shows results for PCE data. In all cases, maximum likelihood estimation relies on the Kalman filter. For comparison, the table also includes the RMSE corresponding to the ARMA(1,1); the weighted ARMA_W; two unconstrained VAR models estimated on core, food, and energy inflation series alone (the VAR_3) or in combination with interest rates data (the $\text{VAR}_{4,3}$); and the random walk RW (see Section 2.3 for details). Moreover, we report RMSEs for SPF and University of Michigan Survey forecasts.[14]

The Baseline $\text{DTSM}_{3,3}$ Case We first look at the RMSEs for the $\text{DTSM}_{3,3}$ estimated on the yields' principal components. The results are particularly favorable for both total and core CPI inflation. On these two series, the 1.33% and 0.46% RMSEs (expressed in percent per annum) are systematically lower than the RMSE produced by each of the univariate models. For total inflation, the $\text{DTSM}_{3,3}$ produces 15% and 32% improvements over the RMSEs for the ARMA and RW models, respectively. Similarly, there are 19% and 22% declines in the RMSEs for core inflation. The model improves the RMSE for food inflation by at least 9% relative to the ARMA model, while it is at par with the ARMA for energy.

Next, we compare the the $\text{DTSM}_{3,3}$ to benchmark models that rely on the distinct core, food, and energy inflation series. The ARMA_W improves on the ARMA RMSEs for total in-

[13]Out-of-sample results for the longer 1962-2011 estimation period are in the Online Appendix.

[14]Thomas (1999), Mehra (2002), and Souleles (2004) document systematic biases in survey forecasts. Along similar lines, Ang, Bekaert, and Wei (2007) show that, while there are some significant biases in inflation survey forecasts, these biases must be small, relative to the total amount of forecast error in predicting inflation. In fact, they find that raw survey forecasts outperform bias-corrected forecasts. Given their findings, we report results based on raw survey forecasts.

flation slightly. Yet, the $\mathrm{DTSM}_{3,3}$ outperforms it by approximately 11%. The unconstrained VAR_3 performs similar to the ARMA_W on total inflation, however it does poorly on core inflation when compared to the ARMA and, especially, our baseline $\mathrm{DTSM}_{3,3}$. When we extend the VAR to include yields' data (the unconstrained $\mathrm{VAR}_{4,3}$), the $\mathrm{DTSM}_{3,3}$ produces a 14% improvement on total CPI inflation, while it is roughly at par with the $\mathrm{VAR}_{4,3}$ on core inflation.

Not surprisingly, professional forecasters do quite well at forecasting inflation (e.g., Ang, Bekaert, and Wei (2007), Faust and Wright (2009)): The SPF does better than each of the univariate models with a 1.34% RMSE for total inflation. Remarkably, the $\mathrm{DTSM}_{3,3}$ is at par with these results.

To assess whether the difference in RMSEs is statistically significant, we choose the ARMA(1,1) as a benchmark (Stock and Watson (1999) and Ang, Bekaert, and Wei (2007)). We test for equal forecast accuracy using the approach of West (1996), which accounts for parameters estimation error.[15] Table 3 shows p-values computed under the null that the RMSE for the ARMA model equals the $\mathrm{DTSM}_{3,3}$ RMSE. The alternative hypothesis is that the RMSE for the ARMA model exceeds the $\mathrm{DTSM}_{3,3}$ RMSE. The test rejects the null and favors the $\mathrm{DTSM}_{3,3}$ for core inflation. For total inflation the test cannot statistically distinguish the $\mathrm{DTSM}_{3,3}$ from the ARMA model. This is not entirely surprising due to the higher volatility of total vs. core inflation.

Table 3, Panel A, further shows that estimation of the $\mathrm{DTSM}_{3,3}$ directly on the yields gives results nearly identical to the estimation on the yields' first four principal components. Moreover, the $\mathrm{DTSM}_{4,3}$, which includes four latent factors, produces RMSEs that are similar to those of the baseline $\mathrm{DTSM}_{3,3}$.

The $\mathrm{DTSM}_{3,2}$ Case The $\mathrm{DTSM}_{3,2}$ does slightly worse, but is close to the $\mathrm{DTSM}_{3,3}$ on core: The RMSE is 0.49%. This still represents a considerable improvement over the ARMA benchmark. In contrast, the $\mathrm{DTSM}_{3,2}$ performance deteriorates considerably on total inflation (1.55% vs. 1.33% for the $\mathrm{DTSM}_{3,3}$) and is at par with the ARMA (1.57%). These results confirm that separating the frequencies in total inflation helps extract predictive content for inflation from yields data. However, in the $\mathrm{DTSM}_{3,2}$ case the improvement is limited to core forecasts. This is not surprising as the latent factors affect core dynamics in a way similar to the $\mathrm{DTSM}_{3,3}$ case. In contrast, the $\mathrm{DTSM}_{3,3}$ does a better job at modeling the crust by treating food and energy as distinct processes.

PCE vs. CPI Inflation Forecasts The results for PCE inflation series are largely consistent with the evidence on CPI inflation. Namely, for total PCE inflation the $\mathrm{DTSM}_{3,3}$ and $\mathrm{DTSM}_{4,3}$ produce a 12.6% decrease in the RMSE compared to the ARMA case. There is an 8.4% improvement in the food inflation RMSE, while the RMSE for energy is roughly

[15] In all tests for equal forecast accuracy, we compare non-nested models. Thus, West (1996) asymptotic results hold. Note in particular that the four-quarter RW that we use here (e.g., Atkeson and Ohanian (2001)) is not nested in the ARMA(1,1).

at par with the ARMA RMSE. On core inflation, the $DTSM_{4,3}$ gives the best results and it outperforms the ARMA by 4.7%. The $DTSM_{3,2}$ does slightly worse than the $DTSM_{4,3}$ on both core and total inflation; yet, it outperforms the ARMA on total. Compared to the unconstrained VAR, the $DTSM_{4,3}$ produces a 15% improvement in the RMSEs for both total and core inflation.

Sensitivity to the Forecasting Period We compute RMSEs over a grid of out-of-sample windows with start date ranging from 1997Q4 to 2000Q4 and end date from 2002Q4 to 2010Q4. For each window in the grid, we compute core and total inflation RMSEs for the baseline $DTSM_{3,3}$ and ARMA models. Figure 2 plots their percentage ratio, $100 \times$ (RMSE $DTSM_{3,3}$/RMSE ARMA $- 1$). That is, negative numbers in the plot signal that the $DTSM_{3,3}$ outperforms the ARMA. The improvement is most visible for the core inflation forecasts in the top panel. In this case, the $DTSM_{3,3}$ does better than the ARMA nearly 99.5% of the times. The reduction in RMSEs is sizeable except for out-of-sample windows that have an early start date, possibly due to the limited length of the estimation period. For total inflation the evidence is more mixed, with the $DTSM_{3,3}$ outperforming the ARMA 61% of the times.

Core and 'Crust' vs. Univariate Inflation Measures Figure 3 illustrates the improvement compared to a $DTSM_{2,1}$ estimated on univariate CPI inflation (either core or total). The level of the RMSE ratios is much higher than those reported in Figure 2, with the ARMA significantly outperforming the $DTSM_{2,1}$ in nearly all cases. This clearly shows that our core and 'crust' framework for modeling inflation shocks produces an improvement in forecasting performance over the $DTSM_{2,1}$ case that is robust to the choice of the out-of-sample window.

Time-Series and No-Arbitrage Restrictions Finally, Figure 4 compares percentage RMSE ratios for the $DTSM_{3,3}$ (left panels) and the unconstrained $VAR_{4,3}$ (right panels) relative to the ARMA benchmark. We estimate the $VAR_{4,3}$ on the first four principal components of CMT yields with maturities up to 10 years, along with core, food, and energy inflation data. In the top panels, we recursively estimate the model with data starting from 1985Q1; in the bottom panels we consider longer samples that start in 1962Q1. In all panels, RMSEs are for total CPI inflation. The color coding and the grid of out-of-sample windows are as in Figures 2-3. Overall, we find that $DTSM_{3,3}$ does well relative to the unconstrained $VAR_{4,3}$, especially when estimation relies on the long sample period.

Long Horizon Inflation Forecasts Table 4 shows out-of-sample RMSEs associated with inflation forecasts at horizons from one quarter to three years. For all inflation series (total, core, food, and energy), the baseline $DTSM_{3,3}$ model outperforms all benchmarks at every forecasting horizon. The results are strongest for core inflation, for which we cannot reject the null hypothesis that the $DTSM_{3,3}$ is lower than the ARMA RMSE. For the other inflation series the $DTSM_{3,3}$ produces RMSEs that are always considerably lower than the ARMA

RMSEs. For comparison with previous studies, that table also shows RMSEs for the RW and the $DTSM_{2,1}$. The baseline $DTSM_{3,3}$ outperforms these benchmarks as well.

5.3 The Determinants of Interest Rates and Inflation

In this section, we examine the determinants of interest rates and inflation. We first decompose the variance of the forecasting errors into components associated with shocks to the latent factors and inflation. We then study the contemporaneous linkage between yields and inflation.

5.3.1 Variance Decomposition

We now investigate what proportion of the variance of the yields and inflation forecasts is explained by shocks to the latent factors versus the inflation factors in the baseline $DTSM_{3,3}$. Table 5 shows variance decompositions for CPI inflation and yields with one-quarter, five- and ten-year maturity, computed as in Hamilton (1994, p. 323-324).[16]

Panel A shows results for CPI inflation. At the short one-year horizon, innovations in CPI inflation explain most of the variation in CPI forecast errors. As the forecasting horizon increases, innovations to the latent factors become prevalent in driving the variance of the errors. In particular, the latent factors explain more than 50% of the unconditional variation of inflation. Moreover, the three latent factors in the $DTSM_{3,3}$ explain close to 70% of the unconditional variance of CPI core inflation, with the first, most persistent, factor accounting for more than 60%.

Panels B-D report the variance decomposition for yields with maturity of one quarter, five and ten years. In all cases, the latent factors account for the majority of the variation in yields' dynamics. For short horizon forecasts, in the baseline $DTSM_{3,3}$ much of the variation is explained by innovations to the less persistent higher-order factors. As the forecasting horizon increases, the first latent factor takes over, especially in long-maturity yields for which the first latent factor explains up to 94% of the unconditional variation.

Taken together, the results in Table 5 suggest that inflation and interest rates share a common structure of latent factors. This is particularly evident in the $DTSM_{3,3}$, in which inflation shocks play a minimal role in explaining yields dynamics.

5.3.2 Unspanned Inflation Risk

Next, we examine the immediate response of the nominal yield curve to a shock in the state vector. Figure 5 displays the B_n coefficients in equation (15) for $DTSM_{3,3}$, annualized and rescaled to correspond to one standard deviation movement in the factors. The loadings $B_n^{\ell k}$, $k = 1, \ldots, 3$, on the latent factors far exceed B_n^{core}, B_n^{food}, and B_n^{energy} in magnitude,

[16] Shocks to the inflation factors are correlated. Thus, as customary we rely on a Cholesky factorization of the covariance matrix when computing the forecasting errors at different horizons. We order the state variables with the latent factors first, then the core, food, and energy inflation factors. Any other ordering of the state variables gives us virtually identical results.

which confirms that shocks to the latent factors are the main driving force in yield changes. As common in the empirical DTSM literature, ℓ^1 plays the role of a 'level' factor that has an even impact on the yield curve (the $B_n^{\ell^1}$ coefficient is fairly flat across yields' maturities). Shocks to ℓ^2 affect the two-year yield the most and have a lower impact on the short and long end of the term structure, while shocks to ℓ^3 mostly affect short term rates. These features are specific to the cascade structure (17), in which factors are ranked based on their frequencies and response functions to latent factors' shocks are hump shaped, except for the highest frequency latent factor (Calvet, Fisher, and Wu (2010)).

In contrast, the loadings on the core, food, and energy variables are close to zero across bond maturities. This suggests that, in the model, the contemporaneous relation between innovations in yields and inflation is tenuous, consistent with the variance decomposition evidence discussed in Section 5.3.1.

To clarify these results, we examine the empirical relation between interest rates and inflation and compare the evidence to the predictions of our model. For each yield maturity n, Table 6 shows results for the OLS regressions:

$$\text{Levels}: \quad y_t^n = \alpha + \beta^c \pi_t^c + \beta^f \pi_t^f + \beta^e \pi_t^e + \varepsilon_t^n \tag{21}$$

$$\text{Changes}: \quad \Delta y_t^n = \beta^c \Delta\pi_t^c + \beta^f \Delta\pi_t^f + \beta^e \Delta\pi_t^e + \varepsilon_t^n\,. \tag{22}$$

We first estimate the regressions for each yield series y_t^n with maturity n equal to one quarter, one, three, five, and ten years against core, food, and energy inflation sampled quarterly from 1985Q1 to 2011Q4. For each regression, we report coefficient estimates and Newey-West heteroskedasticity and autocorrelation robust standard errors (in brackets). Next, we simulate 10,000 samples of quarterly yields with the same maturities as well as core, food, and energy inflation series from the baseline DTSM$_{3,3}$ using the scheme described in Section A.4 of the Online Appendix. We estimate the same regressions on each simulated sample and report mean, 5th, 50th, and 95th percentiles of the estimated coefficients. The results in the left-hand side of the table are for regressions in levels (the table omits the estimate of the intercept α); those in the right-hand side are for regressions in changes.

Table 6 shows that the DTSM$_{3,3}$ predicts a link between inflation and yields that closely matches the features of the data:

1. When estimating regressions of the levels of yields on inflation (equation (21)), we find the coefficient β^c to be significant, while β^f and β^e are not statistically different from zero. The point estimates of these coefficients are close to the mean of the coefficients computed in simulated data and always fall within the simulated 90% confidence intervals.

2. When estimating regressions of the changes in yields on changes in inflation (equation (22)), all coefficients β^c, β^f, β^e are insignificant, with point estimates that are close to zero in magnitude. Model simulations lead to identical conclusions.

3. In the data, the adjusted R^2 coefficients for regressions in levels always exceed 50%, and are nearly 70% for long-maturity yields. Interpreting these results requires caution,

as the regressions include persistent variables; autocorrelation in the residuals could produce spuriously high R^2 coefficients (Granger and Newbold (1974)). Indeed, regressions in changes have adjusted R^2 coefficients that are nearly zero. Model simulations reproduce this evidence closely: 90% confidence intervals include the R^2 estimated in the data. Figure 6 provides a visual illustration of the R^2 distributions.

To accommodate these features, Joslin, Priebsch, and Singleton (2011) propose a DTSM with unspanned macroeconomic risk. They impose restrictions on the model coefficients such that the loadings of the yields (or their linear combinations) on macroeconomic variables are zero. In contrast, we do not impose such conditions a priori. We estimate an unconstrained model and find factor loadings on the inflation series that are nearly zero. The evidence we provide above shows that our model replicates the empirical linkage between yields and inflation data extremely well.[17]

5.4 Real Rates and the Inflation Risk Premium

Here we explore the time-series and term-structure properties of real rates computed using our model and compare them to those of TIPS rates. We then examine the patterns in the model-implied inflation risk premium.

5.4.1 The Time-Series of Real Rates: Model vs. TIPS

Figure 7 shows the one-quarter (spot) real rate estimated with the baseline $\text{DTSM}_{3,3}$, while Figure 8 depicts five- and ten-year real rates. Although we do not include TIPS in the data set used for estimation, Figure 8 also provides a comparison between model-implied real yields and matching-maturity TIPS rates during the sub-sample for which TIPS data are available.[18]

The pattern in the real spot rate is quite intuitive and tightly linked to monetary policy intervention. Just like the Federal funds rate, the spot real yield increases during periods of expansion and declines during recessions. In particular, there is a pronounced rise since the mid-2000s followed by a decline during the most recent crisis.

The long-maturity real yields in Figure 8 show a smoother pattern with a common downward trend. DTSM yields are considerably lower than TIPS rates during the early part of the TIPS sample period. In 1999Q1, the spread is approximately 150 bps at the ten-year maturity. This is consistent with a high liquidity premium embedded in TIPS when their trading began in the late 1990s. As TIPS liquidity conditions improve, the spread narrows and is near zero around 2004. These results are in line with the findings of D'Amico, Kim, and Wei (2010) for the 1999-2007 period and with the evidence that Pflueger and Viceira

[17] The notion of unspanned macroeconomic risk is also supported by the evidence in Duffee (2011a), who detects the presence of a 'hidden' factor that is not explained by the term structure of yields and correlates, albeit weekly, with macroeconomic variables.

[18] The TIPS rates are zero-coupon yields interpolated by the staff at the Federal Reserve Board, using the approach of Gürkaynak, Sack, and Wright (2010).

(2012) gauge from TIPS excess returns. The bottom panel in Figure 8 shows that the five- and ten-year liquidity premia share similar patterns. However, the five-year premium exceeds the ten-year one on average. The two diverge even more during the last recession, indicating that liquidity disruptions had different impacts on the two segments of the TIPS market. Taken together, these results suggest that expected inflation measures backed out from nominal and TIPS yields (break-even inflation rates) can be severely biased. Moreover, the liquidity differential at five- and ten-year maturities can also bias forward measures of expected inflation (e.g., the so-called five-year five-year forward break-even rate).

TIPS rates increase starkly in 2008Q4. As mentioned previously, market participants attribute this pattern to dislocation in TIPS markets following Lehman Brothers' bankruptcy, rather than to an extreme increase in deflationary expectations with a prolonged impact that extends to the five- and ten-year horizons. The model agrees with this interpretation and, as discussed in Section 5.1, it downplays the impact of the big negative shock in energy prices observed in 2008Q4 on expected inflation. Thus, long-maturity real DTSM yields in Figure 8 continue their decline through the recession. In contrast, the real spot rate in Figure 7 has a moderate uptick in 2008Q4, which suggests a short-run expected decline in consumer prices associated with the negative energy shock.

5.4.2 The Term Structure of Real Rates

Figure 9 shows the nominal and real term structures computed using the baseline DTSM$_{3,3}$. The nominal term structure is upward sloping, as is well-known. Of more interest, we find the real term structure to be upward sloping as well. This is consistent with empirical evidence from U.S. TIPS data[19] and the theoretical implications of several asset pricing models. For instance, Campbell and Cochrane (1995) and Wachter (2006) find an upward sloping real term structure in an exchange-economy in which the representative agent displays habit persistence. The long-run risk model of Bansal and Yaron (2004) predicts a downward sloping real term structure; however, Yang (2011) finds the opposite when durable consumption contains a persistent predictable component, while nondurables and services follow a random walk.

5.4.3 The Inflation Risk Premium

Figure 10 displays the five- and ten-year inflation risk premium computed as

$$\mathrm{IRP}_t^n \equiv y_t^n - E_t[\pi_{t+n,n}^{tot}] - y_t^{*n}, \tag{23}$$

where $E_t[\pi_{t+n,n}^{tot}] \equiv \sum_{i=1}^{K_2} \omega_t^i E_t[\pi_{t+n,n}^i] = \sum_{i=1}^{K_2} \sum_{j=1}^{n} \omega_t^i E_t[\pi_{t+j}^i]$ is the time-t expectation of total inflation over the next n periods, $n = 20$ and 40 quarters (5 and 10 years). As in prior studies (e.g., Ang, Bekaert, and Wei (2008) and Buraschi and Jiltsov (2005)), the

[19] Data on nominal and inflation-indexed U.K. government bonds, however, tell a different story. For instance, Pflueger and Viceira (2011; Table 2, Panel B) find downward sloping term structures for *both* nominal and real yields over the 1985/4-2009/12 sample period.

IRP is positive on average and has a downward pattern since the mid-1980s.[20] This could reflect the Federal Reserve's effort to control inflation and its success in shaping market's expectations on consumer price dynamics. In recent years, at times the risk premium shrinks to zero. A notable example is the period of prolonged monetary tightening following 2004. In spite of an increase in nominal and real spot rates (Figure 7), nominal long-maturity yields remained low during that period, a development that Greenspan (2005) famously described as a 'conundrum.' The model fits the shift in the slope of the yield curve and associates it with a reduction in the long term inflation risk premium. This mechanism is at play again towards the end of our sample period, when the inflation risk premium turns negative. These findings support the view that the risk profile of U.S. Treasuries has changed over time. In the early 1980s, long-maturity bonds carry a high risk premium, possibly associated with uncertainty about future inflation. More recently, there are times when Treasuries act as hedges, providing safe-haven protection against recessions in which deflationary risk is perceived to be high.

The risk premia measures in Figure 10 are obtained from a DTSM model estimated on nominal yields and inflation data alone. Yet, the pattern in our estimates is related to the evidence in studies that focus on the comovements between Treasuries and stock market returns. For instance, Campbell, Sunderam, and Viceira (2011) find the covariance between nominal U.S. Treasury bond returns and stock returns to be unusually high in the early 1980s and even negative in the 2000s.

5.5 Energy Pass Through

To expand on the discussion in Section 4, Figure 11 shows various measures of correlation between energy and core inflation. We estimate the $\text{DTSM}_{3,3}$ over samples with start date of 1962Q1 and end dates ranging from 1985Q1 to 2011Q4. For each sample period, in the top left panel we report the estimate for the $\phi_1^{\pi^c,\pi^e}$ coefficient that links lagged realization of energy inflation to core inflation, along with 90% confidence bands. In sample periods ending in the 1980s through the early 2000s, the coefficient is positive and significant. However, it is small in size and declining as the end date of the sample period increases. For a 100bps increase in lagged energy inflation, there is at most a 3-4bps pass through to core inflation.

The right top panel complements these results by showing unconditional and conditional correlations between energy and core inflation. Both measures are positive. The unconditional correlation estimate shares a downward trend with the $\phi_1^{\pi^c,\pi^e}$ coefficient. However, it remains positive over the entire period and shows an uptick when the model is estimated including the most recent data. Such increase is driven by a higher estimate for the conditional correlation. In recent times, shocks to energy inflation show a larger direct impact on

[20]This is at odds with Haubrich, Pennacchi, and Ritchken (2009), who find small fluctuations in the ten-year inflation risk premium around a constant positive level, and a negative two-year inflation risk premium throughout their sample period. Our results also differ from Adrian and Wu (2010), who report a positive inflation risk premium that peaks in fall 2008.

core dynamics.

The two bottom panels of Figure 11 depict the same correlation measures estimated using data samples starting in 1985Q1 and with end dates ranging from 1995Q1 to 2011Q4. The shorter sample size results in a less precise estimate of the $\phi_1^{\pi^c,\pi^e}$. Nonetheless, we observe a similar decrease in the energy pass through on core over time. Moreover, extreme energy shocks in the 2000s weigh more heavily in the estimates for the conditional correlations between core and energy inflation, resulting in a larger uptick in the unconditional correlations at the end of the sample.

These results extend the analysis of, e.g., Clark and Terry (2010), Hooker (2002), and Stock and Watson (2010) to a DTSM setting. One can interpret the decline in the energy pass through as a results of multiple factors. For instance, Stock and Watson (2010) argue that energy is a smaller share of expenditures than it was during the oil price shocks of the 70s, labor union membership has declined sharply over the past forty years, and there has been a shift from production of goods to production of services.

5.6 Nominal Yields Forecasts

Our baseline $\text{DTSM}_{3,3}$ does well at forecasting nominal Treasury yields when compared to other DTSM specifications, time-series models such as the ARMA and the random walk, as well as SPF forecasts. To save on space, we provide detailed results in Section A.6 of the Online Appendix. It is plausible that extending our framework to include other factors (e.g., a measure of real activity or the Cochrane and Piazzesi (2005, 2008) tent-shaped linear combination of forward rates) would further improve the nominal yields' forecasts (e.g., Ang and Piazzesi (2003), Joslin, Priebsch, and Singleton (2010)). Since our focus is on modeling the joint dymanics of inflation and interest rates, we point the reader to those studies for more details.

6 Conclusions

Much of the empirical macro-finance literature finds that financial variables contain little predictive content for consumer price inflation. Nonetheless, this conclusion is at odds with the intuition that the yield curve reflects market participants' expectations of future price dynamics. This leaves us with the challenge to improve conventional models and estimation methods to jointly fit term structure and inflation data. Our DTSM makes a step in this direction.

A key feature of the model is to separately specify the dynamics of the three main components of total inflation: core, food, and energy. These series combine into a measure of total inflation that we use to price Treasury bonds. This approach captures the different degree of persistence and volatility in shocks to the three inflation components. In particular, it downplays the role of short-lived fluctuations in energy prices in determining expectations of future inflation.

When we estimate the model on a panel of nominal Treasury yields and the three inflation measures, we find a considerable improvement in fit compared to DTSM specifications that rely on a single inflation factor (either total or core). The model does especially well at forecasting CPI core inflation and it often outperforms an ARMA(1,1) model on total inflation.

Energy shocks have a limited pass-through on inflation forecasts and interest rates. In contrast, a common structure of latent factors explains most of the variance of the forecasting error for core inflation, as well as for bond yields. Taken together, all this evidence suggests that our framework helps us to extract predictive content from the yield curve to forecast future inflation.

While we do not use market-based expectations of inflation during estimation, the predictions of our model are consistent with such measures. In particular, our inflation forecasts are in line with the Survey of Professional Forecasters and outperform the University of Michigan inflation survey. Moreover, we find that model-implied real rates are nearly identical to TIPS yields except for (1) the early years of TIPS trading and (2) the recent financial crisis, when liquidity problems were prevalent in the U.S. inflation-indexed Treasury market. Remarkably, we reach these conclusions without relying on either TIPS data (as in D'Amico, Kim, and Wei (2010)) or inflation swaps prices (as in Haubrich, Pennacchi, and Ritchken (2009)) for estimation. Finally, the inflation risk premium has a downward sloping pattern; it shrinks to zero in recent years and even turns negative towards the end of our sample period.

A Nominal Bond Prices

The price of a one-period nominal zero-coupon bond is:

$$\begin{aligned} p_t^1 &= E_t\left[m_{t+1}\right] = E_t\left[\exp\left(-r_t^* - \pi_{t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\right] \\ &= E_t\left[\exp\left(-\delta_0 - \delta_1'X_t - \sum_{j=1}^{K_2}\omega^j\pi_{t+1}^j - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\right] \\ &= \exp\left(-\delta_0 - \delta_1'X_t - \Phi_0^\pi - \Phi^\pi X_t - \frac{1}{2}\lambda_t'\lambda_t\right) E_t\left[\exp\left(-(\lambda_t' + \Omega^\pi)\varepsilon_{t+1}\right)\right]. \end{aligned} \tag{24}$$

Since $\varepsilon_{t+1} \sim N(0, I)$, then $E_t[\exp\left(-(\lambda_t' + \Omega^\pi)\varepsilon_{t+1}\right)] = \exp\left(\frac{1}{2}(\lambda_t' + \Omega^\pi)(\lambda_t' + \Omega^\pi)'\right)$. Substituting in equation (24) and rearranging terms we obtain

$$p_t^1 = \exp\left(-\delta_0 - \delta_1'X_t - \Phi_0^\pi - \Phi^\pi X_t + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime} + \Omega^\pi(\lambda_0 + \lambda_1 X_t)\right) = \exp\left(\bar{A}_1 + \bar{B}_1'X_t\right), \tag{25}$$

where $\bar{A}_1 = -\delta_0 - \Phi_0^\pi + \Omega^\pi\lambda_0 + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime}$ and $\bar{B}_1 = -\delta_1' - \Phi^\pi + \Omega^\pi\lambda_1$.

Assume now that equation (13) prices a nominal n-period zero-coupon bond. Then, the same formula prices an $(n+1)$-period bond. To verify this claim, combine equations

(11)-(13):

$$
\begin{aligned}
p_t^{n+1} &= E_t\left[\exp\left(-r_t^* - \sum_{j=1}^{K_2}\omega^j\pi_{t+1}^j - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1} + \bar{A}_n + \bar{B}_n' X_{t+1}\right)\right] \\
&= \exp\left(-\delta_0 - \Phi_0^\pi + \bar{B}_n'\Phi_0 - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_n + \left(-\delta_1' - \Phi^\pi + \bar{B}_n'\Phi\right)X_t\right) \\
&\quad \times E_t\left[\exp\left(\left(-\lambda_t' - \Omega^\pi + \bar{B}_n'\Omega\right)\varepsilon_{t+1}\right)\right] \\
&= \exp\Bigg(-\delta_0 + \bar{A}_n + \bar{B}_n'\left(\Phi_0 - \Omega\lambda_0\right) - \Phi_0^\pi + \frac{1}{2}\bar{B}_n'\Omega\Omega'\bar{B}_n + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime} + \Omega^\pi\lambda_0 - \bar{B}_n'\Omega\Omega^{\pi\prime} \\
&\quad + \left(-\delta_1' - \Phi^\pi + \bar{B}_n'\left(\Phi - \Omega\lambda_1\right) + \Omega^\pi\lambda_1\right)X_t\Bigg). \qquad (26)
\end{aligned}
$$

We collect terms linear in X_t and independent of X_t to obtain the ODEs (14).

B Core, food, and energy weights

Market participants deflate nominal asset prices in equation (12) at the total inflation rate, π_t. In the model that has three inflation factors, we compute π_t as the weighted sum of the core, food, and energy inflation series. That is, $\pi_t = \pi_t^{tot} = \omega_t^c\pi_t^c + \omega_t^f\pi_t^f + \omega_t^e\pi_t^e$, where the weights ω_t^c, ω_t^f, and ω_t^e represent the relative importance of core, food, and energy prices in the total price index at time t. This appendix describes how we construct such weights.

B.1 Consumer price index weights

For the CPI weights we use the relative importance of core, food, and energy in the CPI reported by the Bureau of Labor Statistics (BLS). The relative importance of a component is the percentage share of the expenditure on that component relative to the expenditure on all items within an area. The BLS conducts a Consumer Expenditure Survey to determine how these shares change over time to reflect fluctuations in the consumption patterns of the population. Each year since 1987, the BLS releases the December value of these series based on the core, food, and energy consumption baskets for that year. Monthly fluctuations in prices result in changes in the relative importance shares for these baskets compared to the values reported the previous December. To account for this pattern, we update the value of the December shares to obtain monthly series that reflect the changes in the cost to purchase the same food, core, and energy baskets. The BLS Internet site at http://www.bls.gov/cpi/cpi_riar.htm explains in details how to do that. The BLS does not make relative importance shares broadly available for years prior to 1987. We thank the BLS for sharing such data with us.

B.2 Personal consumption expenditures weights

Similar to the CPI weights, the PCE weights are the shares of the expenditures on the core, food, and energy baskets relative to total personal consumption expenditures. To compute these shares, we use data from the national income and product account (NIPA) Table 2.3.5U, Personal Consumption Expenditures by Major Type of Product and by Major Function. The variables are (1) Personal consumption expenditures; (2) Personal consumption expenditures excluding food and energy; (3) Food and beverages purchased for off-premises consumption; and (4) Energy goods and services.

Figures and Tables

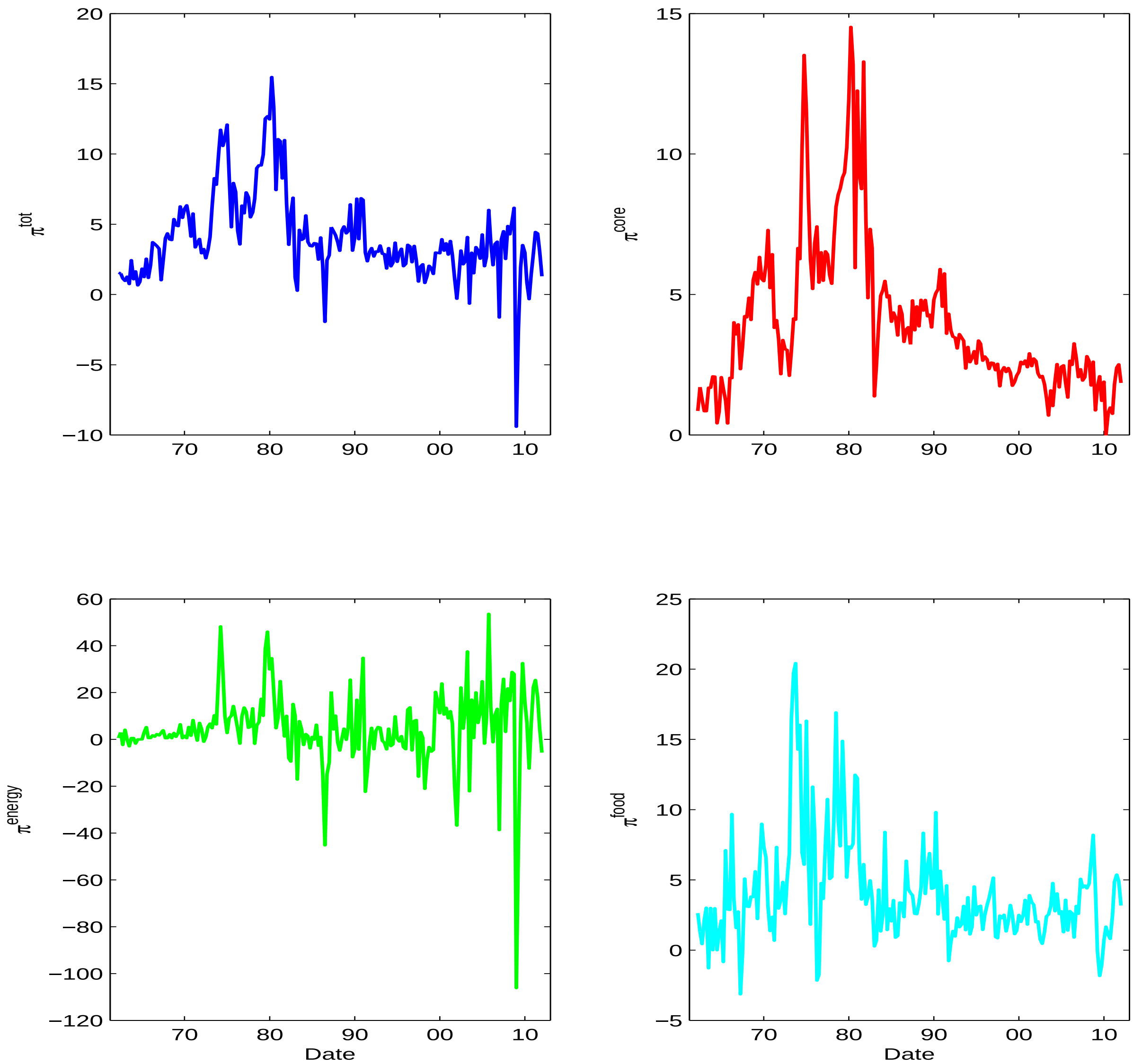
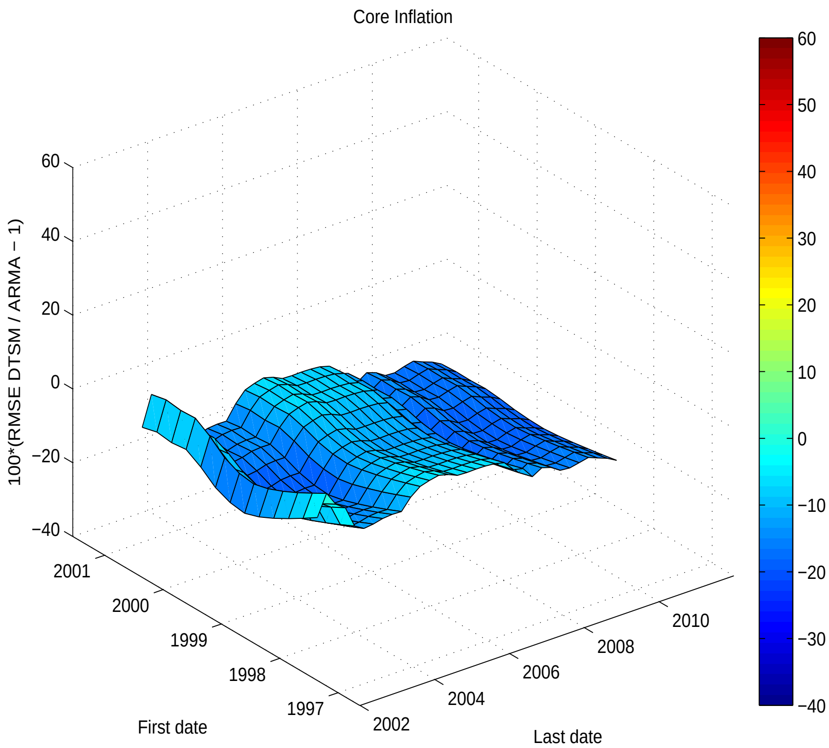


Figure 1: **CPI Inflation Series.** The plots depict total, core, food, and energy quarterly CPI inflation series. The sample period is 1962Q1-2011Q4.



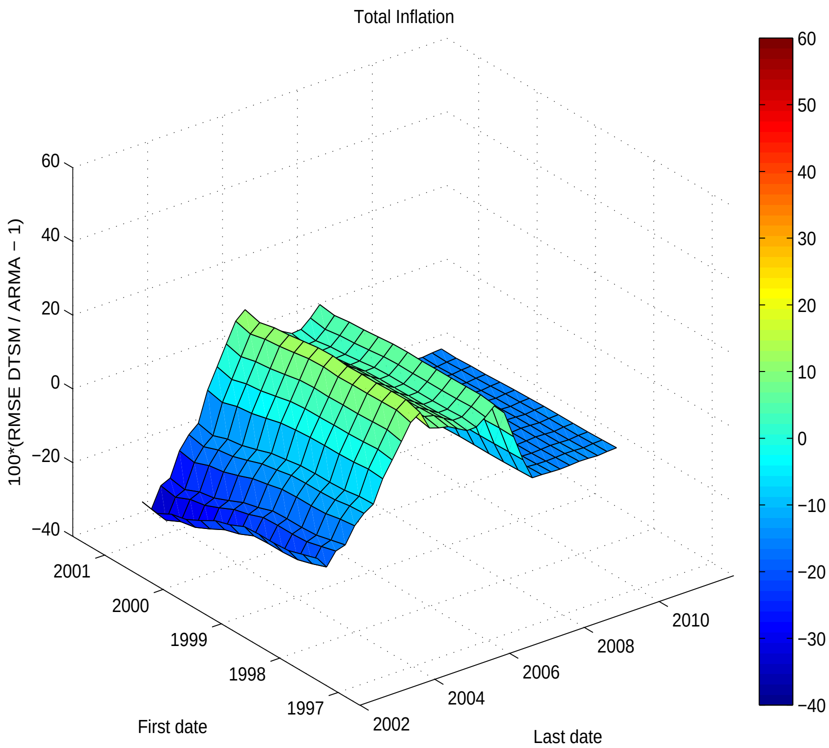


Figure 2: **RMSE Percentage Ratios: DTSM$_{3,3}$.** For the DTSM$_{3,3}$ and ARMA models, we compute RMSEs over a grid of out-of-sample windows with start date ranging from 1997Q4 to 2000Q4 and end date from 2002Q4 to 2010Q4. The figure displays their percentage ratio, $100 \times (\text{RMSE DTSM}_{3,3}/\text{RMSE ARMA} - 1)$. Negative numbers in the plot signal that the DTSM outperforms the ARMA.

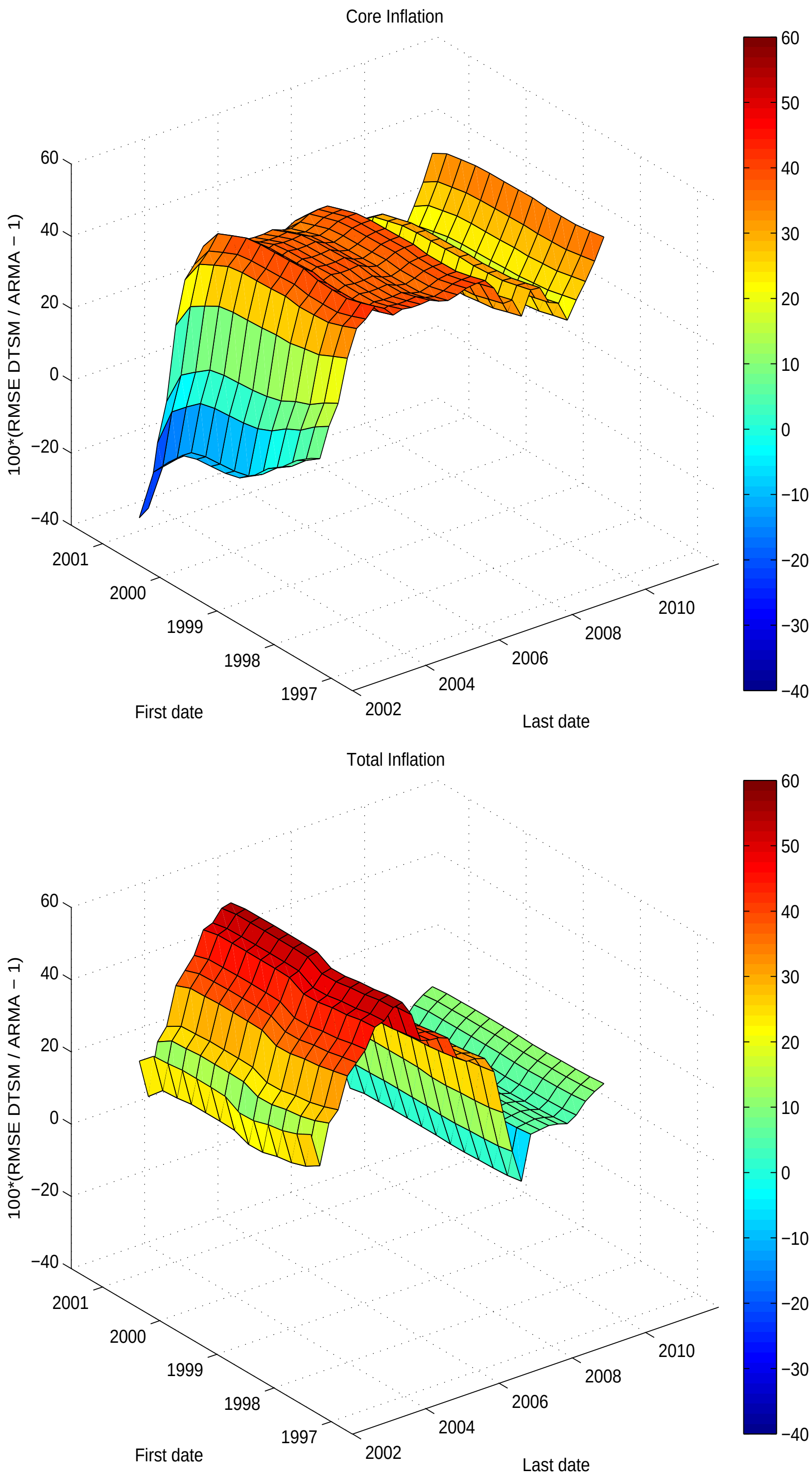


Figure 3: **RMSE Percentage Ratios: DTSM$_{2,1}$.** For the DTSM$_{2,1}$ and ARMA models, we compute RMSEs over a grid of out-of-sample windows with start date ranging from 1997Q4 to 2000Q4 and end date from 2002Q4 to 2010Q4. The figure displays their percentage ratio, $100 \times (\text{RMSE DTSM}_{2,1}/\text{RMSE ARMA} - 1)$. Negative numbers in the plot signal that the DTSM outperforms the ARMA.

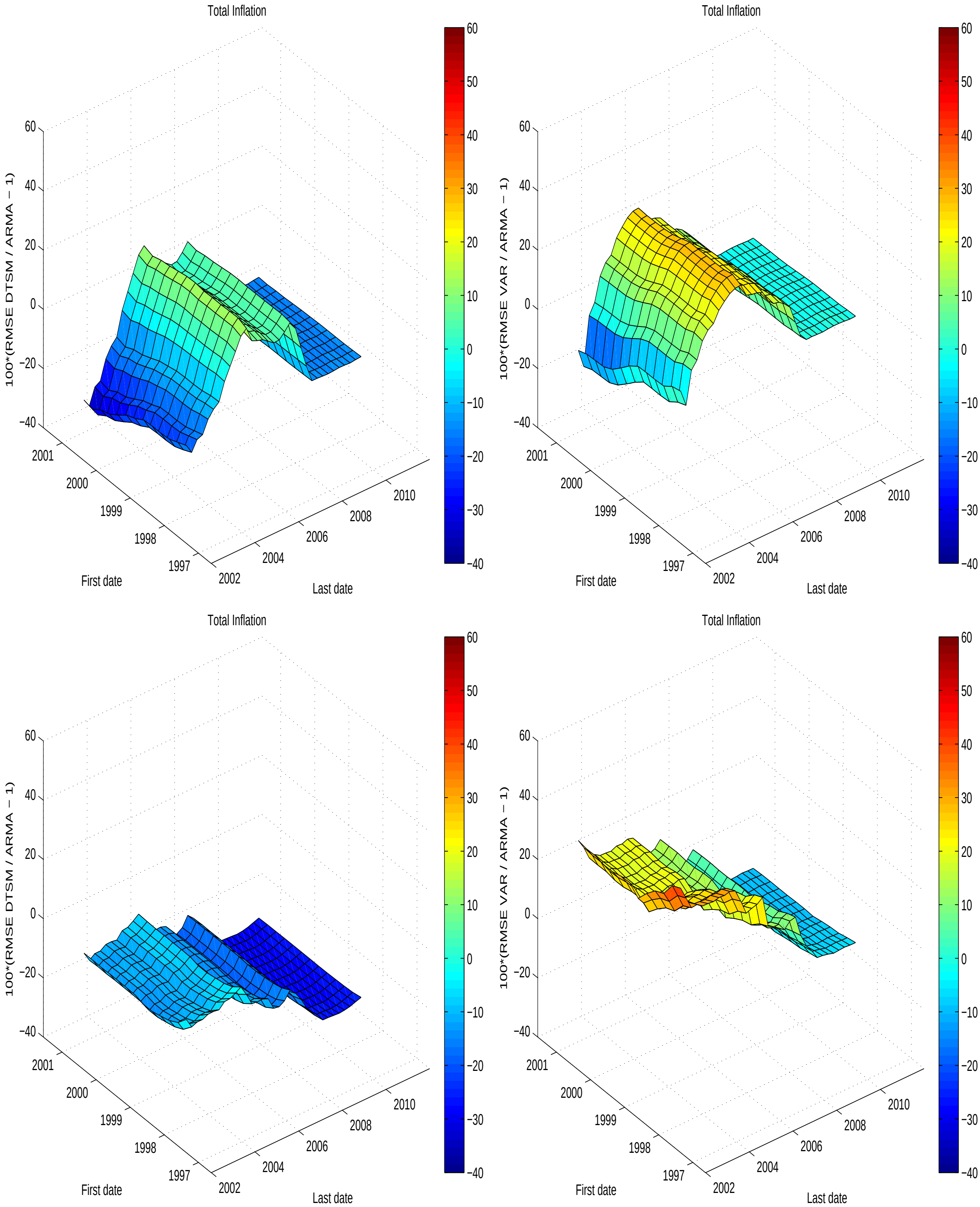


Figure 4: **RMSE Percentage Ratios: $\text{DTSM}_{3,3}$ vs. $\text{VAR}_{4,3}$.** We compute RMSEs for the $\text{DTSM}_{3,3}$, the unconstrained $\text{VAR}_{4,3}$, and the ARMA(1,1) total inflation forecasts over a grid of out-of-sample windows with start date ranging from 1997Q4 to 2000Q4 and end date from 2002Q4 to 2010Q4. The figure displays the percentage ratios $100 \times (\text{RMSE DTSM}_{3,3} \text{ or } \text{VAR}_{4,3}/\text{RMSE ARMA} - 1)$. In the left panels, the percentage ratio has the RMSE for $\text{DTSM}_{3,3}$ in the numerator; in the right panels, it has the RMSE for the $\text{VAR}_{4,3}$. Negative numbers in the plot signal that the $\text{DTSM}_{3,3}$ / $\text{VAR}_{4,3}$ models outperforms the ARMA. In the top panels, the sample period starts in 1985Q1; in the bottom panels, it starts in 1962Q1.

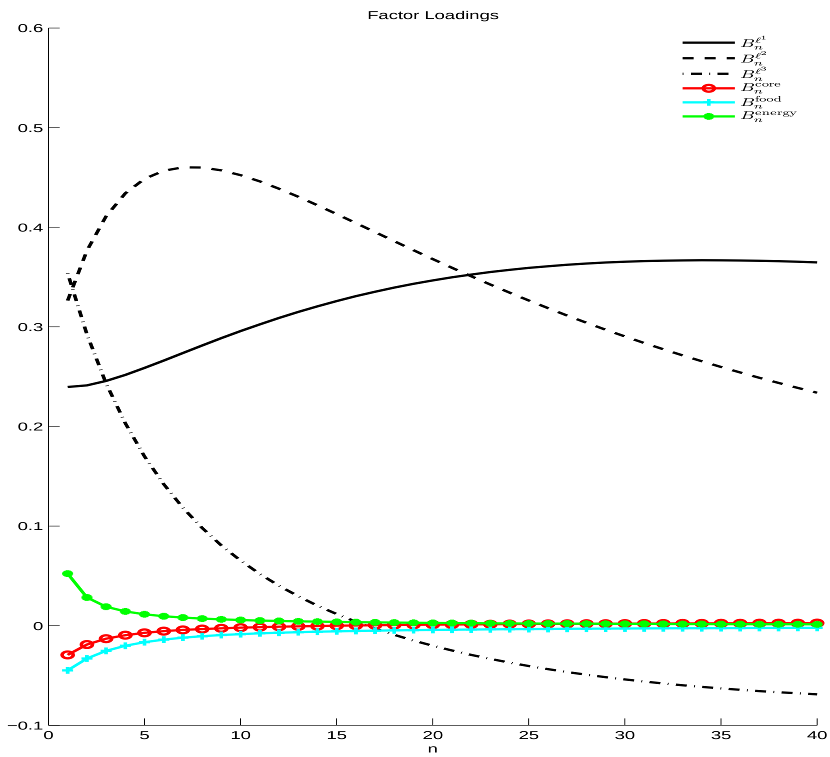


Figure 5: **Factor Loadings.** The plot depicts the factor loadings for nominal yields on the latent factors ($B_n^{\ell k}, k = 1, \ldots, K_1$) and inflation factors (B_n^{core}, B_n^{food}, and B_n^{energy}, where n denotes quarters to maturity) computed with the baseline $\text{DTSM}_{3,3}$. We scale the factor loadings to correspond to one standard deviation movement in the factors and we annualize them by multiplying by 400. The sample period is 1985Q1-2011Q4.

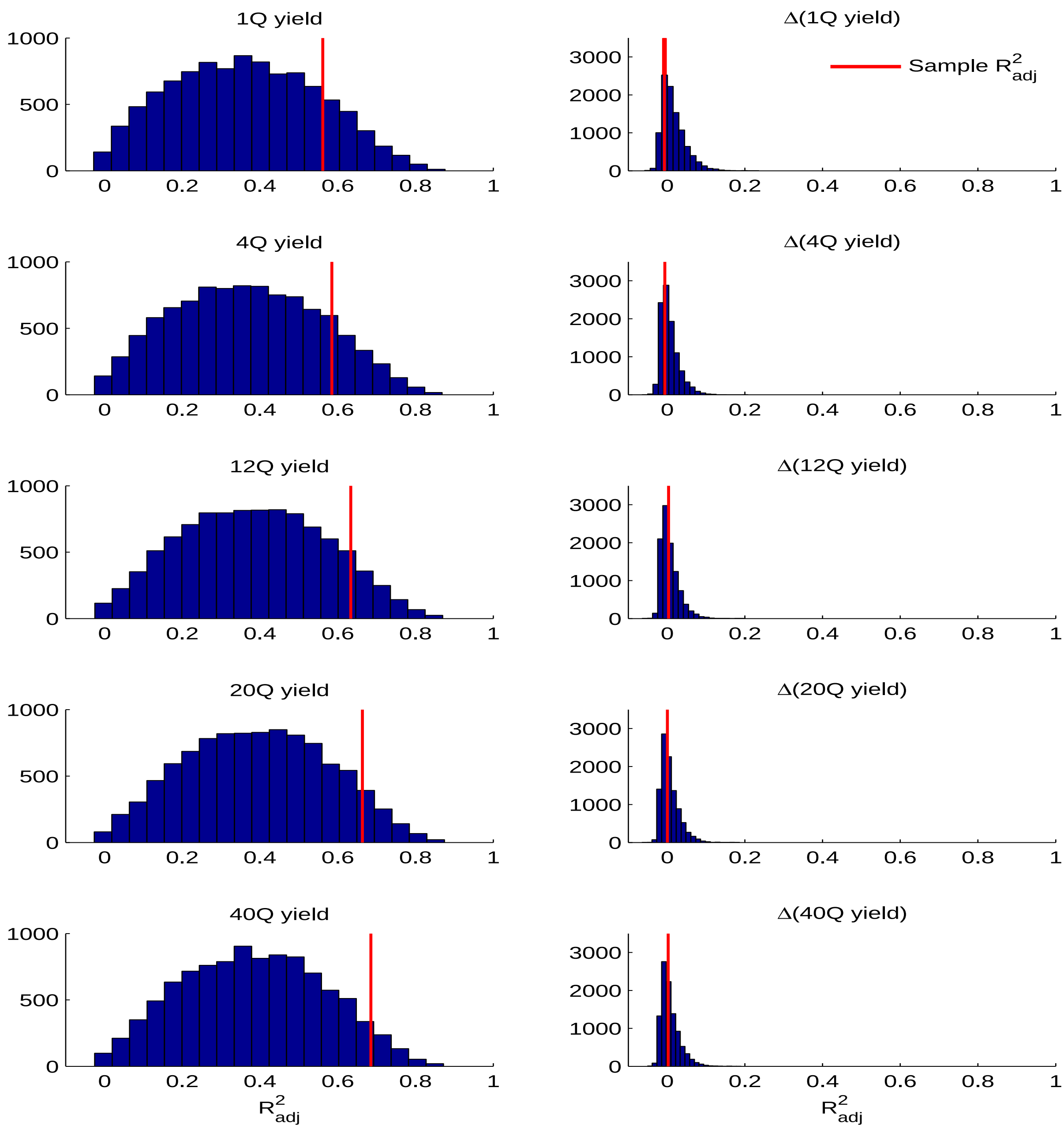


Figure 6: **Linear Spanning Regressions.** For each yield maturity n, the figure shows the adjusted R^2 coefficients for the OLS regressions:

Left panels : $\quad y_t^n = \alpha + \beta^c \pi_t^c + \beta^f \pi_t^f + \beta^e \pi_t^e + \varepsilon_t^n$

Right panels : $\quad \Delta y_t^n = \beta^c \Delta\pi_t^c + \beta^f \Delta\pi_t^f + \beta^e \Delta\pi_t^e + \varepsilon_t^n$.

The vertical red line marks the adjusted R^2 coefficient for the regressions of yields sampled quarterly from 1985Q1 to 2011Q4 against CPI core, food, and energy inflation data. The histograms show the distribution of the adjusted R^2 coefficients for regressions estimated on 10,000 samples of yields and inflation series simulated from the baseline $\mathrm{DTSM}_{3,3}$ using the scheme described in Section A.4 of the Online Appendix. The results in the left panels are for regressions in levels; those in the right panels are for regressions in changes.

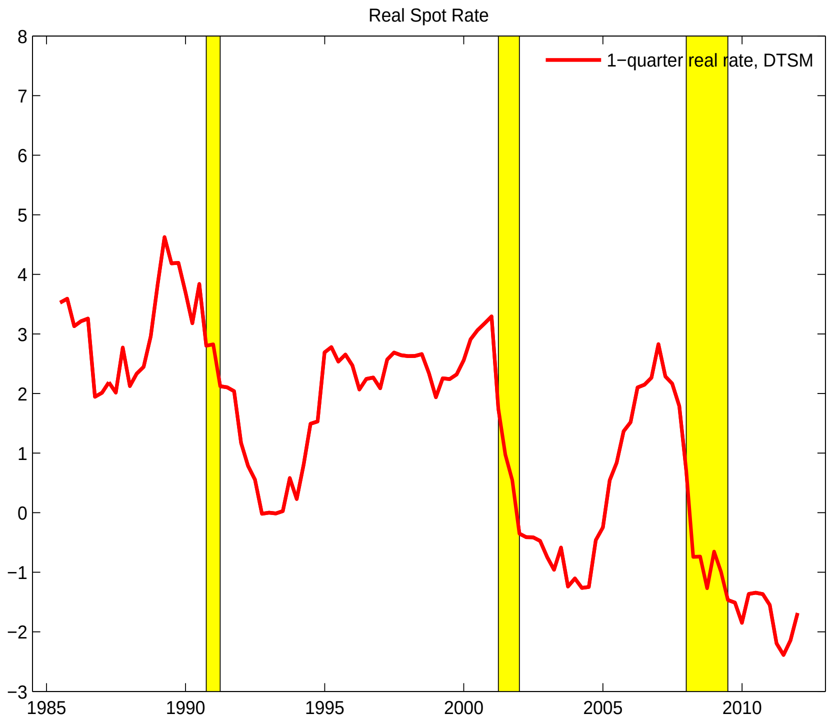


Figure 7: **Real Spot Rate.** The plot shows the model-implied real one-quarter rate computed using the baseline $\mathrm{DTSM}_{3,3}$. The sample period is 1985Q1-2011Q4. The shading corresponds to NBER recessions.

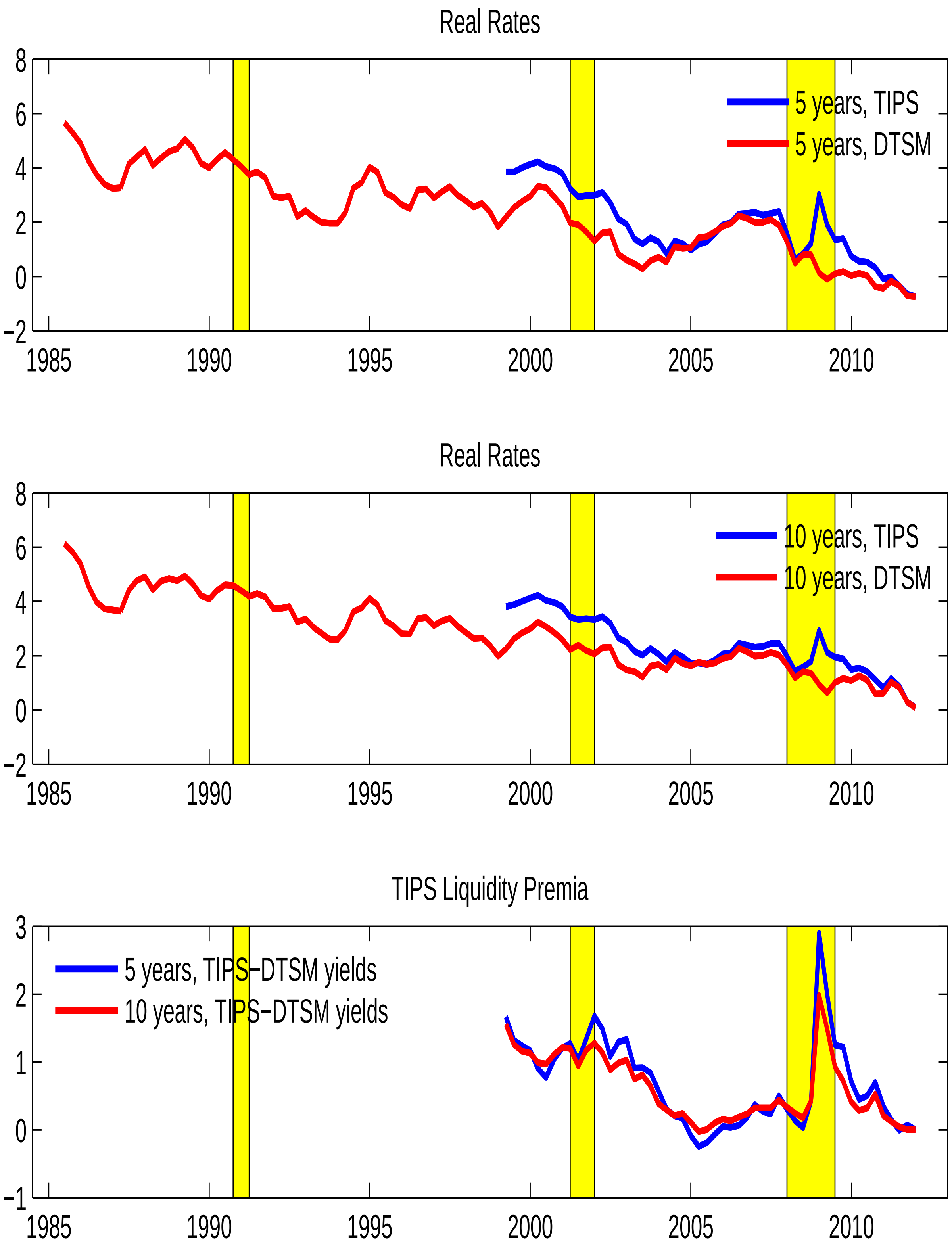


Figure 8: **Real Rates: TIPS vs. DTSM.** The plots contrast real yields with five- and ten-year maturity computed using the baseline $\text{DTSM}_{3,3}$ and TIPS rates with the same maturities. The sample period is 1985Q1-2011Q4. The shading corresponds to NBER recessions.

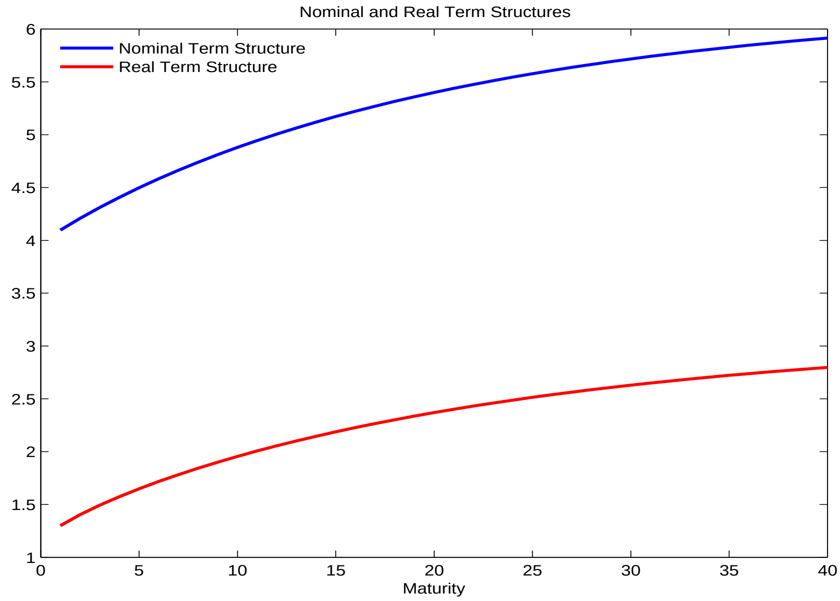


Figure 9: **Nominal and Real Term Structures.** The plot shows the nominal and real term structures computed using the baseline $\text{DTSM}_{3,3}$. The sample period is 1985Q1-2011Q4.

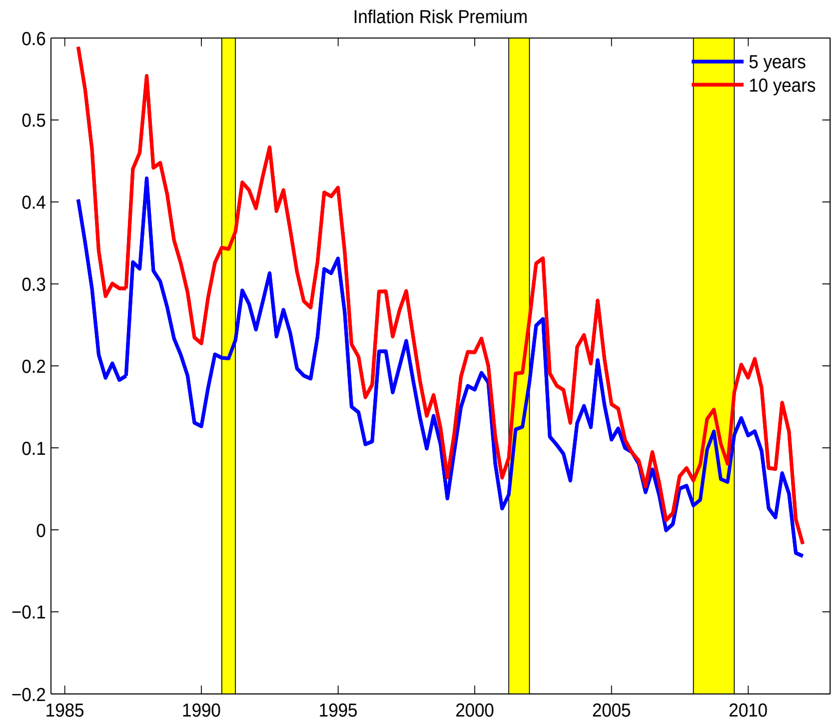


Figure 10: **Inflation Risk Premium.** The plot depicts the five- and ten-year inflation risk premia computed using the baseline $\text{DTSM}_{3,3}$. The sample period is 1985Q1-2011Q4. The shading corresponds to NBER recessions.

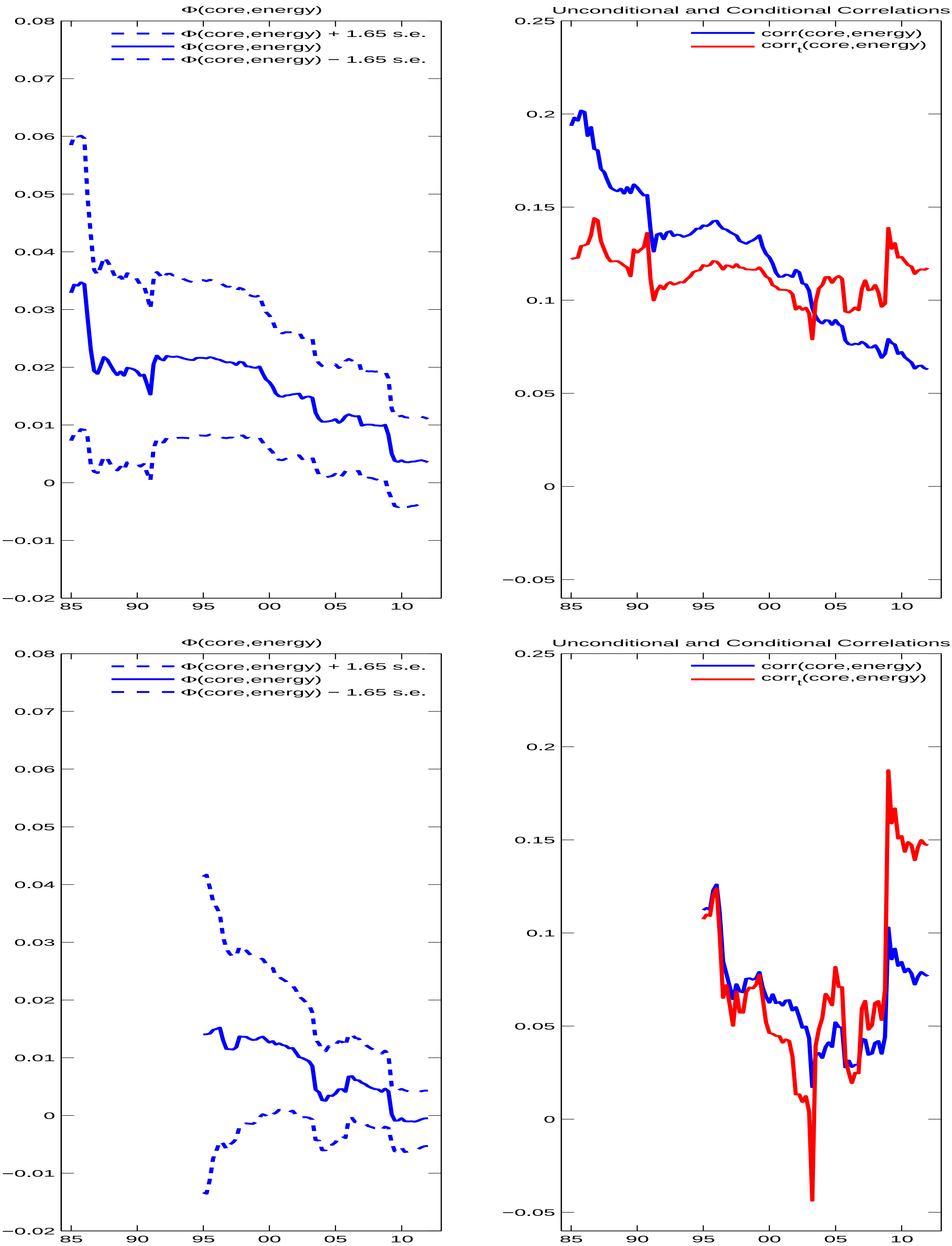


Figure 11: **Energy Pass Through.** The plots depict measures of correlation between energy and core inflation for the baseline $DTSM_{3,3}$. We estimate the model on the first four principal components of CMT yields with maturities up to ten years and CPI inflation data. In the top panels, the sample period starts in 1962Q1 and the end date ranges from 1985Q1 to 2011Q4. In the bottom panels, the sample period starts in 1985Q1 and the end date ranges from 1995Q1 to 2011Q4. The left panels show the estimate for the $\phi_1^{\pi^c,\pi^e}$ coefficient and its 90% confidence bands. The right panels depict conditional and unconditional correlations between core and energy inflation.

Table 1: **Summary Data Statistics.** The table reports summary statistics for CPI and PCE inflation series on core, food, energy and total consumer price indices; as well as CPI and PCE measures of relative importance weights for the core, food, and energy price indices. CPI- and PCE-weighted are the total inflation series computed from their core, food, and energy components using the relative importance weights. Panel A focuses on the 1962Q1-2011Q4 sample period; Panel B uses data from 1985Q1 to 2011Q4.

	Central moments				Autocorrelations		
	Mean	Std. Dev.	Skewness	Kurtosis	AC(1)	AC(2)	AC(3)
Panel A: Long Sample Period: 1962Q1-2011Q4							
CPI	4.05	3.08	0.69	5.99	0.75	0.64	0.65
CPI-weighted	4.08	3.05	0.86	5.51	0.76	0.65	0.67
CPI-core	3.97	2.67	1.58	5.83	0.87	0.81	0.78
CPI-food	4.07	3.75	1.79	7.31	0.63	0.46	0.50
CPI-energy	4.78	15.91	-1.66	15.31	0.30	-0.02	0.03
Weight-core	0.74	0.04	-0.33	1.76	0.98	0.95	0.92
Weight-food	0.18	0.04	0.73	2.35	0.98	0.95	0.92
Weight-energy	0.08	0.02	0.97	2.86	0.97	0.93	0.91
PCE	3.61	2.55	0.91	4.90	0.83	0.74	0.72
PCE-weighted	3.63	2.53	0.99	4.75	0.83	0.75	0.73
PCE-core	3.52	2.18	1.10	3.76	0.92	0.89	0.84
PCE-food	3.59	3.96	1.50	6.07	0.55	0.40	0.45
PCE-energy	4.86	16.45	-1.67	16.19	0.29	-0.02	0.00
Weight-core	0.82	0.04	-0.06	1.35	0.99	0.99	0.98
Weight-food	0.12	0.03	0.25	1.57	0.99	0.98	0.98
Weight-energy	0.06	0.01	0.44	2.72	0.99	0.97	0.95
Panel B: Short sample period: 1985Q1-2011Q4							
CPI	2.84	2.01	-2.26	14.93	0.28	0.02	0.07
CPI-weighted	2.89	1.92	-1.76	11.60	0.29	0.02	0.09
CPI-core	2.80	1.15	0.41	3.00	0.83	0.84	0.79
CPI-food	2.95	1.89	0.68	4.50	0.48	0.35	0.23
CPI-energy	3.30	19.22	-1.87	12.12	0.21	-0.14	-0.08
Weight-core	0.77	0.02	-2.42	8.96	0.90	0.78	0.68
Weight-food	0.15	0.01	0.89	3.87	0.95	0.89	0.84
Weight-energy	0.08	0.01	1.11	3.45	0.90	0.76	0.68
PCE	2.48	1.52	-1.43	9.97	0.42	0.15	0.16
PCE-weighted	2.50	1.47	-1.13	8.21	0.43	0.17	0.18
PCE-core	2.44	1.06	0.71	2.70	0.81	0.75	0.73
PCE-food	2.51	2.23	0.49	3.78	0.41	0.28	0.09
PCE-energy	3.52	20.06	-1.86	12.45	0.20	-0.13	-0.09
Weight-core	0.86	0.02	-0.87	2.90	0.96	0.92	0.88
Weight-food	0.09	0.01	0.55	1.89	0.98	0.97	0.95
Weight-energy	0.05	0.01	0.46	3.48	0.90	0.78	0.69

Table 2: **Expected inflation and real rates.** The top panel reports 2009 CPI forecasts computed using data through 2008Q4. In the last two columns are the annualized 2008Q4 and the 2009 CPI growth rates. The bottom panel reports the five-year real yield computed with data through 2008Q4. In the last two columns are the 2008Q4 five-year TIPS rate and the difference between the five-year nominal yield and the 2008 CPI growth rate. Sections 4 and 5.1 explain the different model specifications.

	2009 CPI forecasts computed as of 2008Q4				Realized	
	$DTSM_{2,1}$ total	$DTSM_{2,1}$ core	$DTSM_{2,3}$	$DTSM_{3,3}$	2008Q4*	2009
CPI	-5.60		0.17	1.49	-9.41	1.47
CPI-core		-0.18	-0.55	1.20	0.87	1.73
CPI-food			4.26	3.29	4.38	-0.57
CPI-energy			-0.22	1.14	-106.19	2.48

	5-year real yield computed as of 2008Q4					
	$DTSM_{2,1}$ total	$DTSM_{2,1}$ core	$DTSM_{2,3}$	$DTSM_{3,3}$	TIPS	5Y nom. yield minus 2008 infl.
5Y real yield	2.92	1.30	0.42	0.68	2.73	0.55

* The 2008Q4 rates are annualized

Table 3: **Forecasts of Annual Inflation Series**. We repeatedly estimate the models using data that start in 1985Q1 and end on dates ranging from 1998Q4 to 2010Q4; we forecast annual inflation out of sample over 1998Q4-2010Q4. Section 4 explains the different model specifications. For each model, the table shows RMSEs in percent per year and p-values for a test of equal forecast accuracy (West (1996)) computed under the null that the RMSE for that model equals the RMSE for the ARMA(1,1), when the alternative is that the RMSE for the ARMA(1,1) exceeds the RMSE for that model.

Panel A: Estimation on CPI data

	Total		Core		Food		Energy	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
ARMA	1.57		0.57		1.55		13.63	
ARMA_W	1.49	0.32						
VAR_3	1.47	0.12	0.67	0.93	1.51	0.40	13.71	0.56
RW	1.95	0.99	0.59	0.73	2.03	0.98	20.40	0.98
Survey forecasts								
U. of M.	1.62	0.63						
SPF	1.34	0.12						
Unconstrained VAR, CMT yields $\leq$ 10Y, OLS estimation on inflation and the yields' PCs								
$\text{VAR}_{4,3}$	1.55	0.42	0.44	0.00	1.45	0.27	14.84	0.87
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$	1.33	0.15	0.46	0.00	1.41	0.18	13.67	0.60
$\text{DTSM}_{4,3}$	1.36	0.14	0.46	0.01	1.42	0.17	13.65	0.54
$\text{DTSM}_{3,2}$	1.55	0.47	0.49	0.01				
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields								
$\text{DTSM}_{3,3}$	1.33	0.14	0.45	0.00	1.42	0.18	13.67	0.62

Panel B: Estimation on PCE data

	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
ARMA	1.19		0.43		1.94		14.48	
ARMA_W	1.06	0.19						
VAR_3	1.08	0.19	0.51	0.99	1.81	0.29	15.08	0.95
RW	1.41	0.98	0.40	0.14	2.56	0.96	21.55	0.98
Unconstrained VAR, CMT yields $\leq$ 10Y, OLS estimation on inflation and the yields' PCs								
$\text{VAR}_{4,3}$	1.22	0.63	0.48	0.92	1.75	0.18	15.94	0.84
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$	1.04	0.20	0.45	0.71	1.75	0.16	14.55	0.64
$\text{DTSM}_{4,3}$	1.04	0.21	0.41	0.33	1.75	0.16	14.53	0.61
$\text{DTSM}_{3,2}$	1.19	0.50	0.50	0.94				
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields								
$\text{DTSM}_{3,3}$	1.04	0.21	0.44	0.65	1.75	0.16	14.56	0.66

Table 4: **Long-Run Inflation Forecasts**. We repeatedly estimate the models using data that start in 1985Q1 and end on dates ranging from 1998Q4 to 2008Q4; we forecast annual inflation out of sample at horizons from one quarter to three years over 1998Q4-2008Q4. Section 4 explains the different model specifications. For each model, the table shows RMSEs in percent per year and p-values for a test of equal forecast accuracy (West (1996)) computed under the null that the RMSE for that model equals the RMSE for the ARMA(1,1), when the alternative is that the RMSE for the ARMA(1,1) exceeds the RMSE for that model.

	CPI				CPI-core				CPI-food			CPI-energy		
	ARMA	RW	$\text{DTSM}_{3,3}$	$\text{DTSM}_{2,1}$	ARMA	RW	$\text{DTSM}_{3,3}$	$\text{DTSM}_{2,1}$	ARMA	RW	$\text{DTSM}_{3,3}$	ARMA	RW	$\text{DTSM}_{3,3}$
1Q	2.75	2.86	2.62	3.01	0.54	0.58	0.56	0.72	1.47	1.59	1.45	26.17	28.67	26.46
		(0.93)	(0.22)	(0.94)		(0.99)	(0.68)	(0.95)		(0.82)	(0.42)		(0.90)	(0.69)
1Y	1.61	1.92	1.36	2.15	0.51	0.52	0.42	0.90	1.54	1.81	1.38	14.24	19.68	14.24
		(0.98)	(0.18)	(0.65)		(0.71)	(0.02)	(0.89)		(0.94)	(0.19)		(0.98)	(0.48)
2Y	1.13	1.48	0.92	1.77	0.58	0.58	0.43	1.02	1.31	1.81	1.07	8.79	14.29	8.79
		(0.98)	(0.20)	(0.61)		(0.77)	(0.02)	(0.63)		(0.99)	(0.09)		(0.99)	(0.50)
3Y	0.93	1.32	0.76	1.69	0.63	0.61	0.51	1.13	0.87	1.45	0.70	6.88	12.60	6.87
		(0.99)	(0.20)	(0.64)		(0.48)	(0.08)	(0.41)		(0.99)	(0.12)		(1.00)	(0.47)

Table 5: **Variance Decomposition**. We use the baseline $\text{DTSM}_{3,3}$ to decompose the proportion of inflation and term structure movements due to shocks to core, food, and energy inflation, and the latent factors $\ell^1, \ldots, \ell^3$. In Panel A, we decompose the variance of core inflation, while Panels B-D show a similar decomposition for the variance of the one-quarter, five- and ten-year yield. The sample period is 1985Q1-2011Q4.

Horizon:	4	20	∞
Panel A: CPI Inflation			
ℓ^1	14.07	49.27	62.81
ℓ^2	0.57	6.75	5.27
ℓ^3	3.67	2.51	1.82
CPI-core	81.68	41.46	30.08
CPI-food	0.00	0.00	0.00
CPI-energy	0.02	0.01	0.01
Panel B: 1-quarter yield			
ℓ^1	21.29	44.27	61.92
ℓ^2	57.70	49.51	33.96
ℓ^3	20.56	6.10	4.04
CPI-core	0.08	0.02	0.01
CPI-food	0.15	0.04	0.03
CPI-energy	0.22	0.06	0.04
Panel C: 5-year yield			
ℓ^1	55.95	77.19	84.37
ℓ^2	43.96	22.78	15.61
ℓ^3	0.09	0.03	0.02
CPI-core	0.00	0.00	0.00
CPI-food	0.00	0.00	0.00
CPI-energy	0.00	0.00	0.00
Panel D: 10-year yield			
ℓ^1	78.75	91.17	93.81
ℓ^2	19.88	8.38	5.87
ℓ^3	1.37	0.45	0.32
CPI-core	0.00	0.00	0.00
CPI-food	0.00	0.00	0.00
CPI-energy	0.00	0.00	0.00

Table 6: **Linear Spanning Regressions.** For each yield maturity n, the table shows results for the OLS regressions:

$$\begin{aligned} \text{Levels :} \quad y_t^n &= \alpha + \beta^c \pi_t^c + \beta^f \pi_t^f + \beta^e \pi_t^e + \varepsilon_t^n \\ \text{Changes :} \quad \Delta y_t^n &= \beta^c \Delta \pi_t^c + \beta^f \Delta \pi_t^f + \beta^e \Delta \pi_t^e + \varepsilon_t^n \,. \end{aligned}$$

We first estimate the regressions for each yield series y_t^n with maturity n equal to one quarter, one, three, five, and ten years against core, food, and energy inflation sampled quarterly from 1985Q1 to 2011Q4. For each regression, we report coefficient estimates and Newey-West heteroskedasticity and autocorrelation robust standard errors (in brackets). Next, we simulate 10,000 samples of yields with the same frequency and maturities as well as core, food, and energy inflation series from the baseline $DTSM_{3,3}$ using the scheme described in Section A.4 of the Online Appendix. We estimate the same regressions on each simulated sample and report mean, 5th, 50th, and 95th percentiles of the estimated coefficients. The results in the left-hand side of the table are for regressions in levels (the table omits the estimate of the intercept α); those in the right-hand side are for regressions in changes.

	Data		Model				Δ Data		Δ Model			
			Mean	5%	50%	95%			Mean	5%	50%	95%
						Panel A: One-quarter yield						
β^c	1.56	(0.21)	1.25	0.43	1.24	2.07	0.04	(0.05)	-0.06	-0.19	-0.06	0.08
β^f	0.06	(0.12)	-0.06	-0.26	-0.06	0.13	-0.00	(0.02)	-0.02	-0.07	-0.02	0.03
β^e	0.00	(0.01)	-0.00	-0.02	-0.00	0.01	0.00	(0.00)	0.00	-0.00	0.00	0.01
R^2_{adj}	56.14%		36.07%	6.47%	35.70%	67.29%	-0.69%		1.65%	-1.81%	0.96%	7.45%
						Panel B: One-year yield						
β^c	1.62	(0.21)	1.22	0.44	1.22	2.01	0.10	(0.04)	-0.02	-0.15	-0.02	0.11
β^f	0.04	(0.12)	-0.05	-0.23	-0.05	0.13	0.00	(0.02)	-0.01	-0.06	-0.01	0.04
β^e	-0.00	(0.01)	-0.00	-0.02	-0.00	0.01	0.00	(0.00)	0.00	-0.00	0.00	0.00
R^2_{adj}	58.50%		36.78%	7.25%	36.47%	67.93%	-0.58%		0.64%	-2.02%	0.11%	5.30%

Table 6, continued

	Data		Model				Δ Data		Δ Model			
			Mean	5%	50%	95%			Mean	5%	50%	95%
Panel C: Three-year yield												
β^c	1.68	(0.20)	1.13	0.44	1.13	1.83	0.11	(0.06)	-0.00	-0.13	-0.00	0.13
β^f	-0.05	(0.11)	-0.04	-0.20	-0.04	0.12	0.00	(0.02)	-0.00	-0.05	-0.00	0.04
β^e	-0.00	(0.01)	-0.00	-0.02	-0.00	0.01	0.00	(0.00)	0.00	-0.00	0.00	0.00
R^2_{adj}	63.34%		38.31%	8.59%	38.11%	68.85%	0.36%		0.67%	-1.93%	0.10%	5.17%
Panel D: Five-year yield												
β^c	1.61	(0.19)	1.06	0.42	1.06	1.68	0.10	(0.06)	0.00	-0.12	0.00	0.13
β^f	-0.08	(0.10)	-0.04	-0.18	-0.04	0.11	0.00	(0.02)	-0.00	-0.05	-0.00	0.04
β^e	-0.00	(0.00)	-0.00	-0.01	-0.00	0.01	0.00	(0.00)	0.00	-0.00	0.00	0.00
R^2_{adj}	66.37%		39.26%	9.77%	39.17%	69.21%	0.07%		0.72%	-1.90%	0.15%	5.30%
Panel E: Ten-year yield												
β^c	1.47	(0.17)	0.89	0.35	0.89	1.42	0.09	(0.06)	0.01	-0.10	0.01	0.11
β^f	-0.09	(0.08)	-0.03	-0.16	-0.03	0.09	0.01	(0.02)	-0.00	-0.04	-0.00	0.04
β^e	-0.00	(0.00)	-0.00	-0.01	-0.00	0.01	0.00	(0.00)	-0.00	-0.00	-0.00	0.00
R^2_{adj}	68.53%		38.31%	9.34%	38.08%	68.25%	0.28%		0.76%	-1.88%	0.17%	5.38%

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