Ambiguity, Information Acquisition and Price Swings in Asset Markets*

Antonio Mele
London School of Economics

Francesco Sangiorgi
Stockholm School of Economics

March 29, 2009

Preliminary draft

Abstract

We study assets markets in which ambiguity averse investors face uncertainty related to the expected value of the asset payoff. In these markets, ambiguity increases the incentives to acquire information, and may lead to complementarities in information acquisition, multiplicity of equilibria and large price swings arising after small changes in the perceived ambiguity in the market.

*We thank Paolo Ghirardato, Emre Ozdenoren and Kathy Yuan for discussions, and retain the usual responsibility for any omission or mistakes.

1 Introduction

"There is a Cloud in Every Silver Lining"

One standard assumption in financial economics is that rational decision makers are able to figure out the probability distribution of the events that affect asset prices. "Ambiguity" is an alternative way to describe uncertainty. It is relevant when some events can not be assigned an obvious probability distribution. Accordingly, the literature on the impact of ambiguity aversion on asset prices is expanding at a fast pace (e.g., Epstein and Wang, 1994, 1995; Cao, Wang and Zhang, 2005; Easley and O'Hara, 2007; Leippold, Trojani and Vanini, 2007; Anderson, Ghysels and Juergens, 2007; Epstein and Schneider 2008; Caskey, 2008; Caballero and Krishnamurthy, 2008; Gagliardini, Porchia and Trojani, 2008; Hansen and Sargent, 2008).

The common assumption in this literature is that financial decision makers are symmetrically informed about the value of the fundamentals that are deemed to affect the asset prices. In this paper, we relax this assumption, and investigate an asset market with heterogenously informed agents. In this market, agents are ex-ante uninformed about the expected value of the asset fundamentals, and display ambiguity aversion. Our departure from the previous literature on ambiguity aversion is the assumption that the very same agents might resolve their ambiguity, by just purchasing information. Those who indeed do so, pay a (constant) cost, as in Grossman and Stiglitz (1980). Those who choose to remain uninformed, instead, attempt to learn about the fundamentals by observing the equilibrium asset price, assuming as usual that noise trading is partly impounded on this price.

We show that agents who are informed and agents who remain uninformed and, hence, ambiguity averse, co-exist, in equilibrium. In fact, we show that a multiplicity of equilibria is likely to occur, as a result of strategic complementarities in the process of information acquisition: the larger the mass of informed agents, the higher the incentives to become informed. Complementaries in information acquisition are at the root of many interesting properties our model generates, such as non-Markovian regimes of expansions, market crashes and varying levels of informational efficiency, with media frenzies, media glooms, and episodes of extreme volatility. These properties are, of course, in common with other models that feature strategic complementarities in information acquisition (e.g., Froot, Scharfstein and Stein, 1992; Veldkamp, 2006; Barlevi and Veronesi, 2000, 2008; Chamley, 2008; García and Strobl, 2008; Hellwig and Veldkamp, 2008). At the same time, the economic rationale behind our model is quite distinct.

The economic mechanism at work in our model is the following. In a market with ambiguity, information acquisition stems out from two opposing forces. On the one hand, there is a standard strategic substitutability effect, by which an increase in the number of informed agents leads to more informative prices, which reduces the incentives to acquire information.

On the other hand, our model uncovers a new effect, related to the ambiguity about the asset value. In the presence of such ambiguity, the uninformed agents trade less and, in some cases, even exit the market, as the mass of informed agents increases. This reduced market participation leads asset prices to be misaligned with fundamentals, even more so than in markets without ambiguity, and makes informed agents benefit from this mispricing. These effects lead informed traders to be better off than the uninformed, as the mass of informed traders increases.

The paper is organized as follows. In the next section, we develop the model and the equilibrium. Section 3 describes the process of information acquisition and Section 4 analyzes the properties of the model. One appendix contains technical details omitted from the main text.

2 Model

2.1 Agents and assets

We consider a market for a risky asset, with payoff equal to $f = \theta + \epsilon$, where $\theta \sim N(\mu_0, \omega_\theta)$ and $\epsilon \sim N(0, \omega_\epsilon)$. Without loss of generality, we set, $\mu_0 = 0$. As in Grossman and Stiglitz (1980), the market is populated by a continuum of agents, with a fraction λ of informed agents and a fraction of $1 - \lambda$ of uninformed agents. Informed agents observe θ at cost $c > 0$. The asset supply is $z \sim N(\mu_z, \omega_z)$ and prevents information to be fully revealed in equilibrium. A riskless asset is also available for trading, which is in perfectly elastic supply, and yields a rate of return equal to zero. All agents have negative exponential utility, with constant absolute risk aversion τ.

Our point of departure from Grossman and Stiglitz (1980) is the assumption that all agents are ex-ante uncertain about the expected value of the fundamental. Although they are unable to assess what μ_0 is, they believe it belongs to some interval, $\mu_0 \in [\underline{\mu}, \bar{\mu}]$, where for some $\Delta\mu \geq 0$, we assume that $\underline{\mu} = -\frac{1}{2}\Delta\mu$ and $\bar{\mu} = \frac{1}{2}\Delta\mu$. The length of this interval, $\Delta\mu$, measures the degree of ambiguity that investors face in the market. We assume that agents display ambiguity aversion in that their preferences are in the form of the maxmin expected utility, as in Gilboa and Schmeidler (1989) (see below). We initially take the value of λ as given, although a fundamental purpose of the paper is to determine this value endogenously, as a result of the information acquisition process.

2.2 Informed agents

By observing the realization of θ, informed agents resolve their ambiguity straight away, and choose portfolio holdings so as to maximize,

$$v_I(\theta) = E\left(-e^{-\tau W_I} \middle| \theta, p\right),$$

where $W_I = (f - p)x_I - c$, p is the observed asset price and, finally, x_I is asset demand, given by:

$$x_I(\theta, p) = \frac{E(f|\theta, p) - p}{\tau Var(f|\theta, p)} = \frac{\theta - p}{\tau \omega_\epsilon}.$$

2.3 Uninformed agents

The uncertainty related to the expected value of the fundamentals, μ_0, leads the uninformed agents to choose portfolio holdings, so as to maximize,

$$v_U(p) = \min_{\mu} E_\mu \left(-e^{-\tau W_U} \middle| p \right) = -e^{-\tau \min_\mu E_\mu(W_U|p) + \frac{1}{2}\tau^2 var(W_U|p)}, \tag{1}$$

where $W_U = (f-p)\, x_U$, and x_U is asset demand. The criterion underlying Eq. (1) is the celebrated maxmin expected utility, introduced by Gilboa and Schmeidler (1989).

We conjecture that for every pair (θ, z), the equilibrium price function is $P(\theta, z)$. Then, we look for an equilibrium in which the uninformed agents sell the asset when the price is sufficiently high and buy the asset when the price is sufficiently low. As we shall show, this search process leads to a simpler problem, in which the uninformed agents' concern is to determine the expectation of the fundamentals in the states of nature in which they buy and sell. Accordingly, let us introduce the following notation,

$$E^{\text{buy}}(f|\, P(\cdot,\cdot) = p) \equiv E_{\underline{\mu}}(f|\, P(\cdot,\cdot) = p), \quad E^{\text{sell}}(f|\, P(\cdot,\cdot) = p) \equiv E_{\bar{\mu}}(f|\, P(\cdot,\cdot) = p).$$

We conjecture that the solution to the uninformed agents' problem is,

$$x_U(p, P(\cdot,\cdot)) = \begin{cases} \dfrac{E^{\text{buy}}(f|\, P(\cdot,\cdot) = p) - p}{\tau Var(f|\, P(\cdot,\cdot) = p)}, & \text{for } p < E^{\text{buy}}(f|\, P(\cdot,\cdot) = p) \\ 0, \quad \text{for } p \in \left[E^{\text{buy}}(f|\, P(\cdot,\cdot) = p), E^{\text{sell}}(f|\, P(\cdot,\cdot) = p)\right] \\ \dfrac{E^{\text{sell}}(f|\, P(\cdot,\cdot) = p) - p}{\tau Var(f|\, P(\cdot,\cdot) = p)}, & \text{for } p > E^{\text{sell}}(f|\, P(\cdot,\cdot) = p) \end{cases} \tag{2}$$

In words, the uninformed agents do not participate in the market if the observed equilibrium price does not take a sufficiently favourable value. This value has to be such that the agents believe that in the worst case scenario, they can actually make profits, on "average." In particular, the uninformed agents enter the market as buyers (sellers) when the price realization, p, is less (larger) than the agents' worst case scenario expectation of the asset value, conditional upon p. Hence, the decision to participate involves a fixed-point problem, in which the expectation of the asset value, conditional on the price realization, is equal to the very same price realization, in equilibrium,

$$E^{\text{buy}}\left(f|\, P(\cdot,\cdot) = \underline{p}\right) = \underline{p} \quad \text{and} \quad E^{\text{sell}}(f|\, P(\cdot,\cdot) = \bar{p}) = \bar{p}. \tag{3}$$

Then, the uninformed agents do not participate in the asset market if the equilibrium price realization, p, is such that $p \in \left[\underline{p}, \bar{p}\right]$. Naturally, the cutoffs $\underline{p}$ and $\bar{p}$ are endogenous, and we shall verify that in equilibrium, they satisfy $\underline{p} < \bar{p}$.

2.4 Equilibrium

We conjecture that the equilibrium price function is, $P(\theta, z) = P(s(\theta, z))$, where $s(\theta, z)$ is the compound signal, defined as,

$$s(\theta, z) = \frac{\lambda}{\tau\omega_\epsilon}\theta - (z - \mu_z). \tag{4}$$

From the market clearing condition,

$$(1-\lambda)\, x_U(p, P(\cdot)) + \lambda x_I(\theta, p) = z, \tag{5}$$

we easily see that the compound signal is observationally equivalent to the equilibrium price. Therefore, the equilibrium in this market is also one in which uninformed agents condition the expectation of the asset value on the compound signal.

We have:

PROPOSITION I. *The equilibrium price is piecewise linear in the compound signal,*

$$P(s) = \begin{cases} \underline{a} + bs, & \text{for } s < \underline{s} \\ a + \dfrac{\tau\omega_\epsilon}{\lambda}s, & \text{for } s \in [\underline{s}, \bar{s}] \\ \bar{a} + bs, & \text{for } s > \bar{s} \end{cases} \tag{6}$$

for some constants $\bar{a}, \underline{a}, a, b$ given in the Appendix. The threshold values for the compound signal, $\underline{s}, \bar{s}$, satisfy:

$$\bar{s} = \frac{\lambda}{\tau\omega_\epsilon}\bar{\mu} + \frac{\omega_s}{\omega_z}\mu_z, \quad \bar{s} - \underline{s} = \frac{\lambda}{\tau\omega_\epsilon}\Delta\mu,$$

where ω_s is the variance of s in Eq. (4). Finally, we have that $\underline{p} < \bar{p}$, where the expressions for $\underline{p}$ and $\bar{p}$ are given in the Appendix.

Figure 1 depicts the equilibrium price in Proposition I. The solid line is the price schedule arising in the presence of ambiguity, $\Delta\mu > 0$. The dashed line is the *benchmark price* in the Grossman and Stiglitz (1980) model. In the top panel, the proportion of informed agents is $\lambda = 0.2$, while in the bottom panel, $\lambda = 0.5$. In equilibrium, the uninformed agents' portfolio choice, as formalized in Eq. (2), reflects the expected returns in the worst case scenario: the uninformed agents buy when $s < \underline{s}$ (sell when $s > \bar{s}$), but less aggressively than they would do in the absence of ambiguity. Such a pessimistic behavior leads to a price lower (higher) than the benchmark for low (high) realizations of the compound signal, s. As the proportion of informed agents, λ, increases, the price impact of uninformed (and ambiguity averse) agents is reduced, and so is the extent of this "mispricing," as illustrated by Figure 1.

When the compound signal, s, lies within the range $[\underline{s}, \bar{s}]$, the uninformed agents do not participate in the market. Proposition I tells us that the non-participation region, $\bar{s} - \underline{s}$, is proportional to the size of the ambiguity in the market, $\Delta\mu$. The proportionality factor is $\frac{\lambda}{\tau\omega_\epsilon}$, and is interpreted as the total risk-bearing capacity of the informed agents, defined as the mass of informed agents, λ, times their trading aggressiveness, $\frac{1}{\tau\omega_\epsilon}$. As the informed risk-bearing capacity increases, prices move towards fundamentals. It now takes more extreme realizations of the compound signal, s, for prices to be favourable enough and induce uninformed agents to trade. Therefore, the non-participation region widens.

The non-participation region is proportional to $\Delta\mu$ for the following reasons. Consider the comparative statics of a change in $\underline{\mu}$ and $\bar{\mu}$. If $\underline{\mu}$ increases, $E^{\text{buy}}\left(f|\, P\left(\cdot,\cdot\right) = \underline{p}\right)$ increases as well, but then the threshold equilibrium price at which the agent does not buy the asset, $\underline{p}$, has to increase. This requires that $\underline{s}$ increase. A similar argument leads to the conclusion that as $\bar{\mu}$ decreases, $\bar{s}$ does necessarily have to decrease as well.

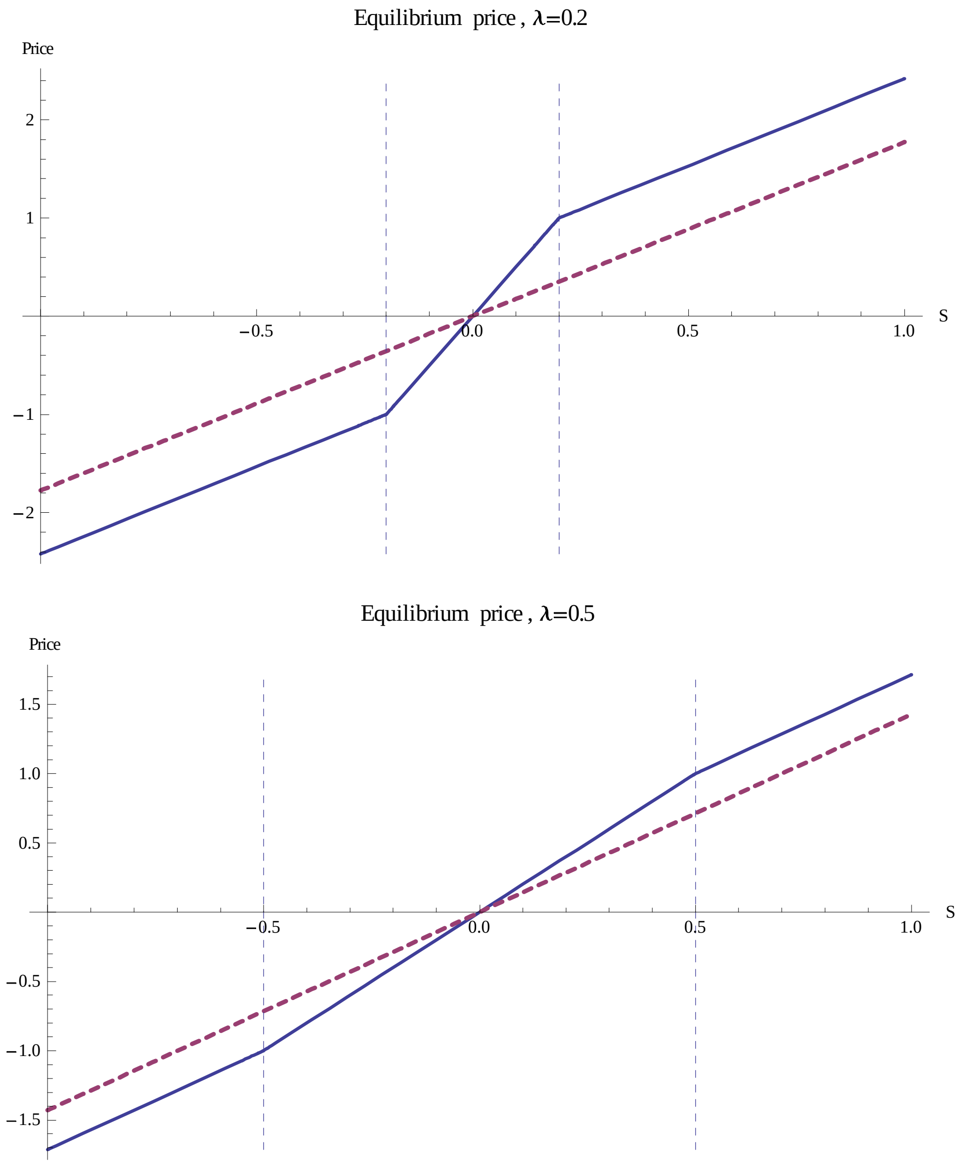

FIGURE 1. This picture depicts the equilibrium asset price in Proposition I, as a function of the compounded signal, s. Both panels compare the price function with the Grossman-Stiglitz linear function (the dashed line), which arises in the absence of ambiguity in the market, $\Delta\mu = 0$. Parameters values are $\Delta\mu = 2$, $\omega_\theta = \omega_\epsilon = \omega_z = \tau = 1$, and $\mu_z = 0$. In the top panel, the proportion of informed agents, $\lambda = 0.2$, and in the bottom panel, $\lambda = 0.5$.

3 Information acquisition

In this section we analyze how ambiguity affects the incentives to acquire fundamental information, and solve for the endogenous fraction of informed agents, λ. As in Grossman and Stiglitz (1980), all agents need to evaluate the ex-ante expected utilities, before deciding whether to become informed or not. However, the process of information acquisition differs from that in Grossman and Stiglitz, in that *all* agents are *ex-ante* ambiguity averse, which leads them to assess future events at the worst case scenarios.

3.1 Uninformed agents

The ex-ante expected utility of a would-be uninformed agent is:

$$\mathcal{U}_U(\lambda) = \min_{\mu} E_{\mu}\left[v_U\left(s\left(\theta, z\right)\right)\right], \tag{7}$$

where $v_U(s)$ is the interim utility for the uninformed agents, defined as

$$v_U(s) = -e^{-\tau \mathcal{C}_U(s)}, \quad \mathcal{C}_U(s) = \min_{\mu} E_{\mu}\left(W_U | s\right) - \frac{1}{2}\tau var\left(W_U | s\right).$$

By Eq. (4), the compound signal s is normally distributed, with mean $\mu_s(\mu)$ and variance ω_s, where,

$$\mu_s(\mu) \equiv \frac{\lambda}{\tau \omega_{\epsilon}}\mu.$$

In the remainder, $\Phi(\cdot; \mu, \omega)$ denotes the cumulative function of a normal variate with mean μ and variance ω. To alleviate the notation, we fix, $\Phi(\cdot) \equiv \Phi(\cdot; 0, 1)$. In the Appendix, we provide a closed-form expression for the unconditional expectation of the interim utility:

$$E_{\mu}\left[v_U(s)\right] = \int_{-\infty}^{\infty} v_U(s)\, d\Phi\left(s; \mu_s(\mu), \omega_s\right).$$

Figure 2 depicts the interim utility, $v_U(s)$, and the density function $d\Phi$ of the compound signal, s. The interim utility achieves its minimum in the non-participation region, where the interim certainty equivalent $\mathcal{C}_U(s)$ is flat at zero. Then, it is monotonically increasing, and symmetric, as the compound signal moves away from the non-participating thresholds $\underline{s}$ and $\bar{s}$. The next proposition provides the solution to the problem in Eq. (7):

Proposition II. *Let $\mu_z \geq 0$. Then, the ex-ante expected utility of the uninformed agents,*

$\mathcal{U}_U(\lambda,\mu)$, *is minimized at,*

$$\mu_U(\lambda)=\min\left\{\frac{\tau\omega_\epsilon}{\lambda}\frac{\omega_s}{\omega_z}\mu_z,\bar{\mu}\right\}.$$

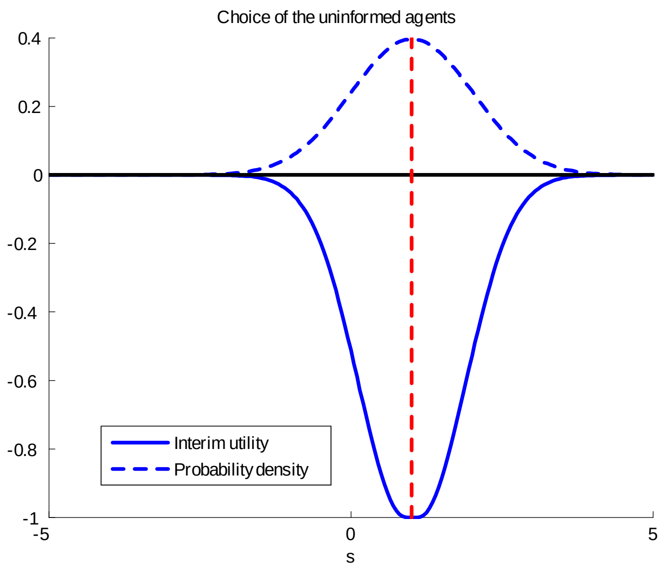

FIGURE 2. This picture depicts the identification and assessment of the worst case scenario made by uninformed agents. The worst case scenario occurs over the non-participation region, $[\underline{s},\bar{s}]$, where the interim utility attains its minimum. Accordingly, the interim utility is given the largest probability weight at $\hat{s}=\frac{1}{2}(\underline{s}+\bar{s})$. The vertical dashed line connects the probability density to the interim utility at the point $\hat{s}$. Parameters values are $\Delta\mu=2$, $\omega_\theta=\omega_\epsilon=\omega_z=\tau=1$, $\lambda=0.1$, and $\mu_z=1$. The resulting value of $\hat{s}$ is 1.01.

The economic mechanism underlying Proposition II is the following. The uninformed agents attach the largest probability to the occurrence of the worst events, and choose μ in such a way that the expected value of the signal, $\mu_s(\mu)$, is as close as possible to the midpoint in the non-participation region, $\hat{s}=\frac{1}{2}(\bar{s}+\underline{s})$. Naturally, $\mu_U(\lambda)$ is increasing in the average supply, μ_z: following an increase in μ_z, for markets to clear, the probability the uninformed agents enter as buyers (sellers) must increase (decrease) and as a result, the non-participation region shifts to the right.

3.2 Informed agents

The ex-ante expected utility for a would-be informed agent is,

$$\mathcal{U}_I(c,\lambda) = \min_{\mu} E_\mu \left[v_I(\theta, s(\theta, z)) \right], \tag{8}$$

where $v_I(\theta, s)$ is the interim utility for any informed agent, defined as

$$v_I(\theta, s) = -e^{-\tau(\mathcal{C}_I(\theta,s)-c)}, \quad \mathcal{C}_I(\theta, s) = \frac{1}{2}\frac{(\theta - P(s))^2}{\tau\omega_\epsilon},$$

and the equilibrium price, $P(s)$, is as in Eqs. (6) of Proposition I.

The Appendix provides a closed-form expression for the unconditional expectation of the interim utility,

$$E_\mu \left[v_I(\theta, s(\theta, z)) \right] = e^{\tau c} \sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}} \cdot E_\mu \left[\bar{v}_I(s; \mu) \right], \tag{9}$$

where $\bar{v}_I(s; \mu)$ is some negative function.

3.3 The indifference condition

An equilibrium with endogenous information acquisition is defined in the usual way, as the fraction of informed agents, $\lambda^* \in [0, 1]$, that makes any agent ex-ante indifferent whether to be informed or not, $\mathcal{U}_I(c, \lambda^*) = \mathcal{U}_U(\lambda^*)$, or,[1]

$$\frac{\mathcal{U}_I(c,\lambda^*)}{\mathcal{U}_U(\lambda^*)} = \underbrace{e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}}_{\text{Grossman-Stiglitz effect}} \cdot \underbrace{\frac{E_{\mu_I}\left[\bar{v}_I(s;\mu)\right]}{E_{\mu_U}\left[v_U(s)\right]}}_{\text{Ambiguity aversion effect}} = 1, \tag{10}$$

where μ_I and μ_U solve the two problems in Eqs. (7) and (8).

The left hand side of Eq. (10) is the value of information, evaluated at λ^*. It is the product of two terms. The first term is the usual value of information in the Grossman and Stiglitz (1980) model, the benchmark without ambiguity, $\Delta\mu = 0$. It summarizes the usual trade-off between the cost of acquiring information and its benefits, in terms of the informational advantage over the uninformed fringe. The effect of ambiguity on the incentives to acquire fundamental information is captured by the additional term in Eq. (10), which leads to what we label as the "ambiguity aversion effect." The next propostion explains how the value of information is affected by this ambiguity effect:

[1] Non-interior equilibria are also defined in the usual way, as $\lambda^* = 0$ such that $\mathcal{U}_I(c, 0) < \mathcal{U}_U(0)$ and $\lambda^* = 1$ such that $\mathcal{U}_I(c, 1) > \mathcal{U}_U(1)$.

Proposition III. *Information is more valuable in a market with ambiguous fundamentals ($\Delta\mu > 0$) than in a market without ambiguity* ($\Delta\mu = 0$).

The additional benefits of collecting fundamental information, due to the presence of ambiguous fundamentals, can be better understood by comparing the welfare of both types of agents to a benchmark without ambiguity. First, *for any* realization of the fundamentals, uninformed agents trade lower quantities than if there was no ambiguity (or if they were ambiguity neutral), as explained in Section 2. Therefore, by giving up investment opportunities, uninformed agents experience lower expected utility. Such a welfare reduction is actually reinforced from an ex-ante perspective: while assessing the outcomes arising from being uninformed at the trading stage, agents attach the largest probability weight to those future states in which participation is the lowest, as formalized in Proposition II and illustrated in Figure 2.

Second, informed investors benefit from the mispricing induced by the price impact of uninformed ambiguity-averse investors, as illustrated in Figure 1: they can buy at lower prices and sell at higher prices, thus making higher profits.

Since the value of information increases with ambiguity, we immediately obtain the following result on the amount of resources spent on collecting information:

Corollary 1. *Information is purchased by more agents in the presence of ambiguity than in the benchmark case without ambiguity.*

4 Complementarities in information acquisition: multiple equilibria and large price swings

Complementarities in information acquisition arise when the incentives to acquire information become stronger with the size of informed agents. This section analyzes conditions under which this situation is likely to occur, and their asset pricing implications.

4.1 Complementarities in information acquisition

The following proposition identifies sufficient conditions under which ambiguity leads to complementarities in the process of information acquisition:

Proposition IV. *Let $\Delta\mu > 0$. Then, there exists a level of the average supply $\bar{\mu}_z > 0$, such that there are complementarities in information acquisition for all $\mu_z > \bar{\mu}_z$.*

As the fraction of informed agents λ increases, there are two opposing forces on the incentives to acquire information. The first relates to the standard strategic substitutability effect, which is well-known since Grossman and Stiglitz (1980): more informed trading increases price efficiency, which reduces the informational advantage of the informed agents above the uninformed. This effect is still present in our model, as the first term in Eq. (10) is monotonically increasing in λ. Our analysis uncovers a second effect, specific to the presence of ambiguity and captured by the second term in Eq. (10): the volume of uninformed trading decreases with the mass of informed agents, which makes uninformed agents worse off ex-ante. Proposition IV shows that the ambiguity aversion effect may dominate the strategic substitutability effect, thereby generating strategic complementarities in the process of information acquisition.

The role the average asset supply, μ_z, plays in generating strategic complementarities is subtle. The informed agents' ex-ante utility is also decreasing in λ, because a reduction in the mass of uninformed agents reduces the extent of the mispricing informed agents benefit from (see Figure 1). This effect could counter-balance the net effect of an increase in λ on relative welfare, but it becomes less relevant for larger values of the asset supply. In this case, the expected gains arising out of mispricings are lower. Consider, for example, the case in which the market is only populated by uninformed investors, in which case the mispricing is the highest. If the average asset supply is sufficiently high, agents will be buyers most of the time. With uninformed investors holding the positive supply and being price setters, prices reflect low expected payoffs, $\underline{\mu}$, and therefore are particularly low. In this case, the worst case scenario for an agent considering, ex-ante, to become informed, is that payoffs are indeed low on average (i.e $\mu_I = \underline{\mu}$), so that the perceived mispricing (and the benefits from it) vanish. If the ex-ante perceived mispricing is low to start with, then, as λ increases, the shrinkage in the ex-ante utility of the informed investors is weak, compared to the loss in the ex-ante utility of the uninformed. As a result, the ambiguity aversion effect in Eq. (10) decreases with λ for μ_z large enough, inducing strategic complementarities.

4.2 Multiple equilibria

Information complementarities may lead to multiple equilibria, as illustrated in Figure 3. Intuitively, a sufficiently high value of the asset supply may lead to strategic complementarities, when the proportion of informed agents, λ, is low, as we explained in the extreme case in which $\lambda = 0$. However, when λ is high, the market is so efficient that strategic substitutability may dominate over the ambiguity effect in Eq. (10), as in Figure 3.

Figure 3 displays the value of information, as a function of λ, obtained for two different degrees of ambiguity, $\Delta\mu$.[2] The solid line, which corresponds to $\Delta\mu = 1$, leads to three equilibria. Two of these, $\lambda^* = \lambda_U$ and $\lambda^* = \lambda_S$, are interior equilibria: the leftmost equilibrium (λ_U) is unstable, and the rightmost (λ_S) is stable. The third, and stable, equilibrium is that with $\lambda^* = 0$. As $\Delta\mu$ increases, the value of information increases, for each λ, and shifts the leftmost (unstable) equilibria to the left, and the rightmost (stable) equilibria to the right. When $\Delta\mu$ is sufficiently high, there remains one equilibrium only, and stable. The dashed line in Figure 3, which corresponds to $\Delta\mu = 1.30$, depicts an example of such a situation.

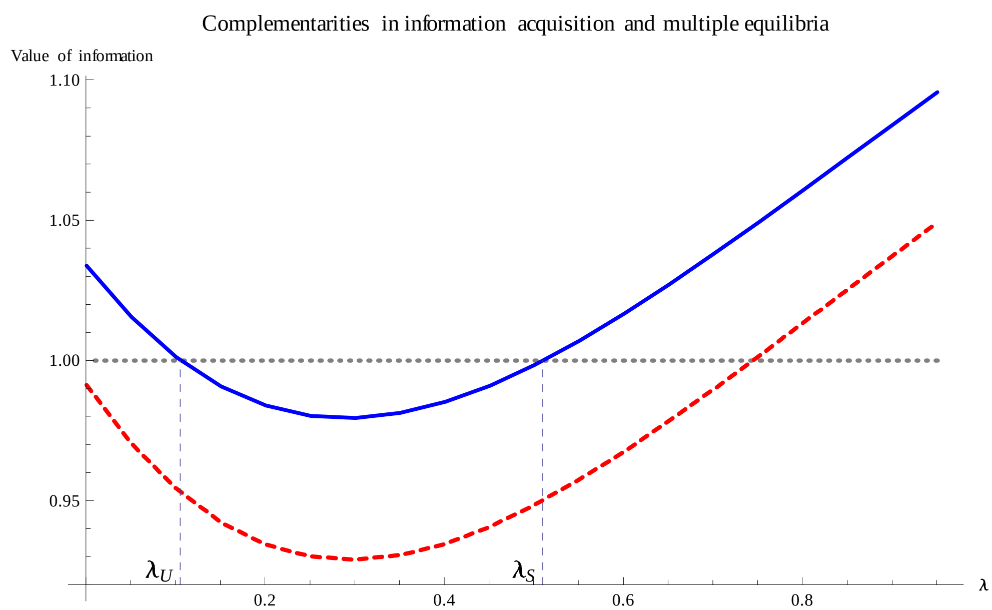

FIGURE 3. This picture depicts the value of information, $\frac{\mathcal{U}_I(c,\lambda)}{\mathcal{U}_U(\lambda)}$, as a function of the fraction of informed agents, λ, for a given cost of information, c. Parameters values are $\omega_\theta = \omega_\epsilon = \omega_z = \tau = \mu_z = 1$, and $c = 0.5$. The solid line is the value of information for $\Delta\mu = 1$, and the dashed line is the value of information for $\Delta\mu = 1.30$.

Figure 4 depicts the proportion of agents who acquire information, as a function of the size of ambiguity, $\Delta\mu$. We can interpret changes in $\Delta\mu$ as those that result in a repetition of a one-period economies. When ambiguity is low, say $\Delta\mu = 0.5$, the market is in its "media gloom" regime. If ambiguity increases, say to 1.30, the proportion of agents who become informed

[2]Note that due to negative exponential utility, lower values of the ratio in Eq. (10) mean higher values of information.

increases by a discrete change: from zero, to nearly 75%, a "media frenzy" regime. As $\Delta\mu$ decreases back to, say, 0.8, the market for information precipitates again. The model, then, generates path-dependence: for any size of ambiguity $\Delta\mu$ between the two vertical dashed lines, the number of informed agents can be either zero or positive, according to the previous values of $\Delta\mu$. Accordingly, given that the value of information increases with $\Delta\mu$, the jump size in the proportion of informed agents is larger when we head towards times of higher uncertainty than when we move back to times of decreased uncertainty.

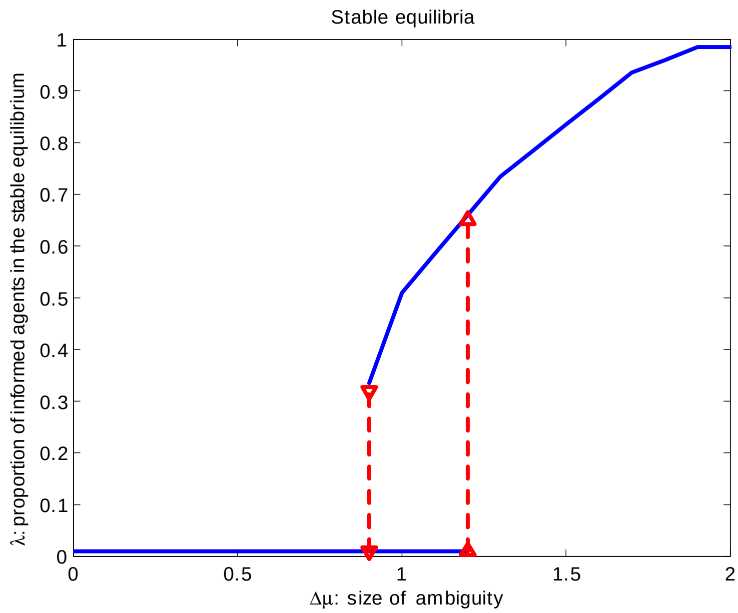

FIGURE 4. Media frenzies and glooms. Parameters values are $\omega_\theta = \omega_\epsilon = \omega_z = \tau = \mu_z = 1$, and $c = 0.5$.

4.3 Crashes and rebounds

Figure 5 depicts the equilibrium price as a function of the size of ambiguity. The left panel shows the unconditional expectation of the price, and the two panels on the right hand side depict the equilibrium price when $\theta = 0$, and $s = -(z - \mu_z) = -3$ (right, top) and $s = -(z - \mu_z) = 3$ (right, bottom). Therefore, the right top panel corresponds to a negative liquidity shock, and the right top to a situation of a positive liquidity shock. In the presence of a negative liquidity shock, when the size of ambiguity gets sufficiently large, the benefits to become informed increase so much that a positive fraction of agents decide to acquire information (just as in the unconditional case), and a price rebound obtains. If ambiguity lowers, the benefits to being informed decrease, the market for information dries up, and all the agents stay uninformed, generating a crash.

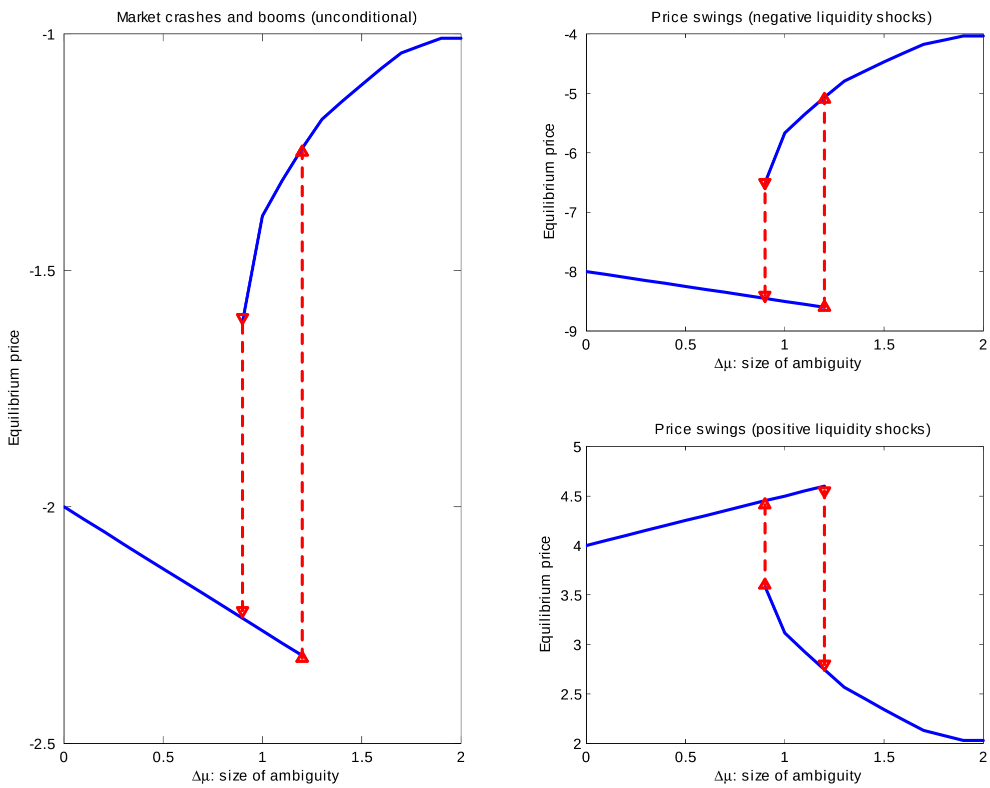

FIGURE 5. Market crashes and booms. Parameters values are $\omega_\theta = \omega_\epsilon = \omega_z = \tau = \mu_z = 1$, and $c = 0.5$. Bad times, $s = -3$. Good times, $s = 3$.

In the presence of a positive liquidity shock, when $\Delta\mu$ is sufficiently small, the market for information dries up, thus making the price totally uninformative. As the size of ambiguity gets sufficiently large, a positive fraction of agents then decide to acquire information, which leads to a crash. If $\Delta\mu$ gets small, the market for information dries up again, entailing a price rebound.

References

Anderson, E., E. Ghysels and J. Juergens (2007): "The Impact of Risk and Uncertainty on Expected Returns." *Journal of Financial Economics*, forthcoming.

Barlevy, G., and P. Veronesi (2000): "Information Acquisition in Financial Markets." *Review of Economic Studies* 67, 79-90. [Corrigendum, 2008, *Review of Economic Studies*]

Bossaert, P., P. Ghirardato, S. Guarneschelli and W. Zame (2007): "Ambiguity in Asset Markets: Theory and Experiment." Working paper, Collegio Carlo Alberto, Turin.

Caballero,R. J., and A. Krishnamurthy (2008): "Collective Risk Management in a Flight to Quality Episode." *Journal of Finance* 63, 2195-2230.

Cao, H., T. Wang, and H. Zhang (2005): "Model Uncertainty, Limited Market Participation and Asset Prices." *Review of Financial Studies* 18, 1219-51.

Caskey, J. (2008): "Information in Equity Markets with Ambiguity-Averse Investors." *Review of Financial Studies*, forthcoming.

Chamley, C. (2008): "On Acquisition of Information in Financial Markets." *Review of Economic Studies*, forthcoming.

Easley, D., and M. O'Hara (2007): "Ambiguity and Non-Participation: the Role of Regulation." *Review of Financial Studies*, forthcoming.

Ellsberg, D. (1961): "Risk, Ambiguity and the Savage Axioms." *Quarterly Journal of Economics* 75, 643-69.

Epstein, L., and M. Schneider (2008): "Ambiguity, Information Quality and Asset Pricing." *Journal of Finance* 63, 197-228.

Epstein, L., and T. Wang (1994): "Intertemporal Asset Pricing under Knightian Uncertainty." *Econometrica* 62, 83-322.

Epstein, L., and T. Wang (1995): "Uncertainty, Risk-Neutral Measures and Security Price Booms and Crashes," *Journal of Economic Theory* 67, 40-82.

Froot, K. A., D. S. Scharfstein, and J. C. Stein (1992): "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation." *Journal of Finance* 47, 1461-1484.

Gagliardini, P., P. Porchia and F. Trojani (2008): "Ambiguity Aversion and the Term Structure of Interest Rates." *Review of Financial Studies*, forthcoming.

García D. and G. Strobl (2008): "Relative Wealth Concerns and Complementarities in Information Acquisition." Working paper, University of North Carolina at Chapel Hill.

Gilboa, I. and D. Schmeidler (1989): "Maxmin Expected Utility with a Non-Unique Prior." *Journal of Mathematical Economics* 18, 141-153.

Gollier, C. (2005): "Does Ambiguity Aversion Reinforce Risk Aversion? Applications to Portfolio Choices and Asset Prices." Working Paper, University of Toulouse.

Grossman, S. J. and J. E. Stiglitz (1980): "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70, 393-408.

Hansen, L. P. and T. J. Sargent (2008): *Robustness.* Princeton NJ: Princeton University Press.

Hellwig, C. and L. Veldkamp (2008): "Knowing What Others Know: Coordination Motives in Information Acquisition." *Review of Economic Studies*, forthcoming.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005): "A Smooth Model of Decision Making under Ambiguity." *Econometrica* 73, 1849-92.

Knight, F. (1921): *Risk, Uncertainty and Profit.* Boston, MA: Houghton Mifflin.

Leippold, M., F. Trojani, and P. Vanini (2007): "Learning and Asset Pricing under Uncertainty." *Review of Financial Studies*, forthcoming.

Veldkamp, L. (2006): "Media Frenzies in Markets for Financial Information," *American Economic Review* 96, 577-601.

Appendix

Proof of Proposition I. By the market clearing condition, Eq. (5), the equilibrium price arising when the uninformed agents do not participate is:

$$P(s) = -\frac{\tau\omega_\epsilon}{\lambda}\mu_z + \frac{\tau\omega_\epsilon}{\lambda}s,$$

which is the second line in Eqs. (6). Next, we compute the uninformed agents' expectation of the asset payoff, in the states of nature in which these agents participate. Using $\omega_s = \left(\frac{\lambda}{\omega_\epsilon\tau}\right)^2 \omega_\theta + \omega_z$, straight forward computations leave:

$$E^{\text{buy}}\left(f \mid S = s\right) = \frac{\tau^2\omega_\epsilon^2\omega_z}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}\underline{\mu} + \frac{\lambda\tau\omega_\epsilon\omega_\theta}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}s \tag{A1}$$

$$E^{\text{sell}}\left(f \mid S = s\right) = \frac{\tau^2\omega_\epsilon^2\omega_z}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}\bar{\mu} + \frac{\lambda\tau\omega_\epsilon\omega_\theta}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}s \tag{A2}$$

Next, we plug Eqs. (A1)-(A2) into the demand schedule, Eq. (2), replace the result into the market clearing condition, Eq. (5), conjecture the piece-wise linear price function in Eqs. (6), and solve for undetermined coefficients, obtaining,

$$\begin{aligned}
\bar{a} &= \underline{a} + \frac{\Delta\mu\,(1-\lambda)\,\tau^2\omega_z\omega_\epsilon^2}{\lambda^2\omega_\theta + \lambda\tau^2\omega_\theta\omega_z\omega_\epsilon + \tau^2\omega_z\omega_\epsilon^2} \\
a &= -\frac{\tau\omega_\epsilon}{\lambda}\mu_z \\
\underline{a} &= \frac{-\lambda^2\mu_z\tau\omega_\epsilon\omega_\theta + \left(\underline{\mu}\,(1-\lambda)\,\omega_\epsilon - \mu_z\tau\omega_\epsilon\left(\omega_\epsilon + \omega_\theta\right)\right)\tau^2\omega_z\omega_\epsilon}{\lambda^2\omega_\theta + \lambda\tau^2\omega_\theta\omega_z\omega_\epsilon + \tau^2\omega_z\omega_\epsilon^2} \\
b &= \frac{\left(\lambda\omega_\theta + \omega_z\tau^2\omega_\epsilon\left(\omega_\theta + \omega_\epsilon\right)\right)\tau\omega_\epsilon}{\lambda^2\omega_\theta + \lambda\tau^2\omega_\theta\omega_z\omega_\epsilon + \tau^2\omega_z\omega_\epsilon^2}
\end{aligned}$$

Finally, we determine the threshold for the compound signal, $\underline{s}$ and $\bar{s}$. We use the cutoff conditions in Eq. (3). As for $\underline{s}$, consider the first equation, $E^{\text{buy}}\left(f \mid P(\cdot,\cdot) = \underline{p}\right) = \underline{p}$. For $s \leq \underline{s}$, the conjectured price function is linear in s. Therefore, we solve for $\underline{p}$ by equivalently solving for $\underline{s}$ in the following condition,

$$E^{\text{buy}}\left(f \mid S = \underline{s}\right) = \underline{p} = \underline{a} + b\underline{s},$$

where $E^{\text{buy}}\left(f \mid S = \underline{s}\right)$ is given by Eq. (A1), and the third equality holds by the first line of the conjectured price function in Eqs. (6). We do the same to determine $\bar{s}$, by solving,

$$E^{\text{sell}}\left(f \mid S = \bar{s}\right) = \bar{p} = \bar{a} + b\bar{s},$$

where $E^{\text{sell}}\left(f \mid S = \bar{s}\right)$ is given by Eq. (A2). The expressions for $\underline{s}$ and $\bar{s}$ given in Proposition I then follow by simple computations. Finally, we need to compute the threshold prices $\bar{p}$ and $\underline{p}$. We plug Eqs. (A1)-(A2) into Eq. (3), use the price function in Eqs. (6), and obtain,

$$\bar{p} = \bar{\mu} + \frac{\lambda\omega_\theta}{\tau\omega_\epsilon\omega_z}\mu_z, \quad \underline{p} = \underline{\mu} + \frac{\lambda\omega_\theta}{\tau\omega_\epsilon\omega_z}\mu_z.$$

The previous expressions confirm that $\underline{p} < \bar{p}$. ■

Derivation of the utilities for the would-be uninformed and informed agents.

Would-be uninformed agents. By Eqs. (A1)-(A2), we have,

$$x_U(s) = \begin{cases} \dfrac{E^{\text{buy}}(f|s) - P(s)}{\tau\omega_{f|s}} = \dfrac{\delta}{\tau\omega_{f|s}}(\underline{s} - s), & \text{for } s < \underline{s} \\ 0, & \text{for } s \in [\underline{s}, \bar{s}] \\ \dfrac{E^{\text{sell}}(f|s) - P(s)}{\tau\omega_{f|s}} = \dfrac{\delta}{\tau\omega_{f|s}}(\bar{s} - s), & \text{for } s > \bar{s} \end{cases}$$

where $\omega_{f|s}$ is the variance of f conditional on s, $P(s)$ is the equilibrium price in Eqs. (6) of Proposition I, and:

$$\omega_{f|s} = \omega_\epsilon + \frac{\omega_z\omega_\theta}{\omega_s}, \quad \delta = \frac{\tau^3\omega_z\omega_\epsilon^3\left(\lambda^2\omega_\theta + \tau^2\omega_z\omega_\epsilon^2 + \tau^2\omega_z\omega_\theta\omega_\epsilon\right)}{\left(\lambda^2\omega_\theta + \lambda\tau^2\omega_z\omega_\theta\omega_\epsilon + \tau^2\omega_z\omega_\epsilon^2\right)\left(\lambda^2\omega_\theta + \tau^2\omega_z\omega_\epsilon^2\right)}. \tag{A3}$$

Accordingly, the interim utility is,

$$v_U(s) = -e^{-\tau\mathcal{C}_U(s)} = \begin{cases} -\exp\left(-\dfrac{1}{2}\dfrac{\delta^2}{\omega_{f|s}}(s - \underline{s})^2\right), & \text{for } s < \underline{s} \\ -1, & \text{for } s \in [\underline{s}, \bar{s}] \\ -\exp\left(-\dfrac{1}{2}\dfrac{\delta^2}{\omega_{f|s}}(s - \bar{s})^2\right), & \text{for } s > \bar{s} \end{cases} \tag{A4}$$

Integrating over the distribution of the compound signal, s, leaves

$$E_\mu[v_U(s)] = -\int_{-\infty}^{\infty} e^{-\tau\mathcal{C}_U(s)} d\Phi(s; \mu_s(\mu), \omega_s) \equiv \sum_{\ell\in\{\text{buy,np,sell}\}} J_\mu^\ell,$$

where

$$\begin{aligned} J_\mu^{\text{buy}} &= -\int_{-\infty}^{\underline{s}} e^{-\tau\mathcal{C}_U(s)} d\Phi(s; \mu_s(\mu), \omega_s) \\ J_\mu^{\text{np}} &= -\int_{\underline{s}}^{\bar{s}} d\Phi(s; \mu_s(\mu), \omega_s) \\ J_\mu^{\text{sell}} &= -\int_{\bar{s}}^{\infty} e^{-\tau\mathcal{C}_U(s)} d\Phi(s; \mu_s(\mu), \omega_s) \end{aligned}$$

A direct computation of these integrals yields,

$$\begin{aligned}
J_\mu^{\text{buy}} &= -\kappa \exp\left(-\frac{\delta^2 \left(\underline{s}-\mu_s\left(\mu\right)\right)^2}{2\left(\omega_{f|s}+\delta^2\omega_s\right)}\right)\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\underline{s}-\mu_s\left(\mu\right)\right)\right)\\
J_\mu^{\text{np}} &= -\left[\Phi\left(\frac{\bar{s}-\mu_s\left(\mu\right)}{\sqrt{\omega_s}}\right)-\Phi\left(\frac{\underline{s}-\mu_s\left(\mu\right)}{\sqrt{\omega_s}}\right)\right]\\
J_\mu^{\text{sell}} &= -\kappa \exp\left(-\frac{\delta^2 \left(\bar{s}-\mu_s\left(\mu\right)\right)^2}{2\left(\omega_{f|s}+\delta^2\omega_s\right)}\right)\left[1-\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\bar{s}-\mu_s\left(\mu\right)\right)\right)\right],
\end{aligned}$$

where

$$\kappa = \sqrt{\frac{\omega_{f|s}}{\omega_{f|s}+\delta^2\omega_s}}.$$

■

Would-be informed agents. Let $\mu_{\theta|s}(\mu)$ and $\omega_{\theta|s}$ denote the conditional expectation and variance of θ given s, which are easily shown to be:

$$\mu_{\theta|s}\left(s;\mu\right) = \frac{\tau^2\omega_\epsilon^2\omega_z}{\lambda^2\omega_\theta+\tau^2\omega_\epsilon^2\omega_z}\mu + \frac{\lambda\tau\omega_\epsilon\omega_\theta}{\lambda^2\omega_\theta+\tau^2\omega_\epsilon^2\omega_z}s, \qquad \omega_{\theta|s} = \frac{\omega_z\omega_\theta}{\omega_s}.$$

We have,

$$E_\mu\left[v_I\left(\theta, s\left(\theta,z\right)\right)\right] = e^{\tau c}\int_{-\infty}^{\infty} E_\mu\left[\left.v_I\left(\theta,s\right)\right|s\right]d\Phi\left(s;\mu_s\left(\mu\right),\omega_s\right) \tag{A5}$$

where,

$$E_\mu\left[\left.v_I\left(\theta,s\right)\right|s\right] = e^{\tau c}\int_{-\infty}^{\infty} v_I\left(\theta,s\right)d\Phi(\theta;\mu_{\theta|s}\left(s;\mu\right),\omega_{\theta|s}).$$

Computing the integrals yields,

$$E_\mu\left[\left.v_I\left(\theta,s\right)\right|s\right] = -e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}\exp\left(-\frac{1}{2}\frac{\left(\mu_{\theta|s}\left(s;\mu\right)-P\left(s\right)\right)^2}{\omega_{f|s}}\right), \tag{A6}$$

where $P(s)$ is the equilibrium price in Eqs. (6) of Proposition I. We have,

$$\mu_{\theta|s}\left(s;\mu\right)-P\left(s\right) = \frac{\omega_z}{\omega_s}\left(\mu-\underline{\mu}\right)+\delta\left(s-\underline{s}\right), \tag{A7}$$

where δ is as in Eq. (A3). Substituting Eq. (A7) into Eq. (A6) leaves,

$$E_\mu\left[\left.v_I\left(\theta,s\right)\right|s\right] = e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}\bar{v}_I\left(s;\mu\right) \tag{A8}$$

where, for $\hat{s} = \frac{1}{2}(\underline{s} + \bar{s})$,

$$\bar{v}_I(s;\mu) = \begin{cases} -\exp\left(-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(s - \underline{s} - \frac{\hat{\delta}}{\delta}\frac{\lambda}{\tau\omega_\epsilon}(\mu - \underline{\mu})\right)^2\right), & \text{for } s < \underline{s} \\ -\exp\left(-\frac{1}{2}\frac{\hat{\delta}^2}{\omega_{f|s}}\left(s - \hat{s} - \frac{\lambda}{\tau\omega_\epsilon}\mu\right)^2\right), & \text{for } s \in [\underline{s}, \bar{s}] \\ -\exp\left(-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(s - \bar{s} + \frac{\hat{\delta}}{\delta}\frac{\lambda}{\tau\omega_\epsilon}(\bar{\mu} - \mu)\right)^2\right), & \text{for } s > \bar{s} \end{cases} \tag{A9}$$

and

$$\hat{\delta} = \frac{\tau\omega_\epsilon\omega_z}{\lambda\omega_s}.$$

Finally, substituting Eq. (A8) into Eq. (A5), and integrating, leaves Eq. (9) in the main text, with

$$E_\mu[\bar{v}_I(s;\mu)] = \sum_{\ell\in\{\text{buy},\text{np},\text{sell}\}} I_\mu^\ell,$$

where

$$I_\mu^{\text{buy}} = -\kappa\exp\left(-\frac{\delta^2\left(\frac{\omega_s}{\omega_z}\mu_z + \gamma_0(\mu - \underline{\mu})\right)^2}{2\left(\omega_{f|s} + \delta^2\omega_s\right)}\right)\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z - \gamma_1(\mu - \underline{\mu})\right)\right)$$

$$I_\mu^{\text{np}} = -\hat{\kappa}\exp\left(-\frac{\hat{\delta}^2(\frac{\omega_s}{\omega_z}\mu_z)^2}{2(\omega_{f|s} + \hat{\delta}^2\omega_s)}\right)\left[\Phi\left(\frac{\hat{\kappa}}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z + \gamma_2(\bar{\mu} - \mu)\right)\right) - \Phi\left(\frac{\hat{\kappa}}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z - \gamma_2(\mu - \underline{\mu})\right)\right)\right]$$

$$I_\mu^{\text{sell}} = -\kappa\exp\left(-\frac{\delta^2\left(\frac{\omega_s}{\omega_z}\mu_z - \gamma_0(\bar{\mu} - \mu)\right)^2}{2\left(\omega_{f|s} + \delta^2\omega_s\right)}\right)\left[1 - \Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z + \gamma_1(\bar{\mu} - \mu)\right)\right)\right]$$

and:

$$\hat{\kappa} = \sqrt{\frac{\omega_{f|s}}{\omega_{f|s} + \hat{\delta}^2\omega_s}}, \quad \gamma_0 = \left(\frac{\hat{\delta}}{\delta} - 1\right)\frac{\lambda}{\tau\omega_\epsilon}, \quad \gamma_1 = \left(1 + \delta\frac{\hat{\delta}\omega_s}{\omega_{f|s}}\right)\frac{\lambda}{\tau\omega_\epsilon}, \quad \gamma_2 = \left(1 + \hat{\delta}\frac{\hat{\delta}\omega_s}{\omega_{f|s}}\right)\frac{\lambda}{\tau\omega_\epsilon}.$$

■

Proof of Proposition II. We claim that $\mu_s = \hat{s} \equiv \frac{1}{2}(\underline{s} + \bar{s})$, or equivalently, that for all $\epsilon > 0$,

$$-\Delta_\epsilon\mathcal{U}_U \equiv -\int_{-\infty}^{\infty} v_U(s)\,\Delta_\epsilon\varphi(s)\,ds > 0, \tag{A10}$$

where $\Delta_\epsilon\varphi(s) \equiv \varphi(s;\hat{s},\omega_s) - \varphi(s;\hat{s}+\epsilon,\omega_s)$, and $\varphi(\cdot,\mu,\sigma^2)$ denotes the Normal density function, with mean μ and variance σ^2. Note that the function $v_U(s)$ is symmetric about $\hat{s}$, so that Proposition II follows once, we show that the inequality in (A10) holds true for each $\epsilon > 0$.

We have:

$$\Delta_\epsilon \varphi(s) = \begin{cases} f\left(s - \hat{s} - \frac{1}{2}\epsilon\right) \geq 0 & \text{for all } s \in \left(-\infty, \hat{s} + \frac{1}{2}\epsilon\right] \\ -f\left(\hat{s} + \frac{1}{2}\epsilon - s\right) \leq 0 & \text{for all } s \in \left[\hat{s} + \frac{1}{2}\epsilon, \infty\right) \end{cases}$$

where we have defined:

$$f(x) \equiv \frac{1}{\sqrt{2\pi\omega_s}} \left[e^{-\frac{1}{2\omega_s}\left(x + \frac{1}{2}\epsilon\right)^2} - e^{-\frac{1}{2\omega_s}\left(x - \frac{1}{2}\epsilon\right)^2} \right].$$

Next, define the two functions,

$$h_1(s) \equiv \begin{cases} e^{-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}(\underline{s} - s)^2} & \text{for all } s \in (-\infty, \underline{s}] \\ 1 & \text{for all } s \in \left[\underline{s}, \hat{s} + \frac{1}{2}\epsilon\right) \end{cases}$$

and

$$h_2(s) \equiv \begin{cases} 1 & \text{for all } s \in \left[\hat{s} + \frac{1}{2}\epsilon, \bar{s}\right) \\ e^{-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}(s - \bar{s})^2} & \text{for all } s \in (\bar{s}, \infty] \end{cases}$$

In terms of h_1 and h_2, we have, $-v_U(s) = h_1(s)\,\mathbb{I}_{\left\{s \leq \hat{s} + \frac{1}{2}\epsilon\right\}} + h_2(s)\,\mathbb{I}_{\left\{s \geq \hat{s} + \frac{1}{2}\epsilon\right\}}$, where $\mathbb{I}_{\{\cdot\}}$ denotes the indicator function, and the expression for $\Delta_\epsilon \mathcal{U}_U$ in (A10) is,

$$\begin{aligned} -\Delta_\epsilon \mathcal{U}_U &= \int_{-\infty}^{\hat{s} + \frac{1}{2}\epsilon} h_1(s) f\left(s - \hat{s} - \frac{1}{2}\epsilon\right) ds - \int_{\hat{s} + \frac{1}{2}\epsilon}^{\infty} h_2(s) f\left(\hat{s} + \frac{1}{2}\epsilon - s\right) ds \\ &= \int_{-\infty}^{\hat{s} + \frac{1}{2}\epsilon} h_1(s) f\left(s - \hat{s} - \frac{1}{2}\epsilon\right) ds - \int_{-\infty}^{\hat{s} - \frac{1}{2}\epsilon} h_1(s) f\left(s - \hat{s} - \frac{1}{2}\epsilon\right) ds \\ &> 0, \end{aligned}$$

where the second equality follows by the symmetry of $v_U(s)$ about $\hat{s}$. ■

Proof of Proposition III. Consider the indifference condition in Eq. (10). We wish to show that for $\Delta\mu > 0$,

$$\frac{\mathcal{U}_I(c, \lambda)}{\mathcal{U}_U(\lambda)} < e^{\tau c} \sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}.$$

Because $E_{\mu_I}[\bar{v}_I(s; \mu)]$ and $E_{\mu_U}[v_U(s)]$ are both strictly negative, the previous inequality holds true if:

$$E_{\mu_I}[\bar{v}_I(s; \mu)] > E_{\mu_U}[v_U(s)], \tag{A11}$$

where we define, as in the main text:

$$\mu_I \in \arg\min_\mu E_\mu[\bar{v}_I(s; \mu)], \quad \mu_U \in \arg\min_\mu E_\mu[v_U(s)].$$

To show that (A11) is true, suppose the contrary, i.e. that:

$$E_{\mu_I}[\bar{v}_I(s; \mu)] \leq E_{\mu_U}[v_U(s)]. \tag{A12}$$

By direct comparison of Eq. (A4) and Eq. (A9), we have that $0 > \bar{v}_I(s, \mu) \geq v_U(s)$ for all $\mu \in [\underline{\mu}, \bar{\mu}]$, and $s \in \mathbb{R}$, the second inequality being strict on some open set in $\mathbb{R}$. As a consequence, we must have the inequality, $E_{\mu_I}[\bar{v}_I(s; \mu)] > E_{\mu_I}[v_U(s)]$ which, combined with (A12), yields,

$$E_{\mu_I}[v_U(s)] < E_{\mu_I}[\bar{v}_I(s; \mu)] \leq E_{\mu_U}[v_U(s)],$$

contradicting that μ_U minimizes $E_\mu [v_U (s)]$. ■

Proof of Corollary 1. Let $\lambda^*(\Delta\mu)$ solve the indifference condition:

$$\frac{\mathcal{U}_I (c, \lambda)}{\mathcal{U}_U (\lambda)} = 1.$$

Assume now that $\lambda^*(0) \geq \lambda^*(\Delta\mu)$, for some $\Delta\mu > 0$. By Proposition III, this can not be the case as we would have

$$\frac{\mathcal{U}_I (c, \lambda^*(\Delta\mu))}{\mathcal{U}_U (\lambda^*(\Delta\mu))} < 1.$$

■

Proof of Proposition IV. We wish to show that

$$\frac{\mathcal{U}_I (c, 0)}{\mathcal{U}_U (0)} > \frac{\mathcal{U}_I (c, 1)}{\mathcal{U}_U (1)},$$

or

$$\frac{\mathcal{I}_0}{\mathcal{J}_0} \frac{\mathcal{J}_1}{\mathcal{I}_1} > \sqrt{\frac{\omega_{\ f|s,\lambda=0}}{\omega_{\ f|s,\lambda=1}}}, \tag{A13}$$

where $\omega_{\ f|s,\lambda=0} = \lim_{\lambda\to 0} \omega_{\ f|s}$, $\omega_{\ f|s,\lambda=1} = \lim_{\lambda\to 1} \omega_{\ f|s}$, and,

$$\mathcal{I}_{\lambda^*} = \lim_{\lambda\to\lambda^*} \sum_{\ell\in\{\text{buy},\text{np},\text{sell}\}} I^\ell_{\mu_I}, \quad \mathcal{J}_{\lambda^*} = \lim_{\lambda\to\lambda^*} \sum_{\ell\in\{\text{buy},\text{np},\text{sell}\}} J^\ell_{\mu_U}, \quad \lambda^* \in \{0, 1\}.$$

We now proceed with determining $\mathcal{I}_{\lambda^*}$ and $\mathcal{J}_{\lambda^*}$ for $\lambda^* \in \{0, 1\}$, and for μ_z sufficiently large. Then, we shall prove that (A13) holds true for μ_z sufficiently large. We shall need the results recorded in the next two lemmas.

Lemma 1. *There exists a $\hat{\mu}_z > 0$ such that for all $\mu_z \geq \hat{\mu}_z$, we have that* $\arg\min_\mu \left(I^{\text{buy}}_\mu + I^{\text{sell}}_\mu\right) = \underline{\mu}$.

Proof. We have,

$$I^{\text{buy}}_\mu = -c_0 \exp\left(-\frac{\left(\tau\omega_f\mu_z + \left(\mu - \underline{\mu}\right)\right)^2}{2\left(\omega_f + \tau^2\omega_f^2\omega_s\right)}\right) \Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z - \tau\omega_z\left(\mu - \underline{\mu}\right)\right)\right)$$

$$I^{\text{sell}}_\mu = -c_0 \exp\left(-\frac{\left(\tau\omega_f\mu_z - (\bar{\mu} - \mu)\right)^2}{2\left(\omega_f + \tau^2\omega_f^2\omega_s\right)}\right) \left[1 - \Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z + \tau\omega_z\left(\bar{\mu} - \mu\right)\right)\right)\right]$$

where $c_0 = \left(1 + \tau^2\omega_f\omega_s\right)^{-\frac{1}{2}}$. It is easy to show that $\mu \mapsto I^{\text{buy}}_\mu$ is increasing. We are left to show that with μ_z sufficiently large, we have that $\mu \mapsto I_\mu^{\text{sell}}$ is increasing as well. We have,

$$
\begin{aligned}
c_0^{-1}\frac{\partial}{\partial\mu} I_\mu^{\text{sell}} &= -\frac{\partial}{\partial\mu}\exp\left(-\frac{\left(\tau\omega_f\mu_z-(\bar{\mu}-\mu)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right)\left[1-\Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z+\tau\omega_z\left(\bar{\mu}-\mu\right)\right)\right)\right] \\
&\quad+\exp\left(-\frac{\left(\tau\omega_f\mu_z-(\bar{\mu}-\mu)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right)\frac{\partial}{\partial\mu}\left[1-\Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z+\tau\omega_z\left(\bar{\mu}-\mu\right)\right)\right)\right] \\
&= \exp\left(-\frac{\left(\tau\omega_f\mu_z-(\bar{\mu}-\mu)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right) \\
&\quad\times\left(\frac{\tau\omega_f\mu_z-(\bar{\mu}-\mu)}{\omega_f+\tau^2\omega_f^2\omega_s}+\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z+\tau\omega_z\left(\bar{\mu}-\mu\right)\right)\right)^2\right)\frac{c_0}{\sqrt{\omega_z}}\tau\omega_z\right).
\end{aligned}
$$

This expression is positive for all $\mu_z : \tau\omega_f\mu_z-(\bar{\mu}-\mu)\geq 0$, which it does when $\mu_z \geq \frac{\Delta\mu}{\tau\omega_f}$. ■

Lemma 2. *There exists a* $\bar{\mu}_z > 0$ *such that for all* $\mu_z \geq \bar{\mu}_z$ *we have that* $\arg\min_\mu\left(J_\mu^{\text{buy}}+J_\mu^{\text{np}}+J_\mu^{\text{sell}}\right)=\bar{\mu}$.

Proof. Follows directly by Proposition II, once we set $\bar{\mu}_z=\frac{\lambda\omega_z}{\tau\omega_\epsilon\omega_s}\bar{\mu}$. ■

We are now ready to compute $\mathcal{I}_0$, $\mathcal{J}_0$, $\mathcal{I}_1$ and $\mathcal{J}_1$.

- As for $\mathcal{I}_0$, note that, clearly, $\mathcal{I}_0=\min_\mu(I_\mu^{\text{buy}}+I_\mu^{\text{sell}})$. Therefore, by Lemma 1, and a simple computation, we have that for all $\mu_z\geq\hat{\mu}_z$, and with $\hat{\mu}_z$ as in the proof of Lemma 1,

$$
\begin{aligned}
\mathcal{I}_0 &= I_{\underline{\mu}}^{\text{buy}}+I_{\underline{\mu}}^{\text{sell}} \\
&= -c_0\left[\exp\left(-\frac{\mu_z^2\tau^2\omega_f c_0^2}{2}\right)\Phi\left(\frac{\mu_z}{\sqrt{\omega_z}}c_0\right)+\exp\left(-\frac{\left(\Delta\mu-\mu_z\tau\omega_f\right)^2c_0^2}{2\omega_f}\right)\Phi\left(-\frac{\mu_z+\Delta\mu\tau\omega_z}{\sqrt{\omega_z}}c_0\right)\right],
\end{aligned}
$$

  where the second equality follows by a simple computation.

- As for $\mathcal{J}_0$ and $\mathcal{I}_1$, it is easily seen that $\mathcal{J}_0$ and $\mathcal{I}_1$ are independent of μ. They are,

$$
\mathcal{J}_0=-c_0\exp\left(-\frac{\mu_z^2\tau^2\omega_f c_0^2}{2}\right),\quad \mathcal{I}_1=-c_1\sqrt{\frac{\omega_\theta+\tau^2\omega_z\omega_\epsilon\omega_f}{c_2}}\exp\left(-\frac{1}{2}\tau^2\omega_\epsilon c_1^2\mu_z^2\right),
$$

  where $\left(1+\tau^2\omega_z\omega_\epsilon\right)^{-\frac{1}{2}}$ and $c_2=\tau^2\omega_z\omega_\epsilon^2+\omega_\theta$.

- As for $\mathcal{J}_1$ we have, by Lemma 2 and a direct computation, that for all $\mu_z\geq\bar{\mu}_z$,

$$
\begin{aligned}
\mathcal{J}_1 &= -c_1\sqrt{\frac{\omega_\theta+\tau^2\omega_z\omega_\epsilon\omega_f}{c_2}} \\
&\quad\times\left[\exp\left(-\frac{\tau^2\omega_\epsilon c_1^2}{2}\frac{\left(\Delta\mu\tau\omega_z\omega_\epsilon-\mu_z c_2\right)^2}{c_2^2}\right)\Phi\left(\rho_0\mu_z+\rho_1\right)+\exp\left(-\frac{\tau^2\omega_\epsilon c_1^2}{2}\mu_z^2\right)\Phi\left(-\rho_0\mu_z\right)\right] \\
&\quad-\left[\Phi\left(\rho_2\mu_z+\rho_3\right)-\Phi\left(\rho_2\mu_z+\rho_4\right)\right], \qquad \text{(A14)}
\end{aligned}
$$

for some constants $\rho_0 > 0$, $\rho_2 > 0$, ρ_1, ρ_3, ρ_4 independent of μ_z.

We are now ready to show that the inequality in (A13) holds true. Consider the following ratios:

$$\begin{aligned}
\frac{\mathcal{I}_0}{\mathcal{J}_0} &= \Phi\left(\frac{\mu_z}{\sqrt{\omega_z}}c_0\right) + \exp\left(\frac{c_0^2}{2\omega_f}\left(2\mu_z\tau\omega_f - \Delta\mu\right)\Delta\mu\right)\Phi\left(-c_0\frac{\mu_z + \Delta\mu\tau\omega_z}{\sqrt{\omega_z}}\right) \\
\frac{\mathcal{I}_1}{\mathcal{J}_1} &= -c_1\sqrt{\frac{\omega_\theta + \tau^2\omega_z\omega_\epsilon\omega_f}{\tau^2\omega_z\omega_\epsilon^2 + \omega_\theta}}\left[\exp\left(\frac{\tau^2\omega_\epsilon c_1^2}{2}\mu_z^2\right)\mathcal{J}_1\right]^{-1}
\end{aligned}$$

where $\mathcal{J}_1$ is as in Eq. (A14). We claim that,

$$\lim_{\mu_z\to\infty}\frac{\mathcal{I}_0}{\mathcal{J}_0} = 1, \quad \lim_{\mu_z\to\infty}\frac{\mathcal{I}_1}{\mathcal{J}_1} = 0.$$

The first limit holds by the property of the cumulative distribution function, and by the L'Hôpital's rule. To show that the second limit holds, note that by the expression for $\mathcal{J}_1$ in Eq. (A14), we only need to show that

$$\lim_{\mu_z\to\infty}\exp\left(\frac{\tau^2\omega_\epsilon c_1^2}{2c_2^2}\left(c_2^2\mu_z^2 - \left(\Delta\mu\tau\omega_z\omega_\epsilon - c_2\mu_z\right)^2\right)\right) = \infty,$$

which is easily seen to be true. ■