Disclosure, Contracting and Competition in Financial Markets

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Abstract

This paper studies competition in the financial service market for large hedge funds. Hedge funds have to be secretive about their asset strategies since these strategies are their sole source of profit. They need to implement their strategy through trading brokers, which might front-run and decrease hedge fund's profit. Hedge funds also interact with prime brokers, which provide loans. We compare two institutionally different situations. In the first institutional framework, all prime brokers are dedicated. We define dedicated prime brokers as prime brokers that do not have a trading desk. In the second case, all prime brokers are dual. Dual prime brokers have their own trading desk and good examples are investment banks. Dual prime brokers can serve as trading brokers for hedge funds, internalizing partially the competition effect of front-running. We find that both ex-ante and interim, hedge funds prefer a monopolist dedicated prime broker to a monopolist dual prime broker. In a monopolistic situation, a dedicated prime broker can make more money from a profitable hedge fund than a non profitable one as it extracts some of the hedge fund profits by charging for credit services. A monopolist dual prime broker internalizes the competition effect of front-running and the relationship generates a higher surplus, which accumulates to the dual prime broker. We then allow for ex-ante competition among prime brokers, which is equivalent to assuming long term prime brokerage relations. Under ex-ante competition, hedge funds receive a proportion of the ex-ante relationship surplus, which we define as the sum of expected ex-ante hedge funds and prime brokers profits. In this case hedge funds prefer dual prime brokers to dedicated prime brokers. We alternatively assume interim direct competition between the two types of prime brokers. We prove that there exists an equilibrium when hedge funds have to interact with both types. We then conjecture that there exist an equilibrium in which both types of prime brokers are active, although hedge funds do not have to contract with both types. We conclude that hedge funds need not worry about the effectiveness of the "Chinese wall" for investment banks if they can have long term relationships with investment banks.

In the last decade or so, the hedge fund industry increased exponentially. In the mid 80's, there were less than a hundred hedge funds in existence, due to the hedge funds' previous loss of reputation. This was due to the large losses incurred in the bear market of $1973-1974$. However,

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the impressive results of some hedge funds managers[1] offered hedge funds a restored credibility[2], which caused the number of hedge funds to increase to 500 by 1990. Currently there are about $8,000$ hedge funds with an estimated 1.2 trillion dollars of assets[3]. This increase in their assets under management was parallelled by the growth of their strategies area, which covers a variety of markets and market events, from M&A arbitrage to Fixed Income arbitrage. Nowadays, there is no financial market in which hedge funds are not active players and at the same time they are also the ones with the highest turn-over. Their initial success attracted many wealthy investors and their good track record explains the massive in-flow of funds into this industry. Such a booming hedge fund industry created a huge demand for specialized services.

Hedge funds have to raise funds, use a prime broker and trade through a trading broker. There are at least two types of prime brokers. The first type is represented by the dedicated prime brokers, whose only business is to act as settlement agent, provide custody for assets, provide financing for leverage, and prepare daily account statements for hedge funds. The second type consists of dual prime brokers, which provide the same type of services as dedicated prime brokers but have their own trading desk. Good examples of dual brokers are investment banks that offer both prime brokerage services and have their own trading desks. Hedge funds have to be secretive about their strategies since these strategies are their sole source of profit. Disclosing trading information to other traders (including the trading broker) decreases hedge funds profits. They funds need to trade with trading brokers to access the asset market and these trading brokers might front run.[4]

We analyze the interaction between hedge funds and prime brokers. There is an obvious potential leak of information between the prime brokerage and trading departments within a dual prime broker. Initially, working with a dual prime broker might seem undesirable due to this leakage of information. We show that hedge funds can use the leakage of information to their benefit. Dual prime brokers can be used to replace the trading brokers. Dual prime brokers front-run but they also internalize partially the competition effect of front-running. If hedge funds can enter into long term exclusive relations with their dual prime brokers, they can receive a proportion of the relationship surplus. In this case they prefer dual prime brokers to dedicated prime brokers because the surplus

[1]Including, among others, George Soros, Michael Steinhart, and Julian Robertson.

[2]For a brief history of hedge funds, see http://www.capmgt.com/brief_history.html.

[3]See the example the latest "Testimony Concerning Hedge Funds" on the S.E.C. web site at http://www.sec.gov/news/testimony/ts051606sfw.htm.

[4]To be exact, trading brokers might dual trade simultaneously.

generated with dual prime brokers is higher.

Another goal of the paper is to rationalize the interim existence and survival of both dedicated and dual prime brokers in the same financial services market place. If we assume that hedge funds have to interact with the two types of prime brokers, we prove the existence of an equilibrium. When we relax this assumption, we conjecture an equilibrium in which hedge funds work with both types of prime brokers.

We contribute to the growing body of theoretical work on hedge funds by focusing on the informational content of their interaction with prime brokers. Most credit agreements between prime brokers and hedge funds provide both capital to hedge funds and potentially valuable trading information to prime brokers. When prime brokers are dual (investment banks), they can easily use the information to their benefit. This suggests a secondary benefit for investment banks to extend credit to their trading competitors. Information is key in financial markets and we prove that it is also key in the interaction between prime brokers and hedge funds.

The novelty of this paper consists in the focus on the information transmission issues arising in the interaction between price-affecting hedge funds and brokers. Our paper adds to multiple strands of the literature. First, the issue of efficient sale of information was addressed in Admati and Pfleiderer (1990). We show that hedge funds can choose to efficiently "sell" information to dual prime brokers in order to obtain a better prime brokerage deal. Second, a number of recent papers considered the interaction between investment banks (strategic traders) and hedge funds (arbitrageurs). Brunnermeier and Pedersen (2005) explored the opportunity of so called "predatory trading". Attari, Mello and Ruckes (2005) stressed the importance of timing the access to additional sources of capital of the otherwise financially constrained arbitrageurs. We add to this body of literature by focusing on the informational dimension of the contracts between hedge funds and prime brokers. Third, there is a "limits to arbitrage" literature, initiated by Shleifer and Vishny (1997). While we do not explicitly account for it, we think that in a repeated interaction framework, information transmission about the capital endowment to prime brokers can lead to a "limits to arbitrage" type situation.

We suggest a simple and tractable framework inspired by the work of Attari, Mello and Ruckes (2005) to address the issues pointed out above. The financial market is populated with four potential types of traders. First, uninformed liquidity traders have a demand composed of a pure random part and a part associated with their perceived under or over-valuation of the asset. Second, trading brokers dual trade after receiving orders from hedge funds. Third, non dedicated prime brokers can trade strategically, internalizing the effect of their trade on the price. Fourth, hedge funds that have access to better information than other market participants are subject to potentially binding capital constraints. The uncertainty regarding the hedge funds from the prime brokers's perspective has to do with the accuracy of hedge fund's information. The hedge funds are also behaving strategically, forming rational expectations about both the impact of their trades on the price and also about the prime brokers' optimal trading strategy. Trading brokers are the only ones capable of placing orders in the asset market. A dual prime broker can potentially serve also as a trading broker for hedge funds.

After describing the setup, the paper presents the equilibrium concept that will be used to analyze different market configurations. Section 2 analyzes the institutional case of dedicated prime brokers. Prime brokers are shown to optimally "invest" in hedge funds, as the loan repayments are their main source of income. We first analyze the monopolist dedicated prime broker case. The monopolist prime broker extracts almost all the surplus generated from trading, leaving some informational rents. The section continues by computing the maximal surplus that can be obtained by working with dedicated prime brokers. We show that the surplus is maximized in the case of perfect competition among dedicated prime brokers. The reason is that increased competition among dedicated prime brokers diminishes any potential distortion in the hedge fund's optimal trading.

Section 3 analyzes the institutional case of dual prime brokers. Prime brokers cannot credibly commit not to trade after inferring information from hedge funds. Dual prime brokers find optimal to finance hedge funds, which are their trading competitors. Dual prime brokers want to provide the right incentives for information extraction. If one non dedicated prime broker has all the bargaining power, she will design a contract such that she can distinguish between hedge funds types. We characterize the solution to the monopolist dual prime broker's problem. We then compute the maximal surplus that can be obtained by working with dual prime brokers. We show that the surplus is maximized by the monopolist dual prime broker. The monopolist dual prime broker wants hedge funds to disclose all the information and not trade at all. This implies that there is little or no distortion in the dual prime broker trade. Since dual prime brokers have their own trading desk, the potential distortion associated with trading brokers vanishes. Therefore, the relationship surplus is maximized.

One of the main results of the paper is the outcome of a comparative statics exercise, presented in a proposition, where the choice is between the dedicated and dual institutionally constrained prime brokerage markets. For monopoly situations, dedicated prime brokers are preferred to dual ones. For competitive situations, long term relations with dedicated prime brokers make hedge funds worse off than dual ones. First, monopolist dedicated prime brokers maximize hedge funds payoff from trading. Second, monopolist dual prime brokers internalize the effect of hedge funds' access to capital. More trading for hedge funds diminishes the dual prime brokers trading profit. Even if prime brokers pay lending fees, dual prime brokers are less willing to finance hedge funds. Section 4 looks at equilibria when both types of prime brokers are allowed by market regulation. If we assume that hedge funds have to interact with the two types of prime brokers, we prove the existence of an equilibrium. When we relax this assumption, we conjecture an equilibrium in which hedge funds work with both types of prime brokers. Section 5 concludes and provides suggestions for future research.

1 The Model Setup

1.1 The Markets and the Market Participants

We will consider two markets: the asset and the prime brokerage (credit) market. We are interested in the interaction between the two. In the asset market, there is only one trading date for the risky asset. The asset's value v is realized after all the trading is done. There are four types of traders in this market: liquidity traders, hedge funds, trading brokers and dual prime brokers. We assume that there is only one liquidity trader and one large hedge fund. The demand of the liquidity trader is $\epsilon + \beta\left(E\left[v\right] - p\right)$, where ϵ is the realization of a liquidity need random variable with mean 0, $\beta > 0$ is the market depth parameter, p is the price of the risky asset and $E\left[v\right]$ is the liquidity trader's expectation about the asset value. The liquidity trader's demand is a limit order, contingent on the realization of the price. The liquidity trader's order has two components: one that relates to the perceived mispricing and purely random one, generated by either consumption smoothing or some other reason. All the other traders submit market orders. Their demand is not conditional on the price. The fundamental incentives to trade in the asset market for hedge funds and brokers are the same. They trade because they are better informed than the liquidity trader. Through her order, the hedge fund provide information to the trading broker which dual-trades. We assume that by institutional design, simultaneous dual-trading (SDT) is allowed. The trading broker infers information from the hedge fund's order. If present, the dual prime broker can also infer information by observing the hedge fund order and might dual-trade. Another institutional assumption about the asset market specifies the existence of margin requirements. The hedge fund is required to post collateral proportional to her order. If we denote the hedge fund's order by θ^E, then an amount of at least $M \cdot |\theta^E|$ has to be posted as collateral. Once posted, collateral cannot be recuperated. Since we assume that the hedge fund has available capital K_0[5], this imposes the constraint that $M \cdot |\theta^E| \leq K_0$ under the absence of any lending. We assume that the hedge fund is the only capital constrained asset and credit market participant. All other players are assumed to have enough capital to meet any margin requirements or other capital adequacy criteria.

The credit market has two types of players - hedge funds and prime brokers. Whereas hedge funds are borrowers, prime brokers are creditors. The reasons to trade on the credit market are the expected ones - hedge funds need capital to finance their asset market position and prime brokers lend capital in exchange of future promised repayments. The hedge fund plays a role in both markets. The presence on the credit market is rationalized by the prospect of trading in the asset market and the institutional requirement of collateral posting. Since the credit market opens first at $t = 1$, any potential contracts are written before the asset market opens at $t = 2$. See also Figure 1 for a visual representation of the time line.

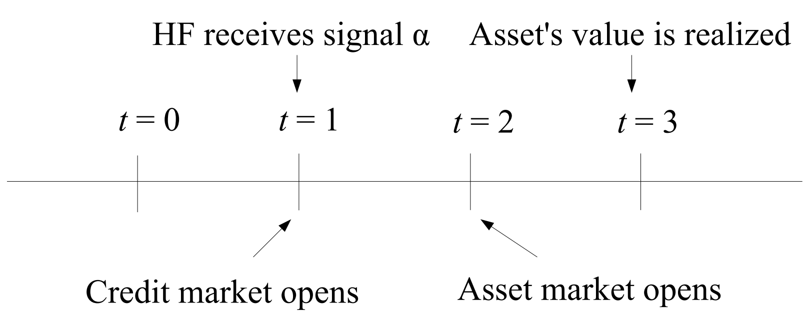

Figure 1: Time line.

[5]We will normalize it to 0 for analytical tractability

1.2 Informational Structure

The hedge fund's information set is and remains at least as fine as any other market participant. Only the hedge fund receives an informative signal about the expected value of the asset. All other market participants share the same beliefs about the expected value of the asset, namely $E[v]$. Before the credit and asset market open, at $t = 1$ the hedge fund receives a signal α that can take one of the following 4 values in the set $A = \{-\alpha_2, -\alpha_1, \alpha_1, \alpha_2\}$. We use the term "high" for a hedge fund that received a signal in $\{-\alpha_2, \alpha_2\}$ and "low" for one that received a signal in $\{-\alpha_1, \alpha_1\}$. All these signals allow the hedge fund to form expectations about the asset value. We assume that $E[v|\alpha = \pm\alpha_2] = E[v] \pm 2\Delta$ while $E[v|\alpha = \pm\alpha_1] = E[v] \pm \Delta$ with $\Delta > 2M$. We assume that $\Pr[\alpha_2|\alpha \in \{\alpha_1, \alpha_2\}] = Pr[-\alpha_2|\alpha \in \{-\alpha_1, -\alpha_2\}] = \nu$.

Hedge funds select the amount of capital to be borrowed from prime brokers, potentially contingent on the signal. By choosing a particular contract from the ones offered in the credit market, hedge funds disclose information to participating prime brokers. We will denote by $\alpha^{PB} \in \mathcal{A}^{\mathcal{PB}}$ the hedge fund's signal as inferred by prime brokers. After selecting the amount of capital to be borrowed, the hedge fund decides about her trading order[6] θ_α^E and she informs the trading broker(s) about her desired trade, while posting the required collateral of at least $M \cdot |\theta_\alpha^E|$. After extracting valuable information from hedge fund's order, trading brokers choose their own orders[7], denoted by $\theta_{\alpha^B}^B$. We will denote by $\alpha^B \in \mathcal{A}^{\mathcal{B}}$ the hedge fund's signal as inferred by the trading broker. The asset market equilibrium price is determined by equating aggregate demand and supply, which we assume equals zero. This is equivalent to:

$$\epsilon + \beta (E[v] - p) + \theta_\alpha^E + \theta_{\alpha^B}^B = 0$$

which gives us the market clearing price:

$$p = E[v] + \frac{\epsilon + \theta_\alpha^E + \theta_{\alpha^B}^B}{\beta}$$

The price is a function of the realization of ϵ and of α. Each strategic market participant forms expectations about the price yet the realization of the price will likely differ from these expectations.

[6] From now on, we will use $(\cdot)^E$ to denote quantities or expectations for the hedge fund, $(\cdot)^B$ for the trading broker and $(\cdot)^{PB}$ for the creditor or prime broker. Note that $E[\cdot]^E = E[\cdot|\alpha]$, $E[\cdot]^B = E[\cdot|\alpha^B]$ and that $E[\cdot]^B = E[\cdot|\alpha^{PB}]$

[7] When dual prime brokers serve as trading brokers, they will have two sources of information: the contract selected by the hedge fund and her order. Here we assume that there is one trading broker, which is not a dual prime broker.

In this section we presented the information structure and the main differences between hedge funds and other asset market participants - access to better information and potential lack of capital.

1.3 The Players' Strategies and Payoffs

All the credit and asset market participants are assumed to be risk neutral and therefore maximize expected profits. We first analyze the asset market. The liquidity trader's limit order is a function only of the random component ϵ and of the equilibrium price p. While the liquidity trader does not internalize the effect of her trading on the equilibrium price, she serves as the market balancing force when considering the effect of the other types of asset market players: hedge funds, trading brokers and potentially trading dual prime brokers. As opposed to the liquidity trader, who submits limit orders, we will restrict these strategic types of market participants to submit market orders. By doing this, we escape the potential complication of the informational content of the price, as in Kyle (1985). Hedge funds and brokers behave strategically, internalizing the effect of their market order on the expected equilibrium price and implicitly on their expected profits. Consider a hedge fund that received signal α and has capital D. The hedge fund chooses θ_α^E to maximize

$$\theta_\alpha^E \cdot E\left[v - p|\alpha\right]$$

subject to the collateral constraint that $M \cdot \left|\theta^E\right| \leq D$.

The credit contract written between the hedge fund and the prime broker(s) specifies a menu of pairs $\{D, T\}$ where D represents the amount of credit extended by the prime broker and T is the promised repayment from the hedge fund, after the asset market closes and the value of the asset is realized. Note that we do not assume limited liability for the hedge fund and this simplifies the computations.[8]

The trading broker maximizes

$$\Pi_{\alpha^B}^B = \theta_{\alpha^B}^B \cdot E\left[v - p|\alpha^B\right]$$

where $\theta_{\alpha^B}^B$ is the trading broker's order contingent on the realization[9] of α^B.

For a dedicated prime broker which offered a contract $\{D, T\}_{\alpha^{PB} \in \mathcal{A}^{PB}}$ and observes the hedge fund

[8]Since the prime broker is risk neutral, the fact that the debt is risk free helps us but we could also accommodate the limited liability case.

[9]Recall that α^B is the hedge fund's type as inferred by the trading broker

choice of contract, the expected profit is

$$\Pi^{PB}_{lending,\alpha^{PB}} = E\left[T - D|\alpha^{PB}\right]$$

A dual prime broker that is also a trading broker will maximize

$$\Pi^{PB}_{global,\alpha^{PB}} = \Pi^{PB}_{lending,\alpha^{PB}} + \Pi^{PB}_{trading,\alpha^{PB}}$$

$$= E\left[T - D|\alpha^{PB}\right] + \theta^{PB}_{\alpha^{PB}} \cdot E\left[v - p|\alpha^{PB}\right]$$

The hedge fund's payoff is

$$\Pi^{E}_{\alpha} = \theta^{E}_{\alpha} \cdot E\left[v - p|\alpha\right] - T + D$$

1.4 The Equilibrium concepts

An equilibrium involves both the asset and the credit markets and all the possible type of players. We start by defining an equilibrium in the asset market at time $t = 2$. Then, we define the credit market equilibrium at time $t = 1$.

1.4.1 Defining Asset Market Equilibrium

There are four types of strategic players in the asset market. First, the liquidity traders will place an order $\epsilon + \beta\left(E\left[v\right] - p\right)$. Second, the hedge fund strategic order is θ^{E}_{α}. Third, the trading broker strategic order is given by $\theta^{B}_{\alpha^{B}}$. If the hedge fund interacts with a dual prime broker, this broker specifies an order $\theta^{PB}_{\alpha^{PB}}$. An asset market equilibrium is defined as a 4-uple $\left\{D_{\alpha^{PB}}, \theta^{E}_{\alpha}, \theta^{B}_{\alpha^{B}}, \theta^{PB}_{\alpha^{PB}}\right\}$ where

$$\theta^{E}_{\alpha} = \text{argmax } \Pi^{E} \text{ s.t. } M \cdot |\theta^{E}_{\alpha}| \leq D_{\alpha^{PB}}$$

$$\theta^{B}_{\alpha^{B}} = \text{argmax } \Pi^{B}$$

and

$$\theta^{PB}_{\alpha^{PB}} = \text{argmax } \Pi^{PB}_{trading}$$

We require that the hedge fund, the trading broker and the dual prime brokers are rational and that they update their beliefs using Bayes' rule. In the asset market, we will refer to a pooling equilibrium as the equilibrium characterized by $\theta^E_{\alpha_1} = \theta^E_{\alpha_2} = |\theta^E_{-\alpha_1}| = |\theta^E_{-\alpha_2}|$. A separating equilibrium will be characterized by anti-symmetry $\theta^E_{\alpha_1} = |\theta^E_{-\alpha_1}| \neq \theta^E_{\alpha_2} = |\theta^E_{-\alpha_2}|$.

1.4.2 Defining Credit Market Equilibrium

In the credit market, prime brokers are lending capital to hedge funds for margin requirements. Prime brokers offer a menu of loans and promised repayments $\{D_{\alpha^{PB}}, T_{\alpha^{PB}}\}_{\alpha^{PB} \in \mathcal{A}^{\mathcal{PB}}}$. Prime brokers behave strategically and they realize that the loan size $D_{\alpha^{PB}}$ affects the hedge fund's trading. An equilibrium in the credit market is a pair $\{D_{\alpha^{PB}}, T_{\alpha^{PB}}\}$. The menu of pairs offered by dedicated prime brokers is a solution to their maximization problem

$$\{D_{\alpha^{PB}}, T_{\alpha^{PB}}\}_{\alpha^{PB} \in \mathcal{A}^{\mathcal{PB}}} = \text{argmax } E\left[\Pi^{PB}_{lending}\right]$$

Dual prime brokers offer a menu of pairs that maximize

$$\{D_{\alpha^{PB}}, T_{\alpha^{PB}}\}_{\alpha^{PB} \in \mathcal{A}^{\mathcal{PB}}} = \text{argmax } E\left[\Pi^{PB}_{global}\right]$$

In the credit market, we will refer to a pooling equilibrium as the equilibrium characterized by constant $D_{\alpha^{PB}}$ and $T_{\alpha^{PB}}$ across all possible α^{PB}. A separating equilibrium is characterized by the fact that $card\left(\mathcal{A}^{\mathcal{PB}}\right) \geq 2$ and that $D_{\alpha_1^{PB}} \neq D_{\alpha_2^{PB}}$.

1.4.3 Defining Global Equilibrium

A global equilibrium can be described by $\left\{D_{\alpha^{PB}}, \theta^E_{\alpha}, \theta^B_{\alpha^B}, \theta^{PB}_{\alpha^B}, T_{\alpha^{PB}}\right\}$ where $\{D_{\alpha^{PB}}, T_{\alpha^{PB}}\}$ is an equilibrium on the credit market and $\left\{D_{\alpha^{PB}}, \theta^E_{\alpha}, \theta^B_{\alpha^B}, \theta^{PB}_{\alpha^{PB}}\right\}$ is an equilibrium on the asset market.

1.5 The Hedge Fund' Strategy versus Multiple Trading Brokers

In this section, we analyze how the hedge fund decides to "slice" her global asset order among different trading brokers. We show that hedge funds minimize the number of traders which infer information. This result allows us to assume from now on that hedge funds interact with only one trading broker. We define a pooling trading equilibrium as an equilibrium in which all types of hedge funds place a similar order with the trading brokers. A separating trading equilibrium is one in which different types place different orders with the trading brokers.

All trading brokers are assumed to be similar in their main characteristic: they all dual-trade. First, assume that the hedge fund decides to split her total order $\theta_{\alpha}^{E}=\frac{D_{\alpha PB}}{M}$ into N equal parts. Each of the N trading brokers, indexed by $i\in\{1,2,..,N\}$ observe the same order $\theta_{\alpha,i}^{E}=\frac{D_{\alpha PB}}{M\cdot N}$ and they have to form expectations about the signal received by the hedge fund and about the number of other trading brokers. In any separating trading equilibrium each trading broker correctly infers the signal and the number of competing trading brokers. We can therefore compute each trading broker's optimal dual-trade and the corresponding profit for the hedge fund. Trading broker i chooses how much to trade in her own account, maximizing

$$\max_{\theta_{\alpha^B,i}^{B}} \theta_{\alpha^B,i}^{B}\cdot E_i^B\left[v-p\right]$$

The symmetric solution to the above concave problem is

$$\theta_{\alpha^B,i}^{B}=\frac{\beta}{N+1}E_i^B\left[v-p\right]-\frac{N}{N+1}\theta_{\alpha}^{E}$$

The key element is the trading broker's ability to perfectly infer the hedge fund's type and the number of competing trading brokers[10]. The hedge fund, anticipating this best response from trading brokers, chooses an order which maximizes her trading profits.

Lemma 1. *In any separating trading equilibrium in the asset market, the hedge fund will always choose not to split her global trade. For a fixed total trade θ_{α}^{E}, the hedge fund profit is strictly decreasing in the number of trading brokers N. The equilibrium number of trading brokers $N_{low}^{*}=N_{high}^{*}$ will always equal 1.*

Proof. See the Appendix. □

The lemma deserves some qualifications. The above result is intuitive. The more informed hedge fund, although has the first mover advantage in choosing her trade, prefers informing less trading

[10] The hedge fund is better off if trading brokers believe that there are more of them then they really are. This makes them less aggressive in their dual-trading.

brokers. The trading profit diminishes when more trading brokers are informed.

Now we look at a pooling trading equilibrium[11], when hedge funds of different types choose to split their trades such that trading brokers cannot infer their type exactly. Assume that the credit market separates the hedge funds into two types which is equivalent to setting $\mathcal{A}^{\mathcal{PB}} = \{(\pm\alpha_2), (\pm\alpha_1)\}$. If the high type has access to funds D^{high} and a low type has only D^{low}, the only way of pooling is to find a common divisor S of $\frac{D^{high}}{M}$ and $\frac{D^{low}}{M}$. The number of trading brokers used by the high type hedge fund will be N_{high} such that $N_{high} \cdot S = \frac{D^{high}}{M}$ while the low type will only use N_{low} such that $N_{low} \cdot S = \frac{D^{low}}{M}$. The following lemma describes the only situation when such a pooling trading equilibrium can arise.

Lemma 2. *The only possible pooling equilibrium can arise only when* $S = \frac{D^{low}}{M}$ *and implicitly* $N^*_{low} = 1$. *Unless* D^{high} *is a multiple of* D^{low}, *such a pooling equilibrium does not survive the refining process.*[12]

Proof. We eliminate any pooling trading equilibrium in which $S < \frac{D^{low}}{M}$. To see why, recall that the trading profits for both types are strictly decreasing in the number of trading brokers. Such a pooling trading equilibrium would not be refining-proof. The survival of the pooling trading equilibrium with $S = \frac{D^{low}}{M}$ depends on the high type hedge fund. On one hand pooling makes the dual-trading brokers less aggressive in their own trading and this seems beneficial to the high type hedge fund. On the other hand, pooling requires splitting the global trade into multiple smaller trades which is equivalent to informing more trading brokers. Unless D^{high} is a multiple of D^{low}, at least one trading broker will infer exactly when facing a high type hedge fund. This is strictly worse for the high type hedge fund as compared to the separating trading equilibrium case. Here there will be multiple trading brokers with mixed beliefs and one with perfect information. In a separating trading equilibrium there is only one perfectly informed dual-trading broker. It becomes obvious that the high type would rather reveal her type to only one trading broker and thus separate in trading. The rest of the proof can be seen in the Appendix. □

The current discussion relies on the fact that both D^{high} and D^{low} are pre-determined. The fact that these quantities are actually endogenous becomes more transparent when discussing the

[11] We still assume a separating equilibrium in the credit and asset market.

[12] Since the condition for the pooling trading equilibrium existence survival is unlikely, we can safely assume that there is only one trading broker.

equilibrium in the credit market. Here, as opposed to other market microstructure models, the hedge fund cannot disguise herself as an uninformed trader.[13] The only option available to the high type is to claim being a low type through pooling.

1.6 Optimal Trading with Full Access to Capital

In this subsection, we define the hedge fund optimal trading with unrestricted access to capital. We assume here that capital has a constant marginal cost. This is a useful benchmark case. First, we need to compute the order placed by the front-running trading broker. Assume that the trading broker observes an order θ_α^E. The trading broker infers the hedge fund's type and chooses $\theta_{\alpha^B}^B$ to maximize

$$\theta_{\alpha^B}^B \cdot \left(E^B[v] - E^B[p]\right)$$

which is equivalent to

$$\theta_{\alpha^B}^B \cdot \left(E^B[v] - E[v] - \frac{\theta_{\alpha^B}^B + \theta_\alpha^E}{\beta}\right)$$

This is a quadratic expression in $\theta_{\alpha^B}^B$ and reaches a maximum at

$$\theta_{\alpha^B}^{B*} = \frac{\beta}{2}\left(E^B[v] - E[v]\right) - \frac{1}{2}\theta_\alpha^E$$

The hedge fund anticipates front-running and incorporates the trading broker's best response into her trading objective function. Therefore, the hedge fund trading profit is

$$\theta_\alpha^E \cdot \left(E^E[v] - E[v] - \frac{\theta_{\alpha^B}^B + \theta_\alpha^E}{\beta}\right)$$

which, after accounting for the trading broker's best response, becomes

$$\frac{1}{2}\theta_\alpha^E \cdot \left(E^E[v] - E[v] - \frac{\theta_\alpha^E}{\beta}\right)$$

[13] We acknowledge that this might seem a short-coming, but in practice prime and trading brokers know the identity of their clients.

Recall that $\theta_\alpha^E = \frac{D}{M}$ and that the hedge fund has to raise capital to post as collateral. With full access to capital, the profit for the hedge fund then becomes

$$\Pi_\alpha^E = \begin{cases} \frac{D}{2\beta M^2} \cdot (2\Delta\beta M - D) - T + D, \text{if } \alpha = \pm\alpha_2 \\ \frac{D}{2\beta M^2} \cdot (\Delta\beta M - D) - T + D, \text{if } \alpha = \pm\alpha_1 \end{cases}$$

Since $T = D$, the profit is a simple quadratic function which is maximized at

$$D^* = \begin{cases} D^{high*} = \beta M\Delta, \text{if } \alpha = \pm\alpha_2 \\ D^{low*} = \beta M\frac{\Delta}{2}, \text{if } \alpha = \pm\alpha_1 \end{cases}$$

The corresponding optimal trading order is

$$\theta_\alpha^{E*} = \begin{cases} sign\,(\alpha) \cdot \beta\Delta, \text{if } \alpha = \pm\alpha_2 \\ sign\,(\alpha) \cdot \beta\frac{\Delta}{2}, \text{if } \alpha = \pm\alpha_1 \end{cases}$$

1.7 Long Term Relationships with Prime Brokers

This section looks at the possibility of long term relations between hedge funds and prime brokers. So far, we allowed only for short term relations between hedge funds and prime brokers. This implicitly restricted the competition between prime brokers to be short term oriented. The Appendix provides a more detailed discussion about short term competition among prime brokers.

Long term relationships allow for competition among prime brokers at time $t = 0$ before any lending and trading takes place. We denote with $\Gamma^{dedicated}$ the ex-ante sum of hedge funds and dedicated prime brokers profits, while Γ^{dual} is the corresponding quantity for the dual prime brokers case. We compute $\Gamma^{dedicated}$ and Γ^{dual} as the maximal surpluses that can be generated by the relation between hedge funds and prime brokers.[14] In reduced form, we parameterize ex-ante competition by $\lambda \in [0, 1]$. In case of monopoly, $\lambda = 0$ and in case of perfect competition $\lambda = 1$. At time $t = 0$, before the realization of hedge fund's type, assume that a prime broker agrees to pays the hedge fund $\lambda \cdot \Gamma^{dedicated/dual}$ in exchange of prime brokerage exclusivity. By accepting this payment, the hedge fund enters into an exclusive long term relation with the prime broker. This implies that the hedge fund cannot use the services of any other prime broker. We assume that the payment cannot

[14] We vary the degree of competitiveness and search for the best surplus.

be used by hedge funds as collateral.

2 Dedicated Prime Brokers

This section analyzes the case of dedicated prime brokers. They cannot trade in the asset market due either to regulation or to their lack of a trading department. We start by considering the case of a monopolist dedicated prime broker. The prime broker chooses the optimal screening process. We characterize the optimal screening schedule and then compute the ex-ante relationship surplus corresponding to this case, $\Gamma^{dedicated}$.

2.1 Monopolist Dedicated Prime Broker

This corresponds to a situation in which one dedicated prime broker has all the bargaining power in the credit market. The prime broker offers a schedule of contracts that maximizes her lending profit. As we prove later, the prime broker does not infer any trading valuable information only from the hedge fund's willingness to accept a specific credit contract.

In case of separating equilibrium in the credit market, the prime broker infers whether the hedge fund belongs to one of the two pseudo-types[15] $\{\alpha_1, -\alpha_1\}$ or $\{\alpha_2, -\alpha_2\}$. The prime broker offers a menu $\left\{\left(D^{high}, T^{high}\right), \left(D^{low}, T^{low}\right)\right\}$. In case of separating equilibrium in the credit market, the trading broker infers the hedge fund's type. The trading broker observes the direction and size of the order and therefore infers exactly the type.[16] If the hedge fund considers rejecting the menu suggested by the monopolist prime broker, her outside option payoff becomes 0.[17]

The asset market equilibrium is also important for the monopolist problem. If both "high" and "low" types are credited in the separating equilibrium in the credit market, the hedge fund's asset market order is

$$\theta_\alpha^E = \begin{cases} \pm\frac{D^{high}}{M}, \text{if } \alpha = \pm\alpha_2 \\ \pm\frac{D^{low}}{M}, \text{if } \alpha = \pm\alpha_1 \end{cases}$$

[15]This is equivalent to setting $\mathcal{A}^{\mathcal{PB}} = \{(\pm\alpha_2), (\pm\alpha_1)\}$

[16]This implies immediately that $\mathcal{A}^{\mathcal{B}} = \mathcal{A}$.

[17]The hedge fund's initial capital K_0 is zero and therefore, absent any loan, the maximum order becomes 0, since $M \cdot |\theta^E| \leq K_0 = 0$. The outside option is type-independent here.

The trading broker's order is also a function of the hedge fund's type

$$\theta^B_{\alpha^B} = \begin{cases} \pm\left(\beta\Delta - \frac{D^{high}}{2M}\right), \text{if } \alpha = \pm\alpha_2 \\ \pm\left(\frac{\beta\Delta}{2} - \frac{D^{low}}{2M}\right), \text{if } \alpha = \pm\alpha_1 \end{cases}$$

We obtain the hedge fund's expected profit, which is symmetric [18]

$$\Pi^E_\alpha = \begin{cases} \frac{D^{high}}{2\beta M^2}\cdot\left(2\Delta\beta M - D^{high}\right) - T^{high} + D^{high}, \text{if } \alpha = \pm\alpha_2 \\ \frac{D^{low}}{2\beta M^2}\cdot\left(\Delta\beta M - D^{low}\right) - T^{low} + D^{low}, \text{if } \alpha = \pm\alpha_1 \end{cases}$$

We incorporated the asst market equilibrium outcome into the hedge fund's profit. The next step is to find the solution to the monopolist prime broker's problem. We introduce new notation to make the discussion transparent. If we define $\hat{\Pi}^E_\alpha = (2\beta M^2)\cdot\Pi^E_\alpha$ and $\hat{T}^{high/low} = T^{high/low}\cdot(2\beta M^2)$ the profit becomes

$$\hat{\Pi}^E_\alpha = \begin{cases} D^{high}\cdot\left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - \hat{T}^{high}, \text{if } \alpha = \pm\alpha_2 \\ D^{low}\cdot\left(\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low}, \text{if } \alpha = \pm\alpha_1 \end{cases}$$

We have to specify the profit that a high type can obtain by pretending to be a low type. We present here the case when the hedge fund receives one of the signals $\pm\alpha_2$ and she pretends receiving the corresponding signal $\pm\alpha_1$. The analyzed deviation is both in the credit and asset market.[19] The trading broker observes the order $\theta^E_\alpha = \pm\frac{D^{low}}{M}$ and makes her own order $\theta^B_{\alpha^B} = \pm\left(\frac{\beta\Delta}{2} - \frac{D^{low}}{2M}\right)$. The profit from deviating[20] is

$$\hat{\Pi}^E_{\alpha,\hat{\alpha}} = \begin{cases} -(D^{high})^2 - \hat{T}^{high} + 2\beta M^2 D^{high} < 0, \text{if } \alpha = \pm\alpha_1 \text{ and } \hat{\alpha} = \pm\alpha_2 \\ D^{low}\cdot\left(3\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low}, \text{if } \alpha = \pm\alpha_2 \text{ and } \hat{\alpha} = \pm\alpha_1 \end{cases}$$

The prime broker's problem is to maximize her lending profit while offering the right incentives. We present the standard separating monopolist prime broker's problem below. We characterize the set

[18] The expected profit accounts for the contractual transfers to the monopolist prime broker T^{high} and T^{low}.

[19] The high type deviates when getting funds from the prime broker and when placing her order with the trading broker.

[20] We allow only for deviations along the same direction. A hedge fund with a "strong buy" signal considers behaving like a hedge fund with a "buy" signal and does not consider placing a "sell" order.

of equilibria in a lemma, after discussing the potential pooling equilibrium in the credit market.

$$\max_{D^{high},T^{high},D^{low},T^{low}} Pr\left(\pm\alpha_2\right)\cdot\left(\frac{\hat{T}^{high}}{2\beta M^2}-D^{high}\right)+Pr\left(\pm\alpha_1\right)\cdot\left(\frac{\hat{T}^{low}}{2\beta M^2}-D^{low}\right)$$

such that

$$\hat{\Pi}^E_\alpha \geq 0 \text{ if } \alpha\in\{\pm\alpha_2,\pm\alpha_1\}$$
$$\hat{\Pi}^E_\alpha \geq \hat{\Pi}^E_{\alpha,\hat{\alpha}}\ \forall\hat{\alpha}\neq\alpha$$

In case of a pooling equilibrium, the prime broker offers a unique contract[21] $\left\{\left(D^{pool},T^{pool}\right)\right\}$, such that she maximizes her revenues while all types participate. This implies a pooling equilibrium in the asset market. Hedge funds of types α_1 and α_2 will place the same order and this is also true about hedge funds of types $-\alpha_1$ and $-\alpha_2$. The trading broker will only be able to infer the direction of the signal.[22] The hedge fund's order is $\theta^E_\alpha=\pm\frac{D^{pool}}{M}$ for $\alpha\in\{\pm\alpha_1,\pm\alpha_2\}$ while the trading broker's order is $\theta^B_{\alpha^B}=\pm\left(\frac{\beta\Delta}{2}\cdot(1+\nu)-\frac{D^{pool}}{2M}\right)$. The hedge fund's trading profit is

$$\hat{\Pi}^E_\alpha=\begin{cases} D^{pool}\cdot\left(\Delta\beta M\cdot(3-\nu)+2\beta M^2-D^{pool}\right)-\hat{T}^{pool}, \text{if } \alpha=\pm\alpha_2\\ D^{pool}\cdot\left(\Delta\beta M\cdot(1-\nu)+2\beta M^2-D^{pool}\right)-\hat{T}^{pool}, \text{if } \alpha=\pm\alpha_1\end{cases}$$

When considering a pooling equilibrium [23], the prime broker solves

$$\max_{D^{pool},T^{pool}} \frac{\hat{T}^{pool}}{2\beta M^2}-D^{pool}$$

such that

$$\hat{\Pi}^E_\alpha \geq 0 \text{ if } \alpha\in\{\pm\alpha_2,\pm\alpha_1\}$$

We presented both the pooling and separating equilibria framework and the two equilibria are spelled out in the following lemma.

Lemma 3. *There exists a global equilibrium with the monopolist dedicated prime broker inducing a separating equilibrium in the credit market.*

[21] This is equivalent to $\mathcal{A}^{\mathcal{PB}}=\{(\pm\alpha_2,\pm\alpha_1)\}$.
[22] This is equivalent to setting $\mathcal{A}^{\mathcal{B}}=\{(\alpha_2,\alpha_1),(-\alpha_2,-\alpha_1)\}$
[23] Low type hedge funds get pooled with high types and face a more aggressive trading broker, compared to the separating case. High type hedge funds face a trading broker which trades less aggressively compared to the separating case.

Case 1. *When ν is small*[24]*, both high and low types are credited. The loan amounts are $D^{high} = D^{high*} = \beta M \Delta$ and*

$$D^{low} = D^{low*} - \frac{2\nu}{1-\nu} D^{low*} = \frac{1-3\nu}{1-\nu} \beta M \frac{\Delta}{2} < D^{low*}$$

Case 2. *When ν is high, only the high type gets credited with $D^{high} = D^{high*} = \beta M \Delta$ and $T^{high} = \frac{1}{2}\beta\Delta(\Delta + 2M)$.*

Case 3. *The pooling equilibrium has $D^{pool} = \beta M \frac{\Delta}{2}(1-\nu)$.*

Proof. To prove the claim about the separating equilibrium, one proceeds in the standard manner. We start by showing that it is enough to restrict attention to the incentive compatibility constraint for the high type hedge fund and the individual rationality constraint for the low type hedge fund. These two constraints imply the other two. The proof for the pooling equilibrium is more straightforward and is delegated with the details of the proof to the Appendix. □

This result deserves an intuitive interpretation. It might be optimal for the monopolist prime broker to extend credit to all types. Since the prime broker's profit can be decreasing in D^{low}, the prime broker finds sometimes optimal to set D^{low} at the lowest level, which is zero.[25] Here the monopolist dedicated prime broker wants to maximize the hedge fund trading profit since it is the sole source of profit. In the separating equilibrium case, the normal deviation of the low type trading appears, in order to provide truth-telling incentives for the high type.

To conclude this section, we prove that the dedicated prime broker will choose not to trade if her information is restricted to come from the lending activity only.

Lemma 4. *When acting only as a creditor, the dedicated prime broker chooses not to trade in the asset market, even if trading was possible. The result survives changing the degree of competitiveness in the credit market.*

Proof. The prime broker's expectation of the value of the asset is unchanged, even for a separating equilibrium in the credit market. To see this, assume the prime broker observes that the hedge fund

[24] The exact upper bound for ν is $\frac{1}{3}$.

[25] The non-negativity constraint binds.

selects a loan meant for the group $\{\alpha_1, -\alpha_1\}$ or for the group $\{\alpha_2, -\alpha_2\}$. Her expectation of the asset value is

$$E\left[v|\alpha\in\{\alpha_1,-\alpha_1\}\right]=E\left[v|\alpha\in\{\alpha_2,-\alpha_2\}\right]=E\left[v\right]$$

due to the symmetry of the framework. If allowed to trade, the prime broker has an order which is a linear combination of $E^{PB}[v]-E[v]$ and of $E^{PB}\left[\theta_\alpha^E\right]$, which are both zero.[26] We conclude that the prime broker will never choose to trade when acting as a creditor only. $\square$

2.2 The Ex-ante Maximal Surplus for Dedicated Prime Brokers Case

Recall that $\Gamma^{dedicated}$ is the maximal surplus that can be generated by a global equilibrium when hedge funds are facing dedicated prime brokers. The results of lemma 3 allows us to compute a lower bound for $\Gamma^{dedicated}$. We focus on the case when both types are credited, thus imposing a restriction on ν.

Hedge funds profit function is

$$\Pi_\alpha^E=\begin{cases}\frac{D^{high}}{2\beta M^2}\cdot\left(2\Delta\beta M-D^{high}\right)-T^{high}+D^{high}, \text{if } \alpha=\pm\alpha_2\\ \frac{D^{low}}{2\beta M^2}\cdot\left(\Delta\beta M-D^{low}\right)-T^{low}+D^{low}, \text{if } \alpha=\pm\alpha_1\end{cases}$$

while the dedicated prime broker's profit function is

$$\Pi_{lending,\alpha^{PB}}^{PB}=\begin{cases}T^{high}-D^{high}, \text{if } \alpha^{PB}=\pm\alpha_2\\ T^{low}-D^{low}, \text{if } \alpha^{PB}=\pm\alpha_1\end{cases}$$

Summing the two profits gives us

$$\Pi_\alpha^E+\Pi^{PB}lending,\alpha_{PB}=\begin{cases}\frac{D^{high}}{2\beta M^2}\cdot\left(2\Delta\beta M-D^{high}\right), \text{if } \alpha=\pm\alpha_2\\ \frac{D^{low}}{2\beta M^2}\cdot\left(\Delta\beta M-D^{low}\right), \text{if } \alpha=\pm\alpha_1\end{cases}$$

We see that any relationship surplus is entirely generated by trading.[27] If we plug in the solution of the monopolist dedicated prime broker problem, the high type trades optimally but the low type trades sub-optimally. The surplus is maximized when both types trade optimally. We formalize the

[26]If information is inferred only from the lending activity of the prime brokers.

[27]Here we account for dual-trading, since it cannot be avoided with dedicated prime brokers.

argument in the following lemma.

Lemma 5. *In the case of dedicated prime brokers, the maximal relationship surplus is reached in the case of perfect competition.*

Proof. We have to prove that the relationship surplus cannot be higher than

$$\nu\frac{\beta\Delta^2}{2}+(1-\nu)\frac{\beta\Delta^2}{8}$$

which is the surplus obtained under perfect competition. This is reached when hedge fund's trading is optimal for both types. Under any other competitive situations, dedicated prime brokers might want to reach a separate equilibrium, distorting the low type's trading. This implies that $D^{low} < D^{low*}$, which in turn means that

$$\Gamma^{dedicated}=\nu\frac{\beta\Delta^2}{2}+(1-\nu)\frac{\beta\Delta^2}{8}$$

□

3 Dual Prime Brokers

This section analyzes the case when prime brokers can be both creditors and trading brokers for better informed hedge funds. Dual prime brokers cannot commit not to trade after observing hedge funds' orders. This is the other type of prime brokers that we observe in financial markets. We assume that there is no "Chinese wall" between the prime brokerage and trading departments for the dual prime brokers. We start with the monopolist case. We then allow for competition in the dual prime broker market and see the effects for hedge funds.

3.1 Monopolist Dual Prime Broker

A non-dedicate prime broker has the all bargaining power and she is offering a schedule of contracts that maximizes her global profit. The global profit is the sum of the lending and trading profits. If the offered menu is separating in the credit market[28], the prime broker infers exactly the hedge

[28] Recall that in the credit market, separation means offering different loans to high and low type hedge funds

fund's type.[29] To find the optimal separating contract problem, the prime broker has to account account for the hedge fund's outside option.[30]

We describe the asset market equilibrium after the prime broker offers loans to the hedge fund. We consider first the low type hedge fund that has chosen loan size D^{low} with the corresponding transfer T^{low}. The hedge fund's order is $\theta^E_{\pm\alpha_1} = \pm\frac{D^{low}}{M}$, while the prime broker's order is $\theta^{PB}_{\pm\alpha_1^{PB}} = \pm\frac{\beta}{2}\left(E^{PB}\left[v\right] - E[v]\right) \mp \frac{1}{2}\theta^E$ This equals $\theta^{PB}_{\pm\alpha_1^{PB}} = \pm\frac{1}{2}\beta\Delta \mp \frac{1}{2}\frac{D^{low}}{M}$. The hedge fund's profit is

$$\hat{\Pi}^E_\alpha = D^{low}\cdot\left(\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low} \text{ if } \alpha \in \{-\alpha_1, \alpha_1)$$

When dealing with a low type hedge fund, the dual prime broker's global profit is

$$\hat{\Pi}^{PB}_{global,\alpha^{PB}} = \frac{1}{2}\left(\Delta\beta M - D^{low}\right)^2 + \hat{T}^{low} - 2\beta M^2 D^{low} \text{ if } \alpha^{PB} \in \{-\alpha_1, \alpha_1)$$

For a prime broker dealing with a high type hedge fund, the global profit is

$$\hat{\Pi}^{PB}_{global,\alpha^{PB}} = \frac{1}{2}\left(2\Delta\beta M - D^{high}\right)^2 + \hat{T}^{high} - 2\beta M^2 D^{high} \text{ if } \alpha^{PB} \in \{-\alpha_2, \alpha_2)$$

The prime broker's global profit maximization problem is

$$\max_{D^{high},T^{high},D^{low},T^{low}} Pr\left(\pm\alpha_2^{PB}\right)\cdot\hat{\Pi}^{PB}_{global,\pm\alpha_2^{PB}} + Pr\left(\pm\alpha_1^{PB}\right)\cdot\hat{\Pi}^{PB}_{global,\pm\alpha_1^{PB}}$$

such that

$$\hat{\Pi}^E_\alpha \geq 0 \text{ if } \alpha \in \{\pm\alpha_2, \pm\alpha_1\} \;\; (IR)$$

$$\hat{\Pi}^E_\alpha \geq \hat{\Pi}^E_{\alpha,\hat{\alpha}} \; \forall\hat{\alpha} \neq \alpha \; (IC)$$

The solution is presented in the following lemma.

Lemma 6. *There does not exist a global equilibrium with the monopolist dual prime broker inducing a separating equilibrium in the credit market. The global profit is strictly decreasing in D^{high} and in D^{low}. An equilibrium exists only when assuming minimal loan sizes $\underline{D}^{high} \geq 0$ and $\underline{D}^{low} \geq 0$. Now the prime broker's problem has a well defined solution, which will be given by $D^{high} = \underline{D}^{high}$ and by*

[29] This is equivalent to excluding any other trading broker and to setting $\mathcal{A}^{\mathcal{PB}} = \mathcal{A}$.

[30] We maintain the assumption that the hedge fund' pre-loan capital is zero. This simplifies the algebra and the outside option is type independent.

$D^{low} = \underline{D}^{low}$. The transfers are

$$T^{high} = \frac{1}{2\beta M^2}\left\{\underline{D}^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - \underline{D}^{high}\right) + 2\Delta\beta M \underline{D}^{low}\right\}$$

$$T^{low} = \frac{1}{2\beta M^2}\underline{D}^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - \underline{D}^{low}\right)$$

Only the high type enjoys informational rents.

Proof. See the Appendix. □

This result deserves commenting. There is no lending, but only extraction of information. The monopolist dual prime broker wants to offer the smallest positive loans which still induce separating. The global profit function is dominated by the trading profit rather than the lending one. This causes the problem not to have a well defined solution. A solution emerges only after adding constraints on the minimal size of loans.

We conclude this section with a comparison across two monopoly situations.

Lemma 7. *In case of a separating equilibrium in the credit market, high type hedge funds strictly prefer a monopolist dedicated prime broker to a monopolist dual prime broker.*

Proof. Low type hedge funds have the same payoff in both cases, since all her trading profits are extracted by the prime brokers. High type hedge funds have different payoffs, because their trading orders change. When the prime broker is dedicated, the high type's order is optimal, whereas in the second case, the order is suboptimal. The informational rents also differ. High type hedge funds prefer the first case, therefore the strict preference. □

When the dual prime broker is a monopolist, the optimal pooling equilibrium strategy is well defined only if we assume a minimal loan size $\underline{D}^{pool}$. A transfer of

$$T^{pool} = \frac{1}{2\beta M^2}\underline{D}^{pool} \cdot \left(\frac{1}{2}\Delta\beta M + 2\beta M^2 - \underline{D}^{pool}\right)$$

is needed to keep the low type participating. We compare the two monopolist pooling equilibria in the following lemma.

Lemma 8. *In case of a pooling equilibrium in the credit market, high type hedge funds strictly prefer a monopolist dedicated prime broker to a monopolist dual prime broker.*

Proof. Low type hedge funds are indifferent. High type hedge funds have the trading profits diminished because the monopolist dual prime broker lowers the pooling credit size to $\underline{D}^{pool}$. □

3.2 The Ex-ante Maximal Surplus for Dual Prime Brokers Case

We defined Γ^{dual} as the maximal surplus that can be generated by a global equilibrium when hedge funds are facing dual prime brokers. The results of lemma 6 allows us to compute an initial guess Γ^{dual}. We focus on the case when both types are credited by assuming minimal loan sizes $\underline{D}^{high} \geq 0$ and $\underline{D}^{low} \geq 0$.

The hedge funds profit function is

$$\Pi_{\alpha}^{E} = \begin{cases} \frac{\underline{D}^{high}}{2\beta M^2} \cdot \left(2\Delta\beta M - \underline{D}^{high}\right) - T^{high} + \underline{D}^{high}, \text{if } \alpha = \pm\alpha_2 \\ \frac{\underline{D}^{low}}{2\beta M^2} \cdot \left(\Delta\beta M - \underline{D}^{low}\right) - T^{low} + \underline{D}^{low}, \text{if } \alpha = \pm\alpha_1 \end{cases}$$

while the dual prime broker's profit function is

$$\Pi_{global,\alpha^{PB}}^{PB} = \begin{cases} \frac{1}{2\beta M^2}\left[\frac{1}{2}\left(2\Delta\beta M - \underline{D}^{high}\right)^2\right] + T^{high} - \underline{D}^{high}, \text{if } \alpha^{PB} = \pm\alpha_2 \\ \frac{1}{2\beta M^2}\left[\frac{1}{2}\left(\Delta\beta M - \underline{D}^{low}\right)^2\right] + T^{low} - \underline{D}^{low}, \text{if } \alpha^{PB} = \pm\alpha_1 \end{cases}$$

Summing the two profits gives us

$$\Pi_{\alpha}^{E} + \Pi^{PB} global, \alpha_{PB} = \begin{cases} \frac{1}{4\beta M^2}\left[(2\Delta\beta M)^2 - (\underline{D}^{high})^2\right], \text{if } \alpha^{PB} = \pm\alpha_2 \\ \frac{1}{4\beta M^2}\left[(\Delta\beta M)^2 - (\underline{D}^{low})^2\right], \text{if } \alpha^{PB} = \pm\alpha_1 \end{cases}$$

We see that the relationship surplus is decreasing in the loan sizes $\underline{D}^{high}$ and $\underline{D}^{low}$. The surplus is maximized when hedge funds do not trade. Therefore, setting $\underline{D}^{high}$ and $\underline{D}^{high}$ arbitrarily small, we obtain that

$$\Pi_{\alpha}^{E} + \Pi^{PB} global, \alpha_{PB} \leq \nu\beta\Delta^2 + (1-\nu)\frac{\beta\Delta^2}{4}$$

The maximal surplus for dual prime brokers is obtained with a monopolist prime broker. Recall that for dedicated prime brokers, the maximal surplus is reached by perfect competition. Here the surplus is maximized when hedge funds refrain from trading but provides information to the monopolist prime broker. The following lemma formalizes the intuition.

Lemma 9. *In the case of dual prime brokers, the maximal relationship surplus is reached in the case of monopoly.*

Proof. We have to prove that the relationship surplus cannot be higher than

$$\nu\beta\Delta^2 + (1-\nu)\frac{\beta\Delta^2}{4}$$

which is the surplus obtained under monopoly. This is reached when hedge fund refrains from trading and transmits the received signal to the dual prime broker. Under any other competitive situations, if two or more dual prime brokers infer the signal, the aggregate surplus will be diminished. This means that

$$\Gamma^{dual} = \nu\beta\Delta^2 + (1-\nu)\frac{\beta\Delta^2}{4}$$

□

We have now all the apparatus required for the main result of the paper.

Proposition 1. *Ex-ante, hedge funds strictly prefer dual prime brokers to dedicated prime brokers for all possible competitive situations except monopoly. Ex-ante and interim, hedge funds strictly prefer a monopolist dedicated prime broker to a monopolist dual prime broker.*

Proof. Let λ be in the interval $(0,1]$. Since $\Gamma^{dual} = 2 \cdot \Gamma^{dedicated}$, then $\lambda \cdot \Gamma^{dual} > \lambda \cdot \Gamma^{dedicated}$. Therefore, hedge funds strictly prefer dual prime brokers. In case of monopoly, $\lambda = 0$ and there is no need for an ex-ante payment to secure exclusivity for prime brokers. Lemmas 7 and 8 discuss this case. □

There is a range of competitive situations for which a fraction of the surplus obtained with a monopolist dual prime broker dominates the whole maximal surplus obtainable when hedge funds have full access to capital. Recall that in the first section of the paper, we defined the optimal trades under full access to capital and dual-trading. The following lemma formalizes this result

Lemma 10. *There exist ex-ante competitive situations such that a fraction of the surplus obtained with a monopolist dual prime broker $\lambda \cdot \Gamma^{dual}$ dominates the entire maximal surplus obtained with dedicated prime brokers $\Gamma^{dedicated}$.*

Proof. The proof is immediate. Recall that $\Gamma^{dual} = 2 \cdot \Gamma^{dedicated}$. Therefore, for $\lambda \in (\frac{1}{2}, 1]$, $\lambda \cdot \Gamma^{dual} > \Gamma^{dedicated}$. □

4 Competition between Different Types of Prime Brokers

We allow dedicated and dual prime brokers to compete directly in the credit market. We analyze two competitive situations. First, we allow hedge funds the liberty of contracting with only one prime broker and we show that in the particular credit market equilibrium we conjecture, both prime brokers are active. Second, we consider a dedicated prime broker competing with a dual one in an intrinsic agency framework. Hedge funds are therefore forced to accept either both contracts or none of them. In the Appendix we discuss the non-existence of direct mechanism equilibria in the two competitive cases. We have to look for equilibria in indirect mechanisms. See the Appendix for further details.

4.1 Delegated Agency Competition

We assume that the prime brokerage market consists of two prime brokers, one dedicated and one dual. Here hedge funds could choose to contract with both, one or none of them. The hedge fund does not use another trading broker if contracts with the dual prime broker. We assume that each prime broker posts a non-linear schedule, such that

$$T_i\left(D_i\right) = \begin{cases} a_1^i + D_i, \text{ for } D_i \leq \widetilde{D} \\ a_2^i + D_i, \text{ for } \widetilde{D} < D_i \leq \widetilde{\widetilde{D}} \\ a_3^i + D_i, \text{ for } D_i > \widetilde{\widetilde{D}} \end{cases} \quad \forall i \in \{A, B\}$$

Here we do not impose the symmetry of schedules and the two prime brokers can offer different schedules. The fact that there is an equilibrium when both prime brokers are contracting with the hedge fund is conjectured in the following lemma.

Conjecture 1. *There exists a global equilibrium which has a pure-strategy equilibrium of the indirect communication delegated common agency game between a dedicated and dual prime broker. Each prime broker offers a tariff*

$$T_i(D_i) = \begin{cases} a_1^i + D_i, \text{ for } D_i \leq \widetilde{D}_i \\ a_2^i + D_i, \text{ for } \widetilde{D} < D_i \leq \widetilde{\widetilde{D}}_i \\ a_3^i + D_i, \text{ for } D_i > \widetilde{\widetilde{D}}_i \end{cases} \quad \forall i \in \{A, B\}$$

In this equilibrium, both prime brokers are active.

4.2 Intrinsic Agency Competition

We maintain the assumption that the prime brokerage market consists of two prime brokers, one dedicated and one dual. For certain reasons[31], hedge funds have to contract with both of them. Hedge funds choose not to use another independent trading broker, since the dual prime broker dual-trades and can do the trading. We assume that each prime broker posts a non-linear schedule,[32] such that

$$T_i(D_i) = a_i + D_i \ \forall i \in \{A, B\}$$

When both types are credited, the hedge fund will have exactly the same profit and orders as in the case when two dedicated prime brokers were competing. This indicates that replacing one of the competing dedicated prime brokers with a dual one does not affect the hedge fund. This is true only when both types are credited. The replacement we suggest above places a stronger constraint on the high type probability required to sustain an equilibrium with no exclusion of the low type. The next lemma formalizes this intuition.

Lemma 11. *There exists a global equilibrium which has a pure-strategy equilibrium of the indirect communication implicit common agency game between a dedicated and dual prime broker in the credit market. Each prime broker offers a two-part tariff* $T_i(D_i) = D_i + a_i \ \forall i \in \{A, B\}$.

Case 1. *When ν is "small"[33], there is an equilibrium in the credit market with both types being*

[31] As mentioned in the appendix,not enough capital or bounded exposure requirements.

[32] We conjecture that there is no direct mechanism credit market equilibrium.

[33] The exact upper bound for ν is $\frac{\beta\left(\frac{\Delta}{2}\right)^2+4\Pi^{PB}_{trading,\pm\alpha_1}}{2\beta\Delta^2-\beta\left(\frac{\Delta}{2}\right)^2+4\Pi^{PB}_{trading\pm\alpha_1}}$.

credited and $a_A + a_B = \frac{\beta}{8}\Delta^2$*. Each type receives her optimal debt level* $D^{high*} = \beta M\Delta$ *and* $D^{low*} = \beta M\frac{\Delta}{2}$*. Only the high type enjoys positive profits net of transfers.*

Case 2. *For any value of* $\nu \in (0,1)$*, there is another credit market equilibrium in which only the high type is credited and* $a_A + a_B = \frac{\beta}{2}\Delta^2$*. The high type will trade at her optimal level* $D^{high*} = \beta M\Delta$*.*

Proof. The proof follows exactly the same steps as the other proofs for the intrinsic common agency equilibria in the Appendix. The only difference is that the dual prime broker, when contemplating a deviation which will exclude the low type, takes into account her global profits, not only the lending ones. This immediately affects the upper bound of ν. □

5 Extensions and Conclusions

We presented a model of competition for dedicated and dual prime brokers in the financial service market for large hedge funds. Hedge funds have to be secretive about their asset strategies since these strategies are their sole source of profit. Hedge funds need to implement their strategy through trading brokers, which front-run and decrease hedge fund's profit. Hedge funds also interact with prime brokers, which provide loans. We compared two institutionally different situations. In the first institutional case, all prime brokers are dedicated. In the second case, all prime brokers are dual. Dual prime brokers serve as trading brokers for hedge funds, internalizing partially the competition effect of front-running. We showed that both ex-ante and interim, hedge funds prefer a monopolist dedicated prime broker to a monopolist dual prime broker. In a monopolistic situation, a dedicated prime broker can more money from a profitable hedge fund than a non profitable one as it extracts some of the hedge fund profits by charging for credit services. A monopolist dual prime broker internalizes the competition effect of front-running and the relationship generates a higher surplus, which accumulates to the dual prime broker. We showed that the total surplus between brokers and hedge funds is higher in the case of monopolist dual prime broker than in the case of a dedicated prime broker that implied using a front-running trading broker. When we assumed ex-ante competition among prime brokers, which is equivalent to assuming long term prime brokerage relations. Hedge funds prefer dual prime brokers to dedicated prime brokers. We then allowed for interim direct competition between the two types of prime brokers. If we assumed that hedge funds have to interact with the two types of prime brokers, we proved the existence of an equilibrium.

When we relaxed this assumption, we conjectured an equilibrium in which hedge funds work with both types of prime brokers. We conclude that hedge funds need not worry about the effectiveness of the "Chinese wall" for investment banks if they can have long term relationships with investment banks.

An interesting extension of the current framework is to allow hedge funds to trade in multiple asset markets. This is the type of situation hedge funds encounter in practice.

Another extension is to allow dual prime brokers to be risk averse. Correlations between the prime broker initial position in the asset marker and the signal received by the hedge fund will cause a "clientele" effect, as pointed out in a similar framework by Ko (2004). Non dedicated prime brokers with different initial positions in the asset markets value differently the trading information. Hedge fund can be better off by wisely choosing among competing prime brokers.

To conclude, we provided a simple and tractable framework in which we address the informational interaction between prime brokers and hedge funds. On one hand, we showed that a competitive market for prime brokerage services is enough to make hedge funds interact with dual prime brokers. On the other hand, a more concentrated market will make the hedge fund prefer to disentangle the prime brokerage from trading brokerage. We contribute to the recent literature by suggesting that hedge funds can benefit from any leakage of information in their prime brokers.

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A Appendix - Oligopoly Discussion

A.1 Oligopoly in the Dedicated Prime Brokerage Market

We address the situation when hedge funds have access to multiple dedicated prime brokers. We assume that there are two competing prime brokers and that in equilibrium, hedge funds have lending relationships with both of them. We assume competition and no communication between the dedicated prime brokers involved in financing the hedge fund. We also require that hedge funds can either accept or reject both offers, that is hedge funds cannot contract with only one prime broker.[34] One reason to require this is that each prime broker has limited capital.[35] Another reason is that regulation might limit how much prime brokers can lend to one hedge fund. Therefore the hedge fund has to contract with both of them. We acknowledge the restrictive nature of this assumption and will relax it later in this section.

We are interested in symmetric and separating equilibria and we denote both menus of contracts offered by prime broker A and B by $\left\{\left(D^{high}_{A/B}, T^{high}_{A/B}\right), \left(D^{low}_{A/B}, T^{low}_{A/B}\right)\right\}$. In equilibrium the high type hedge fund has an aggregate loan of $D^{high}_{A} + D^{high}_{B}$ and the low type hedge fund $D^{low}_{A} + D^{low}_{B}$. Competition makes the requirements for separating equilibria more stringent. High type hedge funds can contemplate deviating by claiming to be low type in front of both prime brokers, A and B. This would be the equivalent of the usual incentive compatibility constraint in a monopolist situation. Now hedge funds can claim to be low type when dealing with prime broker B while

[34] This corresponds to a "syndicated" loan situation.

[35] I thank Prof. Alessandro Pavan for suggesting this interpretation.

telling the truth to prime broker A. Therefore, additional incentive compatibility constraints will restrict the identical contracts offered by the two prime brokers.

We have to specify the equilibrium on the asset market occurring when the hedge fund deviates. This is equivalent to specifying the off-equilibrium path beliefs for the trading broker. Assume that the trading broker observes a "buy" order $\theta_{\alpha}^{E}=\frac{D^{high}+D^{low}}{M}$. We can assume that the trading broker infers a low type as long as the observed order differ from $\theta^{E}=\frac{2D^{high}}{M}$. Alternatively, we can assume that the trading broker thinks the hedge fund to be of type α_1 with probability $1-\nu$ and of type α_2 with probability ν. Therefore, after observing an order of $\theta^{E}=\frac{D_{low}+D_{high}}{M}$, the trading broker's beliefs about the value of the asset becomes $E^{B}\left[v|\theta^{E}\right]=E\left[v\right]+(1+\nu)\Delta$.

A.1.1 Non-Existence of Direct Equilibria

Recall that in the case of the monopolist prime broker in the credit market, we implicitly analyzed only direct mechanisms.[36] To start our analysis and to provide the foundations for the latter discussion, we present a non existence result. This points us in the direction of enlarging the set of mechanisms in order to describe a reasonable set of equilibria for this case. We assumed implicitly that the message space between the hedge fund and the creditor prime broker was at most $M=\left\{\left\{-\alpha_2,\alpha_2\right\},\left\{-\alpha_1,\alpha_1,\right\}\right\}$. Restricting the message space in this manner[37] is not sufficient to find an equilibrium. This is the point of the following lemma.

Lemma 12. *When dedicated prime brokers are competing in the credit market, there does not exist a pure-strategy equilibrium in the direct communication, intrinsic common agency game.*

Proof. See the Appendix, where we only prove non-existence for separating pure-strategy equilibrium. □

A.1.2 Alternative, non-Direct Equilibria with Implicit Agency

The fact that there are no direct mechanism equilibria implies that we have to allow for non-direct mechanisms.[38] We follow the work of Martimort and Stole (2002). They established the methodological steps required to find a subset of the set of equilibria. We employ their approach,

[36] The Revelation Principle holds there.
[37] Which is equivalent to looking only at direct communication.
[38] We are not interested in generating the whole set of equilibria.

and we allow prime brokers to compete through nonlinear pricing functions $\left\{T_{A/B}\left(D_{A/B}\right)\right\}$ defined over the whole real line. Allowing for this type of indirect mechanism guarantees existence of an equilibrium, as pointed out below. To get some intuition we assume that prime broker A posts a schedule as the one above and we present prime broker B's problem. Prime broker B chooses pairs $\left\{\left(D_B^{high}, T_B^{high}\right), \left(D_B^{low}, T_B^{low}\right)\right\}$ to maximize her lending profit. Since the hedge fund has to accept/reject both offers, the participation constraints for prime broker B's problem will be type independent.

Lemma 13. *There exists a global equilibrium such that, in the credit market, a pure-strategy equilibrium of the indirect communication implicit common agency game emerges. Each prime broker offers a two-part tariff* $T_i(D_i) = D_i + a_i \; \forall i \in \{A, B\}$.

Case 1. *When* ν *is "small"*[39], *there is an equilibrium with both types being credited and* $a_A + a_B = \frac{\beta}{8}\Delta^2$. *Each type receives her optimal debt level* $D^{high*} = \beta M \Delta$ *and* $D^{low*} = \beta M \frac{\Delta}{2}$. *Only the high type enjoys positive profits net of transfers.*

Case 2. *For any value of* $\nu \in (0,1)$, *there is another equilibrium in which only the high type is credited and* $a_A + a_B = \frac{\beta}{2}\Delta^2$. *Only the high type trades at the optimal level* $D^{high*} = \beta M \Delta$.

Proof. See part 2 of the Appendix. □

Different types of hedge funds can choose whether to pool or to separate once the two-part tariff is offered. As the previous discussion showed, they choose to pool, for several reasons. First, the low type wants to signal her true type to the trading broker, such that she will get a "softer" competitor in the asset market. Second, the high type hedge fund has a higher marginal profit for each unit of credit and although she prefers not to signal her true type to the trading broker, she is better off separating.

We addressed the case with only two competing prime brokers. This corresponds to an intrinsic common agency duopoly. We analyze below the case with arbitrary N competing dedicated prime brokers.

We look only at best responses functions for each prime broker, allowing us to focus on Nash equilibria of the game. Fixing the other $N-1$ prime brokers' strategies, we can solve the N^{th} prime broker problem for the symmetric equilibrium case. The following lemma formalizes the intuition.

[39] The exact upper bound for ν is $\frac{\left(\frac{\Delta}{2}\right)^2}{2\Delta^2 - \left(\frac{\Delta}{2}\right)^2} = \frac{1}{7}$.

Lemma 14. *There exists a global equilibrium such that, in the credit market, a pure-strategy equilibrium of the indirect communication intrinsic common agency game emerges. Each of the N prime brokers offers a two-part tariff* $T_i(D_i) = D_i + a_i \; \forall i = 1..N$.

Case 1. *For "small"*[40] *values of* ν *there is an equilibrium with all types being credited and* $\sum_{i=1}^{N} a_i = \frac{\beta}{8}\Delta^2$. *Each type receives her optimal debt level* $D^{high*} = \beta M \Delta$ *and* $D^{low*} = \beta M \frac{\Delta}{2}$. *Only the high types enjoy positive profits net of transfers.*

Case 2. *For any value of* $\nu \in (0,1)$, *there is another equilibrium in which only the high type is credited and* $\sum_{i=1}^{N} a_i = \frac{\beta}{2}\Delta^2$. *Only the high type trade and her trade is optimal* $D^{high*} = \beta M \Delta$.

Proof. See part 2 of the Appendix. □

A.1.3 Alternative, non-Direct Equilibria with Delegated Agency

This subsection relaxes the assumption that hedge funds have to contract with both dedicated prime brokers. We still assume competition between the dedicated prime brokers. The hedge fund can potentially contract with only one prime broker and this complicates the problem. We allow the dedicated prime brokers to compete through schedules, but the two-part tariffs assumed in the previous section are not enough to guarantee an equilibrium. We assume that each prime broker posts a non-linear schedule, such that

$$T_i(D_i) = \begin{cases} a_1^i + D_i, \text{ for } D_i \leq \widetilde{D} \\ a_2^i + D_i, \text{ for } \widetilde{D} < D_i \leq \widetilde{\widetilde{D}} \\ a_3^i + D_i, \text{ for } D_i > \widetilde{\widetilde{D}} \end{cases} \quad \forall i \in \{A, B\}$$

We present prime broker B's problem, if we assume that prime broker A posts a schedule as the one above. This helps building the intuition for the existence result below. Prime broker B chooses pairs $\left\{\left(D_B^{high}, T_B^{high}\right), \left(D_B^{low}, T_B^{low}\right)\right\}$ to maximize her lending profit. The participation constraints for prime broker B's problem are type dependent, since the hedge fund is no longer forced to accept both offers. To see this, assume that the hedge fund refuses the prime broker B's offer. High types

[40]The exact upper bound for ν is $\frac{\left(\frac{\Delta}{2}\right)^2}{N\Delta^2-(N-1)\left(\frac{\Delta}{2}\right)^2} = \frac{1}{3N+1}$.

hedge funds can still reach a profit equal to

$$U_{\pm\alpha_2}^{A} = \max_{D_A \geq 0} \Pi_{\pm\alpha_2}^{E}(D_A) - T_A(D_A)$$

Accepting only the schedule of prime broker A gives low types a profit equal to

$$U_{\pm\alpha_1}^{A} = \max_{D_A \geq 0} \Pi_{\pm\alpha_1}^{E}(D_A) - T_A(D_A)$$

Now let us define the high types profit when contracting with both prime brokers as

$$U_{\pm\alpha_2} = \max_{D_A, D_B} \Pi_{\pm\alpha_2}^{E}(D_A + D_B) - T_A(D_A) - T_B$$

The low types profit when contracting with both prime brokers is

$$U_{\pm\alpha_1} = \max_{D_A, D_B} \Pi_{\pm\alpha_1}^{E}(D_A + D_B) - T_A(D_A) - T_B$$

If there is no exclusion of types in equilibrium, the participation constraints are

$$\begin{aligned} U_{\pm\alpha_2} &\geq U_{\pm\alpha_2}^{A} \\ U_{\pm\alpha_1} &\geq U_{\pm\alpha_1}^{A} \end{aligned}$$

These two constraints are type dependent, making the analysis more intricate.

Conjecture 2. *There exists a global equilibrium such that, in the credit market, a pure-strategy symmetric equilibrium of the indirect communication delegated common agency game emerges. Each dedicated prime broker offers a non-linear schedule*

$$T_i(D_i) = \begin{cases} a_1^i + D_i, \text{ for } D_i \leq \widetilde{D} \\ a_2^i + D_i, \text{ for } \widetilde{D} < D_i \leq \widetilde{\widetilde{D}} \\ a_3^i + D_i, \text{ for } D_i > \widetilde{\widetilde{D}} \end{cases} \quad \forall i \in \{A, B\}$$

The equilibrium has $\widetilde{D} = \frac{D^{low*}}{2}$ *and* $\widetilde{\widetilde{D}} = \frac{D^{high*}}{2}$.

A.2 Oligopoly in the Dual Prime Brokers Market

Here imperfectly competing dual prime brokers function as creditors and trading brokers for the hedge fund. Initially, we allowing only two prime brokers, A and B to compete imperfectly. We denote the menus of contracts offered by prime brokers A and B by $\left\{D_{A/B}^{high}, T_{A/B}^{high}\right\}$ and $\left\{D_{A/B}^{low}, T_{A/B}^{low}\right\}$.

A.2.1 Non-Direct Equilibria with Intrinsic Common Agency

We start with a "syndicated" situation which parallels the previous oligopoly discussion. Two dual prime brokers compete in their schedules, knowing that hedge funds are required to either accept or reject both offers. We assume that both prime brokers can perfectly infer hedge funds' type no matter how hedge funds decide to split their trades. The case of two prime brokers competing in contracts resembles the previous oligopoly discussion. The following lemma indicates that we have to look again at indirect mechanisms.

Lemma 15. *When dual prime brokers are competing in the credit market, there does not exist a pure-strategy equilibrium in the direct communication, intrinsic common agency game.*

Proof. See part 2 of the Appendix. □

Recall that in the intrinsic common agency framework, we require that the hedge fund can either accept or reject both offers. This means that hedge funds cannot contract with only one prime broker. We allow the prime brokers A and B to compete through linear pricing functions $\left\{T_{A/B}\left(D_{A/B}\right)\right\}$ defined over the whole real line. After finding the equilibrium in the asset market, we find the hedge fund's profit to be $\Pi_{\pm\alpha_2}^{E} = \frac{\beta}{3}\Delta^2$ and $\Pi_{\pm\alpha_1}^{E} = \frac{\beta}{3}\left(\frac{\Delta}{2}\right)^2$. The prime brokers' trading profit is $\Pi_{trading,\pm\alpha_2}^{PB} = \beta\left(\frac{\Delta}{3}\right)^2$ and $\Pi_{trading,\pm\alpha_1}^{PB} = \beta\left(\frac{\Delta}{6}\right)^2$

Lemma 16. *There exists a global equilibrium which has a pure-strategy symmetric equilibrium of the indirect communication intrinsic common agency game in the credit market. Each prime broker offers a two-part tariff $T_i\left(D_i\right) = D_i + a_i \ \forall i \in \{A, B\}$.*

Case 1. *When $\nu \leq \underline{\nu}$[41], both types are credited and the equilibrium debt levels are given by $D^{high} = \beta M\Delta = D^{high*}$ and $D^{low} = \beta M\frac{\Delta}{2} = D^{low*}$ with $a_A + a_B = \frac{\beta}{3}\left(\frac{\Delta}{2}\right)^2$. Each prime broker has an*

[41] The exact value of $\underline{\nu}$ is $\frac{\Pi_{\pm\alpha_2}^{E}}{2\Pi_{\pm\alpha_2}^{E} - \Pi_{\pm\alpha_1}^{E} + 2\Pi_{trading,\pm\alpha_1}^{PB}}$.

order of $\theta^{PB}_{\alpha^{PB}} = \begin{cases} sign\left(\alpha^{PB}\right)\beta\frac{\Delta}{3}, \text{ if } D = D^{high} \\ sign\left(\alpha^{PB}\right)\beta\frac{\Delta}{6}, \text{ if } D = D^{low} \end{cases}$. *Only the high type enjoys positive profit net of transfers.*

Case 2. *When* $\nu \geq \overline{\nu}$[42]*, there is another equilibrium where only the high type is credited and the equilibrium debt level is* $D^{high} = \beta M\Delta = D^{high*}$*. The participation fees are* $a_A + a_B = \frac{\beta}{3}\Delta^2$*. Each prime broker trades a quantity* $\theta^{PB}_{\alpha^{PB}} = sign(\alpha^{PB})\beta\frac{\Delta}{3}$*, whenever the hedge fund trades. There are no positive profits net of transfers.*

Proof. See part 2 of the Appendix. □

If we allow for more than two dual prime brokers, the indirect mechanism equilibria specifies different participation fees and trading orders. The intrinsic common agency framework with sharing of trading information among prime brokers makes high type hedge funds worse off when competition is increasing. More competing prime brokers cause the hedge funds profits to decrease ceteris paribus. This result is independent of the hedge fund's type and is "fallacy of commons" for the value of information disseminated among multiple prime brokers. As a function of the number of competing dual prime brokers N, the hedge fund's profit is $\Pi^E_{\pm\alpha_2}(N) = \frac{\beta}{N+1}\Delta^2$ and $\Pi^E_{\pm\alpha_1}(N) = \frac{\beta}{N+1}\left(\frac{\Delta}{2}\right)^2$. The prime brokers trading profit is $\Pi^{PB}_{trading,\pm\alpha_2^{PB}}(N) = \beta\left(\frac{\Delta}{N+1}\right)^2$ and $\Pi^{PB}_{trading,\pm\alpha_1^{PB}}(N) = \beta\left(\frac{\Delta}{2(N+1)}\right)^2$. We have the apparatus required to formalize the above intuition.

Lemma 17. *For all* N *there exists a global equilibrium that has a pure-strategy symmetric equilibrium of the indirect communication intrinsic common agency game in the credit market. Each prime broker offers a two-part tariff* $T_i(D_i) = D_i + a_i \ \forall i = 1..N$*.*

Case 1. *When* $\nu \leq \underline{\nu(N)}$[43]*, the equilibrium debt levels are given by* $D^{high} = \beta M \cdot \Delta = D^{high*}$ *and* $D^{low} = \beta M\frac{\Delta}{2} = D^{low*}$ *with* $\sum_N^{i=1} a_i = \frac{\beta}{N+1}\left(\frac{\Delta}{2}\right)^2$*. Each prime broker has an order of* $\theta^{PB}_{\alpha^{PB}} = \begin{cases} sign\left(\alpha^{PB}\right)\beta\frac{\Delta}{N+1}, \text{ if } D = D^{high} \\ sign\left(\alpha^{PB}\right)\beta\frac{\Delta}{2(N+1)}, \text{ if } D = D^{low} \end{cases}$. *Only the high type enjoys positive profits net of transfers.*

[42] The exact value of $\overline{\nu}$ is $\frac{2\Pi^E_{\pm\alpha_1} - \Pi^E_{\pm\alpha_2} + 2\Pi^{PB}_{trading,\pm\alpha_1}}{\Pi^E_{\pm\alpha_2} + 2\Pi^{PB}_{trading,\pm\alpha_1}}$.

[43] The exact value of $\underline{\nu(N)}$ can be found to equal $\frac{\Pi^E_{\pm\alpha_1}}{N\Pi^E_{\pm\alpha_2} - (N-1)\Pi^E_{\pm\alpha_1} + N\Pi^{PB}_{trading,\pm\alpha_1}}$ and note that we depressed the dependence on N of all the functions involved for clarity.

Case 2. *When* $\nu \geq \overline{\nu(N)}$[44]*, the equilibrium debt level is* $D^{high} = \beta M \Delta = D^{high*}$*. Only the high type has access to funds and* $\sum_{N}^{i=1} a_i = \frac{\beta}{N+1}\Delta^2$*. Each prime broker has an order of*

$$\theta^{PB}_{\alpha^{PB}} = sign\left(\alpha^{PB}\right)\beta\frac{\Delta}{N+1},\ \textit{whenever the hedge fund trades.}$$

There are no positive profit net of transfers for the high type hedge fund.

Proof. The proof is similar to the proof for the duopoly case with dual prime brokers and is omitted. □

The lemma shows that the indirect mechanism equilibria we described here and in the previous oligopoly section do not converge to a perfectly competitive equilibrium. We claim that the convergence can be re-established if the described equilibria are "outside options". Hedge fund when negotiating with prime broker N, for example. If hedge funds use the equilibria as a credible treat, the prime broker posts a pricing function that offers the same profit $a_N + \Pi^{PB}_{trading,\pm\alpha}$ as under the collective posting game.

A.2.2 Non-Direct Equilibria with Delegated Common Agency

So far, we assumed a syndicate of dual prime brokers with sharing of trading information. We now assume that prime brokers are well capitalized and the hedge fund can with only one prime broker. We allow the dual prime brokers A and B to compete through non linear pricing functions $\left\{T_{A/B}\left(D_{A/B}\right)\right\}$ defined over the whole real line. We suggest a schedule

$$T_i\left(D_i\right) = \begin{cases} a^i_1 + D_i, \text{ for } D_i \leq \widetilde{D} \\ a^i_2 + D_i, \text{ for } \widetilde{D} < D_i \leq \widetilde{\widetilde{D}} \\ a^i_3 + D_i, \text{ for } D_i > \widetilde{\widetilde{D}} \end{cases} \quad \forall i \in \{A, B\}$$

We conjecture the existence of an equilibrium between dual prime brokers in the credit market.

Conjecture 3. *There exists a global equilibrium that has a pure-strategy symmetric equilibrium of the indirect communication delegated common agency game in the credit market. Each dual prime*

[44] The exact value of $\overline{\nu(N)}$ is $\frac{N\Pi^E_{\pm\alpha_1} - (N-1)\Pi^E_{\pm\alpha_2} + N\Pi^{PB}_{trading,\pm\alpha_1}}{\Pi^E_{\pm\alpha_2} + N\Pi^{PB}_{trading,\pm\alpha_1}}$ and note that again we depressed the dependence on N.

broker offers a non-linear tariff

$$T_i\left(D_i\right)=\begin{cases} a_1^i+D_i,\ \textit{for}\ D_i\leq\widetilde{D'} \\ a_2^i+D_i,\ \textit{for}\ \widetilde{D'}<D_i\leq\widetilde{\widetilde{D'}} \\ a_3^i+D_i,\ \textit{for}\ D_i>\widetilde{\widetilde{D'}} \end{cases} \quad \forall i\in\{A,B\}$$

The equilibrium has $\widetilde{D'}=\beta M\frac{\Delta}{4}=\frac{D^{low*}}{2}$ *and* $\widetilde{\widetilde{D'}}=\beta M\frac{\Delta}{2}=\frac{D^{high*}}{2}$.

B Appendix - Proofs of the Lemmas

B.1 Proof of Lemma 1

We present the proof for the case when the hedge fund receives a "good" signal, that is α is positive. Assume that the hedge fund of type α wants to place a total order θ_α^E and can choose how many trading brokers to work with. Let the number of trading brokers be equal to $N\in\{1,2,..\}$. In a separating equilibrium, each broker $i\in\{1,2,..N\}$ will correctly infer the total number of brokers N and the global order θ_α^E. This makes $\mathcal{A^B}_i=\mathcal{A}$. Therefore, broker i will maximize the following objective function:

$$\max_{\theta_i^B}\theta_i^B\cdot\left(E_i^B[v]-E[v]-\frac{\theta_\alpha^E+\sum_1^N\theta_j^B}{\beta}\right)$$

with the symmetric solution

$$\theta_i^B=\frac{\beta}{N+1}\cdot\left(E^B[v]-E[v]\right)-\frac{1}{N+1}\theta_\alpha^E$$

The hedge fund profit is

$$\frac{1}{N+1}\theta_\alpha^E\cdot\left(E^[v|\alpha]-E[v]-\frac{\theta_\alpha^E}{\beta}\right)$$

For any θ_α^E the profit is strictly decreasing in N and the result remains true for all other hedge fund's types.

B.2 Proof of Lemma 2

Assume that there are two equilibrium levels of trading per broker S^1 and S^2 such that $S^1<S^2$. This means that $N_{low}^1>N_{low}^2$ and $N_{high}^1>N_{high}^2$. For each type, the trading profit for the hedge fund is strictly decreasing in the number of brokers. Both types prefer the equilibrium with the highest S. But S is maximal when $S = \frac{D^{low}}{M}$. This implies that any pooling trading equilibrium has the low type trade with exactly one trading broker. The high type, when D^{high} is not a multiple of D^{low}, splits the order such that $N^*_{high} = \left[\frac{D^{high}}{D^{low}}\right] + 1$. Whereas all the first $N^*_{high} - 1$ brokers cannot infer the hedge fund's type, the last one observes an order less than $\frac{D^{low}}{M}$. This last broker infers that the hedge fund is a high type. In a pooling trading equilibrium the high type trades against a perfectly informed broker and $N^*_{high} - 1$ partially informed brokers which leads to an obviously worse outcome than the separating equilibrium. The low type prefers the separating equilibrium, since the first $N - 1$ brokers pool her with the high type and dual-trade more aggressively.

B.3 Proof of Lemma 3

For the dedicated prime broker's problem, there are multiple IC and IR constraints. Due to the symmetry of the framework, we claim that is sufficient to restrict attention to constraints IR_{α_1}, IR_{α_2}, IC_{α_1} and IC_{α_2}. A separating equilibrium in the credit market makes $\mathcal{A}^{\mathcal{PB}} = \{(\pm\alpha_2), (\pm\alpha_1)\}$. We can rewrite the constraints as

$$D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - \hat{T}^{high} \geq D^{low} \cdot \left(3\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low} \ (IC_{\alpha_2})$$

$$D^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low} \geq -D^{high2} - \hat{T}^{high} + 2\beta M^2 D^{high} \ (IC_{\alpha_1})$$

$$D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - \hat{T}^{high} \geq 0 \ (IR_{\alpha_2})$$

$$D^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low} \geq 0 \ (IR_{\alpha_1})$$

If constraint (IR_{α_1}) is met, then constraint (IC_{α_1}) is immediately met, because the RHS of (IC_{α_1}) is negative. Since (IR_{α_2}) and (IC_{α_1}) imply

$$D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - \hat{T}^{high} \geq D^{low} \cdot \left(3\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low}$$

$$> D^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - D^{low}\right) - \hat{T}^{low} \geq 0$$

then

$$D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - \hat{T}^{high} \geq 0$$

which is exactly (IR_{α_2}). We claim that at the optimum, the constraints (IR_{α_1}) and (IC_{α_2}) must be binding. The argument is a standard one. If $\hat{T}^{low}$ is such that (IR_{α_1}) does not bind, then the prime broker can increase $\hat{T}^{low}$. Her profit increases weakly while the constraint (IC_{α_2}) is relaxed.[45] Therefore, (IR_{α_1}) binds and if (IC_{α_2}) does not bind, the prime broker can increase $\hat{T}^{high}$ and therefore her profit without interfering with the other constraint (IR_{α_1}). For any positive pair of credit amounts, D^{high} and D^{low} both constraints are binding. From (IR_{α_1}) we can determine the level of the first transfer:

$$\hat{T}^{low} = D^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - D^{low}\right)$$

From (IC_{α_2}) we can express the level of the second transfer:

$$\hat{T}^{high} = D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - D^{low} \cdot \left(3\Delta\beta M + 2\beta M^2 - D^{low}\right) + \hat{T}^{low}$$

The prime broker chooses positive D^{high} and D^{low} to maximize her profit:

$$\frac{\nu}{2\beta M^2} \cdot \left[D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - 2\Delta\beta M D^{low}\right]$$

$$+\frac{1-\nu}{2\beta M^2} D^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - D^{low}\right) - \nu D^{high} - (1-\nu) D^{low}$$

First order conditions give us the optimal loan sizes D^{high} and D^{low}. Imposing a positive loan size for the low type gives us $\nu \leq \frac{1}{3}$. The prime broker sets $D^{low} = 0$, whenever $\nu > \frac{1}{3}$. The prime broker's profit from allowing the low type to have access to credit does not cover the cost incurred by having to lower the transfer from the high type in order to maintain her truth telling incentives. When considering crediting only the high type entrepreneur, the prime broker's problem is simpler, since the incentive compatibility constraints are changed. The transfer charged to the high type is

$$T^{high} = \frac{\beta}{2}\Delta^2$$

[45] It makes the high type less willing to pretend to be the low type.

Since the prime broker extends credit only to the high type, the incentive compatible constraint for the high type becomes equivalent to the individual rationality constraint for the high type. Here, the prime broker extracts all the surplus from the relationship.

The proof for the pooling case is simpler. When considering a pooling equilibrium, the prime broker has the advantage that can extend credit to both types without separating them. The only question is that of participation. The participation constraint binds sooner for the low type, as proved by the strict inequality

$$D^{pool} \cdot \left(\frac{3}{2}\Delta\beta M + 2\beta M^2 - D^{pool}\right) - \hat{T}^{pool} > D^{pool} \cdot \left(\frac{1}{2}\Delta\beta M + 2\beta M^2 - D^{pool}\right) - \hat{T}^{pool} \ ,\forall D^{pool} \geq 0$$

The prime broker's profit is maximized when the low type hedge fund is allowed to optimally trade against the broker with mixed beliefs. This occurs when $D^{pool} = \beta M \frac{\Delta}{2}(1-\nu)$.

B.4 Proof of Lemma 6

In the credit market, the monopolist prime broker offers a menu that keeps the hedge fund's incentives in place. We can prove that it is enough to restrict our attention to (IC_{high}) and (IR_{low}). These two binding constraints can be used in the prime broker's objective function. From (IR_{low}) we determine the level of the first transfer:

$$\hat{T}^{low} = D^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - D^{low}\right)$$

From (IC_{high}) we express the level of the second transfer:

$$\hat{T}^{high} = D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - D^{low} \cdot 2\Delta\beta M$$

The fact that the prime broker trades after inferring the hedge fund's type changes the objective function as compared to the previous monopolist situation. The prime broker chooses positive D^{high} and D^{low} to maximize the sum of the lending and trading profit:

$$\frac{\nu}{2\beta M^2} \cdot \left[D^{high} \cdot \left(2\Delta\beta M + 2\beta M^2 - D^{high}\right) - 2\Delta\beta M D^{low}\right] + \frac{1-\nu}{2\beta M^2} D^{low} \cdot \left(\Delta\beta M + 2\beta M^2 - D^{low}\right)$$

$$+\nu \cdot \beta \cdot \left(\Delta - \frac{D^{high}}{2\beta M}\right)^2 + (1-\nu) \cdot \beta \cdot \left(\frac{\Delta}{2} - \frac{D^{low}}{2\beta M}\right)^2 - \nu D^{high} - (1-\nu) D^{low}$$

To prove that there are no positive D^{low} and D^{high} solving the problem, it is enough see that $\frac{\partial \Pi^{PB}_{global}}{\partial D^{low}} < 0 \; \forall D^{low} \geq 0$ and $\frac{\partial \Pi^{PB}_{global}}{\partial D^{high}} < 0 \; \forall D^{high} > 0$. For any loan size $D^{high} = \epsilon > 0$, the monopolist bank can increase her profit by reducing the loan size, for example to $D^{high'} = \frac{\epsilon}{2} > 0$.

B.5 Proof of Lemma 12

We only prove here that a separating and monotonic equilibrium does not exist. First, we introduce some notation. We denote the hedge fund's type by $\underline{\gamma} = \frac{\Delta}{2}$ when $\alpha \in \{\alpha_1, -\alpha_1\}$ and by $\overline{\gamma} = \Delta$ when $\alpha \in \{\alpha_2, -\alpha_2\}$. We denote the trading profit before paying the transfers to the prime brokers by $\Pi^E_{Total}(D_A, D_B, \gamma)$ and with $\Pi^E_{Ind}(D_A, \gamma)$ the following maximum

$$\max_{\gamma' \in \{\underline{\gamma}, \overline{\gamma}\}} \Pi^E_{Total}\left(D_A, D_B(\gamma'), \gamma\right) - T_B(\gamma')$$

where γ' is the type reported by the hedge fund to prime broker B. This is the indirect profit that the hedge fund can obtain once that she sent the optimal message to prime broker B, as a function of the true type and the loan from prime broker A. Now we can define the prime broker's A problem, which solves

$$\max_{\left\{\left(D_A^{high}, T_A^{high}\right), \left(D_A^{low}, T_A^{low}\right)\right\}} \nu \left(T_A^{high} - D_A^{high}\right) + (1-\nu)\left(T_A^{low} - D_A^{low}\right) \text{ such that}$$

$$\begin{aligned}
\Pi^E_{Ind}\left(D_A^{high}, \overline{\gamma}\right) - T_A^{high} &\geq \Pi^E_{Ind}\left(D_A^{low}, \overline{\gamma}\right) - T_A^{low} \; (IC_{high}) \\
\Pi^E_{Ind}\left(D_A^{low}, \underline{\gamma}\right) - T_A^{low} &\geq \Pi^E_{Ind}\left(D_A^{high}, \underline{\gamma}\right) - T_A^{high} \; (IC_{low}) \\
\Pi^E_{Ind}\left(D_A^{high}, \overline{\gamma}\right) - T_A^{high} &\geq 0 \; (IR_{high}) \\
\Pi^E_{Ind}\left(D_A^{low}, \underline{\gamma}\right) - T_A^{low} &\geq 0 \; (IR_{low})
\end{aligned}$$

A fully separating and monotonic equilibrium implies that

$$\Pi^E_{Ind}\left(D_A^{high}, \overline{\gamma}\right) = \Pi^E_{Total}\left(D_A^{high}, D_B^{high}, \overline{\gamma}\right) - T_B^{high}$$

The monotonicity of the equilibrium gives $D_A^{high} > D_A^{low}$ and that

$$\Pi_{Ind}^{E}\left(D_A^{low}, \overline{\gamma}\right) = \Pi_{Total}^{E}\left(D_A^{low}, D_B^{high}, \overline{\gamma}\right) - T_B^{high}$$

From the optimality of prime broker's A offer, (IC_{high}) is binding and this implies

$$\Pi_{Total}^{E}\left(D_A^{high}, D_B^{high}, \overline{\gamma}\right) - T_A^{high} = \Pi_{Total}^{E}\left(D_A^{low}, D_B^{high}, \overline{\gamma}\right) - T_A^{low}$$

Replacing T_A^{high} from the last equality into the prime broker's A objective function give us the optimal level of D_A^{high} which we denote by D_A^{high0}. The optimal level D_A^{high0} satisfies the first order condition $D_A^{high0} = \Delta\beta M - D_B^{high}$ and is of course consistent with the equilibrium requirement that the high type hedge fund always receives $\left\{D_A^{high}, D_B^{high}\right\}$. By symmetry, since the prime broker's B has also to be optimal, we get that

$$\Pi_{Total}^{E}\left(D_A^{high0}, D_B^{high}, \overline{\gamma}\right) - T_B^{high} = \Pi_{Total}^{E}\left(D_A^{high0}, D_B^{low}, \overline{\gamma}\right) - T_B^{low}$$

For any deviation of prime broker A, $D_A^{high'}$, such that $D_A^{high'} > D_A^{high}$ makes

$$\Pi_{Ind}^{E}\left(D_A^{high'}, \overline{\gamma}\right) = \Pi_{Total}^{E}\left(D_A^{high'}, D_B^{low}, \overline{\gamma}\right) - T_B^{low}$$

Let us contemplate such a deviation from prime broker A, while keeping the same allocation $\left(D_A^{low}, T_A^{low}\right)$. The optimality implies that (IC_{high}) is still binding and this yields

$$\Pi_{Total}^{E}\left(D_A^{high'}, D_B^{low}, \overline{\gamma}\right) - T_A^{high'} - T_B^{low} = \Pi_{Total}^{E}\left(D_A^{low}, D_B^{high}, \overline{\gamma}\right) - T_A^{low} - T_B^{high'}$$

If we use the above equality to optimize the prime broker's objective function with respect to D_A^{high}, a contradiction is obtained. The objective function for prime broker A is continuous in D_A^{high} and differentiable to the right of D_A^{high0}. Since the derivative is proportional to $\Delta\beta M - D_B^{high} - D_A^{high}$, then when prime broker B offers a separating contract, it becomes positive for D_A^{high0}. Therefore, prime broker's A can increase her profit by slightly increasing D_A^{high}. This means that D_A^{high0} is not a global optimum for prime broker's A profit, which is a contradiction. This also contradicts the fact that in equilibrium, the high type hedge fund receives a larger loan from each prime broker.

B.6 Proof of Lemma 13

We proceed in a number of steps. First, we compute the prime brokers profits when both types are financed. Second, we compute the profits when only the high type is credited. Third, to see whether the equilibria are well-defined, we look at the incentives to deviate.

Step 1. We assume that prime broker B offers a non-linear schedule such that $T_B(D_B) = a_B + D_B$. The indirect profit function vis-a-vis prime broker's A becomes

$$\Pi^E_{Ind}\left(D_A, \gamma\right) = \max_{D_B} \Pi^E_{Total}\left(D_A, D_B, \gamma\right) - a_B - D_B$$

After substituting the solution, the resulting indirect profit is

$$\Pi^E_{Ind}\left(D_A, \gamma\right) = -a_B + D_A + \frac{\beta}{2}\gamma^2$$

Satisfying (IC_{high}) and (IC_{low}) imposes that $T^A_{low} - D^A_{low} = T^A_{high} - D^A_{high}$, which makes prime broker A indifferent between all pairs $\left(T^A_{low}, D^A_{low}\right)$ and $\left(T^A_{high}, D^A_{high}\right)$, as she obtains the same profit. We can denote the constant profit of prime broker's A by a_A. The last step is to determine the size of a_A. Since (IR_{high}) is implied by (IR_{low}), we only have to worry about this last constraint. This directly implies that $a_A + a_B = \frac{\beta}{8}\Delta^2$.

Step 2. Assume that the two prime brokers coordinate to allow access only to the high type hedge fund. Assume that prime broker B offers a non-linear schedule such that $T'_B(D_B) = a'_B + D_B$. Since prime broker A contemplates posting a schedule allowing trading only for the high type, she will choose a transfer T'_A that will extract all the surplus. We can obviously write $T_A = a'_A + D_A$. Computing the trading profit for the high type results in $a'_A + a'_B = \frac{\beta}{2}\Delta^2$. This case is much simpler, since only the high type will be served. We choose this approach to preserve the symmetry of the framework vis-a-vis the other case.

Step 3. If prime broker B offered a schedule with $a_B = \frac{\beta}{16}\Delta^2$, prime broker A can choose to post a schedule with $a^{notdeviate}_A = \frac{\beta}{16}\Delta^2$ or one with $a^{deviate}_A = \frac{\beta}{2}\Delta^2 - \frac{\beta}{16}\Delta^2$ which allows only the high type to participate. Comparing prime broker A's expected profits in the two cases is equivalent to comparing $\nu\ cdota_A^{deviate}$ to $a_A^{notdeviate}$. This gives us the upper bound for ν. To prove the second case, assume that prime broker B posted a schedule $T'_B(D_B) = a'_B + D_B$ where $a'_B = \frac{\beta}{4}\Delta^2$. The same argument makes prime broker A deciding about the level of a_A. She can choose a_A such that the low type participates or not. We claim that the fee a_A charged when the low type participates is less than zero and therefore this is not an equilibrium. To see that, recall that the trading profit for the low type is $\frac{\beta}{8}\Delta^2$ and therefore a_A can be at most $\frac{\beta}{8}\Delta^2 - a'_B < 0$.

B.7 Proof of Lemma 14

Assume that the initial $N-1$ prime brokers chose their schedules and now we analyze the N^{th} prime broker's problem. We use the same three steps as before.

Step 1. Assume that the initial $N-1$ prime brokers offer non-linear schedules such that $T_i(D_i) = a_i + D_i\ \forall i = 1..N-1$. The indirect profit function vis-a-vis prime broker N becomes

$$\Pi_{Ind}^{E}\left(D_N, \gamma\right) = \max_{D_i, \forall i=1..N-1} \Pi_{Total}^{E}\left(D_N, \sum_{i=1}^{N-1} D_i, \gamma\right) - \sum_{i=1}^{N-1} a_i - \sum_{i=1}^{N-1} D_i$$

After substituting the solution, the resulting indirect profit is

$$\Pi_{Ind}^{E}\left(D_N, \gamma\right) = -\sum_{i=1}^{N-1} a_i + D_N + \frac{\beta}{2}\gamma^2$$

Satisfying (IC_{high}) and (IC_{low}) imposes that $T_{low}^N - D_{low}^N = T_{high}^N - D_{high}^N$, which makes prime broker N indifferent between all pairs $\left(T_{low}^N, D_{low}^N\right)$ and $\left(T_{high}^N, D_{high}^N\right)$. We denote the constant profit of prime broker's N by a_N. The last step is to determine the size of a_N. Since (IR_{high}) is implied by (IR_{low}), we only have to worry about this last constraint. This directly implies that $\sum_{i=1}^{N} a_i = \frac{\beta}{8}\Delta^2$.

Step 2. Assume that the initial $N-1$ prime brokers offer non-linear schedules such that $T'_i(D_i) = a'_i + D_i\ \forall i = 1..N-1$. Since prime broker N contemplates posting a schedule allowing trading only for the high type, she will choose a transfer T'_N that will extract all the high type's surplus. We can write $T'_N = a'_N + D_N$. Computing the trading profit for the high type results in $a'_N + \sum_{i=1}^{N-1} a'_i = \frac{\beta}{2}\Delta^2$.

Step 3. If the first $N-1$ prime brokers offered schedules with $a_i = \frac{\beta}{2N}\left(\frac{\Delta}{2}\right)^2$, prime broker N can choose a schedule with $a_N^{notdeviate} = \frac{\beta}{2N}\left(\frac{\Delta}{2}\right)^2$ or one with $a_N^{deviate} = \frac{\beta}{2}\Delta^2 - \frac{(N-1)\beta}{2N}\left(\frac{\Delta}{2}\right)^2$. This latter schedule allows only the high type to participate. Comparing prime broker A's profit in the two cases is equivalent to comparing $\nu \cdot a_A^{deviate}$ to $a_A^{notdeviate}$. The upper bound for ν is therefore found.

B.8 Proof of Lemma 15

We restrict attention to the non-existence of a separating and monotonic equilibrium. The hedge fund's type is $\underline{\gamma} = \frac{\Delta}{2}$ when $\alpha \in \{\alpha_1, -\alpha_1\}$ and $\overline{\gamma} = \Delta$ when $\alpha \in \{\alpha_2, -\alpha_2\}$. Recall that the trading profit before transfers is $\Pi^E_{Total}(D_A, D_B, \gamma)$ and that $\Pi^E_{Ind}(D_A, \gamma)$ is

$$\max_{\gamma' \in \{\underline{\gamma}, \overline{\gamma}\}} \Pi^E_{Total}\left(D_A, D_B(\gamma'), \gamma\right) - T_B(\gamma')$$

where γ' is the type reported by the hedge fund to prime broker B. This is the indirect profit that the hedge fund can obtain once she sent the optimal message to prime broker B, as a function of the type and the loan from prime broker A. We define the prime broker's A objective function, assuming that the prime broker's trading quantities $\theta_A^{\overline{PB}}$ and $\theta_A^{\underline{PB}}$ are fixed[46]

$$\nu\left(T_A^{high} - D_A^{high}\right) + (1-\nu)\left(T_A^{low} - D_A^{low}\right)$$

$$+\nu\theta_A^{\overline{PB}} \cdot \left(2\Delta - \frac{D_A^{high} + D_B^{high}}{\beta M} - \frac{\theta_A^{\overline{PB}} + \theta_B^{\overline{PB}}}{\beta}\right)$$

$$+(1-\nu)\theta_A^{\underline{PB}} \cdot \left(\Delta - \frac{D_A^{low} + D_B^{low}}{\beta M} - \frac{\theta_A^{\underline{PB}} + \theta_B^{\underline{PB}}}{\beta}\right)$$

which is equivalent to the following profit function

$$\nu\left[T_A^{high} - D_A^{high} \cdot \left(1 + \frac{\theta_A^{\overline{PB}}}{\beta M}\right)\right] + (1-\nu)\left[T_A^{low} - D_A^{low} \cdot \left(1 + \frac{\theta_A^{\underline{PB}}}{\beta M}\right)\right]$$

[46] We assume that the trading quantities are determined separately in the asset market equilibrium. Determining the prime brokers' trading quantities in the current equilibrium makes the algebra less transparent, while only making the argument even stronger. Now the incentives of prime broker A to make the hedge fund lie to prime broker B are even stronger, because of the extra trading profits.

The prime broker A's problem is to maximize the objective function such that

$$\begin{gathered}\Pi^E_{Ind}\left(D_A^{high},\overline{\gamma}\right)-T_A^{high}\geq\Pi^E_{Ind}\left(D_A^{low},\overline{\gamma}\right)-T_A^{low}\ (IC_{high})\\ \Pi^E_{Ind}\left(D_A^{low},\underline{\gamma}\right)-T_A^{low}\geq\Pi^E_{Ind}\left(D_A^{high},\underline{\gamma}\right)-T_A^{high}\ (IC_{low})\\ \Pi^E_{Ind}\left(D_A^{high},\overline{\gamma}\right)-T_A^{high}\geq 0\ (IR_{high})\\ \Pi^E_{Ind}\left(D_A^{low},\underline{\gamma}\right)-T_A^{low}\geq 0\ (IR_{low})\end{gathered}$$

A fully separating and monotonic equilibrium implies that

$$\Pi^E_{Ind}\left(D_A^{high},\overline{\gamma}\right)=\Pi^E_{Total}\left(D_A^{high},D_B^{high},\overline{\gamma}\right)-T_B^{high}$$

The monotonicity of the equilibrium implies that $D_A^{high}>D_A^{low}$ and that

$$\Pi^E_{Ind}\left(D_A^{low},\overline{\gamma}\right)=\Pi^E_{Total}\left(D_A^{low},D_B^{high},\overline{\gamma}\right)-T_B^{high}$$

From the optimality of prime broker A's offer, (IC_{high}) is binding and this implies

$$\Pi^E_{Total}\left(D_A^{high},D_B^{high},\overline{\gamma}\right)-T_A^{high}=\Pi^E_{Total}\left(D_A^{low},D_B^{high},\overline{\gamma}\right)-T_A^{low}$$

Replacing T_A^{high} from the last equality into the prime broker A's objective function and choosing the optimal level of D_A^{high} which we denote by D_A^{high0}. The optimal level D_A^{high0} satisfies the first order condition $D_A^{high0}=\Delta\beta M-D_B^{high}-\theta_A^{PB}\cdot M-\frac{1}{2}\theta_B^{PB}\cdot M$ and is of course consistent with the equilibrium requirement that the high type hedge fund always receives $\left\{D_A^{high},D_B^{high}\right\}$. By symmetry, since the prime broker B's has also to be optimal, we get that

$$\Pi^E_{Total}\left(D_A^{high0},D_B^{high},\overline{\gamma}\right)-T_B^{high}=\Pi^E_{Total}\left(D_A^{high0},D_B^{low},\overline{\gamma}\right)-T_B^{low}$$

For any deviation of prime broker A $D_A^{high'}$ such that $D_A^{high'}>D_A^{high}$ makes

$$\Pi^E_{Ind}\left(D_A^{high'},\overline{\gamma}\right)=\Pi^E_{Total}\left(D_A^{high'},D_B^{low},\overline{\gamma}\right)-T_B^{low}$$

Let us contemplate such a deviation from prime broker A, while keeping the same allocation $\left(D_A^{low}, T_A^{low}\right)$. The optimality still implies that (IC_{high}) is binding and this yields

$$\Pi^E_{Total}\left(D_A^{high'}, D_B^{low}, \overline{\gamma}\right) - T_A^{high'} - T_B^{low} = \Pi^E_{Total}\left(D_A^{low}, D_B^{high}, \overline{\gamma}\right) - T_A^{low} - T_B^{high'}$$

If we use the above equality to optimize the prime broker's objective function with respect to D_A^{high}, a contradiction is obtained. The objective function for prime broker A is continuous in D_A^{high} and differentiable to the right of D_A^{high0}. Since the derivative is proportional to $\Delta\beta M - D_B^{high} - \theta_A^{PB} \cdot M - \frac{1}{2}\theta_B^{PB} \cdot M - D_A^{high}$, then when prime broker B offers a separating contract, it becomes positive for D_A^{high0}. Therefore, prime broker A can increase her profit by slightly increasing D_A^{high}. This means that D_A^{high0} is not a global optimum for prime broker A's profit, which is a contradiction. This contradicts the fact that in equilibrium, the high type hedge fund receives a large loan from each prime broker.

B.9 Proof of Lemma 16

The proof proceeds in the usual three steps. First, we find the equilibrium when both types are credited. Second, we find the equilibrium when only the high type is credited. Third, we compute the prime brokers profits when deviating to see the conditions for the equilibrium survival.

Step 1. Assume that the two prime brokers coordinate to allow access to funds for both types of hedge funds. Let prime broker B offer a non-linear schedule such that $T_B(D_B) = a_B + D_B$ and that, conditional on the realization of the hedge fund's type, her trading orders are θ_B^{PB} and $\theta_B^{\overline{PB}}$. The indirect profit function vis-a-vis prime broker A has to account for the fact that for each type γ, both prime brokers A and B strategically dual-trade. Imposing that both prime brokers dual-trade, allows us to compute $\theta_B^{PB} = \theta_A^{PB} = \frac{\Delta\beta M - D^{low}}{3M}$ and $\theta_B^{\overline{PB}} = \theta_A^{\overline{PB}} = \frac{2\Delta\beta M - D^{high}}{3M}$. The indirect profit is

$$\Pi^E_{Ind}\left(D_A, \gamma\right) = \max_{D_B} \Pi^E_{Total}\left(D_A, D_B, \gamma\right) - a_B - D_B$$

After substituting the solution $D_B(D_A, \gamma) = \gamma\beta M - D_A$ the resulting indirect profit is

$$\Pi^E_{Ind}\left(D_A, \gamma\right) = -a_B + D_A + \frac{\beta}{3}\gamma^2$$

For each type $\gamma \in \{\underline{\gamma}, \overline{\gamma}\}$ the indirect profit depends linearly on D_A, which implies that (IC_{high}) and (IC_{low}) impose that $T^A_{low} - D^A_{low} = T^A_{high} - D^A_{high}$. This makes prime broker A indifferent between all pairs $\left(T^A_{low}, D^A_{low}\right)$ and $\left(T^A_{high}, D^A_{high}\right)$. Prime broker A thus obtains the same lending profit across types, denoted again by a_A. The last step is to determine the size of a_A. Since (IR_{high}) is implied by (IR_{low}), we need to meet this latter constraint. This implies that $a_A + a_B = \frac{\beta}{3}\underline{\gamma}^2$.

Step 2. Assume that the two prime brokers coordinate to allow access only to the high type hedge fund. Assume that prime broker B offers a non-linear schedule such that $T'_B(D_B) = a'_B + D_B$. Since prime broker A contemplates posting a schedule allowing trading only for the high type, her order is $\theta^{\overline{PB}}_B = \theta^{\overline{PB}}_A = \frac{2\Delta\beta M - D^{high}}{3M}$. Since the transfer T'_A extracts all the surplus from the high type, we can obviously write $T_A = a'_A + D_A$. Computing the trading profit for the high type results in $a'_A + a'_B = \frac{\beta}{3}\overline{\gamma}^2$.

Step 3. Assume that prime broker B posts a schedule with $a_B = \frac{\beta}{6}\underline{\gamma}^2$. Prime broker A can post a schedule with a participation fee of $a^{notdeviate}_A = a_B = \frac{\beta}{6}\underline{\gamma}^2$. But deviation implies a fee of $a^{deviate}_A = \frac{\beta}{3}\overline{\gamma}^2 - \frac{\beta}{6}\underline{\gamma}^2$. Recall that prime brokers dual-trade here. Non participation for the low type implies no dual-trade for the prime broker when hedge funds turns out to be of the low type. Lending and dual-trading with both types dominates lending and trading only with the high type when

$$\nu \cdot \left\{\Pi^{PB}_{lending,\pm\alpha_2} + \Pi^{PB}_{trading,\pm\alpha_2}\right\} + (1-\nu) \cdot \left\{\Pi^{PB}_{lending,\pm\alpha_1} + \Pi^{PB}_{trading,\pm\alpha_1}\right\}$$

$$\geq \nu \cdot \left\{\Pi^{PB'}_{lending,\pm\alpha_2} + \Pi^{PB'}_{trading,\pm\alpha_2}\right\}$$

which gives the required upper bound for ν. For the other case, assume that prime broker B posts a schedule meant only for the high type, with $a_B = \frac{\beta}{6}\overline{\gamma}^2$. A deviation for prime broker A means posting a schedule which allows the low type to participate. Comparing again the expected profit from deviating to the one from complying gives us the required lower bound for ν.