

# Limited Capital Market Participation and Human Capital Risk\*

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## Abstract

By introducing a labor market into the neoclassical asset pricing model, this paper shows that limited capital market participation can be an equilibrium outcome. Labor contracts are derived endogenously as part of a dynamic equilibrium in a production economy. Firms write labor contracts that insure workers, allowing agents to achieve a Pareto optimal allocation even when the span of asset markets is restricted to just stocks and bonds. Capital markets facilitate this risk sharing because it is there that firms offload the labor market risk they assumed from workers. In effect, by investing in capital markets investors provide insurance to wage earners who then optimally choose not to participate in capital markets.

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A commonly held view amongst financial economists is that a significant fraction of wealth consists of non-tradable assets, most notably human capital wealth. Indeed, this hypothesis is often used to explain why one of the key predictions of the CAPM does not hold, that all agents hold the same portfolio of risky assets. Because investors should use the capital markets to diversify as much risk as possible, and because non-tradable human capital exposure varies across individuals, investors should optimally choose to hold different portfolios of risky assets. Although this explanation certainly has the potential to explain the cross sectional variation in portfolio holdings, it also necessarily implies wide stock market participation. However, the fact is that the majority of people do not participate in the capital markets. Not only do these individuals appear to eschew the opportunity to partially hedge their human capital exposure, the hedging of human capital risk does not appear to be a primary motivator for the minority of people who actually do participate in capital markets. Instead, the anecdotal evidence suggests that rather than a desire to hedge, what motivates most investors is a willingness take on additional risk because they find the risk return tradeoff attractive.<sup>1</sup> The objective of this paper is to put forward a plausible explanation for these two characteristics of investor behavior.

The most commonly cited explanation for why most people do not participate in capital markets is barriers to entry, although in economies such as the United States it is difficult to accept that significant economic barriers to entry exist.<sup>2</sup> Instead, most researchers cite educational barriers to entry because research has shown that education level is strongly correlated with participation.<sup>3</sup> But the problem with this explanation for limited stock market participation is that it does not address the question of *why* the educational barriers exist at all. After all, we see wide participation in arguably more complicated financial products such as mortgages, auto leases and insurance. In these cases the educational barriers to entry were removed by the motivation to make profits — firms invested considerable resources in educating people so they could sell these products. Given the welfare gain to hedging non-tradable human capital, why does a similar economic motivation to educate consumers to hold stocks apparently not exist?

Market incompleteness may potentially offer an explanation for limited stock market participation. For instance, the asset span might be so “narrow” that the stock market offers little opportunity for Pareto improving trades. Although rarely cited explicitly, this explanation is implicit in the literature on non-traded wealth. But, for this explanation to be credible, one must also then account for why the asset span does not endogenously expand. In fact, the span of traded assets has changed only marginally in recent years, despite the explosion in the number

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<sup>1</sup>For example, considerable resources are devoted to advising people on how to find high return investments whereas advice on investments with good hedging characteristics is largely non-existent.

<sup>2</sup>Andersen and Nielsen (2011) provide evidence suggesting that non-participation does not derive from financial barriers to entry.

<sup>3</sup>Mankiw and Zeldes (1991) document the relation between education and participation and Hong, Kubik, and Stein (2004) document that non-formal education, such as social interaction, is also correlated with participation. Malmendier and Nagel (2009) provide evidence that irrationality might also play a role — investors appear to misestimate the return to investing in capital markets because they put too much weight on their own experience.

of new assets. More importantly, one would not naturally expect incompleteness to result in non-participation. Indeed, the low correlation between human capital and stock market returns documented in Lustig and Van Nieuwerburgh (2008) would suggest that despite the incompleteness, the stock market offers diversification benefits which would imply wide participation. Thus, market incompleteness appears to be an unlikely explanation for limited stock market participation.

If frictions, like barriers to entry and market incompleteness, are not preventing agents from participating, then they must be choosing not to participate. One possibility is that agents' initial endowments and productivities are naturally so close to a Pareto optimal allocation that there is little reason to engage in further trade. But considering the heterogeneity in actual endowments, this explanation seems implausible. A more plausible possibility is that some agents are able to share risk by trading in other markets and therefore trading in stock markets provides little incremental benefit.

Building on this insight, we identify the labor market as one such market and posit that the unwillingness of some individuals to use capital markets is a consequence of the fact that they are able to share enough risk through their wage contracts so that the benefit of trading in capital markets is small. A Pareto optimal allocation can therefore be achieved even with the “narrow” asset span we observe in actual stock markets implying that limited stock market participation is an efficient equilibrium outcome.

We focus on labor markets because they are an ideal place to share risk. The structure of most firms has historically been built around long-term tailored labor contracts between the firm and its workers. Indeed, viewed this way, one might wonder why all risk cannot be optimally shared in labor markets. The reason is that long-term labor contracts are not necessarily efficient for all employees — some employees are better off retaining the flexibility to switch jobs. Because of this labor market mobility, to achieve efficient risk sharing, asset markets are also required.

Our results call into question one of the basic assumptions in asset pricing — that because asset markets do not span labor risk, human capital is not traded. In our equilibrium, the labor market allows workers to trade human capital risk. The implication is that even though risk is shared efficiently, because not all wealth is traded in equity markets, the equity risk premium is not the same as the risk premium for consumption risk. As a result we can generate a substantial equity risk premium even while the risk premium for consumption risk is modest.

By recognizing the important role of labor contracts in the riskiness of equity, we demonstrate why, in an otherwise standard neoclassical model of asset prices, the volatility of equity prices might bear little apparent relation to the volatility of consumption. An important empirical fact that the standard neoclassical model of asset prices appears to be unable to explain is the seeming disconnect between the volatility of consumption and asset prices. Not only is consumption volatility significantly lower than the volatility of asset prices, but the two series behave manifestly differently. For example, average quarterly volatility of the S&P 500 index

is 68% higher during recessions (as identified by the NBER).<sup>4</sup> Yet, the concomitant increase in consumption volatility is much smaller if it exists at all — over the period 1947-2009, the point estimate of the volatility of (seasonally adjusted) quarterly GDP growth in NBER recessions is only 11% higher than in expansions.<sup>5</sup> As we will show, for a reasonable choice of parameter values, our model can quantitatively match this stylized fact. We can also explain a related empirical finding: that equity wealth is considerably more volatile and human capital wealth is less volatile than total wealth.<sup>6</sup> Because human capital wealth is traditionally measured using *wage income*, that is, the income that results once risk sharing has already taken place in the labor market, traditional measures underestimate the volatility of human capital wealth.

In our equilibrium a majority of workers choose not to participate in equity markets because their labor contracts already efficiently share risk. Consequently, these workers choose to remain employed with a single employer. This result implies that such workers consume their wage income, that is, workers that do not change jobs often are more likely to have no other investment income and so for these workers, consumption and wages are identical. Because wages can be measured more accurately than consumption, this result has an important implication — it suggests that the wages of such workers can be used as a proxy for consumption in a test of the consumption CAPM.

The paper is organized as follows. In the next section we provide a brief literature review. In Section 2 we introduce the model. In Section 3, we derive the Pareto efficient outcome and show how it can be implemented as an equilibrium outcome under realistic restrictions. Section 4 discusses the asset pricing implications both in the time series and in the cross section. We discuss the robustness of the model's predictions in Section 5. Section 6 makes some concluding remarks. All proofs are left to the appendix.

## 1 Background

The idea that one role of the firm is to insure its workers' human capital risk dates at least as far back as Knight (1921). Knight takes as a primitive that the job of worker and manager entails taking on different risks and notes that entrepreneurs bear most of the risk. Using this idea, Kihlstrom and Laffont (1979) endogenizes who, in a general equilibrium, becomes an entrepreneur and who becomes a worker. Less risk averse agents choose to be entrepreneurs who then optimally insure workers. However, the wage contract in that paper is exogenously imposed rather than an endogenous response to the desire to optimally share risk and so the

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<sup>4</sup>It is 21.4% in recessions and 12.7% at other times. Quarterly volatility is defined to be the standard deviation of daily returns of the S&P 500 index over the quarter over the period 1962-2009. This difference is highly statistically significant.

<sup>5</sup>Using quarterly data published by the BEA, the volatility of GDP growth in recessions is 4.66% while is 4.19% at other times over the 1947-2009 time period.

<sup>6</sup>See, for example, Lustig, Van Nieuwerburgh, and Verdelhan (2007).

resulting equilibrium is not Pareto efficient.

The papers that first recognized the importance of endogenizing the wage contract, and therefore the ones most closely related to our paper, are Dreze (1989) and Danthine and Donaldson (2002). Like us, Dreze (1989) considers the interaction between a labor and capital market in general equilibrium and focuses on efficient risk sharing. Our point of departure is how we model production — Dreze does not consider the implication of productive heterogeneity. Consequently, there is no natural reason (beyond differences in risk aversion and wealth) for some workers to insure other workers in Dreze’s model. Hence, the model does not explain limited capital market participation or focus on the return to bear labor risk.

Danthine and Donaldson (2002), like us, explicitly model both labor and financial markets with agent heterogeneity. Their model features investors and workers, but, importantly, Danthine and Donaldson (2002) do not allow workers to invest or investors to work and so their model does not address endogenous capital market participation. In that model all workers are insured by investors who are endowed with wealth rather than productivity and hence have a precautionary reason to save which they do by investing in firms. Because this motive is missing in our model, prices must adjust in our model to induce some workers to insure other workers.

Our paper also contributes to the large literature, that started with Mayers (1972), studying the effect of non-tradable wealth in financial markets. The main results in that literature are that investors should no longer hold the same portfolio of risky assets and the single factor pricing relation must be adjusted. Although Fama and Schwert (1977) finds little evidence supporting Mayer’s model, both Campbell (1996) and Jagannathan and Wang (1997) find that adding a measure of human capital risk significantly increases the explanatory power of the CAPM. Santos and Veronesi (2006) find that the labor income to consumption ratio has predictive power for stock returns and can help explain risk premia in the cross section. Because wage contracts provide insurance for human capital risk, our model implies that wages (the typical measure of human capital used in the literature) should have explanatory power for stock returns.

The theoretical predictions of the neoclassical asset pricing model rely on effectively complete markets, so initially researchers were tempted to attribute the failure of those models to market incompleteness. However, Telmer (1993) and Heaton and Lucas (1996) convincingly argue that market incompleteness cannot account for important puzzles such as the apparently high risk premium of the market portfolio. As we show in this paper, quite the opposite intuition might be true. The failure of the models might stem from the fact that agents actually share risk more completely than is supposed in the literature. If labor markets effectively share risk, then because equity holders are the ultimate insurers of human capital risk, they will demand a high risk premium. As we will demonstrate, our results are consistent with the findings in Mankiw and Zeldes (1991) and Brav, Constantinides, and Geczy (2002) in that those who choose not to participate are less wealthy, less educated and more reliant on wage income as their source of wealth. Furthermore, consistent with the anecdotal evidence, the primary motivation for

investing in capital markets is the attractive risk return tradeoff offered, not a desire to hedge human capital risk.

## 2 Model

Like any source of risk, human capital risk has both an idiosyncratic component and a systematic component. Although the idiosyncratic component is likely to be large, especially early in a person's career, we will focus exclusively on the systematic component because we are interested in the implications of how agents share risk in the economy. Idiosyncratic risk, by its very nature, can be diversified away, so there is little reason for any agent to hold this risk in a complete market equilibrium. Consequently, the risk sharing implications of sharing idiosyncratic risk are well understood.<sup>7</sup>

Given our objective to study how systemic risk is shared in the economy, our model must include heterogeneous agents. An important source of individual heterogeneity in the economy is worker flexibility: Some workers only have access to a single production technology while others can choose between production technologies. Building on this insight we model productivity as follows. Our economy consists of a continuum of workers that produce a single, perishable, consumption good, using a technology that is parameterized as follows: The total instantaneous output produced is  $A_t(b + fs)$ .  $A_t$  is a common component and  $(b + fs)$  is an individual component we term an individual worker's production technology, where  $s$  is the variable that captures the current state of the economy. *Inflexible* workers are endowed with a fixed  $b$  and  $f$  while *flexible* workers can choose  $b$  and  $f$  throughout their career (by switching industries).

We model the production technology set as follows. There is a closed set of production technologies (industries),  $\mathcal{P} \subset [\underline{b}, 1] \times [0, \overline{K}]$ , for some  $\overline{K} > 0$  and  $\underline{b} < 1$ . Each inflexible agent only has access to a single production technology in this set,  $(b, f)$ , and produces  $A_t(b + fs)$  of a consumption good, where  $A_t \equiv A_0 e^{rt}$  is the non-stochastic<sup>8</sup> part of production common to all agents. We assume that all inflexible agents have access to technologies with  $b \geq 0$ , to ensure that each individual's production is nonnegative in all states of the world. Flexible agents have access to every production technology in  $\mathcal{P}$ . The production technology set has the properties that  $(\underline{b}, \overline{K}) \in \mathcal{P}$ ,  $(1, 0) \in \mathcal{P}$ ,  $(b, \overline{K}) \in \mathcal{P} \Rightarrow b = \underline{b}$  and  $(1, f) \in \mathcal{P} \Rightarrow f = 0$ .

The dynamics of  $A_t$  are meant to capture overall economic growth and allows us to model recessions as a relative drop in productivity. The stochastic process  $s$  is a diffusion process on

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<sup>7</sup>Although it's not the focus of the paper, Harris and Holmström (1982) makes it clear how agents share idiosyncratic labor risk. That paper shows that most, but not all, of this risk can be removed by the labor contract. Under the optimal labor contract firms insure all agents against negative realizations of idiosyncratic labor risk but agents remain exposed to some positive realizations. Of course, the owners of these firms do not have to expose themselves to this labor risk because by holding a large portfolio of firms, they can diversify the risk away.

<sup>8</sup>It is straightforward to extend our analysis to allow stochastic growth in  $A_t$  as long as innovations in  $A_t$  are independent of innovations in  $s$ .

$\mathbb{R}_+$  that summarizes the state of the world:

$$ds = \mu(s) dt + \sigma(s) d\omega.$$

We will model  $s$  as a mean-reverting square root process,

$$ds = \theta(\bar{s} - s) dt + \sigma\sqrt{s} d\omega, \quad (1)$$

where the condition  $2\theta\bar{s} > \sigma^2$  ensures strict positivity. The mean-reversion introduces a business cycle interpretation and is useful for calibrations, although, as we shall see, much of the theory goes through for general  $\mu(s)$  and  $\sigma(s)$ , so long as  $\mu$  and  $\sigma$  are smooth,  $\sigma$  is strictly positive, and the growth conditions  $|\mu(s)| \leq c_1(1 + s)$ ,  $\sigma(s) \leq c_2(1 + s)$  are satisfied for finite constants,  $c_1$  and  $c_2$ . It is natural to define a recession as states for which  $s < \bar{s}$ , whereas an expansion is when  $s > \bar{s}$ .

Let the inflexible agents be indexed by  $i \in \mathcal{I} = [0, \alpha]$ , where  $0 < \alpha < 1$ , with agent  $i$  working in industry  $(b_i, f_i)$ . Here, we assume that  $b_i$  and  $f_i$  are measurable functions that are nondegenerate in the sense that it is neither the case that the full mass of agents work in industries with  $b = 0$ , nor in industry  $(1, 0)$ . Then the total productivity of all inflexible agents in the economy is

$$A_t K_I(s) = A_0 e^{rt} K_I(s),$$

where

$$K_I(s) \equiv \int_{i \in \mathcal{I}} \frac{1}{\alpha} (b_i + f_i s) di = \bar{b} + \bar{f} s. \quad (2)$$

Note that  $0 < \bar{b} < 1$  and  $0 < \bar{f} < \bar{K}$ .

The rest of the agents in the economy are *flexible* agents, comprising mass  $1 - \alpha$ ,  $i \in \mathcal{F} = (1 - \alpha, 1]$ . Because these agents have access to any production technology in  $\mathcal{P}$  and are free to move between production technologies at any point in time, for a given  $s$ , it is optimal for them to work in an industry  $(b^*, f^*)$ , that solves

$$(b^*, f^*) = \arg \max_{(b, f) \in \mathcal{P}} b + f s, \quad (3)$$

leading to the optimal productivity of flexible agents

$$A_t K_F(s) = A_0 e^{rt} K_F(s),$$

where

$$K_F(s) \equiv b^*(s) + f^*(s)s. \quad (4)$$

Notice that, at any point in time, all flexible agents choose to work in industries that generate

the same output. Furthermore, we have

**Lemma 1** *The optimal production function of flexible agents satisfies:*

- (a)  $K_F(0) = 1$ ,
- (b)  $\lim_{s \rightarrow \infty} \frac{K_F(s)}{s} = \bar{K}$ ,
- (c)  $K_F(s)$  is a convex function of  $s$ .

We next assume that flexible agents can work part time in different industries, i.e., if  $(b_1, f_1) \in \mathcal{P}$  and  $(b_2, f_2) \in \mathcal{P}$ , then  $(\lambda b_1 + (1 - \lambda)b_2, \lambda f_1 + (1 - \lambda)f_2) \in \mathcal{P}$  for all  $\lambda \in [0, 1]$ . This implies that for all  $b \in [\underline{b}, 1]$  there is a  $(b, f) \in \mathcal{P}$ . Now, flexible agents will only consider production technologies on the efficient frontier,  $(b, f(b))$ , where  $f(b) \stackrel{\text{def}}{=} \max\{f : (b, f) \in \mathcal{P}\}$ , and it follows immediately that  $f$  is a strictly decreasing, concave function defined on  $b \in [\underline{b}, 1]$ , such that  $f(\underline{b}) = \bar{K}$  and  $f(1) = 0$ . Going forward, we make the additional technical assumptions that  $f$  is strictly concave, twice continuously differentiable, and that  $f'(\underline{b}) = 0$ , and  $f'(1) = -\infty$ . Under these assumptions it is easy to show that

**Lemma 2**  $K_F(s)$  is a twice continuously differentiable, strictly convex function, such that  $K'_F(0) = 0$  and  $\lim_{s \rightarrow \infty} K'_F(s) = \bar{K}$ .

Lemma 2 ensures that  $K_F(s)$  is a diffusion process (which is, of course, also true of  $K_I(s)$ ). The total output in the economy at time  $t$  is:

$$A_t K_{tot}(s_t) = A_0 e^{rt} K_{tot}(s_t), \quad (5)$$

where

$$K_{tot}(s) \equiv \alpha K_I(s) + (1 - \alpha) K_F(s), \quad (6)$$

implying that  $K_{tot}(s)$  is also a diffusion process.

Figure 1 plots the production function for flexible workers and the average inflexible worker. Note that because  $f(b)$  is concave, flexible workers are always more productive than the average inflexible worker. The next lemma adds the additional observation that worker mobility implies that flexible workers will move into safer jobs in bad times and riskier jobs in good times so that they will have a natural advantage in providing insurance to inflexible workers.

**Lemma 3** *The following results hold for the volatility of the agents' productivity:*

- (a) *For low  $s$ , the volatility of the flexible agent's productivity is lower than that of the inflexible agent:*

$$Vol\left(\frac{dK_F}{K_F}\right) < Vol\left(\frac{dK_I}{K_I}\right).$$

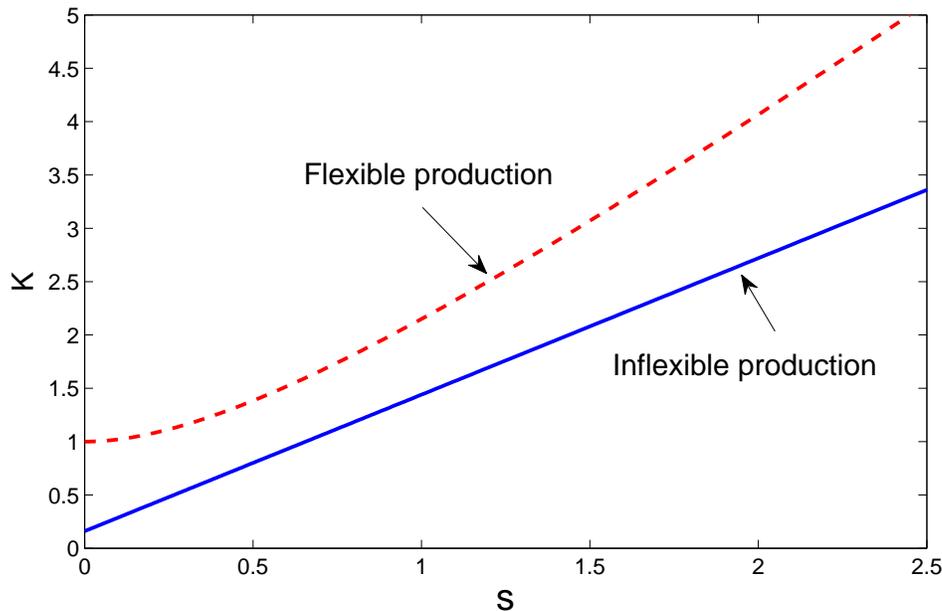


Figure 1: **Average Production of Flexible and Inflexible agents,  $K_F(s)$  and  $K_I(s)$ :** Because  $f(b)$  is concave, flexible workers are always more productive than the average inflexible worker. Of course, in each state there is always an inflexible worker who is as productive as the flexible workers are.

(b) For high  $s$ , the volatility of the flexible agent's productivity is higher than that of the inflexible agent,

$$Vol\left(\frac{dK_F}{K_F}\right) > Vol\left(\frac{dK_I}{K_I}\right).$$

Workers and firms are organized as follows. A worker can choose either to work for himself and produce the consumption good, or he can choose to “sell” his production to a firm and earn a wage instead. Workers are also owners — they are free to invest in firms through the capital markets and consume any dividend payments. In equilibrium, markets must clear — all firms must attract enough investment capital to fulfill their wage obligations. Finally, we assume that all agents are infinitely lived, with constant relative risk-aversion (CRRA), risk-aversion coefficient  $\gamma > 0$ , and expected utility of consumption

$$U_i(t) = E_t \left[ \int_t^\infty e^{-\rho(s-t)} u(c_s) ds \right]. \quad (7)$$

Here,

$$u(c) = \begin{cases} \log(c), & \gamma = 1, \\ \frac{c^{1-\gamma}}{1-\gamma}, & \gamma \neq 1. \end{cases} \quad (8)$$

Throughout the paper, we assume an infinite horizon economy. In our numerical calculations we use a finite horizon set-up, where the horizon is large enough to get convergence to the solution of the infinite horizon economy (i.e., the steady state solution). An advantage of this approach is that it leads to a unique solution, allowing us to avoid the nontrivial issue of defining transversality conditions to rule out bubble solutions in an economy with nonlinear dynamics.

### 3 Equilibrium

We begin by deriving the complete market Pareto optimal equilibrium and then explain how this equilibrium can be implemented. Because  $A_t K_{tot}$  maximizes total output, any Pareto optimal equilibrium must have this output.

#### 3.1 Complete Markets Competitive Outcome

Under the complete markets assumption, a representative agent with utility  $u_r$  exists, such that the solution to the representative agent problem is identical to the solution of the multi-agent problem. Moreover, all agents have CRRA utility functions with the same  $\gamma$ , so  $u_r$  is also of the CRRA form, with the same  $\gamma$ . Thus, in a complete market equilibrium, the value of any asset generating instantaneous consumption flow  $\delta(s_t, t)dt$ , is

$$\begin{aligned} P(s_t) &= \frac{1}{u'_r(A_t K_{tot}(s_t))} E \left[ \int_t^\infty e^{-\rho(\tau-t)} u'_r(A_\tau K_{tot}(s_\tau)) \delta(s_\tau, t) d\tau \right] \\ &= K_{tot}(s_t)^\gamma E \left[ \int_t^\infty e^{-\hat{\rho}(\tau-t)} K_{tot}(s_\tau)^{-\gamma} \delta(s_\tau, t) d\tau \right], \end{aligned} \quad (9)$$

where  $\hat{\rho} = \rho + \gamma r$ . Hence, the total value of human capital of all agents of each type (their total wealth) at time  $t = 0$  is:

$$W_I \equiv \alpha A_0 K_{tot}(s_0)^\gamma E \left[ \int_0^\infty e^{-(\hat{\rho}-r)\tau} K_{tot}(s_\tau)^{-\gamma} K_I(s_\tau) d\tau \right], \quad (10)$$

$$W_F \equiv (1 - \alpha) A_0 K_{tot}(s_0)^\gamma E \left[ \int_0^\infty e^{-(\hat{\rho}-r)\tau} K_{tot}(s_\tau)^{-\gamma} K_F(s_\tau) d\tau \right]. \quad (11)$$

Any Pareto optimal equilibrium features perfect risk sharing — all agents' consumption across states have the same ordinal ranking. Moreover, because of the CRRA assumptions, it is well known that a stronger result applies in our equilibrium — all agents' ratio of consumption across any two states is the same. In other words, every agent consumes the same fraction of total

output in every state:

$$c_I(s, t) = \eta A_t K_{tot}(s) = \eta A_t (\alpha K_I(s) + (1 - \alpha) K_F(s)) \quad (12)$$

$$c_F(s, t) = (1 - \eta) A_t K_{tot}(s) = (1 - \eta) A_t (\alpha K_I(s) + (1 - \alpha) K_F(s)) \quad (13)$$

where  $c_I$  and  $c_F$  is the aggregate consumption of all the inflexible and flexible agents respectively and  $\eta$  is the fraction of the total output consumed by all the inflexible agents. Given that the agents can trade their human capital, from the budget constraint at time 0 it follows that,

$$\eta = \frac{W_I}{W_I + W_F}. \quad (14)$$

We can also view  $\eta$  as a function of the initial state,  $\eta(s_0)$ . The following properties of  $\eta$  follow immediately:

**Lemma 4** *The function  $\eta(s_0)$ , defined by (14) satisfies the following conditions*

(i)  $\eta$  is twice continuously differentiable.

(ii)  $\eta \in \left[ \min \left\{ \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{1}{b}}, \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{K}{f}} \right\}, \alpha \right]$ .

(iii)  $\eta$  is increasing for small  $s_0$  and decreasing for large  $s_0$ .

(iv)  $\lim_{s_0 \rightarrow \infty} \eta(s_0) = \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{K}{f}}$ .

In words, because inflexible agents can never be less productive than flexible agents, the equilibrium fraction of consumption inflexible agents consume is bounded above by  $\alpha$ , the fraction of the economy they comprise. It is bounded below by the inflexible consumption in the state that their share of productivity is minimized, that is either when  $s = 0$  or  $\infty$ . Because inflexible agents share of productivity is lower both for low and high values of the state variable,  $\eta$  has an interior maximum. It therefore follows that if we define  $s^* \stackrel{\text{def}}{=} \min\{\arg \max_{s_0} \eta(s_0)\}$ , then  $0 < s^* < \infty$ . The wealth share of the inflexible agent is thus maximized at a finite value,  $s^*$ , and we denote the wealth share at this point by  $\eta^* \equiv \eta(s^*)$ . Finally, as the state variable goes to infinity, both types of agents productivity converge to a linear function of the state variable and, in addition, the value of insurance becomes negligible, so the equilibrium converges to one with no risk sharing — each agent consumes what he produces.

To solve explicitly for the equilibrium requires computing the expectation in (11), which can be accomplished using standard techniques from dynamic programming:

**Proposition 1** *The price,  $P(s, t)$ , of an asset that pays dividends  $\delta(s, t)$  satisfies the PDE*

$$P_t + \left( \mu(s) - \gamma R(s) \sigma(s)^2 \right) P_s + \frac{\sigma(s)^2}{2} P_{ss} - \left( \hat{\rho} + \gamma \mu(s) R(s) - \frac{\sigma(s)^2}{2} \gamma (\gamma + 1) R(s)^2 + \frac{\sigma(s)^2}{2} \gamma T(s) \right) P + \delta(s, t) = 0 \quad (15)$$

where

$$R(s) = \frac{K'_{tot}(s)}{K_{tot}(s)}, \quad \text{and} \quad T(s) = \frac{K''_{tot}(s)}{K_{tot}(s)}. \quad (16)$$

An immediate implication of this proposition is that the instantaneous risk free interest rate is captured by the term in front of  $P$ :<sup>9</sup>

$$r_s \equiv \hat{\rho} + \gamma \mu(s) R(s) - \frac{\sigma(s)^2}{2} \gamma (\gamma + 1) R(s)^2 + \frac{\sigma(s)^2}{2} \gamma T(s). \quad (17)$$

In general, we will need to solve (15) numerically, which may be nontrivial because it is defined over the whole of the positive real line,  $s \in \mathbb{R}_+$ . It is also not *a priori* clear what the boundary conditions are either at  $s = 0$ , or at  $s = \infty$  where  $P_s$  may become unbounded. We follow Parlour, Stanton, and Walden (2009) and avoid these issues by making the transformation,  $z \stackrel{\text{def}}{=} \frac{s}{s+1}$  to get:

**Proposition 2** *The price,  $P(s, t)$ , of an asset that pays dividends  $\delta(s, t)$ , where  $\delta(s, t) \leq ce^{rt} K_{tot}(s)^\gamma$  for some positive constant  $c$ , and  $t < T$  is*

$$P(s, t) = K_{tot}(s)^\gamma Q \left( \frac{s}{s+1}, t \right),$$

where  $Q : [0, 1] \times [0, T] \rightarrow \mathbb{R}_+$  solves the PDE

$$Q_t + (1-z)^2 \left( \mu \left( \frac{z}{1-z} \right) - \sigma \left( \frac{z}{1-z} \right)^2 (1-z) \right) Q_z + \frac{1}{2} (1-z)^4 \sigma \left( \frac{z}{1-z} \right)^2 Q_{zz} - \hat{\rho} Q + \delta \left( \frac{z}{1-z}, t \right) K_{tot} \left( \frac{z}{1-z} \right)^{-\gamma} = 0, \quad (18)$$

and  $Q(z, T) = 0$ .

Without loss of generality, we assume that  $A_0 = 1$  going forward, since all variables are homogeneous of degree zero or one in  $A_0$ . All the numerical solutions in this paper were derived by

<sup>9</sup>Heuristically, a risk-free zero coupon bond with maturity at  $dt$  will have a price that is almost independent of  $P$ , so  $P_s$  and  $P_{ss}$  are close to zero. Therefore, the local dynamics are  $P_t - r_s P = 0$ , i.e.,  $\frac{dP}{P} = -r_s dt$  so the short term discount rate is indeed  $r_s$ .

solving (18).

## 3.2 Implementation

We now show how the complete market equilibrium can be implemented in an incomplete market economy that uses labor markets as an additional risk sharing tool. Obviously, because our object is to provide insights on how actual markets, which are far from complete, share risk, it is important that we model both asset and labor markets realistically. Hence, we restrict agents' and firms' ability to write and trade contracts in the following ways:

### Restriction 1

- (i) *Binding contracts cannot be written directly between agents.*
- (ii) *Firms may enter into binding contracts with agents subject to the following restrictions:*
  - (1) *Limited liability may not be violated.*
  - (2) *Workers and equity holders cannot be required to make payments.*
- (iii) *Banks may enter into short term debt contracts with agents and firms, paying an interest rate  $r_s$ .*

These restrictions reflect the practical limitations of markets. Because individualized binding contracts cannot trade in anonymous markets, a matching mechanism does not exist that would allow for widespread use of bilateral contracts as a risk sharing device. Perhaps because there are far fewer firms than agents in the economy, so it is easier to match firms and agents, we do observe binding bilateral labor contracts written between agents and firms. However, even these contracts are limited. Both equity and labor contracts are one sided in the sense that typically firms commit to make payments to agents. Agents very rarely commit to make payments to firms and courts rarely enforce such contracts. The only condition under which agents can enter a contract that commits them to make payments is if they take a loan from a bank. Both firms and agents can either borrow or lend from a bank subject to the condition that in equilibrium the supply of loans must equal the amount of deposits. Thus, the span of traded assets consists of debt and equity. As we will see, there is no default in equilibrium so the interest rate banks pay is the risk free rate.

We also impose the following restriction on the industries in which firms operate:

**Restriction 2** *Firms are restricted to operate in only one industry. That is, all workers in a firm must have the same  $b$  and  $f$ .*

In reality, most firms operate in a single industry so most workers switch jobs when they change occupations.<sup>10</sup> Although conglomerates do exist, even these firms typically operate in only a few

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<sup>10</sup>Moscarini and Thomsson (2007) document that 63% of workers who changed occupations also changed employers.

industries. Our results would not change if we allowed firms to operate in a subset of industries. What we cannot allow is a firm that operates in *every* industry.

We assume that there is a (very) small cost to dynamic trading in capital markets:

**Restriction 3** *Dynamic trading in equity markets imposes a utility cost of  $\epsilon = 0^+$  per unit time.*

This restriction captures transaction costs of active trading, as well as the utility cost of designating time and effort to active portfolio rebalancing strategies. The condition implies that an equilibrium outcome that does not require active portfolio trading in asset markets dominates an equilibrium that is identical in real terms, but that *does* require active portfolio trading. For tractability, we do not impose any transaction costs of switching jobs, although it can be argued that such costs are also present, and in fact may be higher than the costs of dynamic trading in asset markets. In Section 5 we will evaluate the importance of this assumption by introducing a cost of switching jobs and argue that the main implications of our model are robust.

We are now ready to describe how the complete markets equilibrium can be implemented under these restrictions. At first glance it might appear as if asset markets are unnecessary. After all, we allow firms to write bilateral contracts with agents, so by serving as an intermediary, firms can effectively allow agents to write bilateral contracts between themselves. For example, firms could hire both types of workers, pool their production and reallocate it by paying wages equal to a constant fraction of total production. However, such contracts alone cannot implement the Pareto optimal equilibrium. The reason is that in such an equilibrium, although risk is efficiently shared conditional on production, total production is not maximized — flexible workers must switch industries in order to maximize their production. But the only way for the firm to pool production and reallocate it would be to extract a commitment of lifetime employment from flexible workers. Such a commitment is suboptimal. Because of the need for worker mobility, both labor and asset markets are required to implement the complete markets equilibrium.

To achieve the complete market equilibrium all inflexible agents sign a binding employment contract with firms in the industry of their specialty that commits both parties to lifetime employment.<sup>11</sup> Agents give up all their productivity and in return receive a wage equal to their Pareto optimal equilibrium allocation,  $\eta(s_0)A_tK_{tot}(s)$ , in every future state  $s$ . Flexible agents either choose to work for themselves, or work for firms and earn wages equal to their productivity. In some states, inflexible wages will exceed productivity. Because firms cannot force investors to make payments, firms require capital to credibly commit to the labor contract. They raise this capital by issuing limited liability equity. In states in which wages exceed productivity, the firm uses this capital to make up the shortfall and does not pay dividends. For the moment we

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<sup>11</sup>In reality, employment contracts that bind workers are not enforceable. However, about half the working population do in fact work for a single employer (see Hall (1982)). As we discuss in Section 5, this suggests that the lifetime employment contract is nevertheless common, that is, that firms use other means to commit employees to lifetime employment.

restrict attention to states in which the capital in the firm is positive.

Flexible agents purchase the equity by borrowing the required capital from the bank. Firms then redeposit the capital in the bank (ensuring that the supply of deposits equals the demand for loans) and pay instantaneous dividend flows equal to

$$A_t \max(\alpha K_I(s) + C_t r_s - \eta K_{tot}(s), 0),$$

where  $A_t C_t$  is the amount of capital owned by the firm at time  $t$  and  $\eta \equiv \eta(s_0)$ . Thus, flexible agents consume

$$A_t \left[ (1 - \alpha) K_F(s) + \max(\alpha K_I(s) + C_t r_s - \eta K_{tot}(s), 0) - C_t r_s \right], \quad (19)$$

where we assume (and later show) that flexible agents always choose to adjust their bank loans to match the capital firms deposit in the bank. Using (6), when dividends are positive, the term in square brackets in (19) becomes

$$(1 - \alpha) K_F(s) + \alpha K_I(s) + C_t r_s - \eta K_{tot}(s) - C_t r_s = (1 - \eta) K_{tot}(s), \quad (20)$$

so flexible agents consume their complete market allocation and  $dC_t = 0$ . Similarly, when dividends are zero we get

$$(1 - \alpha) K_F(s) - C_t r_s + \frac{dC_t}{dt}. \quad (21)$$

Now the stochastic change in firm capital equals the shortfall, that is,

$$dC_t = (\alpha K_I(s) + C_t r_s - \eta K_{tot}(s)) dt.$$

Substituting this expression into (21) gives

$$(1 - \alpha) K_F(s) - C_t r_s + (\alpha K_I(s) + C_t r_s - \eta K_{tot}(s)) = (1 - \eta) K_{tot}(s), \quad (22)$$

so the flexible agent consumes his complete markets allocation in every state in which the firm's capital is positive.

Finally, consider the first time that either the value of the firm drops to zero or the firm's capital drops to zero. In such a state the firm can raise additional capital by issuing new equity (either by repurchasing existing equity for zero and issuing new equity to raise capital, or if the equity is not worth zero, issuing new equity at the market price). Hence by always issuing new capital in this state, the firm can ensure that neither its capital, nor its value, will drop below zero and that it never pays negative dividends. Thus, in this equilibrium both agents always consume their complete markets allocation which is Pareto optimal. This implies that flexible agents cannot be better off by following a different borrowing policy, justifying our assumption

that they will always choose to borrow the amount firms deposit in the bank. Moreover, since this outcome implies a passive investment strategy for inflexible, as well as flexible, agents, this equilibrium implementation is optimal under the assumption of a small but positive cost of active rebalancing.<sup>12</sup>

The following proposition summarizes these results:

**Proposition 3** *The following implementation leads to the complete market Pareto efficient outcome:*

- *Flexible workers either work for themselves or for a firm which pays the instantaneous wage  $w_F = A_t K_F(s_t)$ .*
- *Inflexible workers work for publicly traded firms, which pay instantaneous wages equal to a constant multiple of aggregate production. In aggregate, firms pay the inflexible wage*

$$w_I = \eta A_t K_{tot}(s_t).$$

- *In states in which inflexible productivity plus interest on bank deposits exceeds wages, firms pay dividends equal to*

$$A_t[\alpha K_I(s) + C_t r_s - \eta K_{tot}(s)]. \quad (23)$$

*and retain capital  $A_t C_t$  with  $dC_t = 0$*

- *In states in which inflexible productivity plus interest on bank deposits does not exceed wages, firms pay no dividends and reduce capital to make wage payments*

$$dC_t = (\alpha K_I(s) + C_t r_s - \eta K_{tot}(s))dt. \quad (24)$$

- *The flexible workers own all the equity in the stock market. They pay for this equity by borrowing the capital from banks. Firms redeposit the capital in banks. Flexible workers optimally adjust their borrowing to ensure that at all times the supply of deposits equals the demand for loans.*
- *Whenever: (1) the price of the firm drops to zero, the firm raises new capital by repurchasing old equity for nothing and issuing new equity or (2) the amount of capital drops to zero, the firm raises new capital by issuing new equity at the market price.*

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<sup>12</sup>In fact, if in addition one assumes a (small) one-time cost of stock market participation, it is easy to show that this optimal implementation is *unique*, since it minimizes the fraction of the population that participates in the market.

There are two important distinguishing characteristics of this solution that reflect reality. First, it features limited capital market participation — only flexible workers participate in capital markets. Indeed, because job mobility precludes flexible workers from sharing risk in labor markets they must participate in capital markets for any risk sharing to take place. Without understanding the importance of the labor market, one might naively look at inflexible workers' wealth and conclude that because this wealth is not traded in asset markets, they would be better off using asset markets to hedge some of this exposure. But, in equilibrium inflexible workers choose not to further hedge their human capital risk exposure because it is not beneficial. In addition, flexible workers choose to hold equity, not because of a desire to hedge — they choose to increase the riskiness of their position — but because of the compensation they receive in terms of a high equity risk premium.

The implication, that inflexible workers choose not to participate in markets, is consistent with one of the most robust findings in the literature — that wealth, education and intelligence are positively correlated with stock market participation (see Mankiw and Zeldes (1991) and Grinblatt, Keloharju, and Linnainmaa (2010)). Clearly, flexible workers are wealthier in our model, but more importantly, if productive flexibility derives from education or intelligence, then they are likely to have higher I.Q. scores and be better educated. In fact, Christiansen, Joensen, and Rangvid (2008) show that the degree of *economics* education is casually (positively) related to stock market participation. They interpret this result as evidence that non-participation derives from educational barriers to entry. But their results are also consistent with flexibility. Not all education provides productive flexibility so we would expect to see variation in the type of education and stock market participation. Their study clearly documents this variation. Finally, note that non-participation in capital markets implies that inflexible workers also do not hold bonds, that is, they choose not to save. This result might help to explain the low savings rate observed in the U.S. — the reason workers choose not to save is that their labor contracts effectively do the saving for them.<sup>13</sup>

Another implication of limited stock market participation is that inflexible workers consumption is financed solely from their wage income. In the neoclassical model, the consumption of any optimizing agent should price assets, so this observation implies that inflexible workers' wages should explain asset returns. Because wages are measured much more accurately than consumption, the implication of our model is that the wage income of workers who do not switch jobs should do a better job explaining asset returns than their consumption. In contrast, *aggregate* wages, which include wages of both flexible and inflexible workers, are considerably less informative about expected returns, in line with the findings of Fama and Schwert (1977).

The second distinguishing characteristic of our solution is that firm equity can be thought of as an option-like claim on total consumption. We therefore expect the volatility of equity

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<sup>13</sup>Introducing a life-cycle dimension in our model would not change this, as long as the employer pays the Pareto optimal consumption share at all times.

returns to exceed the volatility of total consumption. Because we do not have idiosyncratic risk in our setting, this volatility imparts risk — equity is considerably more risky than total consumption.

That equity can be viewed as an option is well known. However, normally this insight is derived using financial leverage. In our case the firm has no debt, indeed it actually holds cash. In a standard setting this would mean that equity would not have option characteristics, indeed, because of the cash, equity would be *less* risky than the firm’s assets. In our setting, it is not financial leverage that gives equity option-like characteristics, but the operating leverage resulting from wage commitments. Notice that this operating leverage is considerably more risky than the typical kind of operating leverage studied in the literature. Typically, firms have the option to shut down — if a firm is losing money then it can reduce its scale or shut down altogether. However, in our case firms optimally choose to give up this option — they commit to continue to pay wages even when, *ex post*, the value maximizing decision would be to shut down and pay out the remaining capital to equity holders. In other words, the firm can not recoup its cash in bad states, because the capital is already “owned” by the workers through their labor contracts. Effectively investors *choose* to make their capital investments completely irreversible.

Because of this operating leverage, asset returns vary with the business cycle in a highly nonlinear fashion. This means that the unconditional link between real variables and asset returns can look quite weak even though they are instantaneously perfectly correlated. In fact, as we shall see, our results are in line with the findings in Duffee (2005), that consumption and equity returns are only weakly related in bad times, but are highly correlated in good times.

## 4 Asset Pricing Implications

Although the primary focus of our model is capital market participation, our equilibrium features novel asset pricing implications. Because flexible agents insure inflexible agents and use equity as the means to accomplish this transfer, equity is primarily an insurance contract in our model. This insurance imparts non-linearities in the price of equity. In this section we show how these non-linearities lead to characteristics that have the potential to at least partially explain some important asset pricing puzzles.

### 4.1 Parameterization

To study the non-linearities in equity prices, we must pick a set of values to parameterize the model. In this subsection we explain how we chose these parameters. It is important to appreciate that we are not picking the parameter set to show that the model can match important moments in the data. To expect a model as stylized as ours to explain all the important moments in the data is naive. To begin with, firms in our model consist exclusively

of labor, they have no capital. Agents all have the same utility function (CRRA) and are able to risk share perfectly. Labor markets are frictionless and agents have no alternative sources of wealth. None of these assumptions are realistic and all are likely to affect the magnitudes of the key moments. Consequently, our goal for this section is much less ambitious. We simply show that the same forces that explain limited market participation give rise to non-linearities in asset prices reminiscent of some important asset pricing puzzles.

We begin by assuming that flexible workers make up 1/3 of the working population, implying that 2/3 of the population choose not to participate directly in capital markets, in line with the estimates reported in Haliassos and Bertaut (1995), Guiso, Haliassos, Jappelli, and Claessens (2003), Hong, Kubik, and Stein (2004), Christiansen, Joensen, and Rangvid (2008) and Malmendier and Nagel (2009). In addition  $f$  has the form:

$$f(b) = \frac{(1-b) \left( 3\bar{K}\sqrt{\bar{K}-b\bar{K}} - (1-b)\sqrt{\bar{K}-b\bar{K}} + 2\bar{K}^2 \right)}{\left( \sqrt{\bar{K}-b\bar{K}} + (1-b) \right) \left( \sqrt{\bar{K}-b\bar{K}} + \bar{K} \right)}, \quad b \in [1-K, 1]. \quad (25)$$

implying that the total production of flexible agents is

$$K_F(s) = 1 + \bar{K} \frac{s^2}{s+1}. \quad (26)$$

We assume that flexible workers' limiting productive sensitivity to the state variable,  $\bar{K}$ , is 2.3 and that the average inflexible worker has  $\bar{b} = 0.16$ , and  $\bar{f} = 1.28$ . Thus,

$$K_I(s) = 0.16 + 1.28s. \quad (27)$$

We plotted these two production functions in Figure 1. Note that (27) is also the production of the average or representative firm in the economy. Consequently we define the market portfolio as the equity claim on this firm. These choices, together with the other parameter choices described below imply that inflexible agents have approximately 50% of the wealth in the economy (recall that they make up 66% of the economy).

The state process,  $s$ , evolves according to (1), with parameter values  $\theta = 0.003$ ,  $\sigma = 6\%$  and  $\bar{s} = 0.67$ . The economy is thus in a recession when  $s < 0.67$  and when  $s > 0.67$  it is in a boom. With these parameters, the unconditional probability that  $0 < s_t < 2.5$  is over 99%, so we focus on this range. We start the economy at  $s_0 = 0.8$ . The long-term growth rate of the economy is  $r = 1.2\%$ , with volatility of about 4%, which is in line with what was used in Mehra and Prescott (1985). We pick a relative risk aversion coefficient of 8.5, within the range Mehra and Prescott (1985, p. 154) consider reasonable, and impatience parameter  $\rho = 2\%$ .

The initial capital the firm raises is arbitrary in our model — because we have an effectively complete asset market, the Modigliani-Miller proposition implies that the firm's capital structure

Variable	Symbol	Value
Average Inflexible constant production	$\bar{b}$	0.16
Average Flexible Limiting Production Sensitivity	$\bar{K}$	2.3
Impatience Parameter	$\rho$	2%
Risk Aversion	$\gamma$	8.5
Long-term Growth Rate	$r$	1.2%
% of population Inflexible	$\alpha$	67%
Inflexible variable production	$\bar{f}$	1.28
State Variable Volatility	$\sigma$	6%
Initial $s$	$s_0$	0.8
Mean reversion speed	$\theta$	0.003
Long-term mean	$\bar{s}$	0.67
Initial Capital	$C_0$	0.98

Table 1: **Parameter Values**

is irrelevant. Of course, in a world with frictions the amount of capital raised will be affected by a tradeoff between the benefits (e.g., lower transaction costs) and costs (e.g., increased taxes and agency costs). We pick a level of initial capital for the representative firm that ensures that it almost never needs to return to capital markets<sup>14</sup> and to match the price volatility of the market, which we set to 15.4%. This leads to initial capital of  $C_0 = 0.98$ . Table 1 summarizes these parameter choices.

## 4.2 Implications for Aggregate Market

We calculate the equilibrium by solving (18) numerically. To compute  $W_i$ , we set  $\delta = K_i(s)$  for each agent type  $i \in \{I, F\}$  in (15). Table 2 summarizes this equilibrium. We then compute  $\eta$ , the inflexible agents' wealth share, by solving (14) and get 51.2%.

Variable	Symbol	Value
Risk Free Rate	$r_s$	3.8%
Firm Expected Return	$r_e$	8.4%
Firm Volatility	$\sigma_p$	15.4%
Consumption Volatility	$\sigma_c$	4.5%
Wealth fraction	$\eta$ ,	51.2%
Probability $s < 2.5$		99%
Unconditional correlation	$\rho_{pc}$	0.45
Equity premium	$r_e - r_s$	4.6%

Table 2: **Equilibrium Moment Values at  $s_0$**

<sup>14</sup>The expected time to refinancing is over 1,000 years.

Of course, since our model is a complete market with a diffusion risk structure, instantaneous asset pricing moments are defined by a standard stochastic discount factor relationship, with all the restrictions that these imply. For example, standard bounds on the instantaneous market Sharpe ratio hold in our model. Nevertheless, because the time-varying operational leverage associated with the representative firms' wage contracts, and the associated nonlinearities of dividend payments, the model can give rise to realistic unconditional asset pricing moments. In this equilibrium, the risk free rate is 3.8% and the firm's expected return is 8.4%, leading to an equity risk premium of 4.6%. The model also delivers realistic second moments. Market volatility is 15.4% whereas consumption volatility is only 4.5%. More interesting is the unconditional correlation between equity returns and consumption. Because this is a standard neo-classical model, consumption prices assets, so the instantaneous correlation between equity returns and consumption is either 1 or -1 (as we will shortly see, equity values can be decreasing in the state variable). However, the average correlation across all states is only 0.45, that is, the unconditional correlation (what empiricists typically measure) is substantially lower than the instantaneous correlation.

The instantaneous equity premium is  $r_e - r_s = \rho_{pc}\gamma\sigma_c\sigma_p$ . If we use *unconditional* moments to evaluate this expression we get  $0.45 \times 8.5 \times 4.5\% \times 15.4\% = 2.7\%$ , which is quite a bit lower than the actual unconditional equity premium of 4.6%. This disparity occurs because the unconditional estimate of the correlation is not a good proxy for the actual variable of interest, the instantaneous correlation. The correlation between consumption and returns is different in expansions and contractions. The estimated correlation in our calibration conditional on being in a contraction ( $s < 0.67$ ) is 0.12, whereas the estimated correlation conditional on being in an expansion ( $s > 0.67$ ) is 0.99. This disparity is in line with the results in Duffee (2005) that the correlation between stock returns and consumption growth is low (about 0) in bad times, and high (about 0.6) in good times.

The value of the market (that is, the equity claim on the representative firm) is equal to the amount of cash held plus the value of inflexible worker average productivity minus the value of the wage commitment.<sup>15</sup> As the top panel of Figure 2 demonstrates, this price function is highly nonlinear in  $s$ . Its option-like qualities are self evident. The function is insensitive to the state variable for low values of  $s$  — it is actually slightly decreasing for very low  $s$ . It then increases rapidly in a convex fashion for low to high  $s$ .

To understand the dynamic behavior of the market across the business cycle, note that as the state of the economy worsens, dividend payouts decrease, reflecting the fact that wages exceed productivity. Consequently, the value of the firm drops sharply. When the economy deteriorates further, productivity continues to drop but agents propensity to consume does not drop by as much. The reason is that because the state variable is mean reverting, agents understand that the current state is temporary. They therefore anticipate that the economy is likely to

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<sup>15</sup>Flexible worker productivity and wages can be ignored because flexible workers always earn their productivity.

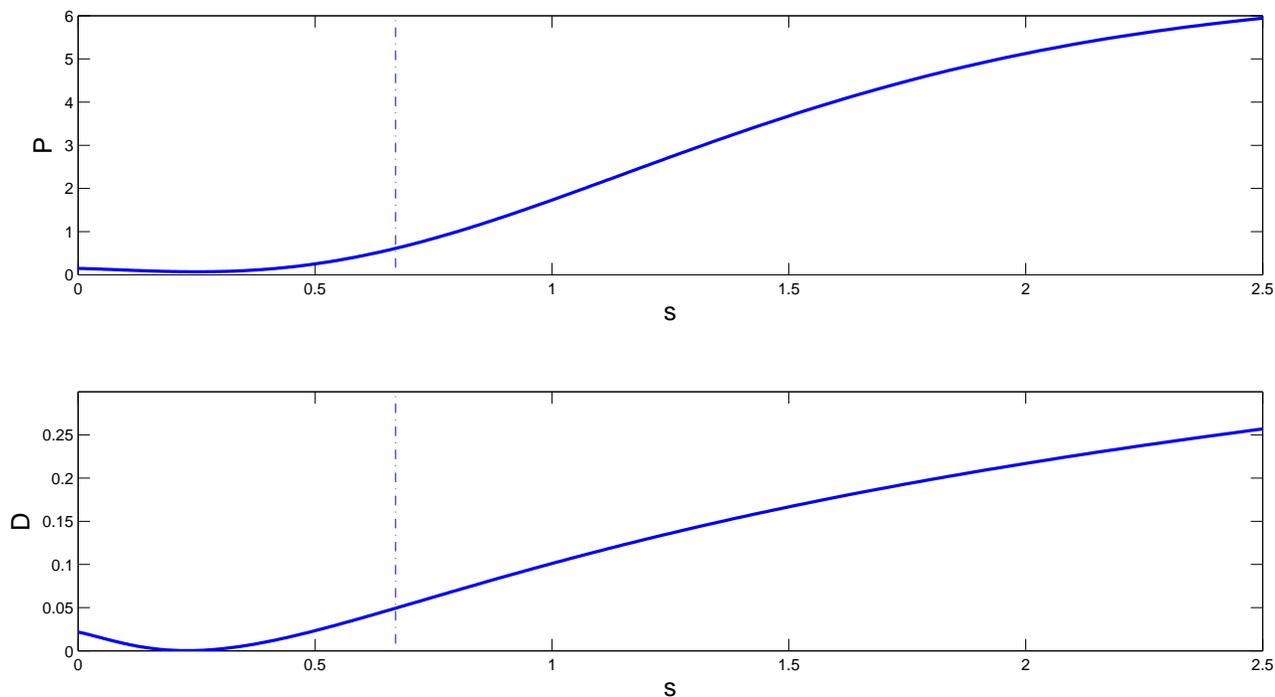


Figure 2: **Value of equity (upper panel) and total dividends (lower panel) as a function of  $s$ .** The vertical dashed line marks  $\bar{s}$ , the average value of the state variable.

improve and because they want to smooth consumption, they have a propensity to borrow to consume. In equilibrium net borrowing is zero, so interest rates have to rise to clear markets, as is evident in Figure 3. Because the firm holds cash, this increase in interest rates generates interest income for the firm. Eventually, the increase in interest rates dominates the decrease in worker productivity, so that dividends begin to increase again, as is evident in the lower panel in Figure 2. The result is that for very low values of the state variable the value of the firm is actually decreasing in  $s$ .<sup>16</sup> As a result the instantaneous correlation between consumption and equity returns flips from 1 to -1, giving rise to the effects noted in the prior paragraphs. Clearly, our correlation results depend critically on the presence of this region which in turn derives from mean reversion in the state variable — without it the value of equity would not be decreasing in  $s$  at low values. Note that  $D$  and  $P$  are both nonnegative within the range of the plots, and therefore there is no need for refinancing within this range.<sup>17</sup>

<sup>16</sup>In steady state, the probability that the state variable will be in this region is 25%.

<sup>17</sup>For  $s > 2.5$ , there will be a point at which dividends turn negative, so that refinancing might be needed at some point.

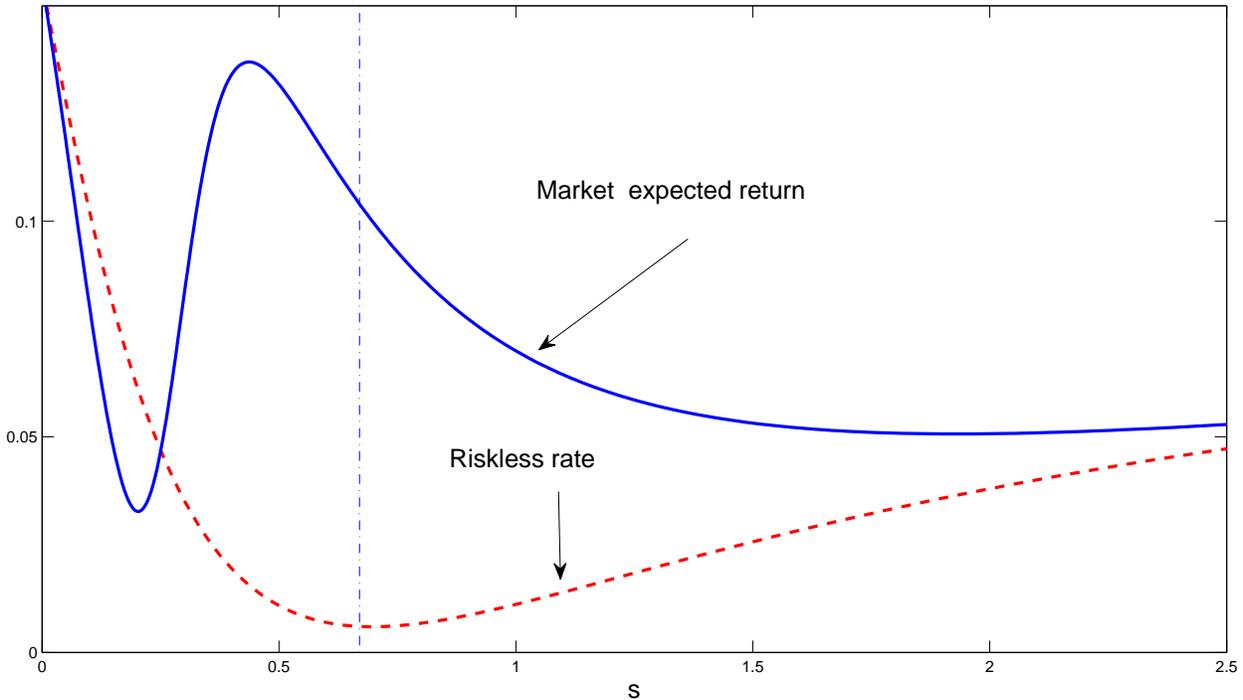


Figure 3: **Expected return:** Expected return on the firm's stock (solid blue line) and the (short) risk free rate (dotted red line). The vertical dashed line marks  $\bar{s}$ , the average value of the state variable.

The overall message is that whereas in normal times, the performance of the stock market is well aligned with the state of the economy, in the bad states of the world, the link between the economy and equity is weaker. Figure 3 plots the expected return of the market, together with the risk-free rate. The most striking element in the plot is the difference in behavior of the equity risk premium in expansions and contractions. When  $s > \bar{s}$  the equity risk premium is decreasing in the state variable, which makes intuitive sense. Equity is an insurance contract, for high values of the state variable the insurance contract is not very risky. In contractions, the equity risk premium continues to increase as the state deteriorates, reaching a maximum of about 12%. But then, as the value of equity becomes less sensitive to the state of the economy, the risk premium begins to drop and, for low enough values of  $s$ , actually becomes negative. At the mean point of state variable,  $s = \bar{s} = 0.67$ , expected stock returns are about 10%, which is close to the unconditional expected return of 8.4%. However, at  $\bar{s}$ , the equity risk premium is over 9%, which is substantially higher than the unconditional equity premium of 4.6%.

It is informative to compare our results with those in Danthine and Donaldson (2002).

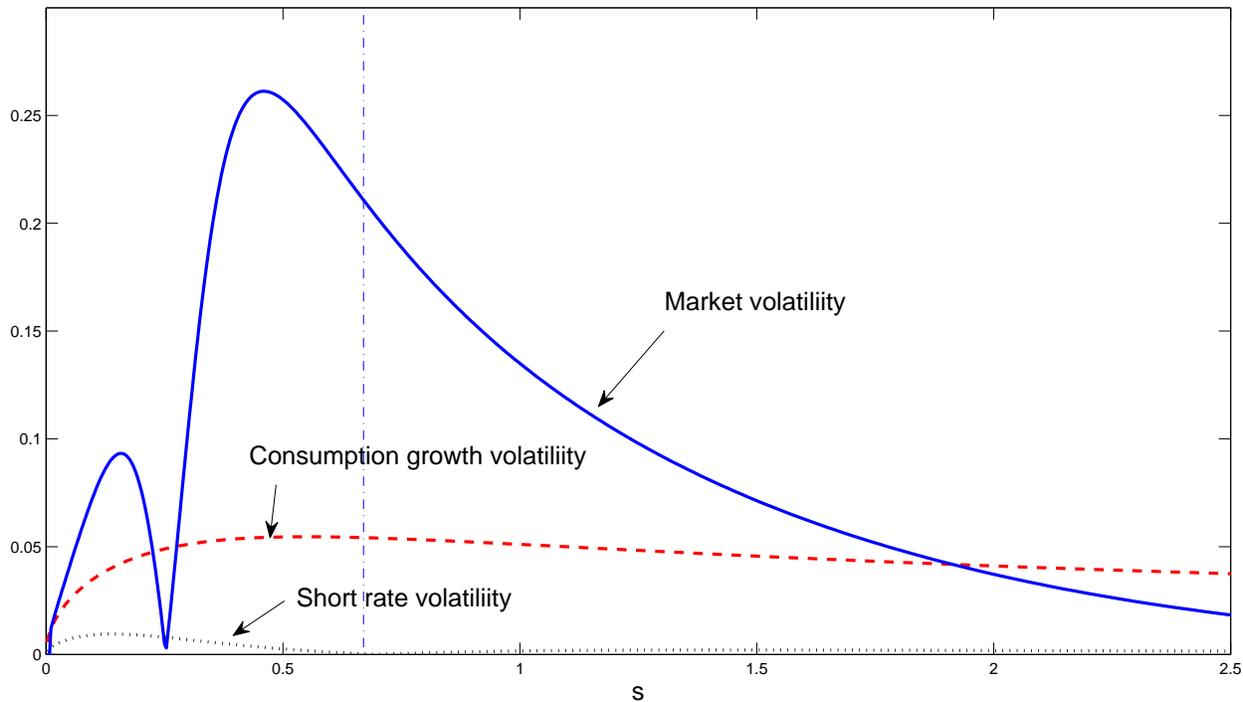


Figure 4: **Volatility:** The blue solid curve is firm volatility ( $\frac{V'}{V}\sigma(s)$ ), the red dash-dotted curve is consumption growth volatility ( $\frac{K'_{tot}}{K_{tot}}\sigma(s)$ ), and the black dashed line is interest rate volatility. The vertical dashed line marks  $\bar{s}$ , the average value of the state variable.

Although Danthine and Donaldson (2002) also model the effect of labor markets on asset prices, they are unable to generate a significant market risk premium in a model without frictions.<sup>18</sup> The reason is that in their model, investors are not workers and hence have a precautionary reason to invest. In our model flexible workers must be induced to invest, that is, take on additional risk in equilibrium. That implies that the return on equity (the means by which flexible workers are enticed to take on this risk) has to be high enough to induce this behavior. This is a key insight in our model — rather than a place to hedge risk and smooth consumption, asset markets are a place where investors are enticed to take on extra risk.

Because we do not have idiosyncratic risk in our model, an increase in the risk premium must be associated with an increase in volatility. Figure 4 confirms this insight. The volatility of the firm initially explodes in bad states, in line with the empirical evidence cited in the introduction.

<sup>18</sup>They introduce two frictions in their model: (1) Adjustment costs, which provides only a modest increase in the risk premium and (2) changes in the “bargaining power” of workers and investors that effectively prevents perfect risk sharing.

But then it actually starts decreasing, reflecting the fact that the value of equity becomes less sensitive to the state variable (see Figure 2, top panel), until it reaches zero close to 0.2. For even lower  $s$ , price is decreasing in  $s$ , and so volatility increases. Finally, volatility decreases again as  $s$  approaches 0 and as the volatility of  $ds$  becomes negligible, once again decreasing the equity volatility. Thus, the volatility of the firm is a nonlinear function of  $s$ , with regions of very high volatility in bad states of the world. More importantly, the same is not true of the volatility of consumption growth (dashed line in Figure 4) — it is virtually unaffected by the level of  $s$ . Our model therefore delivers an almost complete disconnect between consumption volatility and asset volatility. In line with the empirical evidence, consumption growth volatility is low and virtually unaffected by the business cycle, yet equity volatility is high and much more sensitive to the business cycle.

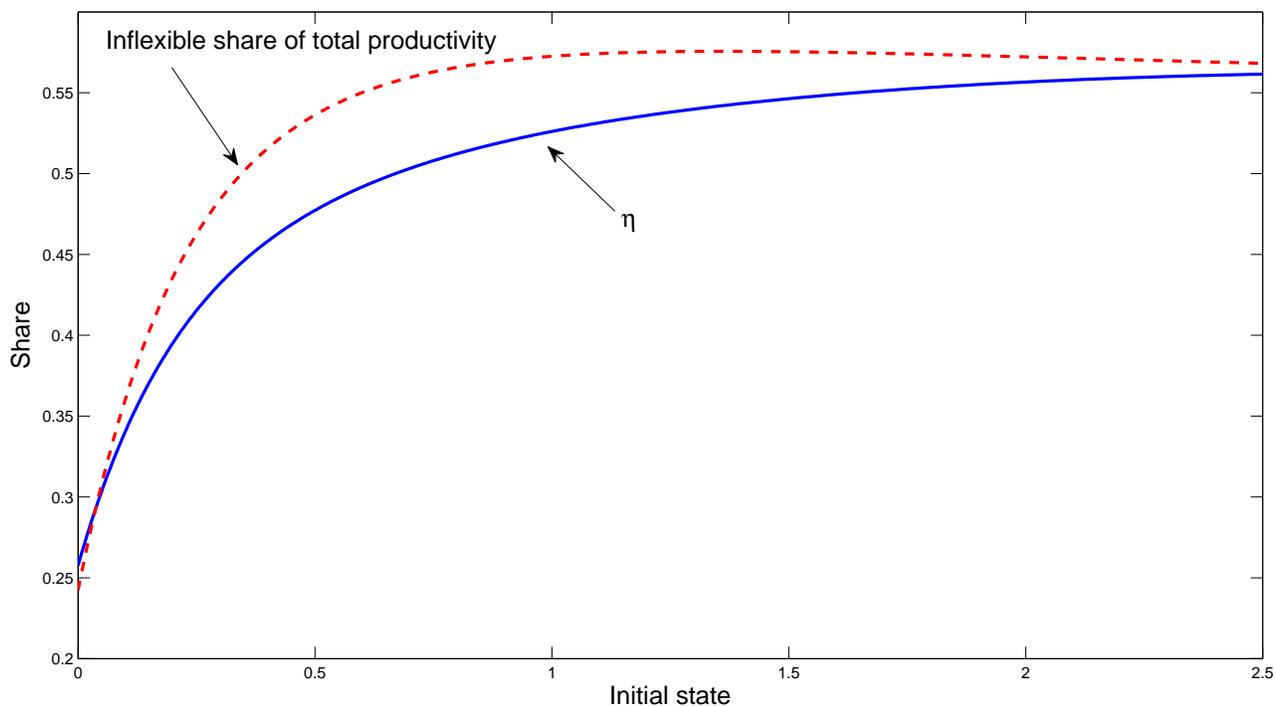


Figure 5: **Sensitivity of the Equilibrium to the Initial State:** The solid curve is the equilibrium share of wealth of the inflexible worker at  $t = 0$ ,  $\eta$ , as a function of the initial  $s_0$ . The dashed curve is the inflexible agent's aggregate share of total productivity at time 0.

Finally, it makes sense to consider how the equilibrium changes as a function of the initial state. Figure 5 plots the equilibrium share of total consumption of inflexible agents,  $\eta(s_0)$ . What is clear is that for most values of the state variable, the equilibrium consumption share is

	$\bar{b}$	$\bar{f}$	$K$	$\sigma$	$r$	$\alpha$	$\gamma$	$\theta$	$\bar{s}$
$r_s$	0.472	-1.172	-0.793	-2.505	2.684	-2.116	-1.722	-0.326	-0.160
$\sigma_p$	-0.419	0.939	-0.155	1.239	-0.669	1.431	0.613	0.082	-0.023
$\sigma_c$	-0.113	0.279	-0.189	0.596	0	0.505	0	0.0845	0.046
$\rho_{pc}$	0.0004	-1.849	-0.046	-3.571	1.956	-3.789	-1.849	0.079	0.820
$r_e$	0.249	-0.733	-0.274	-0.919	1.203	-1.699	-0.536	0.108	0.079
$r_e - r_s$	0.065	-0.370	0.155	0.391	-0.021	-1.438	0.425	0.467	0.277

Table 3: **Comparative statics:** Elasticity of equilibrium variables in a local neighborhood of equilibrium. Each column represents the effect of a 1% change in the parameter listed in the column head on the equilibrium variable listed in the row head. For example, the top left element is  $\frac{\partial r_s/r_s}{\partial \bar{b}/\bar{b}} = 0.472$ , i.e., that a one percent increase in  $\bar{b}$  (from 0.16 to 0.1616) approximately leads to a 0.47 percent increase in  $r_s$  (from 3.8% to 3.817%).

insensitive to the initial state. The figure also shows the initial aggregate instantaneous relative productivity,  $\frac{K_I}{K_{tot}}$ , of inflexible agents.  $\frac{K_I}{K_{tot}}$  reaches its maximum at  $s_0 \approx 1.2$ , whereas  $\eta^* \approx 0.54$  occurs at  $s_0 = s^* \approx 2.5$ . The reason  $\eta$  is maximized to the right of the point where the inflexible agent's relative productivity is maximized is that the value of insurance is greater in the bad states than in the good states. Hence, inflexible agents are willing to pay more for insurance at the point where their production share is maximized than at points to the right, so  $\eta$  continues to increase.

### 4.3 Comparative statics

Since the equilibrium production, and thereby the other equilibrium properties of the model, is nonlinear it is difficult to draw global inferences about the sensitivity of the equilibrium to the parameter choices, but it is straightforward to do a local, comparative static, analysis. In Table 3, we show the elasticity of equilibrium variables with respect to the parameters of the model. Each column represents the effect of a 1% change in the parameter listed in the column head on the equilibrium variable listed in the row head. For example, the top left element is  $\frac{\partial r_s/r_s}{\partial \bar{b}/\bar{b}} = 0.472$ , i.e., a one percent increase in  $\bar{b}$  (from 0.16 to 0.1616) leads to approximately an 0.47 percent increase in  $r_s$  (from 3.8% to 3.817%). We stress that the approximation is only valid for small changes of parameters.

The equilibrium is most sensitive to changes in the volatility of the state variable,  $\sigma$ , and the fraction of inflexible workers,  $\alpha$ . Given that the equilibrium is determined by the price that risk averse agents are willing to pay for consumption in different states of the world, it is not surprising that the sensitivity with respect to  $\sigma$  is high. Similar sensitivity arises in the standard Lucas model, where, e.g., the market price of risk is a quadratic function of  $\sigma$ . Similarly, the sensitivity with respect to  $\alpha$  arises because  $\alpha$  determines the amount of insurance that is provided in bad states by the flexible agents. The equilibrium is relatively less sensitive

to the technology parameters ( $\bar{b}$ ,  $\bar{f}$  and  $K$ ) and the parameters governing the mean reversion ( $\theta$  and  $\bar{s}$ ).

It is interesting to compare the sensitivity of equity volatility,  $\sigma_p$ , and consumption volatility,  $\sigma_c$  to small changes in the parameters. Because our equilibrium features complete risk sharing, it is perhaps not surprising that consumption volatility is not very sensitive to changes in the underlying parameters. But what is perhaps more surprising is that equity volatility is considerably more sensitive. Note also that the equity risk premium is particularly sensitive to  $\alpha$ , the fraction of inflexible agents. Obviously, the ratio of flexible to inflexible agents is critical determinant of the cost of insurance.

## 5 Robustness

Although our model is stylized and many of our assumptions are restrictive, the nature of these assumptions does not mean that we cannot learn from the model. As we demonstrate in this section, most of our key assumptions can be relaxed without changing the main conclusion in the paper — that limited capital market participation is a consequence of the role labor markets play in sharing risk.

### 5.1 Idiosyncratic Risk

We restrict attention to systematic uncertainty not because we believe idiosyncratic uncertainty is unimportant (it is surely more important early in a worker’s life than systematic uncertainty), but because it is straight forward to see that introducing idiosyncratic uncertainty does not alter the main conclusions of the paper. In presence of idiosyncratic worker and thus firm risk it is clear that the firm can still offer the same Pareto optimal compensation (based on aggregate production) to its workers. Because idiosyncratic risk is not priced in the market, the firm’s value maximizing strategy stays the same in the presence of such risk. The main difference is that the firm may have to refinance at another point in time (because its value may reach zero because of idiosyncratic shocks) and there will be an additional source of cross sectional variation.

### 5.2 Risk Sharing and Non-Participation

In our model all agents optimally share risk. Because inflexible workers achieve this solely in the labor market they do not need to participate in asset markets and hence we are able to derive the stark prediction of complete risk sharing even with significant non-participation. In practice agents do appear to face uninsurable risks and, as a consequence, the evidence for complete risk-

sharing is weak.<sup>19</sup> In light of these facts, it is natural to question whether our model actually does explain limited market participation.

It is important to appreciate that our results do not depend on complete risk sharing, merely that for many participants asset markets provide *no more* opportunity to share risk than labor markets. Here, the evidence is not definitive. For example, Guvenen (2007) rejects perfect risk sharing for stockholders but cannot reject the hypothesis that non-stockholders share risk perfectly. Guvenen regards this finding as puzzling but it clearly supports our insight that the reason non-stockholders choose not to invest is that they have already shared risk optimally in the labor market. In addition Guiso, Pistaferri, and Schivardi (2005) demonstrate in a sample of Italian firms that wage contracts completely insure transitory idiosyncratic shocks to firm performance and partially insure permanent shocks.

Almost all the studies that reject complete risk sharing study the change in consumption that results from an idiosyncratic shock to income (see, e.g., Nelson (1994) and Hayashi, Altonji, and Kotlikoff (1996)). For the significant fraction of people who rely exclusively on their labor contracts for risk sharing, a (negative) idiosyncratic shock to income is by definition a risk that is not insured in the labor market. So the fact that the studies find evidence that individuals do not use asset markets to offset this risk is at least consistent with the idea that asset markets do not provide additional risk sharing opportunities. Indeed, Cochrane (1991) is particularly informative on this question because that study does not use idiosyncratic shocks to income as a measure of idiosyncratic risk. Consistent with our insights, that study finds strong evidence of full insurance for one measure — temporary illness — suggesting that labor markets do in fact provide significant insurance opportunities to non-participants.

Other evidence that may appear to be inconsistent with our results are the studies that have found that the consumption patterns of participants and non-participants differ.<sup>20</sup> Clearly, in our model the consumption of participants and non-participants are identical — both consume a fixed fraction of total production. But this is an artifact of the assumption of CRRA preferences. It is quite possible to have an economy with complete risk sharing without participants and non-participants having identical consumption dynamics. If we go outside of the CRRA setting, different covariances between individual consumption and asset returns arise because of wealth effects. Specifically, if flexible workers are richer, and also closer to risk-neutral, their consumption covaries more with asset returns (since they take on proportionally more risk), even if there is perfect risk sharing. The same argument holds for consumption volatility of different

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<sup>19</sup>For example, although Mace (1991) does not reject full insurance, Cochrane (1991) rejects it for long illness and involuntary job loss but not for spells of unemployment, loss of work due to strike and involuntary move.

<sup>20</sup>For example, Mankiw and Zeldes (1991) find that participants have a higher covariance and correlation with asset prices than that of non-participants, and that consumption volatility of participants is higher than that of nonparticipants and Vissing-Jorgensen (2002) finds that elasticities of intertemporal substitutions differ for asset holders and non-asset holders although the difference is not statistically significant (at the 5% confidence level) in either study. Malloy, Moskowitz, and Vissing-Jorgensen (2009) find that only looking at stock-holders gives a better calibration of the moments of stock market returns.

agents. Furthermore, if we go outside of the setting with diffusion processes, similar results are obtained for the instantaneous correlation of consumption of different agents.

Finally note that because non-participants are, on average, less wealthy, there are also good reasons to expect their consumption to be measured considerably less accurately than participant consumption. For example, poorer individuals are more likely to pay for a larger fraction of their consumption (e.g., house cleaning services) using their own labor implying that a greater portion of their consumption is not measured by consumption expenditures.<sup>21</sup> Hence, the evidence in both Mankiw and Zeldes (1991) (who measure correlation between consumption growth and excess asset returns) and Vissing-Jorgensen (2002) (who measures the elasticity of intertemporal substitution) that participant consumption is better able to reconcile asset pricing anomalies, might simply be reflective of the greater degree of measurement error in non-participant consumption.

### 5.3 Lifetime Employment

In our model we allow agents to commit to lifetime employment with a single firm. In reality, although firms often commit to employ agents, rarely do agents *explicitly* commit to stay with firms. However, if we introduce a cost to switching jobs, we can support our equilibrium even with a restriction on agent commitment. When there is a cost to switching jobs, workers will only choose to switch jobs when the benefits exceed the costs of switching. Because  $\eta(s_0)$  is flat around  $s^*$  (see Figure 5), only a small cost of switching jobs is required to ensure that inflexible agents will not have an incentive to switch jobs for most starting values of  $s$ , making an explicit commitment unnecessary. For instance, in our example, if the cost of moving is greater than 5% of wealth, the representative inflexible worker will *never* choose to switch jobs because  $\eta = 0.512$  and  $\eta^* \approx 0.54$ , making an explicit commitment not to quit unnecessary. The same argument is not true for flexible workers — as is evidenced by Figure 1, by switching jobs they experience a substantial productivity increase, so flexible workers will continue to switch jobs, albeit less frequently.

To quantify how a small switching cost would affect our equilibrium, we can consider the problem in partial equilibrium by taking the pricing kernel of the friction free economy as exogenous, and characterizing the optimal choices of an agent who faces a cost to switch jobs. With a cost of switching of just 1% of net wealth, the expected time for the average inflexible agent to make his first job switch is 50 years. And because each switch brings the agent closer to  $\eta^*$ , the second job switch takes even longer — over a period of 500 years we found that the inflexible agent switched an average of once every 150 years. Consequently, the cost to the firm in our equilibrium of not obtaining an explicit commitment to lifetime employment from the inflexible agent is trivial.

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<sup>21</sup>Aguilar and Hurst (2005) have demonstrated in a different context how ignoring this measurement problem can lead to misleading conclusions.

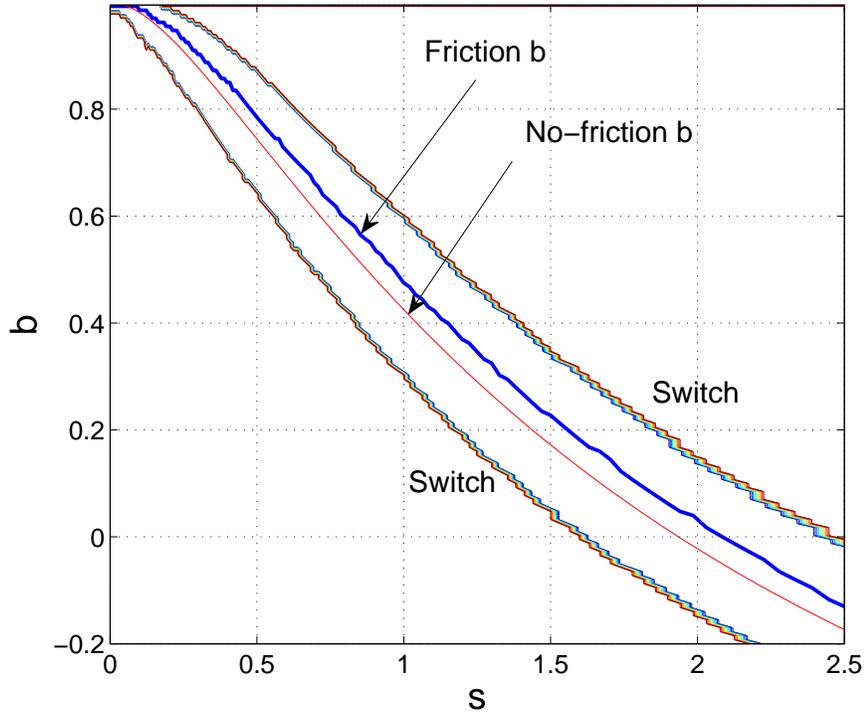


Figure 6: **Switching strategy of a flexible agent who faces switching costs of 1% of net wealth:** The blue line corresponds to the chosen  $b$  as a function of  $s$ , the state in which the switch occurred. For a given  $b$  the states within the switch-lines are states where the costs outweigh the benefits of a switch. When a switch-line is touched the flexible agent switches to the  $b$  (on the blue line) that corresponds to the current state. The red line is the  $b$  chosen by a flexible agent who faces no switching costs.

The introduction of this switching cost also does not affect the welfare or investment behavior of flexible agents. Figure 6 plots the effect of a switching cost of 1% on the behavior of flexible agents. The blue line marks the chosen  $b$  for a given level of the state variable  $s$ . Given any point on the blue line, the two lines labeled “Switch” show the minimum change in  $s$  that triggers a job-switch. When a switch is made, a new  $b$  on the blue line is chosen at the new level of  $s$ . Because the benefits of switching are larger for flexible agents, their frequency of job switching is a factor of 10 higher than the average inflexible agent — the flexible agent switches about once every 15 years. Of course, this rate of job switching is much lower than the equilibrium without switching costs. Nevertheless, this drop in the frequency of job switching hardly affects the welfare of flexible agents — the net decrease in wealth (that is the direct cost of switching as well as the cost of the resulting suboptimal job match) is less than 2% of the wealth of a flexible agent who faces no switching costs. More importantly, investment decisions are almost

identical to decisions without frictions. The average dividend the flexible agent receives is only 0.4% lower than what he would receive if he did not face the switching cost. The reason is that switches, although rare, occur in exactly the states of the world when they are worth the most, i.e., in the bad states when insurance is very valuable and in the really good states when a flexible agent can choose an extremely productive industry. Not being able to switch in states where the value of switching is low is not worth much, hence the effect of the switching cost on the flexible agent's investment strategy is marginal.

Note that flexible agents optimally respond to the introduction of switching costs in two ways. First, they switch jobs far less often. Second, they choose different jobs than they would if there were no switching costs, as is evidenced in Figure 6 (red line). As the figure illustrates, flexible agents choose a safer industry (one with higher  $b$ ) when they face a switching cost. Because the agent facing frictions anticipates that she will not immediately switch again if the economy continues to deteriorate she chooses to get extra insurance by picking a safer industry than an unrestricted agent.

Given the choice, inflexible workers would like to commit to firms. Consequently, in an economy in which explicit commitment is outlawed, inflexible workers have an incentive to increase the costs of switching and thereby implicitly commit to firms. Deferred compensation contracts (like pension funds and stock vesting periods) are a form of implicit commitment because they increase the cost of switching. In addition many union contracts explicitly tie wages to seniority with the firm, making a job switch very costly and thereby implicitly committing workers to firms. Hence, the maintained assumption in our parameterization, that inflexible workers face the same costs of switching as flexible workers, is likely unrealistic — flexible workers likely face lower costs. In line with these observations, there is convincing empirical evidence suggesting that in fact a large fraction of workers do indeed implicitly commit to firms. For example, Hall (1982) finds that after an initial job search in which employees might work for short periods for different employers, half of all men then work the rest of their lives for a single employer.

## 5.4 Unemployment

In light of current unemployment rates, one may question the relevance of a model in which firms choose to commit never to fire employees. Such a commitment is optimal in our model because workers always have positive productivity and so the model always features full employment. In practice, however, it is natural to expect that in some states of the world some inflexible workers may have negative productivity, that is, the value of their productivity is less than the effort expended. In such situations, it is optimal for the firm to fire such workers and pay them a severance package.

We also ignore the role of the government in sharing risk. Were we to include a government that supplied unemployment insurance, it would not be optimal for the firm to provide insurance

in the unproductive states. Instead we would observe states in which the firm fires workers without severance and instead workers collect unemployment insurance. Clearly, the government would have to tax workers to finance this insurance, but so long as the government insurance is restricted to only a subset of states, there would still be a role for the firm to provide labor market insurance in the remaining states. Because risk is shared through a combination of government insurance and the labor contract, inflexible workers have little reason to participate in equity markets.

Finally, it is worth emphasizing that our model does not feature full insurance; all workers are worse off in bad states. Thus even with efficient risk sharing, the widespread economic suffering that is characteristic of a downturn is not inconsistent with our model. Both workers and investors suffer losses. Indeed, the high profile bankruptcies of both General Motors and Chrysler provide strong anecdotal evidence of the benefits of labor market risk sharing. In the year preceding the bankruptcy, all three U.S. automakers burned through billions of dollars of shareholder equity by continuing to produce and sell cars even when it did not appear in their economic interest to do so.<sup>22</sup> The only plausible explanation for this behavior is the companies' commitments to their labor force.<sup>23</sup> The auto companies are certainly not alone. Many, if not most, companies operated at a loss during recessions, indicating that if companies had more flexibility to curtail production, unemployment rates would be substantially higher during recessions.

## 6 Conclusion

Our objective in this paper is to demonstrate the potential importance of explicitly modeling labor markets within the neoclassical asset pricing model. We show that under realistic assumptions that restrict the span of allowable contracts in both labor and asset markets, neither asset markets nor labor markets alone can share risk efficiently. Together the two markets can share risk efficiently, and as a result the model has the potential to shed new light on some of the most important normative challenges faced by the neoclassical asset pricing model.

Our main contribution shows that when the two markets co-exist, a large fraction of agents will optimally choose not to participate in capital markets. These agents share risk solely in labor markets. Agents who do participate in equity markets ultimately bear the risk insured by the firm's labor contracts implying that equity is relatively risky — the price of market risk is a highly non-linear function of the state. As a result the model also delivers a very large disparity in the volatility of consumption and the volatility of asset prices, a time varying dependence between consumption growth and asset returns, and a seemingly high market risk premium.

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<sup>22</sup>The combined losses of G.M. and Ford totaled \$46 Billion in 2008 which exceeded the combined market value of both companies at the beginning of the year by \$18 Billion.

<sup>23</sup>One might argue that these companies followed this strategy because they anticipated a government bailout, but this logic cannot explain Ford's strategy.

To many readers it is surprising that our model can deliver a high equity risk premium in what is otherwise a standard neoclassical complete market model. What is important to appreciate is that there is a distinction in our model between the premium for bearing market risk and the premium for bearing consumption risk. In our model, like any neoclassical complete market model, the premium for bearing consumption risk is low. However, the premium for bearing market risk — that is, for investing in equity markets — is high and attributable to the operating leverage imparted by the insurance in the firm’s labor contracts. This operating leverage is higher than the operating leverage typically cited in the literature because the firm commits to operate even in states when the ex post value maximizing strategy is to shut down. In effect, equity is an option on consumption, and hence the market risk premium is much larger than the risk premium for bearing consumption risk. We believe that this mechanism offers one plausible explanation for the high operating leverage ratios needed to explain the high observed market risk premium.

A central feature of our paper is that equity holders insure workers. Although outside of our model, one would expect workers to differ in their attitudes to risk and consequently use risk as one criteria for selecting where to work. If they do this, one might expect a clientele effect to result — some firms specialize in providing more insurance than others and hence attract more risk averse workers. If this is the case one would expect differences in job tenure across firms to explain firm riskiness.

A new insight in this paper is that inflexible workers just consume their wages, that is, the wages of inflexible workers is identical to their consumption and so should price assets. Because non-participants also do not switch jobs, labor mobility could be used to disentangle the wages of participants and non-participants.<sup>24</sup> In light of the fact that wages are measured much more accurately than consumption, this insight opens up the possibility of using wage changes of workers who choose not to switch jobs often to test the consumption CAPM.

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<sup>24</sup>A measure of labor mobility across firms (which should be closely related to our flexibility notion) is constructed in Donangelo (2009), using survey data from the Bureau of Labor Statistics between 1988-2008.

## Appendix

**Proof of Lemma 1:** (a) Follows since  $f$  is decreasing and  $f(\underline{b}) = \bar{K}$ .

(b) Clearly,  $\bar{K}s \leq C_F(s) \leq \bar{K}s + 1$  for all  $s$ , since the lower bound can be realized by choosing  $b(s) = 1$ , and the upper bound follows from the constraint that  $b \leq 1$ . (b) therefore immediately follows.

(c) Follows since  $b + f(b)s$  is (weakly) convex as a function of  $s$  for each  $b$  and the maximum of a set of convex functions is convex. ■

**Proof of Lemma 2:** The flexible worker solves  $\max_{b \in [\underline{b}, 1]} b + f(b)s$ . The first order condition is  $f'(b) = -\frac{1}{s}$ , and since  $f'$  is a continuously differentiable, strictly decreasing, mapping from  $[\underline{b}, 1]$  onto  $(-\infty, 0]$ , the implicit function theorem implies that there is a unique, decreasing, continuously differentiable solution to the first order condition,  $b^*(s)$ , such that  $b^*(0) = 1$  and  $\lim_{s \rightarrow \infty} b^*(s) = \underline{b}$ . Since the second order condition is  $f''(b)s < 0$ , this function indeed yields the maximal strategy,  $K_F(s) = b^*(s) + f(b^*(s))s$ .

Now,  $K'_F(s) = b^{*'}(s) + b^{*'}(s)f(b^*(s))s + f(b^*(s)) = 0 + f(b^*(s))$ , so  $K'_F(0) = f(b^*(0)) = f(1) = 0$ , and  $K'_F(\infty) = f(b^*(\infty)) = f(\underline{b}) = \bar{K}$ . Moreover,  $K''_F(s) = b^{*''}(s)f'(b^*(s)) = -\frac{b^{*'}(s)}{s}$ , which is continuous and positive, so  $K_F$  is indeed strictly convex and twice continuously differentiable. ■

**Proof of Lemma 3:** We have

$$Vol \left( \frac{dK_F}{K_F} \right)^2 = \left( \frac{K'_F}{K_F} \sigma(s) \right)^2 dt,$$

whereas

$$Vol \left( \frac{dK_I}{K_I} \right)^2 = \left( \frac{K'_I}{K_I} \sigma(s) \right)^2 dt = \left( \frac{\bar{f}}{\underline{b} + \bar{f}s} \sigma(s) \right)^2 dt.$$

From Lemma 2,  $\frac{K'_F}{K_F}$  converges to 0 for small  $s$ , whereas  $\frac{\bar{f}}{\underline{b} + \bar{f}s}$  converges to  $\frac{\bar{f}}{\underline{b}} > 0$ , so for small  $s$ , the inflexible worker's productivity indeed has higher volatility.

For large  $s$ , we have

$$\frac{dK_I}{K_I} = \frac{\bar{f}}{\underline{b} + \bar{f}s} = \frac{1}{\frac{\underline{b}}{\bar{f}} + s}.$$

Moreover, from Lemma 2 it follows that

$$\frac{dK_F}{K_F} = \frac{f(b^*(s))}{b^*(s) + f(b^*(s))s} = \frac{1}{\frac{b^*(s)}{f(b^*(s))} + s}.$$

The inequality therefore follows if  $\frac{b^*(s)}{f(b^*(s))} < \frac{\underline{b}}{\bar{f}}$ , but since  $b^*(s) \rightarrow \underline{b}$  for large  $s$  (from the proof of Lemma 2) and therefore  $f(b^*(s)) \rightarrow \bar{K}$ , the flexible worker's productivity is indeed riskier for large  $s$ . The lemma is proved. ■

**Proof of Proposition 1:** From (9), the price at  $t = 0$  of a general asset, paying an instantaneous dividend stream,  $g(s, t)dt$ , where  $g$  is a continuous function, is

$$P(s) = K_{tot}^\gamma E \left[ \int_0^\infty e^{-\rho t} \frac{g(s, t)}{K_{tot}(t)^\gamma} dt \right] \stackrel{\text{def}}{=} K_{tot}^\gamma Q(s, 0), \quad (28)$$

where

$$Q(s, t) \stackrel{\text{def}}{=} E \left[ \int_t^\infty e^{-\rho(\tau-t)} \frac{g(s, \tau)}{K_{tot}(\tau)^\gamma} d\tau \right].$$

From Feynman-Kac's formula (see, e.g., Karatzas and Shreve (1991)) it follows that  $Q \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}_+)$ , and that  $Q$  satisfies the PDE

$$Q_t + \mu(s)Q_s + \frac{\sigma(s)^2}{2}Q_{ss} - \hat{\rho}Q + \frac{g}{K_{tot}^\gamma} = 0. \quad (29)$$

Since  $K_{tot}$  is smooth, it follows that  $P$  is also smooth and since  $Q = \frac{P}{K_{tot}^\gamma}$ , it follows that  $Q' = \frac{P'}{K_{tot}^\gamma} - \gamma \frac{PK'_{tot}}{K_{tot}^{\gamma+1}}$  and  $Q'' = \frac{P''}{K_{tot}^\gamma} - 2\gamma \frac{P'K'_{tot}}{K_{tot}^{\gamma+1}} - \gamma \frac{PK''_{tot}}{K_{tot}^{\gamma+1}} + \gamma(\gamma+1) \frac{P(K'_{tot})^2}{K_{tot}^{\gamma+2}}$ . By plugging these expressions into (29), and defining  $R(s) = \frac{K'_{tot}}{K_{tot}}$ , and  $T(s) = \frac{K''_{tot}}{K_{tot}}$ , we arrive at (15). ■

**Proof of Proposition 2:** Equation (18) follows directly from (29) and the transformation  $s = \frac{z}{1-z}$ .

The PDE is of parabolic type, and typically both boundary conditions at  $z = 0$  and  $z = 1$ , and a terminal condition at  $t = T$  are needed for such PDEs to be well-posed. However, the second order ( $Q_{zz}$ ) term vanishes at the boundaries, and using the same solution method as in Parlour, Stanton, and Walden (2009), where a similar transformation is made, it is straightforward to show that no boundary condition are needed at  $z = 1$  or  $z = 0$ , since the term in front of  $Q_z$  is positive at  $z = 0$  (i.e., it is an outflow boundary) and zero at  $z = 1$ . ■

## References

- AGUIAR, M., AND E. HURST (2005): "Consumption versus Expenditure.," *Journal of Political Economy*, 113(5), 919 – 948.
- ANDERSEN, S., AND K. M. NIELSEN (2011): "Participation Constraints in the Stock Market: Evidence from Unexpected Inheritance Due to Sudden Death," *Review of Financial Studies*, 24(5), 1667–1697.
- BRAV, A., G. M. CONSTANTINIDES, AND C. C. GECZY (2002): "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence," *Journal of Political Economy*, 110(4), 793–824.
- CAMPBELL, J. Y. (1996): "Understanding Risk and Return," *Journal of Political Economy*, 104(2), 298–345.
- CHRISTIANSEN, C., J. S. JOENSEN, AND J. RANGVID (2008): "Are Economists More Likely to Hold Stocks?," *Review of Finance*, 12(3), 465–496.
- COCHRANE, J. H. (1991): "A Simple Test of Consumption Insurance," *The Journal of Political Economy*, 99(5), 957–976.
- DANTHINE, J.-P., AND J. B. DONALDSON (2002): "Labour Relations and Asset Returns," *The Review of Economic Studies*, 69(1), 41–64.
- DONANGELO, A. (2009): "Human Capital Mobility and Expected Stock Returns," University of California, Berkeley.
- DREZE, J. H. (1989): "The Role of Securities and Labor Contracts in the Optimal Allocation of Risk-Bearing," in *Risk, Information and Insurance*, ed. by H. Louberge. Kluwer Academic Publishers.
- DUFFEE, G. (2005): "Time Variation in the Covariance between Stock Returns and Consumption Growth," *Journal of Finance*, pp. 1673–.
- FAMA, E. F., AND G. W. SCHWERT (1977): "Human capital and capital market equilibrium," *Journal of Financial Economics*, 4(1), 95 – 125.
- GRINBLATT, M., M. KELOHARJU, AND J. LINNAINMAA (2010): "IQ and Stock Market Participation," Discussion paper, UCLA, Presented at the 2010 WFA.
- GUIO, L., M. HALIASSOS, T. JAPPELLI, AND S. CLAESSENS (2003): "Household Stockholding in Europe: Where Do We Stand and Where Do We Go?," *Economic Policy*, 18(36), 125–170.
- GUIO, L., L. PISTAFERRI, AND F. SCHIVARDI (2005): "Insurance within the Firm," *Journal of Political Economy*, 113, 1054–87.
- GUVENEN, F. (2007): "Do Stockholders Share Risk More Effectively than Nonstockholders?," *Review of Economics and Statistics*, 89(2), 275–288.
- HALIASSOS, M., AND C. C. BERTAUT (1995): "Why do so Few Hold Stocks?," *The Economic Journal*, 105(432), 1110–1129.
- HALL, R. E. (1982): "The Importance of Lifetime Jobs in the U.S. Economy," *The American Economic Review*, 72(4), 716–724.
- HARRIS, M., AND B. HOLMSTRÖM (1982): "A Theory of Wage Dynamics," *Review of Economic Studies*, 49(3), 315–333.
- HAYASHI, F., J. ALTONJI, AND L. KOTLIKOFF (1996): "Risk-Sharing Between and Within Families," *Econometrica*, 64, 261–294.

- HEATON, J., AND D. J. LUCAS (1996): "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, 104(3), 443–487.
- HONG, H., J. D. KUBIK, AND J. C. STEIN (2004): "Social Interaction and Stock-Market Participation," *The Journal of Finance*, 59(1), 137–163.
- JAGANNATHAN, R., AND Z. WANG (1997): "The Conditional CAPM and the Cross-Section of Expected Returns," *Journal of Finance*, 51, 3–53.
- KARATZAS, I., AND S. E. SHREVE (1991): *Brownian Motion and Stochastic Calculus*. Springer-Verlag, New York, NY.
- KIHLSTROM, R. E., AND J.-J. LAFFONT (1979): "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion," *Journal of Political Economy*, 87(4), 719–48.
- LUSTIG, H., AND S. VAN NIEUWERBURGH (2008): "The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street," *Rev. Financ. Stud.*, 21(5), 2097–2137.
- LUSTIG, H. N., S. VAN NIEUWERBURGH, AND A. VERDELHAN (2007): "The Wealth-Consumption Ratio: A Litmus Test for Consumption-Based Asset Pricing Models," *SSRN eLibrary*.
- MACE, B. J. (1991): "Full Insurance in the Presence of Aggregate Uncertainty," *The Journal of Political Economy*, 99(5), 928–956.
- MALLOY, C. J., T. J. MOSKOWITZ, AND A. VISSING-JORGENSEN (2009): "Long-Run Stockholder Consumption Risk and Asset Returns," *The Journal of Finance*, 64(6), 2427–2479.
- MALMENDIER, U., AND S. NAGEL (2009): "Depression Babies: Do Macroeconomic Experiences Affect Risk-Taking?," .
- MANKIW, N. G., AND S. P. ZELDES (1991): "The Consumption of Stockholders and Nonstockholders," *Journal of Financial Economics*, 29, 97–112.
- MAYERS, D. (1972): "Nonmarketable assets and capital market equilibrium under uncertainty," in *Studies in the Theory of Capital Markets*, ed. by M. C. Jensen, pp. 223–248. Praeger.
- MEHRA, R., AND E. C. PRESCOTT (1985): "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145–161.
- MOSCARINI, G., AND K. THOMSSON (2007): "Occupational and Job Mobility in the US," *Scandinavian Journal of Economics*, 109(4), 807–836.
- NELSON, J. (1994): "On Testing for full Insurance using Consumer Expenditure Survey Data," *Journal of Political Economy*, 102, 384–394.
- PARLOUR, C., R. STANTON, AND J. WALDEN (2009): "Banking and Asset Pricing in a Flexible Tree Economy," Working paper, UC Berkeley.
- SANTOS, T., AND P. VERONESI (2006): "Labor Income and Predictable Stock Returns," *Rev. Financ. Stud.*, 19(1), 1–44.
- TELMER, C. I. (1993): "Asset-Pricing Puzzles and Incomplete Markets," *Journal of Finance*, 48(5), 1803–1832.
- VISSING-JORGENSEN, A. (2002): "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, 110(4), 825–853.