Financial Contracting, Limited liability and Bubbles

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Abstract

We study a two-period economy with a representative consumer and financial intermediaries that issue equities and bonds to finance their investment in two risky assets. We show that limited liability per se cannot lead to bubbles because price system can prevent misallocation of resources (and bubbles) by pricing equities and bonds of the intermediaries that will implement investment. On the other hand, contractual incompleteness may allow the intermediaries to exploit limited liability by taking excessive risk and bid up the price of the riskier asset to be too high. Consequently, the riskier asset is over-produced in the economy, and it destroys welfare of the consumers. Our results imply that private incentives for monitoring cannot be strong enough to prevent bubbles.

1 Introduction

This paper discusses the existence and welfare implications of rational bubbles in financial markets. We propose a model of bubbles induced by financial intermediaries with limited liability and debt financing. Limited liability and debt induce intermediaries to bid up the price of an asset so that it becomes overvalued relative to another less risky asset. This leads to overproduction of the bubble asset and a welfare loss.

Discussions of past financial crises have pointed to a link between asset price inflation and excess credit. For example, Acharya and Naqvi (2012) note "from 2002 to 2007, the ratio of debt to national income went up from 3.75 to one, to 4.75 to one. During this same period, house prices grew at an unprecedented rate of 11% per year...". Reinhart and Rogoff (2008) argue that historically, real estate lending has been linked to bubbles. Stein (1995) proposed a model of this link: more debt allows credit-constrained borrowers to raise more funds, and this

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increased funding, invested in a fixed quantity of assets, causes the price of those assets to rise (Stein (1995) focuses on real estate; subsequently others have developed similar analyses for general asset markets as we describe below). However, his focus is on amplification of shocks into asset prices, and does not address the issue of overvaluation per se. If investors are credit rationed, an expansion of credit will drive up prices by allowing them to buy assets they want to buy; however, presumably they should only want to buy assets that are not overvalued..If they do not want to buy the assets anyway, a relaxation of the credit rationing will not increase investor demand for the assets.

In order to argue that leverage could generate bubbles, we need to consider whether the resulting investments are good value for the leveraged investors. If there is a bubble in asset prices, why do investors want to use leverage to buy those assets? The corporate finance literature suggests a mechanism whereby leverage could cause asset prices to rise above fair value. Leverage encourages risk-shifting (also known as asset substitution) (e.g., Jensen and Meckling (1976)). If assets are in limited supply (or are supplied with finite elasticity) and many firms overinvest in risky assets because of risk-shifting, the increased demand could drive up asset prices. This argument is made in Allen and Gale (2000)'s seminal contribution. However, this still leaves open the question of why investors are willing to supply capital in the first place to levered intermediaries who will then use that capital to engender over-valuation. In Allen and Gale (2000), debt is inelastically supplied by investors who do not have alternatives such as equity investment.

This paper has three main goals. First, we develop the risk shifting argument in a model where financial intermediaries raise equity and debt from investors. We show that bubbles will arise when both (a) financial contracting is imperfect and does not allow equity investors to control leverage - either by clauses written into contracts, or by intervention in managerial decisions and (b) where management has preferences that differ from investors'. When the agency problem is severe enough, risky assets will be overpriced in equilibrium.

Second, we provide a model with an explicit analysis of the welfare implications of bubbles. In our model assets are supplied by producers in response to prices. Bubbles cause overproduction of riskier assets. In equilibrium, consumers are worse off compared to the first-best benchmark in which financial contracting allows control over leverage.

Third, we provide empirical predictions. A bubble in our model is an asset whose price exceeds the value given by applying the consumers' pricing kernel to the cash flows (Allen and Gale (2000)'s model does not make this prediction because their bubbles are defined by reference to a pricing kernel from a different economy, which is the first best benchmark with the same preferences, endowments and production possibilities but without institutional frictions).

Consider a two-period economy with a representative consumer, who want to consume a part of his endowment today and invest the rest for future consumption, and a representative producer, who can transform consumption goods into two real assets that produce risky payoffs in the future. One asset is riskier and the other is less risky. There exist ex-ante homogeneous intermediaries that issue financial securities (equities and bonds) to finance their investment in the assets. We assume that some of the intermediaries have bad management who raise extra debt for empire-building rather than maximizing equity value due to imperfect managerial contracts. Protected by limited liability toward downside risk, levered intermediaries would have incentive to take excessive risk and consequently bidding up the price of the riskier asset above its intrinsic value. However, limited liability per se cannot lead to bubbles because price system can prevent bubbles by pricing equities and bonds of the intermediaries that will implement investment. The equilibrium depends on the severity of contractual incompleteness in the economy. If contractual incompleteness is not so severe, there is no bubble in equilibrium because financial securities are correctly priced. On the other hand, severe contractual incompleteness prevents financial securities from being correctly priced, thus this enables bad management to exploit limited liability by taking excessive risk and bid up the price of the risky asset to be too high. Consequently, the riskier asset is over-produced in the economy, and it destroys welfare of the consumers.

We extend the intuition of Modigliani and Miller (1958)'s Theorem by showing that leverage is irrelevant to equity value in the absence of bubbles, but leverage lowers equity value in the presence of bubbles. It is because limited liability enables levered intermediaries to hold overpriced assets. We find that it is impossible to prevent bubbles by private monitoring efforts. That is, the self-regulation of the market cannot achieve efficiency in the presence of agency problem in the economy. Although shareholders' decision of not monitoring bad management creates welfare-destroying spillover effects, such negative externality to the economy is not included in their optimization problems. Instead, they are willing to monitor bad management only when the increase in equity value is greater than the cost of monitoring. However, no one would want to monitor bad management in the absence of bubbles because leverage is irrelevant to equity value. This result is parallel with the paradox of Grossman and Stiglitz (1980) who find that informationally efficient market is impossible because no one would want to acquire costly information once the market is efficient. Therefore, our result implies a necessity of public monitoring efforts or public intervention to mitigate contractual incompleteness.

Bubbles are not easy to detect empirically. Long lived assets with growing cash flows derive a significant portion of their value from cash flows far into the future, hence, it is difficult to make inferences from a comparison of prices with ex-post cash flows (e.g., West (1998).) If two assets have identical cash flow entitlements, but have different prices, then the more expensive one could be viewed as a bubble. Such assets have been identified in a few instances, for example, Royal Dutch/Shell shares (see Lamont and Thaler (2003).) In most cases, of course, comparisons between two assets do not allow the cash flows to be matched in such a precise fashion. However, asset pricing theory does assert that higher-risk assets should earn higher expected returns. If this fails, the riskier assets are overvalued relative to the lower risk assets. Greenwood and Hanson (2013) find that corporate bonds have lower expected returns than US government securities in some periods. Hence, a methodology do exist that has the potential to examine the empirical predictions of the model.

Our model predicts that bubbles are more likely in the following cases: if the assets are predominantly held by intermediaries which are more opaque, if the assets are riskier, if they are held by intermediaries with weak corporate governance or with more dispersed shareholders, if the assets are in relatively inelastic supply, if the assets cannot be easily distinguished from similar assets with different risk levels, and if the assets are typically associated with leverage and whose leverage is easy to hide (such as SIVs).

As noted above the most closely related paper is Allen and Gale (2000). There is of course a large literature on bubbles. Brunnermeier and Oehmke (2012) and Scherbina (2013) are excellent surveys. Here we comment on a selection of papers to explain the motivation for our analysis.

What is a bubble? The classic definition of bubble is an asset whose price exceeds its fundamental value. Some authors describe bubbles in terms of additional properties: for example, that the motive for agents holding the asset is to resell it, or that the asset price follows a sequence of gradual increase followed by sudden bursting (Brunnermeier and Oehmke (2012)). In this paper, we do not impose additional conditions, and simply define a bubble as an asset that is overpriced relative to the value given by applying the consumers' pricing kernel. The numerical example we provide to illustrate our results also has a stronger property that an asset whose payoffs are a mean-preserving spread of another assert's payoffs has a higher price in equilibrium.

In finite horizon settings, backward induction generally rules out bubbles. An agent who holds an overvalued asset up until the terminal date (whether over a time period of a discrete-time model, or an interval of time in a continuous-time model) will be unwilling to hold the asset. Hence theoretical models of bubbles have studied infinite-horizon settings. Fiat money in the classic overlapping-generations model can be viewed as a bubble since it has a positive price but pays no dividend (although the discount rate in this economy is also zero; see Dow and Gorton (1993)). More generally, if we take a correctly-priced asset in an infinite horizon economy and add on a component to the price that grows at the risk-free rate, the resulting price will generate returns that make investors willing to hold the asset. The same is true if we add on any risky price path whose expected return compensates for its risk. However, the value of the bubble component is non-zero, so this may violate a transversality condition - for example, that in an exchange economy, the aggregate wealth of consumers should equal the value of the endowment. Hence bubbles will arise only in economies with particular features where this problem does not arise, such as overlapping generations models Tirole (1985) (Kocherlakota (1992) studies an economy with infinitely-lived agents which has special features resembling an overlapping generations model). In Santos and Woodford (1997) assets in zero net supply may display bubbles because their value does not affect aggregate wealth.

With imperfect information, it is natural for an asset to be overvalued in the sense that the value conditional on one agent's information might exceed the value conditional on other agents' information or conditional on the joint information of all the agents (Allen, Morris, and Postlewaite (1993)). These issues are not the subject of the paper. Furthermore, our paper focuses exclusively on rational bubbles. A substantial literature studies how irrational agents may cause bubbles. Both the literature on bubbles under imperfect information and on irrational bubbles are surveyed in Scherbina and in Brunnermeier and Oehmke.

Our analysis is based on intermediaries who raise capital from agents and invest in assets. As the process of intermediation induces inefficient bubbles, there must be reasons why the agents choose to supply capital to intermediaries rather than invest directly on their own behalf. As in most models of separation of ownership and control, these reasons are not explicitly modeled. The underlying reasons could include free rider problems with multiple shareholders resulting from risk diversification, or difficulties of contractual enforcement between agents with capital and agents with managerial ability. Plausibly most assets held by banks are assets that the ultimate capital providers do not have the expertise to hold.

The organization of the paper is as follows. Section 2 describes the basic model. Section 3 solves the equilibrium. Section 4 investigates comparative statics with numerical examples. Section 5 study an extension of the model where costly monitoring is possible. Section 6 concludes.

2 Model

2.1 Basic Setup

We consider a simple two period ($t = 1, 2$) economy with a single consumption good. There are two possible states of the world $\omega \in \{H, L\}$ where state H is realized with probability ρ and state L is realized with probability $1 - \rho$. The state is realized in period $t = 2$. We assume that there are three types of participants in the economy: a representative consumer, a representative producer, and intermediaries in the financial sector.

The representative consumer can be considered as a continuum of consumers in unit measure. The consumer is endowed with w unit of the consumption good in period $t = 1$, but there is no endowment in period $t = 2$. w may be interpreted as the wage for the labor provided by the consumer when he is young. We assume that the consumer has a time-separable concave utility function

$$u(c_1) + \beta u(c_2)$$

where β is a discount factor, and $u(\cdot)$ is a twice differentiable function with $u'(\cdot) > 0, u''(\cdot) < 0$ and $u'(0) = \infty$. The consumer may consume the initial endowment when young, but can also invest in financial assets issued by the financial sector in order to consume when he is old.

The financial sector consists of a continuum of intermediaries in unit measure. The intermediaries are established and invest in the real assets in period $t = 1$. They pay dividends to their shareholder (who is actually the consumer) by liquidating all the assets in period $t = 2$. We let $\mathcal{J}$ denote the set of intermediaries. To finance investment, intermediaries create financial contracts in the form of equities and bonds, and sell them to the consumer in a securities market where short sales are allowed.

We assume that there are two types of management: (a) good management, and (b) bad management. We let $\mathcal{J}^G$ and $\mathcal{J}^B$ denote the set of intermediaries that are operated by good and bad management, respectively. We also let λ denote the portion of bad management, i.e., $\lambda = \int_{j \in \mathcal{J}^B} dj$. We assume that the management type of each intermediary is not known to the consumer although λ is common knowledge. We assume that both good and bad management derives utility from their compensation based on equity value. Unlike good management, however, bad management also derives utility from private benefit of controlling assets. Therefore, bad management has an incentive of maximizing equity value as well as building empires by financing as much as possible. We assume that bad management always prioritize empire building to equity value maximization. Such conflicts of interest can be understood in the context of classical agency problems in the presence of contractual incompleteness. That is, we can interpret that managerial contracts fail to prevent agency problems with a probability of λ. Therefore, λ is the parameter of contractual incompleteness in the economy. We endogenize this by allowing costly monitoring by shareholders in the extension of the model in Section 5.

Each intermediary inelastically supplies the shares of equities in unit measure to the securities market in the beginning of period $t = 1$. Given the choice of monitoring, management decides the number of bonds they will supply to the securities market in the end of period $t = 1$. After financing with equity and bonds, the intermediaries can invest in two types of real assets in which the consumer cannot directly invest. Then, the real assets are supplied by the producer, and they are traded by the intermediaries. We assume that short sales are not allowed for the real assets. Short sale constraints in the real asset market can be justified by the possibility of default on stock-lending contracts.

We index each real asset by $k \in \{R, S\}$ where R stands for 'Riskier' and S stands for 'Safer'. Asset k produces a state-contingent payoff $\pi_k(\omega)$ in state ω as a liquidation value. The assets produce strictly higher payoffs in state H than they do in state L. We assume that[1]

$$0 \leq \pi_R(L) < \pi_S(L) < \pi_S(H) < \pi_R(H).$$

We denote q_S and q_R to be the price of asst S and R in the real asset market, respectively.

The representative producer can be considered as a continuum of competitive manufacturing or agricultural firms in unit measure. We assume that the producer is a profit-maximizing firm whose shares are owned by the consumer. We normalize the number of shares to unit measure. For each asset $k \in \{R, S\}$, let $\bar{z}_k$ denote the amount of asset k created by the producer. The cost of producing $\bar{z}_k$ unit of asset k is given by a function $g_k(\bar{z}_k)$ for each $k \in \{R, S\}$. For example, an input $g_R(\bar{z}_R)$ of the consumption good is necessary to produce $\bar{z}_R$ of asset R. We assume that the cost function g_k is twice-differentiable and convex such that $g'_k(\cdot) > 0$, $g''_k(\cdot) > 0$ and $g'_k(0) = 0$ for each $k \in \{R, S\}$. Notice that production technology has decreasing returns to scale due to the convex cost function. Finally, we assume that the producer does not own any initial endowment and there does not exist any agency problem in management. The producer's profit Π is given by the revenue less the cost as follows:

$$\Pi = q_S\bar{z}_S + q_R\bar{z}_S - g_S(\bar{z}_S) - g_R(\bar{z}_R). \tag{1}$$

In the end of period $t = 1$, the profit generated by producing the real assets is immediately distributed to the shareholders as dividends.

The timeline of the model is summarized by Figure 1.

[1] In the numerical examples in Section 4, we will also assume that the payoffs on asset R are a mean-preserving spread of those on asset S, i.e.,

$$E[\pi_S(\omega)] = E[\pi_R(\omega)],$$

although this is not necessary for our analytical results. In this case, under a risk-averse or risk-neutral preference and in a standard complete-markets model without intermediation or frictions, asset R must have a lower price in equilibrium (because the state-price for state $\omega = H$ must be lower than the state-price for state $\omega = L$). In our numerical example, we show that asset R may instead have a higher price in equilibrium.

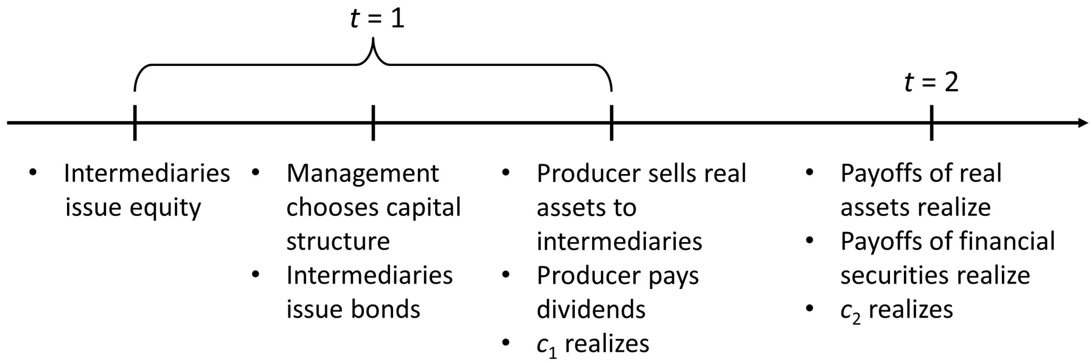

Figure 1: Timeline of the model

2.2 Financial Securities

We consider standard financial contracts as follows: The shareholders of an intermediary are residual claimants who only receive payoffs when the intermediary is solvent (i.e., the total asset value is greater than or equal to what it owes to the bondholders). The bondholders receive the face value of bonds if the intermediary is solvent. In case the intermediary is insolvent, however, they receive the liquidated value pro rata to their ownership of the bonds.

We assume that equity contracts cannot have covenants on leverage nor investment choices, and it is also not possible for the intermediaries to make commitments. Because all the intermediaries are ex-ante identical, all the intermediaries offer unit share of their equity at an identical price p_e that is endogenously determined in the securities market.

We let b^j denote the number of outstanding bonds issued by intermediary j. For simplicity, we assume that an intermediary can choose only two levels of $b^j \in \{0, 1\}$. Therefore, an intermediary with $b^j = 0$ is unlevered, and an intermediary with $b^j = 1$ is levered.[2] Intermediary j offers unit share of bonds at the price of p_b^j that is endogenously determined in the securities market. We assume that the leverage ratio of levered intermediaries is fixed to $\bar{l}$, i.e.,

$$\frac{p_b^j}{p_e} = \bar{l}, \quad \text{for all } j \in \mathcal{J} \text{ with } b^j = 1. \tag{2}$$

Consequently, the face value of bonds is determined so that the leverage ratio of a levered intermediary is equal to $\bar{l}$.

[2] One could interpret that unlevered and levered intermediaries in our model represent financial institutions with low and high leverage, respectively.

We let $\theta_a^j(\omega)$ denote the total asset value of intermediary j in state ω:

$$\theta_a^j(\omega) = \pi_S(\omega) z_S^j + \pi_R(\omega) z_R^j, \tag{3}$$

where z_S^j and z_R^j are the quantities of asset S and R, respectively. Then, the payoff for each unit of bond in period $t = 2$ is given by

$$\theta_b^j(\omega) = \max(f, \theta_a^j(\omega)).$$

where f is the face value of bonds. The payoff for each unit of equity in state ω is given by

$$\theta_e^j(\omega) = \max(\theta_a^j(\omega) - \theta_b^j(\omega), 0). \tag{4}$$

2.3 Optimization Problems

A. Consumer's Problem

We let x_e denote the shares of each intermediary's equity held by the consumer. We also let x_b^j denote the shares of intermediary j's bonds held by the consumer. Therefore, the consumer's choice of equity and bond holdings can be represented by the pair $\big(x_e, (x_b^j)_{j \in \mathcal{J}}\big)$.

Given his own choice of equity holdings x_e in the beginning of period $t = 1$ and the intermediaries' supply of bonds $(b^j)_{j \in \mathcal{J}}$, the consumer solves the following problem in order to maximize the expected utility of life-time consumption:

$$\Gamma(x_e, (b^j)_{j \in \mathcal{J}}) = \max_{(x_b^j)_{j \in \mathcal{J}}} \quad u(c_1) + \beta E\big[\big(c_2(\omega)\big)\big], \tag{5}$$

subject to[3]

$$c_1 = w + \Pi - p_e x_e - \int_{j \in \mathcal{J}} p_b^j x_b^j dj, \tag{6}$$

$$c_2(\omega) = \left(\int_{j \in \mathcal{J}} \theta_e^j(\omega) dj\right) x_e + \int_{j \in \mathcal{J}} \theta_b^j(\omega) x_b^j dj, \text{ for all } \omega \in \{H, L\}. \tag{7}$$

In the beginning of the period $t = 1$, he solves

$$\max_{x_e} \quad E\big[\Gamma(x_e, (b^j)_{j \in \mathcal{J}})\big], \tag{8}$$

[3]Notice that we can omit the non-negativity constraint for the consumption streams because of the Inada condition that we have assumed. Also, the consumer's budget constraints always bind because his preference is locally non-satiated.

B. Management's Problem

(1) Portfolio Choice

Shareholder's valuation for unit share of intermediary j's equity is given by $E[m(\omega)\theta_e^j(\omega)]$ where $m(\omega) = \beta\frac{u'(c_2(\omega))}{u'(c_1)}$ is the price kernel of the consumer in state $\omega \in \{H, L\}$. Given the choice financing, management of intermediary $j \in \mathcal{J}$ solves the following problem:

$$\Psi^j(b^j) = \max_{z_S^j, z_R^j} \quad E\big[m(\omega)\theta_e^j(\omega)\big], \tag{9}$$

subject to

$$q_S z_S^j + q_R z_R^j = p_e + p_b^j b^j, \tag{10}$$

$$z_S^j \geq 0, z_R^j \geq 0. \tag{11}$$

Let $r_k(\omega)$ denote the return of the real asset $k \in \{S, R\}$ in state ω, i.e.,

$$r_k(\omega) = \frac{\pi_k(\omega)}{q_k} - 1.$$

Then, it is immediate from management's problem in Eq (9) that an unlevered intermediary would strictly prefer holding an asset k over asset k' whenever $E[m(\omega)r_k(\omega)] > E[m(\omega)r_{k'}(\omega)]$ for any $k, k' \in \{S, R\}$. On the other hand, a levered intermediary would strictly prefer holding an asset k over asset k' whenever $\pi_k(H)r_k(H) > \pi_{k'}(H)r_{k'}(H)$. That is, levered intermediaries only care about the return in state H unlike unlevered intermediaries that care about the returns in both states.

(2) Leverage Choice

Let b^{j*} be the optimal choice of debt by management. Remember that bad management always prefers more financing in order to increase asset value under control. Therefore, bad management always issues bonds, i.e.,

$$b^{j*} = 1 \text{ for all } j \in \mathcal{J}^B.$$

On the other hand, good management issues bonds only when leverage, i.e.,

$$b^{j*} = \underset{b^j}{\text{argmax}} \quad \Psi^j(b^j) \text{ for all } j \in \mathcal{J}^G.$$

C. Producer's Problem

By optimally choosing the quantity of the real assets to be produced $(\bar{z}_S, \bar{z}_R)$, the producer solves the following problem:

$$\Pi = \max_{\bar{z}_S, \bar{z}_R} \quad q_S \bar{z}_S + q_R \bar{z}_R - \big[g_S(\bar{z}_S) + g_R(\bar{z}_S) \big], \tag{12}$$

subject to

$$\bar{z}_S \geq 0, \bar{z}_R \geq 0.$$

3 Equilibrium

As we will show, equilibrium has the following features. The equilibrium depends on the severity of the agency problem. If the agency problem (or the severity of contractual incompleteness) is not severe, there is no bubble, and each intermediary is indifferent to leverage. This is so because the marginal intermediary which chooses leverage is good management. If the agency problem is severe, there is a bubble and intermediaries chooses to be levered if and only if they have bad management. This is because the marginal intermediary which chooses leverage is bad management. The price of the riskier asset is higher than the value given by the pricing kernel applied to its payoffs. If there is a bubble, the levered intermediaries hold all the risky asset (and, depending on the values of the exogenous parameters, may also hold some of the safer asset) and the unlevered intermediaries hold the safer asset only. Because the debt is provided at fair market prices and holding the bubble asset is less profitable than holding the safer (undervalued) asset, equity in the levered intermediaries is less valuable than equity in the unlevered intermediaries. When buying equity, investors do not know whether any given intermediary has bad management and will choose to be levered or not.

3.1 Definition of Equilibrium

We first define equilibrium for the exchange economy in a standard manner.

Definition 1. *An equilibrium is a pair consisting of allocations* $\big((x_e, (x_b^j)_{j\in\mathcal{J}}), (b^j, z_S^j, z_R^j)_{j\in\mathcal{J}}, (\bar{z}_S, \bar{z}_R)\big)$ *and prices* $\big((p_e, (p_b^j)_{j\in\mathcal{J}}), (q_S, q_R)\big)$ *such that (i)* $\big(x_e, (x_b^j)_{j\in\mathcal{J}}\big)$ *solves the consumer's problem, (ii)* $\big((b^j, z_S^j, z_R^j\big)$ *solves intermediary j's management's problem for all* $j \in \mathcal{J}$*, (iii)* $(\bar{z}_S, \bar{z}_R)$ *solves the producer's problem, (iv)* $\big(p_e, (p_b^j)_{j\in\mathcal{J}}\big)$ *clears the securities market for all* $j \in \mathcal{J}$*, i.e.*

$$x_e^j = 1, \text{ for all } j \in \mathcal{J}, \tag{13}$$

$$x_b^j = b^j, \text{ for all } j \in \mathcal{J}, \tag{14}$$

and (v) (q_S, q_R) *clears the asset market, i.e.,*

$$\int_{j\in\mathcal{J}} z_S^j dj = \bar{z}_S, \tag{15}$$

$$\int_{j\in\mathcal{J}} z_R^j dj = \bar{z}_R. \tag{16}$$

We define that there is a bubble when an asset's market price is higher than its fundamental value according to the consumer's pricing kernel $m(\omega)$ in each state $\omega \in \{H, L\}$. Then, we can categorize potential equilibria according to the overvaluation of the real assets.

Definition 2. *An equilibrium is a* ***no-bubble equilibrium*** *if* $q_k = E[m(\omega)\pi_k(\omega)]$ *for all* $k \in \{S, R\}$, *and a* ***bubble equilibrium*** *if there exists* $k \in \{S, R\}$ *such that* $q_k > E[m(\omega)\pi_k(\omega)]$.

3.2 Solving Equilibrium

A. Security Prices

Because the consumer can infer the equilibrium investment strategy of the intermediaries given their choice of leverage, the price in the securities market reflects the true value of the intermediary's debt. From the F.O.C of the consumer's problem in Eq (5), we have the following Euler equation for intermediary j's bonds:

$$p_b^j = E[m(\omega)\theta_b^j(\omega)]. \tag{17}$$

On the other hand, the price of equity cannot reflect the exact value of its claim on assets because (a) the intermediaries are ex-ante identical, and (b) demand cannot be conditioned on the leverage. Using the constrained Envelope theorem, we can show that the F.O.C of the consumer's problem in Eq (8) yields the following Euler equation:

$$p_e = E\left[m(\omega)\int_{j\in\mathcal{J}} \theta_e^j(\omega)dj\right], \quad \text{for all } j \in \mathcal{J}. \tag{18}$$

That is, the equity price of each intermediary j is given by the average value of all the equity in the market.

Although the equity price of each intermediary may not reflect the exact value of assets, the total value of all the financial securities should reflect the total value of assets under management in the entire financial sector.

Lemma 1. *In equilibrium, the total value of equity and bonds is equal to the total value of all* *the real assets:*

$$\int_{j\in\mathcal{J}}(p_e + p_b^j b^j)dj = E\Big[m(\omega)\big(\pi_S(\omega)\bar{z}_S + \pi_R(\omega)\bar{z}_R\big)\Big]. \tag{19}$$

Proof. See appendix. □

We let μ denote the portion of levered intermediaries, i.e., $\mu = \int_{j\in\mathcal{J}} \mathbb{1}_{(b^j=1)}dj$. Using Eq (2) and (19), we have

$$\mu p_e(1+\bar{l}) + (1-\mu)p_e = E[m(\omega)(\pi_S(\omega)\bar{z}_S + \pi_R(\omega)\bar{z}_R)]. \tag{20}$$

Solving Eq (20) for p_e gives us the equity price of each intermediary given μ:

$$p_e = \frac{E[m(\omega)(\pi_S(\omega)\bar{z}_S + \pi_R(\omega)\bar{z}_R)]}{1+\mu\bar{l}}. \tag{21}$$

B. Real Asset Prices

Integrating each intermediary's budget constraint, Eq (10), over all the intermediaries and applying the market clearing conditions in Eq (15) and (16) yields

$$q_S\bar{z}_S + q_R\bar{z}_R = \int_{j\in\mathcal{J}}(p_e + p_b^j b^j)dj. \tag{22}$$

where $1-\lambda$ is the portion of intermediaries that implement costly monitoring. Therefore, Lemma 1 implies that

$$q_S\bar{z}_S + q_R\bar{z}_R = E\Big[m(\omega)\big(\pi_S(\omega)\bar{z}_S + \pi_R(\omega)\bar{z}_R\big)\Big]. \tag{23}$$

Notice that the total value of the portfolio held by the intermediaries should be equal to the amount they have financed from the consumer. Then, Eq (23) implies that there cannot be bubbles on both assets because overvaluation for one asset means undervaluation for the other asset. From the management's problem of Eq (9), we can deduce that only the riskier asset can be overvalued relative to the safer asset. First, suppose that asset R is overvalued. As long as asset R still yields a higher return in state H than asset S does, such prices would be sustainable in equilibrium because levered intermediaries would strictly prefer asset R to asset S. Now, suppose instead that asset S is overvalued relative to asset R. Then, unlevered intermediaries would strictly prefer asset R to asset S. Furthermore, levered intermediaries would also strictly prefer asset R to asset S because asset R yields a higher return in state H than asset S does if asset S is relatively overvalued. Because no intermediary wants to hold asset S, it cannot be an equilibrium.

We can also show that there cannot be overvaluation of the riskier asset if and only if the equity price of each intermediary is equal to its intrinsic value.

Lemma 2. *There is no bubble in equilibrium if and only if the equity prices are correct, i.e.,*

$$p_e = E[m(\omega)\theta_e^j(\omega)], \text{ for all } j \in \mathcal{J}. \tag{24}$$

Proof. See appendix. □

Because the bonds are always correctly price, Eq (24) implies that all the financial securities that will implement investment are correctly priced. Therefore, this prevents bubbles because levered intermediaries do not have more funds than the total value of portfolios they will eventually hold. Furthermore, this further implies that perfect equity contracts that are contingent on future leverage choices (e.g., covenants) would be able to exclude bubbles because financial securities would perfectly priced in that case.

Finally, we can bound the size of a bubble: the management of a levered intermediary that cares about the payoff in the high state but not in the low (default) state is willing to overpay for the riskier asset, but not beyond the point where it costs as much to purchase high-state payoffs for the risky asset as for the safer asset.

Lemma 3. *In equilibrium,* $q_R \leq \frac{\pi_R(H)}{\pi_S(H)} q_S$.

Proof. See appendix. □

C. Leverage Choices

Using Lemma 2, we can obtain an extended version of Modigliani and Miller (1958)'s Theorem.

Lemma 4. *(Extended Modigliani-Miller Theorem) (a) In the absence of a bubble, the equity value of each intermediary is indifferent to leverage, i.e.,*

$$\Psi^j(0) = \Psi^j(1), \text{ for all } j \in \mathcal{J}.$$

(b) In the presence of a bubble, the equity value of a levered intermediary is strictly lower than that of an unlevered intermediary, i.e.,

$$\Psi^j(0) > \Psi^j(1), \text{ for all } j \in \mathcal{J}.$$

Proof. In a no-bubble equilibrium, equity value of all the intermediaries are identical regardless of leverage due to Lemma 2. This proves the first part. In a bubble equilibrium, however, unlevered intermediaries only hold asset S that is undervalued. That means all the asset R is held by levered intermediaries. Because the value of bonds is correct, it implies that the equity of an unlevered intermediary should be higher than a levered intermediary given their portfolio choices. This proves the second part. $\square$

As a corollary of Lemma 4, we deduce that good management never chooses leverage in the presence of bubbles.

Corollary 1. *In the presence of bubbles, an intermediary is levered if and only if it has bad management, i.e.,* $\mu = \lambda$.

D. Production of Real Assets

The F.O.C of the producer's problem of (12) gives

$$g_k'(\bar{z}_k^*) = q_k, \quad \text{for all } k \in \{S, R\} \tag{25}$$

The supply of the real assets in equilibrium, $(\bar{z}_S^*, \bar{z}_R^*)$, satisfies Eq (25) given the pair of the real asset prices (q_S, q_R). Remember that the marginal cost is zero for producing an initial unit of any real asset, i.e,. $g_k'(0) = 0$. Therefore, the supply of both assets are positive whenever the prices of the real assets are positive. Furthermore, the producer's profit Π is strictly positive whenever at least one asset is in positive supply. This is because the cost function $g_k(\cdot)$ is increasing in the number of outputs, i.e., $g_k'(\cdot) > 0$. Therefore, the consumer receives a positive amount of dividends in period $t = 1$ whenever some of the real assets are traded at non-zero prices.

E. Pricing Kernel

From the market clearing condition of the securities market, $x_e^j = 1$ and $x_b^j = b^j$ in equilibrium. From Eq (6) and (7),

$$c_1 = w + \Pi - \int_{j \in \mathcal{J}} (p_e + p_b^j b^j) dj, \tag{26}$$

$$c_2(\omega) = \int_{j \in \mathcal{J}} (\theta_e^j(\omega) dj + \theta_b^j(\omega) x_b^j) dj = \int_{j \in \mathcal{J}} \theta_a^j(\omega) dj, \quad \text{for all } \omega \in \{H, L\}. \tag{27}$$

Therefore, substituting Eq (1) and (22) into Eq (26) gives the equilibrium level of consumption in period $t = 1$:

$$c_1^* = w - g_S(\bar{z}_S) - g_R(\bar{z}_R), \tag{28}$$

and substituting Eq (3) into Eq (27) and applying the market clearing conditions gives the equilibrium level of consumption in period $t = 2$:

$$c_2^*(\omega) = \pi_S(\omega)\bar{z}_S + \pi_R(\omega)\bar{z}_R, \text{ for all } \omega \in \{H, L\}. \tag{29}$$

From Eq (28), Eq (29), we can derive a unique pricing kernel given the supply of real assets $(\bar{z}_S, \bar{z}_R)$ as follows:

$$m(\omega) = \beta \frac{u'\big(\pi_S(\omega)\bar{z}_S + \pi_R(\omega)\bar{z}_R\big)}{u'\big(w - g_S(\bar{z}_S) - g_R(\bar{z}_R)\big)}, \text{ for all } \omega \in \{H, L\}. \tag{30}$$

F. Existence of Bubbles

In a no-bubble equilibrium, unlevered intermediaries are indifferent between asset S and R, but levered intermediaries strictly prefer asset R. This implies that the amount financed by levered intermediaries should not exceed the market capitalization of the risky asset because levered intermediaries would invest all the financed money into the riskier asset. Therefore, we have

$$\lambda p_e(1 + \bar{l}) \leq E[m(\omega)\pi_R]\bar{z}_R. \tag{31}$$

Substituting Eq (21) into Eq (31) yields

$$\mu \leq \bar{\mu},$$

where

$$\bar{\mu} = \frac{E[m(\omega)\pi_R(\omega)]\bar{z}_R}{E[m(\omega)\pi_S(\omega)]\bar{z}_S(1 + \bar{l}) + E[m(\omega)\pi_R(\omega)]\bar{z}_R}. \tag{32}$$

Because bad management always takes leverage, we conclude that the portion of bad management should be less or equal to the threshold in order to have a no-bubble equilibrium, i.e., $\lambda \leq \bar{\mu}$. If so, the equilibrium portion of levered intermediaries can be any number between λ and $\bar{\mu}$, i.e., $\mu^* \in [\lambda, \bar{\mu}]$. Because the marginal intermediary that raise debt is operated by good management, leverage does not lower the equity value of the intermediary.

In a bubble equilibrium, unlevered intermediaries only hold asset S because of its undervaluation. On the other hand, levered intermediaries only hold asset R if $q_R < \frac{\pi_R(H)}{\pi_S(H)} q_S$, but would be indifferent between asset S and R if $q_R = \frac{\pi_R(H)}{\pi_S(H)} q_S$.

Because unlevered intermediaries only hold asset S and levered intermediaries only hold asset R if $q_R < \frac{\pi_R(H)}{\pi_S(H)} q_S$, the market capitalization of asset S and R should be equal to the total financing of unlevered and levered intermediaries, respectively. Therefore, we have

$$q_S = \frac{(1-\mu)p_e}{\bar{z}_S}, \tag{33}$$

$$q_R = \frac{\mu p_e(1+\bar{l})}{\bar{z}_R}. \tag{34}$$

Substituting Eq (33) and (34) into the equation $q_R \leq \frac{\pi_R(H)}{\pi_S(H)} q_S$ yields

$$\mu \leq \hat{\mu},$$

where

$$\hat{\mu} = \frac{\pi_R(H)\bar{z}_R}{\pi_S(H)\bar{z}_S(1+\bar{l}) + \pi_R(H)\bar{z}_R}. \tag{35}$$

Notice that the first threshold is strictly lower than the second threshold, i.e., $\bar{\mu} < \hat{\mu}$.

Because only bad management takes leverage in a bubble equilibrium, we have $\mu^* = \lambda$. Then, we have $q_R = \frac{\pi_R(H)}{\pi_S(H)} q_S$ if $\lambda \geq \hat{\mu}$. Therefore, we can obtain the unique equilibrium price of asset S and R given λ as follows:

$$q_S = \begin{cases} E[m(\omega)\pi_S(\omega)], & \text{if } 0 \leq \lambda \leq \bar{\mu}; \\ \frac{1-\lambda}{1+\lambda\bar{l}} E[m(\omega)(\pi_S(\omega) + \pi_R(\omega)\frac{\bar{z}_R}{\bar{z}_S})], & \text{if } \bar{\mu} < \lambda \leq \hat{\mu}; \\ \frac{1-\hat{\mu}}{1+\hat{\mu}\bar{l}} E[m(\omega)(\pi_S(\omega) + \pi_R(\omega)\frac{\bar{z}_R}{\bar{z}_S})], & \text{otherwise;} \end{cases} \tag{36}$$

and

$$q_R = \begin{cases} E[m(\omega)\pi_R(\omega)], & \text{if } 0 \leq \lambda \leq \bar{\mu}; \\ \frac{\lambda(1+\bar{l})}{1+\lambda\bar{l}} E[m(\omega)(\pi_S(\omega)\frac{\bar{z}_S}{\bar{z}_R} + \pi_R(\omega))], & \text{if } \bar{\mu} < \lambda \leq \hat{\mu}; \\ \frac{\hat{\mu}(1+\bar{l})}{1+\hat{\mu}\bar{l}} E[m(\omega)(\pi_S(\omega)\frac{\bar{z}_S}{\bar{z}_R} + \pi_R(\omega))], & \text{otherwise;} \end{cases} \tag{37}$$

Notice that q_S and q_R are continuous in the pair $(\bar{z}_S, \bar{z}_R)$. Therefore, the existence of equilibrium can be established using Eq (25).

Now, we are ready to present our main result in this section. We show that limited liability can lead to bubbles in the presence of contractual incompleteness. However, this result also implies that limited liability per se cannot be the cause of bubbles in the absence of other frictions.

Theorem 1. *(i)* (Existence) *There exists an equilibrium in the economy. In equilibrium, there exists a bubble on asset R if and only if there exists enough contractual incompleteness in the economy, i.e., $\lambda > \bar{\mu}$. (ii)* (Uniqueness) *The equilibrium is unique in terms of (q_S, q_R) and $(\bar{z}_S, \bar{z}_R)$ if $u(\cdot)$ is a homogeneous function of any degree and $g_S(\cdot)$ and $g_R(\cdot)$ are also homogeneous functions where $g_S(\cdot)$ and $g_R(\cdot)$ are of an identical degree.*

Proof. See appendix. □

The result gives an immediate implication about the relation between the size of a bubble and contractual incompleteness.

Corollary 2. *All other things being equal, the size of a bubble tends to get bigger as contractual incompleteness becomes more severe up to a certain point, i.e.,* $\frac{\partial q_R}{\partial \lambda} > 0$ *for* $\lambda \in (\bar{\mu}, \hat{\mu})$.

The result also gives an immediate implication about the relation between the leverage ratio and bubbles.

Corollary 3. *All other things being equal, bubbles are more likely to arise when the leverage ratio is higher, i.e.,* $\frac{\partial \bar{\mu}}{\partial l} < 0$.

3.3 Welfare

The first-best would be achieved by solving a benevolent social planner's problem. The social planner solves the following problem:

$$\max_{\bar{z}_S, \bar{z}_R} \quad u(c_1) + \beta E[u(c_2(\omega))], \tag{38}$$

subject to

$$\begin{aligned} c_1 &= w - g_S(\bar{z}_S) - g_R(\bar{z}_R), \\ c_2(\omega) &= \pi_S(\omega)\bar{z}_S + \pi_R(\omega)\bar{z}_R, \quad \text{for all } \omega \in \{H, L\}. \end{aligned}$$

Then, the first-best is unique because the solution of Eq (38) is always interior and the objective function is strictly concave. The first-best is achieved when

$$g'(\bar{z}_k^{FB}) = E\big[m(\omega)\pi_k(\omega)\big], \ \text{ for all } k \in \{S, R\}$$

Notice that this is true in the market solution Eq (25) if and only if there is no bubble, i.e., $q_k = E\big[m(\omega)\pi_k(\omega)\big]$ for all $k \in \{S, R\}$. Furthermore, Eq (25) implies that there is overproduction of asset k whenever there is a bubble on asset k because $g'(\cdot) > 0$. As was shown in the previous section, there can only be a bubble on asset R. Therefore, there would be an oversupply of the riskier asset and an undersupply of the safer asset in the economy because of a bubble. This deviation from the first-best destroys the welfare of the consumer.

Theorem 2. *The equilibrium welfare is inferior to the first-best whenever there exists a bubble. In a bubble equilibrium, there exists too large supply of asset R and too little supply of asset S relative to the efficient level of asset supply, i.e.,* $\bar{z}_R^* > \bar{z}_R^{FB}$ *and* $\bar{z}_S^* < \bar{z}_S^{FB}$.

4 Comparative Statics

In this section, we provide a comparative statistics using numerical examples. We assume that the utility function is given by a CRRA function, i.e., $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and the cost function is given by a quadratic adjustment function, i.e., $g_k(\bar{z}_k) = \frac{a_k}{2}\bar{z}_k^2$ for all $k \in \{S, R\}$. We choose parameter values as follows: $\pi_S(H) = 6, \pi_S(L) = 4, \pi_R(H) = 9, \pi_R(L) = 1, \gamma = 0.5, \beta = 1, \rho = 0.6, \omega = 5, a_S = 1, a_R = 1$. Notice that this setup guarantees a unique equilibrium due to Theorem 1 because $u(\cdot)$ and $g_k(\cdot)$'s are all homogeneous. It is also worth mentioning that the parameters are set so that asset S and R have an identical expected payoffs, i.e., $E[\pi_S(\omega)] = E[\pi_R(\omega)]$. Then, the payoffs on asset R is a mean-preserving spread of those on asset S, thus the risk-averse consumer should strictly prefer asset S to asset R. Therefore, overvaluation of asset R relative to asset S shows an obvious mispricing regardless of the consumer's pricing kernel.

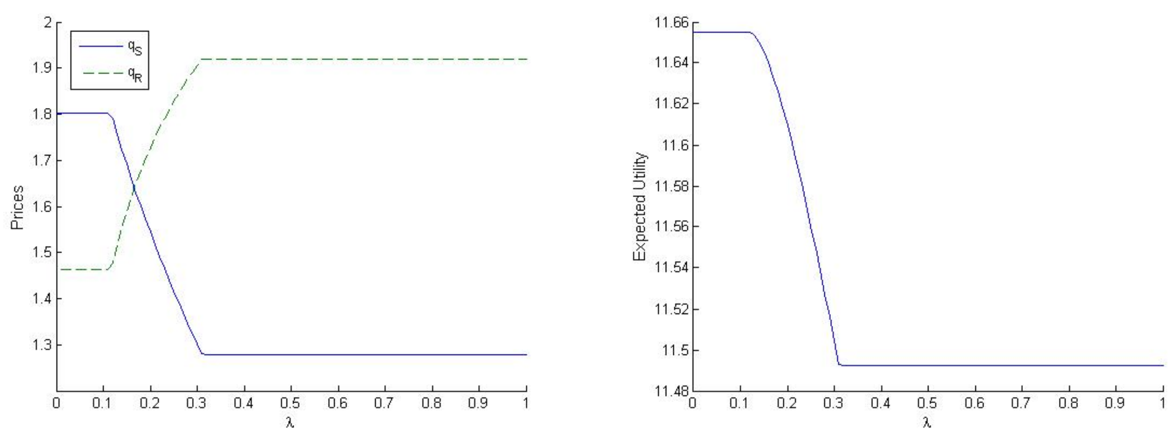

Figure 2: The left panel plots equilibrium asset prices with respect to contractual incompleteness. The right panel plots equilibrium expected utility of the consumer with respect to contractual incompleteness. The leverage ratio $\bar{l}$ is fixed to 3.

The left panel of Figure 2 shows that there is no bubble when there is not enough contractual incompleteness (or agency problem). But, the price of asset R quickly rises above its fundamental value as soon as there is enough contractual incompleteness, and it reaches its maximum level as contractual incompleteness becomes more severe. This is in line with the prediction by Corollary 2. On the right panel, the welfare of the economy is measured by the expected utility of the consumer. As Theorem 2 predicts, there exists steep welfare reduction in the degree of contractual incompleteness because the existence of bubbles distorts the optimal allocation of resources in production. Therefore, our result predicts that improved monitoring such as stricter risk management would help reducing asset bubbles thereby improving welfare.

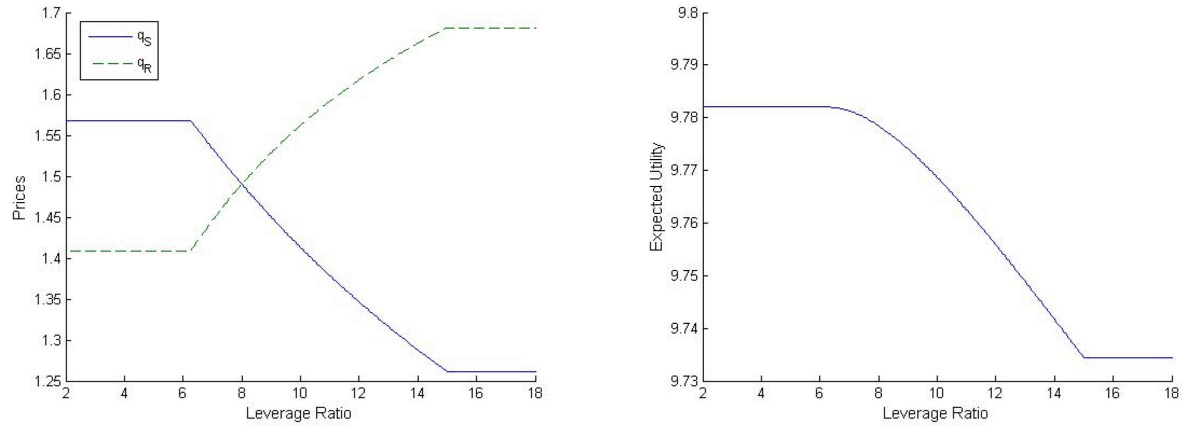

Figure 3: The left panel plots equilibrium asset prices with respect to leverage ratio. The right panel plots equilibrium expected utility of the consumer with respect to leverage ratio. The parameter of contractual incompleteness λ is fixed to 0.1.

Figure 3 shows comparative statics with leverage ratio. As in the case of contractual incompleteness, we can observe that a bubble arises as the parameter increases. This is in line with the prediction by Corollary 3. The welfare of the economy steeply decreases as the leverage ratio goes up in a bubble equilibrium. Therefore, this result implies that capping the leverage ratio of financial institutions may reduce asset bubbles and increase welfare by preventing excessive risk-taking by the financial sector.

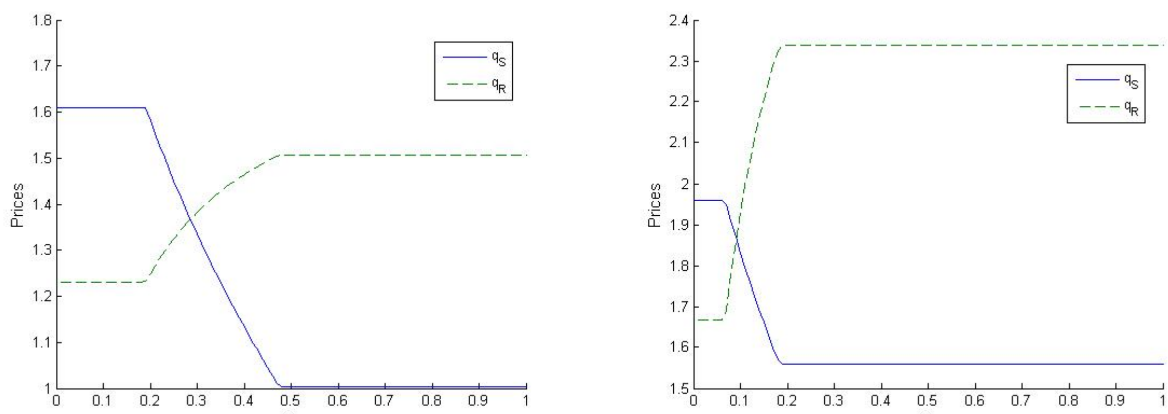

Figure 4: The left and right panel plots equilibrium asset prices with respect to contractual incompleteness when $a_R = 0.5$ and $a_R = 2$, respectively. The leverage ratio $\bar{l}$ is fixed to 3.

Figure 4 shows comparative statics with the elasticity of asset supplies. Our theory predicts that there will be a bigger bubble for the assets with a relatively inelastic supply. This is captured by the quadratic adjustment cost a_R. Therefore. the asset supply is less elastic for higher a_R because its adjustment cost is higher.

5 Extensions

Can own efforts of the financial sector improve welfare by mitigating bubbles? We have shown that entrenched management would want to take excessive leverage in order to build empires, and this results in excessive risk-taking that causes bubbles in the market. Therefore, reducing agency conflicts by costly efforts such as monitoring or improving contractual completeness would be able to prevent or at least mitigate welfare-destroying price distortions in the market. In this section, we extend our model by incorporating costly monitoring in order to answer whether monitoring can be effective tools of preventing bubbles.

Now, suppose that agency problem of bad management can be prevented when monitored by good management. That is, the shareholders of an intermediary with bad management can avoid potential agency conflicts by merging with another intermediary with good management. We assume that the organization of the newly created intermediary by the merger is restructured so that bad management is subordinated by good management. Then, there does not exist agency problem any more in the newly intermediary. We further assume that the merger can be implemented by an equity swap where the exchange ratio is favorable to the ones with good management such that the shareholders with bad management transfer $\alpha \in [0, 1)$ unit of shares to the shareholders with good management. Therefore, shareholders of an intermediary with bad management face a trade-off between improved governance and the loss of ownership by α unit of shares. The loss of ownership by $(1-\alpha)$ share is interpreted as the cost of monitoring bad management.

From Theorem 1, it is clear that there will only exist a no-bubble equilibrium when there is not enough contractual incompleteness, i.e., $\lambda \leq \bar{\mu}$. Now, we will focus on the case with enough contractual incompleteness, i.e., $\lambda > \bar{\mu}$. Notice that shareholders of intermediary $j \in \mathcal{J}^G$ always prefer a merger whenever $\alpha > 0$ because it strictly increases their share of cash flow, i.e., $(1+\alpha)\Psi^j(b^j) > \Psi^j(b^j)$. On the other hand, shareholders of intermediary $j \in \mathcal{J}^B$ prefer a merger only when

$$(1-\alpha)\Psi^j(0) > \Psi^j(1).$$

However, the extended Modigliani-Miller Theorem (Lemma 4) states that in a no-bubble equilibrium, there is no difference in equity values between levered and unlevered intermediaries, i.e., $\Psi(0) = \Psi(1)$. In that case, the shareholders of an intermediary with bad management do not have any incentive to spend the monitoring fee because $(1-\alpha)\Psi(0) < \Psi(1)$ whenever $\alpha > 0$. Therefore, there cannot exist a no-bubble equilibrium if $\lambda > \bar{\mu}$. This is parallel with the paradox of Grossman and Stiglitz (1980) who find that informationally efficient market is impossible because no one would want to acquire costly information once the market is efficient. In our model, it is impossible not to have any bubble in the market because no one wants to monitor management once the market becomes void of bubbles. Consequently, there always exists a bubble in equilibrium. Because the extended Modigliani-Miller Theorem states that an unlevered intermediary is more value than a levered one in a bubble equilibrium, the following condition is satisfied:

$$(1-\alpha)\Psi^j(0) = \Psi^j(1), \quad \text{for all } j \in \mathcal{J}^B. \tag{39}$$

That is, the shareholders of intermediaries with bad management should be indifferent between doing the costly monitoring or not.

We denote ν to be the portion of intermediaries with bad management that are merged with other intermediaries with good management. Effectively there are $(1-\lambda)+\nu$ of intermediaries that have good management. Therefore, the portion of levered intermediaries is given by $\mu = \lambda - \nu$ in equilibrium. The first-best is achieved whenever ν is greater than or equal to $\bar{\nu} = \bar{\mu} - \lambda$. By the aforementioned argument, the equilibrium ν is determined by Eq (39), but this is always strictly lower than $\bar{\nu}$. Therefore, we find that the equilibrium level of monitoring is always lower than the socially optimal level. We summarize the argument by the following theorem:

Theorem 3. *If $\lambda > \bar{\mu}$, it is impossible to have a no-bubble equilibrium by the financial sector's own efforts whenever monitoring is costly (i.e., $\alpha > 0$).*

Theorem 3 states that the self-regulation of the market cannot achieve efficiency in the presence of agency problem. When monitoring is collectively neglected by shareholders, it creates welfare-destroying distortions in asset prices. However, shareholders do not care about such negative externalities to the economy. Therefore, our result demonstrates that seemingly-irrational asset overvaluation can actually stem from contractual frictions in the economy and the paradoxical nature of private incentives of reducing such frictions. As for regulatory implications, our result shows that public intervention on monitoring is necessary for stabilizing asset prices because private incentives cannot be aligned to mitigate agency problem on their own.

Remark. *Although there may be many alternative mechanisms of monitoring bad management, in this section we have particularly assumed that bad management can be monitored by good management through a merger. This is to avoid any distortionary effect of monitoring* *efforts on asset prices. That is, if monitoring causes deadweight loss in the economy, such cost should be included as a part of supplying the real assets to the consumer. Although it could be an interesting problem in itself, it would complicate the first-best level of resource allocations. By assuming that monitoring requires a transfer of wealth from shareholders with bad management to shareholders with good management, we sidestep the issue of changing the level of the first-best allocation. We leave exploring distortionary effect of monitoring on asset prices for future research.*

6 Conclusions

We propose a model of bubbles induced by financial intermediaries with limited liability and debt financing. For this, we study a two-period economy with a representative consumer and two assets where one is riskier and the other is less risky. There exist ex-ante homogeneous intermediaries that issue equities and bonds to finance their investment in the assets. We assume that some of the intermediaries have bad management who raise extra debt for empire-building rather than maximizing equity value due to imperfect managerial contracts. Protected by limited liability toward downside risk, levered intermediaries would have incentive to take excessive risk and consequently bidding up the price of the riskier asset above its intrinsic value. However, limited liability per se cannot lead to bubbles because price system can prevent bubbles by pricing equities and bonds of the intermediaries that will implement investment. The equilibrium depends on the severity of contractual incompleteness in the economy. If contractual incompleteness is not so severe, there is no bubble in equilibrium because financial securities are correctly priced. On the other hand, severe contractual incompleteness prevents financial securities from being correctly priced, thus this enables bad management to exploit limited liability by taking excessive risk and bid up the price of the risky asset to be too high. Consequently, the riskier asset is over-produced in the economy, and it destroys welfare of the consumers. Using comparative statics, we find that high leverage ratio or inelasticity of asset supply can also contribute to creating bubbles along with agency problem in the economy.

Finally, we find that it is impossible to prevent bubbles by private monitoring efforts. That is, the self-regulation of the market cannot achieve efficiency in the presence of agency problem in the economy. Although shareholders' decision of not monitoring bad management creates welfare-destroying spillover effects, such negative externality to the economy is not included in their optimization problems. Instead, they are willing to monitor bad management only when the increase in equity value is greater than the cost of monitoring. Paradoxically, no one would want to monitor bad management in the absence of bubbles because leverage is irrelevant to equity value. Therefore, our result implies a necessity of public monitoring efforts or public intervention to mitigate contractual incompleteness.

7 Appendix

Proof of Lemma 1

Proof. Eq (17) implies that

$$\int_{j\in\mathcal{J}} p_b^j b^j dj = E\Big[m(\omega)\int_{j\in\mathcal{J}} \theta_b^j(\omega) b^j dj\Big]. \tag{40}$$

By using Eq (18) and (40), we get

$$\int_{j\in\mathcal{J}} (p_e + p_b^j b^j) dj = E\Big[m(\omega)\int_{j\in\mathcal{J}} (\theta_e^j(\omega) + \theta_b^j(\omega) b^j) dj\Big] = E\Big[m(\omega)\int_{j\in\mathcal{J}} \theta_a^j(\omega) dj\Big]. \tag{41}$$

Then, substituting Eq (3) into Eq (41) yields

$$\begin{aligned}\int_{j\in\mathcal{J}} (p_e + p_b^j b^j) dj &= E\Big[m(\omega)\int_{j\in\mathcal{J}} (\pi_S^j(\omega) z_S^j + \pi_R^j(\omega) z_R^j) dj\Big] \\ &= E\Big[m(\omega)\Big(\pi_S^j(\omega)\int_{j\in\mathcal{J}} z_S^j dj + \pi_R^j(\omega)\int_{j\in\mathcal{J}} z_R^j dj\Big)\Big]\end{aligned}$$

Applying the market clearing condition for the real asset market to the above equation gives us the desired result. □

Proof of Lemma 2

Proof. Suppose that the equity prices are correct but there exists a bubble. Because the bond prices are always correct, the total asset value of intermediary j should be equal to the sum of its equities and bonds:

$$p_e + p_b^j b^j = E[m(\omega)\pi_S(\omega)] z_S^j + E[m(\omega)\pi_R(\omega)] z_R^j. \tag{42}$$

On the other hand, the value of the portfolio should be equal to the total amount of financing:

$$p_e + p_b^j b^j = q_S z_S^j + q_R z_S^j. \tag{43}$$

Using Eq (42) and (43), we get

$$(E[m(\omega)\pi_S(\omega)] - q_S) z_S^j + (E[m(\omega)\pi_R(\omega)] - q_R) z_R^j = 0. \tag{44}$$

Therefore, integrating Eq (44) across all the intermediaries yields

$$(E[m(\omega)\pi_S(\omega)] - q_S)\int_{j\in\mathcal{J}} z_S^j dj + (E[m(\omega)\pi_R(\omega)] - q_R)\int_{j\in\mathcal{J}} z_R^j dj = 0. \tag{45}$$

Because $\bar{z}_k = \int_{j\in\mathcal{J}} z_k^j dj$ are strictly positive for both $k \in \{S, R\}$ in any equilibrium,[4] Eq (45) is true if and only if $q_k = E[m(\omega)\pi_k(\omega)]$ for all $k \in \{S, R\}$, i.e., there cannot exist any bubble. This contradicts. Now, suppose that there is no bubble but the equity prices are not correct. Then, we have

$$q_S z_S^j + q_R z_S^j = E[m(\omega)\pi_S(\omega)]z_S^j + E[m(\omega)\pi_R(\omega)]z_R^j. \tag{46}$$

Furthermore, Eq (43) should still be true regardless of the existence of bubbles. Eq (46) and (43) imply that

$$p_e + p_b^j b^j = E[m(\omega)\pi_S(\omega)]z_S^j + E[m(\omega)\pi_R(\omega)]z_R^j. \tag{47}$$

Because the bond prices are always correct, any intermediary j's equity value should be correct due to Eq (47). This contradicts. Therefore, this finishes the proof. □

Proof of Lemma 3

Proof. In case $E[m(\omega)\pi_R(\omega)] < q_R$, an unlevered intermediary would not want to hold any asset R, thus only a levered intermediary would want to hold it. However, a levered intermediary would be willing to hold asset R only when $\frac{\pi_R(H)}{q_R} \le \frac{\pi_S(H)}{q_S}$. Therefore, it is impossible to have an equilibrium with $q_R > \frac{\pi_R(H)}{\pi_S(H)} q_S$ because no intermediary would want to hold asset R at the given price. □

Proof of Theorem 1

Proof. (i) *Existence*: From Section 3.2, we know that there exist a unique pair of prices (q_S, q_R) given $(\bar{z}_S, \bar{z}_R)$. Furthermore, q_k's are continuous functions of $(\bar{z}_S, \bar{z}_R)$ for all $k \in \{S, R\}$. Notice that $g_k(\cdot)$ and $g_k'(\cdot)$ for all $k \in \{S, R\}$ are invertible because they are monotone increasing continuous functions. The feasible set of $(\bar{z}_S, \bar{z}_R)$ is given by $Z = \left[0, g_S^{-1}(w)\right] \times \left[0, g_R^{-1}(w)\right]$.

[4] Because the payoff of the assets are strictly positive, it is immediate that q_S and q_R are strictly positive in equilibrium from the intermediaries' optimization problem. Furthermore, the marginal cost of production is zero (i.e., $g'(0) = 0$) when $\bar{z}_k = 0$ for all $k \in \{S, R\}$. Consequently, it is also immediate that $\bar{z}_S$ and $\bar{z}_R$ should be both positive in equilibrium from the producer's optimization problem.

Then, we define functions $F_k : Z \to \mathbb{R}^2$ such that

$$F_k(\bar{z}_S, \bar{z}_R) = (g'_k)^{-1}(q_k), \text{ for all } k \in \{S, R\}. \tag{48}$$

Due to Eq (25), the following should be true in equilibrium:

$$F_k(\bar{z}_S, \bar{z}_R) = \bar{z}_k \text{ for all } k \in \{S, R\}. \tag{49}$$

Notice that both $F_S(\cdot)$ and $F_R(\cdot)$ are continuous function of $(\bar{z}_S, \bar{z}_R)$ on a compact set Z. By the Brouwer fixed point theorem, there exist a fixed point for $F_S(\cdot)$ and $F_R(\cdot)$, which is a solution $(\bar{z}_S, \bar{z}_R)$ for the equation system of Eq (49).

(ii) *Uniqueness*: Now, we let $g_S(\cdot), g_R(\cdot)$ and $u(\cdot)$ be homogeneous functions. Then, their derivatives are also homogeneous. Furthermore, $g'_S(\cdot)$ and $g'_R(\cdot)$ have an identical degree because $g_S(\cdot)$ and $g_R(\cdot)$ have an identical degree. Let V be the ratio of the fundamental value of asset S to R, i.e., $V = \frac{E[m(\omega)\pi_S(\omega)]}{E[m(\omega)\pi_R(\omega)]}$. Then, we can show that V is uniquely determined by $\frac{\bar{z}_S}{\bar{z}_R}$ as follows:

$$V = \frac{\rho\left(\frac{u'\left(\pi_S(H)\frac{\bar{z}_S}{\bar{z}_R}+\pi_R(H)\right)}{u'\left(\pi_S(L)\frac{\bar{z}_S}{\bar{z}_R}+\pi_R(L)\right)}\right)\pi_S(H) + (1-\rho)\pi_S(L)}{\rho\left(\frac{u'\left(\pi_S(H)\frac{\bar{z}_S}{\bar{z}_R}+\pi_R(H)\right)}{u'\left(\pi_S(L)\frac{\bar{z}_S}{\bar{z}_R}+\pi_R(L)\right)}\right)\pi_R(H) + (1-\rho)\pi_R(L)}$$

Also, notice that V is monotone decreasing in $\frac{\bar{z}_S}{\bar{z}_R}$. Dividing Eq (36) by (37) for each separate three cases yield

$$\frac{q_S}{q_R} = \begin{cases} V, & \text{if } 0 \le \lambda \le \bar{\mu}; \\ \frac{1-\lambda}{\lambda(1+\bar{l})}\frac{\bar{z}_R}{\bar{z}_S}, & \text{if } \bar{\mu} < \lambda \le \hat{\mu}; \\ \frac{1-\bar{\mu}}{\bar{\mu}(1+\bar{l})}\frac{\bar{z}_R}{\bar{z}_S}, & \text{otherwise}; \end{cases} \tag{50}$$

where $\bar{\mu}$ and $\hat{\mu}$ are also uniquely determined given $\frac{\bar{z}_S}{\bar{z}_R}$ such that[5]

$$\bar{\mu} = \frac{1}{V\frac{\bar{z}_S}{\bar{z}_R}(1+\bar{l})+1}, \tag{51}$$

$$\hat{\mu} = \frac{\pi_R(H)}{\pi_S(H)\frac{\bar{z}_S}{\bar{z}_R}(1+\bar{l})+\pi_R(H)}. \tag{52}$$

Eq (50) implies that $\frac{q_S}{q_R}$ is a monotone decreasing continuous function of $\frac{\bar{z}_S}{\bar{z}_R}$ such that $\frac{q_S}{q_R}$ decreases from a positive number to zero as $\frac{\bar{z}_S}{\bar{z}_R}$ goes from zero to ∞. Because $g_S(\cdot)$ and $g_S(\cdot)$

[5]This is immediate from Eq (32) and (35).

are homogeneous, Eq (48) implies

$$\frac{F_S(\bar{z}_S, \bar{z}_R)}{F_R(\bar{z}_S, \bar{z}_R)} = \frac{(g'_S)^{-1}(\frac{q_S}{q_R})}{(g'_R)^{-1}(1)}. \tag{53}$$

Using Eq (25) and (53), we derive the following equation that should hold in equilibrium:

$$\frac{\bar{z}_S}{\bar{z}_R} = \frac{(g'_S)^{-1}(\frac{q_S}{q_R})}{(g'_R)^{-1}(1)}. \tag{54}$$

The left-hand side of Eq (54) increases from zero to ∞ as $\frac{\bar{z}_S}{\bar{z}_R}$ goes from zero to ∞. On the other hand, the right-hand side decreases from a positive number to zero as $\frac{\bar{z}_S}{\bar{z}_R}$ goes from zero to ∞. Therefore, there exists a unique solution for Eq (54), and this yields a unique $\frac{q_S}{q_R}$ from (50).

We will now prove that there exist a unique pair of allocations $(\bar{z}_S, \bar{z}_R)$ and prices (q_S, q_R) given the ratios $\frac{\bar{z}_S}{\bar{z}_R}$ and $\frac{q_S}{q_R}$. We let $\hat{z}$ be the ratio $\frac{\bar{z}_S}{\bar{z}_R}$. Now we fix $\hat{z}$ because it is given. Then, we can represent the pricing kernel in Eq (30) given $\bar{z}_R \in \big[0, (g_R)^{-1}\big(\frac{w g_R(1)}{g_S(\hat{z})+g_R(1)}\big)\big]$ as follows:

$$m(\omega) = \beta \frac{u'\big(\pi_S(\omega)\hat{z}\bar{z}_R + \pi_R(\omega)\bar{z}_R\big)}{u'\big(w - g_R(\bar{z}_R)\big(\frac{g_S(\hat{z})}{g_R(1)} + 1\big)\big)}, \quad \text{for all } \omega \in \{H, L\}. \tag{55}$$

Notice that $m(\omega)$ decreases as $\bar{z}_R$ increases. Then, Eq (54) implies that q_R decreases as $\bar{z}_R$ increases. Using Eq (25), we get

$$\bar{z}_R = g'_R(q_R). \tag{56}$$

The left-hand side of Eq (54) increases from zero to ∞ as $\frac{\bar{z}_S}{\bar{z}_R}$ goes from zero to ∞. On the other hand, the right-hand side decreases from a positive number to zero as $\frac{\bar{z}_S}{\bar{z}_R}$ goes from zero to $(g_R)^{-1}\big(\frac{w g_R(1)}{g_S(\hat{z})+g_R(1)}\big)$. Therefore, there exists a unique solution for Eq (56), thus this allow us to obtain a unique solution for the other three variables q_S, q_R and $\bar{z}_S$. □

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