Opacity, Credit Rating Shopping and Bias*

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First draft: July 6, 2010. This draft: February 27, 2013.

Abstract

We develop a rational expectations model where a debt issuer purchases credit ratings(s) to provide useful information to investors and attract demand. The issuer purchases rating(s) sequentially and decides which to disclose. Our analysis emphasizes the importance of opacity about the contacts between the issuer and rating agencies, leading to potential asymmetric information by investors about which ratings have been obtained. Issuers purchase too many ratings in some circumstances, allowing rating agencies to extract rents due to the issuer's value of the option to cherry pick which ratings to announce. While the equilibrium forces the disclosure of ratings when the market knows that these have been produced, uncertainty can emanate from the rating process over whether ratings have been obtained that were not disclosed. Absent disclosure (the opaque case), ratings bias would arise whenever the equilibrium involves publication of fewer ratings than the number available to the issuer. However, investors adjust asset pricing to eliminate the potential bias under rational expectations.

*The authors wish to thank for helpful comments Anat Admati, Sudipto Bhattacharya, Mike Fishman, William Fuchs, Todd Milbourn, Uday Rajan, Marcelo Rezende, Gunter Strobl and participants at presentations of earlier versions of the paper at the Bank of Japan, Carnegie Mellon University, the Federal Reserve Bank of New York, the Financial Research Association meetings, the Financial Intermediation Research Society (Sydney), the London School of Economics, the Midwest Finance Association meetings, the National Bureau of Economic Research meeting on "The Economics of Credit Rating Agencies," the Notre Dame Conference on Current Topics in Financial Regulation, the Norwegian School of Economics and Business Administration (NHH), the Stockholm School of Economics, the University of British Columbia, the University of Iowa and the University of Tokyo. The authors also thank the Sloan Foundation for financial support.

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1. Introduction

In the aftermath of the financial crisis there has been considerable spotlight on the accuracy of credit ratings and the potential for upward bias in these, especially given that the issuer pays for ratings and can selectively publish and disclose those that would be available for the marketplace to consider in evaluating complex financial instruments.[1] This context raises fundamental questions about the nature of equilibrium. To what extent are communications prior to the disclosure of ratings publicly available? Under what conditions do the ratings disclosed reflect selective disclosure in equilibrium and bias to which investors would like to adjust? This is central to understanding the nature of credit ratings that are disclosed and the implications for asset pricing.

While ratings bias emerges in the literature under a variety of assumptions in which investors react myopically to ratings, it is important to address whether the incentives to shop for ratings and disclose selectively disappear under rationality. For example, is the rationality of investors sufficient to guarantee unbiased ratings? Would the enforcement of mandatory disclosure of the issuer's interaction with the rating agency ensure unbiased ratings? Traditional frictions–such as asymmetric information in which the issuer has better information exogenously than investors or moral hazard in which the rating agency has incentives to distort the ratings to attract additional ratings fees–would potentially lead to bias. Instead, we will focus on a more subtle type of asymmetric information.

A possible source of ratings bias is the ability of issuers to obtain preliminary (or indicative) ratings from rating agencies without being required to disclose these contacts. Of course, disclosure about such ratings could take a variety of forms such as mandatory disclosure of the information provided by the rating agency, disclosure of the contact of a particular rating agency by the issuer in the specific context (e.g., the underlying information might be complex and it could be too difficult to mandate disclosure of the fundamental information), or, as has been the case historically, the contact could be viewed as private. Indeed, the economics of the context suggests the universality of such contacts, as when the costs of obtaining indicative ratings are sufficiently low, which often has mirrored the past practice. This discussion suggests a number of policy issues, including what types of disclosure should be required and the incentives between the stages of purchasing an indicative rating and disclosing that rating. It also emphasizes the importance of the form of equilibrium. As our analysis illustrates, under rational expectations disclosure of contacts by the issuer with the rating

[1] An interesting empirical analysis that documents the potential subjectivity in ratings is Griffin and Tang (2012). The potential for bias in ratings and more specifically, the apparent inverse relationship between ratings standards and the success of a rating firm is illustrated dramatically by a statistic in Lucchetti (2007), who reports that Moody's market share "dropped to 25% from 75% in rating commercial mortgage deals after it increased standards." The ongoing relevance and pervasiveness of ratings shopping in the aftermath of the financial crisis is illustrated by Neumann (2012).

agency is very powerful and can eliminate ratings bias.

We develop a rational expectations framework in which the issuer can help convey information to the market by using ratings agencies to improve the precision of the market's information. We examine the impact of transparency at the ratings stage. In addition to mandatory disclosure of all ratings, we focus upon two alternative disclosure regimes—one in which all contacts for ratings are transparent and an opaque alternative in which the contacts are not disclosed. If the disclosure of indicative ratings already purchased is costless, then in the transparent case all purchased ratings are disclosed, implying that ratings shopping and ratings bias do not arise. In the spirit of the literature on voluntary disclosure of information (e.g., Grossman and Hart (1980), Grossman (1981) and Milgrom (1981)), when it is common knowledge that a set of ratings have been purchased, then all of those must be disclosed in equilibrium to avoid a harsh inference about any undisclosed ratings.

In contrast, in the absence of any disclosure requirement about ratings contacts (the "opaque" case) we show how endogenous uncertainty can originate from the rating process, as a result of which investors do not know whether ratings are not being disclosed because they were not obtained and therefore unavailable, or because the ratings were sufficiently adverse. As a consequence, the issuer can avoid a completely adverse inference and discretionary or selective disclosure arises in equilibrium. Ratings shopping and ratings bias would then obtain in the opaque case whenever the equilibrium entails publication of fewer ratings than the number of indicative ratings purchased—as the issuer would then choose selectively which ratings to publish, choosing the highest ratings obtained.[2]

In our setting we consider a game in which there are two rating agencies and the issuer decides sequentially whether to purchase the second rating after observing the first rating. If the first rating is sufficiently high (i.e., over a relevant threshold), then the issuer would conclude that it doesn't need to expend resources for an additional rating, while if the first rating is lower than the threshold, then the issuer purchases a second indicative rating that it would disclose if that second indicative rating were over the same threshold. Although both ratings may be disclosed if sufficiently close, only a single rating is disclosed when either rating is sufficiently strong. When a single rating is disclosed in this equilibrium, because of the common threshold applied to both rating agencies the investor is unable to distinguish the cases in which this rating was obtained from the first rating agency or from the second.[3] Of course, there is rating bias in the scenario when a single rating is disclosed. A single rating is disclosed only if above the threshold, which makes it more likely to select ratings whose

[2] Neumann (2012) points out that it is unusual for a rating firm to publish a detailed report for a deal that it didn't ultimately rate. Publications of such reports would be typical in a transparent world, but not in an opaque setting.

[3] Because of the assumption of symmetric costs and precision, the investor lacks information about the identities of the first vs. second rating agency when the issuer initially randomizes between the two.

realizations are high, conditional on the fundamental value of the asset. This selection effect and bias would be particularly strong in the situation in which the second rating was over the threshold, but the first rating was below the threshold.

A key insight of the opaque framework is that when information about which ratings are purchased is private, issuers cannot refrain from shopping for more ratings and disclosing selectively, unless rating fees are sufficiently high. These incentives result in inefficient overproduction of information, enabling ratings agencies to extract rents.[4] Somewhat paradoxically, issuers are worse-off because of their private information. In fact, in the spirit of Akerlof (1970), for some parameter values the adverse selection can be so severe that the market breaks down, even if trade is ex-ante efficient.

In an extension of our basic model, we depart from the assumption of verifiable information and consider a setup in which rating agencies may offer biased ratings to attract business from the issuer. Our results in this framework suggest that the very nature of an equilibrium with rating shopping and selective disclosure may exacerbate rating agencies' incentives to "adjust" ratings above their model output. In addition, our analysis provides novel predictions on the interaction between rating shopping and rating adjustment. For example, for ratings above a given threshold, assets with lower model-implied valuations receive larger adjustments, a prediction that is consistent with the pattern documented empirically by Griffin and Tang (2012) in the context of CDOs.

Our paper contributes to the literature on discretionary disclosure as the source of asymmetric information arises endogenously through the equilibrium choices in our setting (e.g., compare to Dye (1985), Shin (2003) and Acharya, DeMarzo and Kremer (2011)), rather than being exogenously specified, as would be traditional in many frameworks with asymmetric information. In these papers the source of uncertainty (whether the manager has a signal or is uninformed) is exogenous. Two papers in this literature, which point to some contrasts with our specification, are those of Matthews and Postlewaite (1985) and Shavell (1994). In their settings whether sellers obtain information on their products is private information, as in our opaque regime. As testing is costless in Matthews and Postlewaite (1985), full disclosure is universal (we would have full disclosure in our setting if the costs were zero). However, under some conditions in Matthews and Postlewaite (1985) testing does not occur because the full disclosure rule acts as a commitment not to test, unlike in our transparent regime. As the cost of collecting information is random in Shavell (1994), not all sellers decide to obtain information and those who do disclose selectively. In contrast to these papers, in our setup the issuer can obtain information from multiple sources, and the cost of information (the rating fee) is not exogenously specified, but determined as a strategic variable by the rating agencies. It is important to

[4] For a related result on how voluntary disclosure results in socially excessive incentives to acquire information, see Shavell (1994). In our context we model a market for information.

emphasize that our framework highlights the empirical implications for ratings shopping (especially, see Section 5), unlike the earlier theoretical literature on disclosure.

Much of the focus of earlier theoretical papers on credit rating agencies had been on situations in which ratings shopping and selective disclosure arise without rational expectations, such as Bolton, Freixas and Shapiro (2012), Sangiorgi, Sokobin and Spatt (2009), and Skreta and Veldkamp (2009). We contrast the distinctive empirical implications of our framework with these myopically-oriented models of ratings shopping at the start of Section 5.

A number of papers now adopt rational expectations to focus on specific aspects of ratings inflation and regulatory policy rather than shopping, including Opp, Opp and Harris (2010)–who examine information acquisition and the impact of rating contingent regulation by a monopoly rating agency in the face of asset complexity, Mathis, McAndrews and Rochet (2009)–who examine an alternative payment model and Fulghieri, Strobl and Xia (2010)–who study the impact of unsolicited ratings on ratings standards and reputation. Faure-Grimaud, Peyrache and Quesada (2009) identify the optimal contract between a rating agency and a firm and conditions under which this results in the firm owning its rating. In their setup, ratings reveal the asset value perfectly, so there is no incentive for a firm to purchase multiple ratings and ratings bias does not arise. Manso (2012) develops a tractable framework that analyzes feedback effects in which ratings influence the probability of survival of the borrower, ratings competition can lead to downgrades and multi-notch downgrades can occur in response to small shocks to fundamentals. Kartasheva and Yilmaz (2012) analyze the determinants of the precision of ratings in equilibrium and examine the implications for a number of policy issues such as ratings standardization, reliance on ratings for regulation and the regulation of ratings fees.

While we are able to show that an equilibrium emerges without ratings bias under specific conditions (such as transparency of ratings contacts), the opaque case makes clear that the absence of bias is not a robust or universal outcome of our model—even under the assumption of rational expectations. Recent empirical evidence in Kronlund (2011) suggests that ratings shopping has distorted the actual ratings on corporate bonds (ratings are relatively higher for issues that are more likely to experience ratings shopping). Yet interestingly, Kronlund's evidence suggests that investors adjust for this in market pricing. In effect, the market pricing reflects the potential winner's curse associated with the choice of rating agencies (also see discussion in Sangiorgi, Sokobin and Spatt (2009)). We note that these broad themes from our theoretical specification are not dependent upon the specific assumption of two rating agencies and would be valid, even if there were many rating agencies.[5]

[5] Becker and Milbourn (2011) study the impact of the introduction of Fitch in the marketplace upon the ratings issued by Moody's and S&P. They find that these ratings are relatively higher after the introduction of Fitch and that competition through shopping intensified then.

Ratings bias and overproduction of information (and excess rents to the rating agencies due to the potential value obtained by issuers from selective disclosure for certain parameters) will arise more generally (for many rating agencies) in the opaque equilibrium, while the transparent equilibrium does not entail ratings bias and is efficient.[6] This leads to an important normative implication of our paper–that issuers should be required to disclose their receipt of preliminary ratings. Indeed, the SEC formally proposed such a rule in the fall of 2009 and such a requirement was discussed by the executive branch and legislators in the debate on financial reform, but the SEC's proposed regulation has not been adopted to date and became a lower regulatory priority once it was not included as part of the Dodd-Frank Act's requirements for credit rating agency regulation (other aspects of Dodd-Frank's requirements for credit rating agencies appear to have squeezed the proposal from the current regulatory agenda).

Ratings shopping has had a number of other impacts on recent policy debates. For example, the New York State Attorney General's 2008 settlement with the three major rating agencies mandating fees at the indicative ratings stage (though not barring them at the disclosure stage) attempted to reduce ratings shopping (Office of New York State Attorney General, 2008). The greater the fee at the indicative stage the fewer the ratings that are obtained and the more limited the scope for selective disclosure and ratings shopping. Additionally, ratings bias (which arises in the opaque scenario) would undercut the appropriateness of reliance on ratings for regulatory purposes.[7] Critics of rating agencies have suggested that ratings shopping and the ability of the issuer to choose its rating agencies represent an important conflict of interest that distorts the ratings process. One of our paper's messages is that ex ante fees reduce (but do not eliminate) the incentive to shop for (or inflate) ratings.

Our paper is organized as follows. Section 2 describes the underlying specification of the model and the disclosure policy in the presence of common knowledge. Section 3 addresses the equilibrium in a transparent market and Section 4 examines the equilibrium in an opaque market. Section 5 considers an extension of the model that allows rating agencies to inflate ratings directly. Section 6 discusses some empirical implications of the model and ties these to the empirical literature on credit ratings. Section 7 concludes. The Appendices contain details omitted from the main text, including proofs.

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[6] As further discussed at the end of Section 4, rating bias and overproduction of information are not confined to the particular equilibrium upon which we focus in the opaque case.

[7] The Dodd-Frank Act mandates that U.S. regulators no longer rely on ratings for regulatory purposes. How domestic (and especially international) regulators implement this in various contexts continues to evolve.

2. Setup

An *issuer* is endowed with one unit of an asset with random payoff

$$X \sim N(\mu_X, \sigma_X^2).$$

The issuer is risk neutral, and has an exogenous holding cost for the asset equal to V. The issuer can either hold the asset or sell it to the market, which is composed of a continuum (with mass equal to one) of risk-averse investors. Investors have CARA preferences with common absolute risk aversion coefficient r. A riskless asset is traded in perfectly elastic supply with net return normalized to zero. All players have rational expectations and zero discount rates. Ex-ante, the issuer and the investors share the same prior information about the asset value. When the asset is priced according to prior information only, the issuer sells it to investors for

$$p_0 = \mu_X - r\sigma_X^2, \tag{1}$$

implying ex-ante gains from trade equal to

$$\Delta := V - r\sigma_X^2.$$

We assume $\Delta > 0$, so that trade is always efficient.

The issuer can influence investors' valuation of the asset by conveying information to the market via *rating agencies* (RAs). We assume there are two such agencies, endowed with the same rating technology: each rating agency can produce at a cost H an unbiased noisy signal, or *rating*

$$S_i = X + \varepsilon_i,$$

for $i = 1, 2$, with ε_i uncorrelated with X, i.i.d. and

$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2).$$

Hence, the two rating agencies have equivalent, but independent technologies. Each RA maximizes profits by setting the fee c_i at which the issuer can purchase its rating, with the constraint that $c_i \geq H$.

The rating process is as follows. The issuer can approach RA i and purchase its rating at fee c_i; in which case the RA produces the rating S_i, which is communicated to the issuer. At this point the issuer owns the rating and can either withhold it or make it public through the rating agency. Only in the latter case would investors observe the rating. A crucial feature of the rating process that plays a major role in our analysis is its degree of transparency. The rating process is defined as *transparent* or *opaque,* depending on whether or not the act of purchasing a rating is observable by investors. If the market is opaque, investors only observe the purchased ratings that the issuer decides to publish voluntarily.

The timing of the model is as follows.

1. RAs simultaneously post fees c_1, c_2. Fees are observed by all players.
2. The issuer shops for ratings. The issuer can shop sequentially, that is, can purchase a first rating, and decide whether to purchase the second rating after observing the value of the first rating.
3. The issuer decides which ratings to disclose (if any).
4. The asset is sold to investors for the price p.
5. The payoff X is realized and consumption takes place.

At any point in the time interval $\tau \in (1, 4)$, the issuer can decide to quit and retain the asset, in which case trade does not occur. The value of the issuer's outside option at time τ is therefore the risk-neutral valuation of the asset (conditional on the information that she has acquired) net of the holding cost V, that is,

$$\mathrm{E}\left(X | \mathcal{I}_{\tau}^{I}\right) - V, \tag{2}$$

where $\mathcal{I}_{\tau}^{I}$ denotes the issuer's information set at τ. We refer to the issuer's participation constraint (PC) as the condition that the outside option (2) is not greater than the expected continuation payoff at all $\tau \in (1, 4)$. For a given pair of rating fees, a strategy for the issuer is a disclosure rule for the purchased ratings at stage 3, a decision rule on the number of ratings to purchase at stage 2 and whether to quit and retain the asset at any point $\tau \in (1, 4)$. The solution concept implemented is the Perfect Bayesian Equilibrium, or *equilibrium* hereafter, for short. The fees are determined as a Nash equilibrium in the game played by the RAs in stage 1, anticipating the issuer's demand function in the following stages. We focus on symmetric equilibria in which RAs set equal fees.

We remark that the setup we consider is one in which there is no ex-ante information asymmetry between the issuer and the investors. There are ex-ante gains from trade and information is potentially valuable in that it increases investors' demand. There is no agency problem on the part of the rating agencies: purchased ratings are unbiased and reported truthfully. Investors have rational expectations and understand the incentives behind the issuer's actions. Consequently, the outcomes of the model are not driven by investors' naïveté.

We specify our framework to focus on the strategic aspects of discretionary disclosure that emerge as a consequence of opacity, building from the microstructure of credit ratings, as the more traditional impact of disclosure cost as a source of discretionary disclosure is well understood (e.g., see Verrecchia (1983)) and isolating the strategic aspects that emerge endogenously is unique to our framework. Disclosure costs per se would be an important ingredient in focusing upon the incentives of rating agencies, but we abstract from the underlying conflict of interest and agency issues that the rating agencies face. We also collapse any differential between the credit rating ultimately adopted by the rating agency and the preliminary or indicative rating conveyed in the first stage (slippage from these initial rating assessments) and any security design (such as tranching) issues that would arise at an ex ante stage. Section 5 extends the basic framework by relaxing the assumption that rating agencies provide an unbiased noisy signal of the asset payoff.

2.1. Value of information, surplus and efficiency

The issuer's payoff from selling the asset to investors equals the asset price net of the costs incurred to purchase ratings. The CARA-normal framework implies that, conditional on the issuer disclosing all purchased ratings, the asset price equals the conditional expectation of the fundamental minus a risk discount that is proportional to the conditional variance. In this case, the expected price equals

$$\begin{aligned} \mathrm{E}(p) &= \mathrm{E}\left[\mathrm{E}\left(X|\,\mathcal{I}^{inv}\right) - r\mathrm{Var}\left(X|\,\mathcal{I}^{inv}\right)\right] \\ &= \mu_X - \underbrace{r\mathrm{E}\left[\mathrm{Var}\left(X|\,\mathcal{I}^{inv}\right)\right]}_{\text{Risk premium}}, \end{aligned} \tag{3}$$

where $\mathcal{I}^{inv}$ denotes the investors' information set. Information is therefore valuable, ex-ante, because it reduces the risk premium. Denote with $\mathrm{Var}\left(X|\,\{S_i\}_{i=0}^{n}\right)$ the variance of X conditional on n ratings. Taking into account the cost of information production, H, the *net* value of information, Ω, is defined as

$$\Omega := \max_{n\in\{0,1,2\}} r\left[\sigma_X^2 - \mathrm{Var}\left(X|\,\{S_i\}_{i=0}^{n}\right)\right] - n \times H. \tag{4}$$

Then, the *potential surplus* is defined as the sum of the ex-ante gains from trade and the net value of information:

$$\Delta + \Omega.$$

Let Π denote the issuer's ex-ante expected equilibrium profits net of the ex-ante value of the outside option (holding the asset), that is

$$\Pi = \mathrm{E}(p) - \mathrm{E}(\text{costs}) - (\mu_X - V), \tag{5}$$

where E(costs) denotes the expected value of the fees that are going to be paid to RAs during the game. RAs profits equal such costs net of the information production cost H. Then, the *equilibrium surplus* is defined as

$$\Pi + \text{RAs profits}.$$

Therefore, we say that an equilibrium is *efficient* if

$$\Delta + \Omega = \Pi + \text{RAs profits},$$

that is, if *i*) trading takes place, and *ii*) the efficient amount of information is produced and transmitted.

2.2. Disclosure policy with common knowledge

Given the assumed ownership structure of ratings, the issuer always has the option of disclosing information selectively. For example, she could purchase one rating and disclose it only if this rating is good enough, or she could purchase both ratings and disclose only the best of the two and so on. Selective disclosure induces a selection bias in the ratings that are published and results in ratings inflation. Skreta and Veldkamp (2009) formalize this intuition under the assumption of naïve investors. With rational investors, it is not obvious that this option is viable in equilibrium. The reason is the standard "unraveling" argument:

Lemma 1 *If investors know for sure that a given rating has been purchased, then in equilibrium the issuer will disclose such rating to investors.*

The idea behind this result is simple and well understood. As a simple illustrative example, let risk aversion be zero and assume instead that in equilibrium the issuer purchases one rating and discloses it only if above a threshold, $\bar{S}$. Then, conditional on no disclosure, the asset price equals $\mathrm{E}\big(X|\, S < \bar{S}\big)$. Clearly, the issuer will disclose the rating S if and only if she gets a better price by disclosing than by not disclosing, that is, if and only if $\mathrm{E}(X|\, S) \geq \mathrm{E}\big(X|\, S < \bar{S}\big)$. Therefore, the threshold $\bar{S}$ has to satisfy $\mathrm{E}\big(X|\, S = \bar{S}\big) = \mathrm{E}\big(X|\, S < \bar{S}\big)$, but this equality is only satisfied for $\bar{S}$ equal to the minimum of the support of S. In other words, the rating is always disclosed, and full disclosure is supported by off-equilibrium "worst case beliefs" that, if the rating is not disclosed, it is equal to the minimum of its support, which in our setup is minus infinity. The unraveling result is undone if there are disclosure costs (e.g., Verrecchia, 1983) or some exogenous source of uncertainty about whether a player has information to disclose (e.g., Dye, 1985). As we do not make any of these assumptions, ratings bias from selective disclosure would not arise in this framework unless some degree of uncertainty arises endogenously in equilibrium.

3. Equilibrium in the transparent market

This section describes the equilibrium of the model under the assumption that the rating process is transparent. This case will provide a benchmark against which we can compare the model's predictions in the opaque case.

3.1. Issuer's strategy for exogenous fees

The model is solved backwards. In the transparent market there is common knowledge of which ratings are purchased. As a consequence, information unravels and all purchased ratings are disclosed. Anticipating full disclosure, the strategy of the issuer consists of the choice of how many ratings to purchase for exogenously given fees.[8] Define

$$c_l := r\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right); \qquad c_h := r\left(\sigma^2_X - \sigma^2_{X|S}\right), \tag{6}$$

where $\sigma^2_{X|S}$ and $\sigma^2_{X|2S}$ denote $\mathrm{Var}\left(X|\left\{S_i\right\}_{i=0}^n\right)$ for $n = 1$ and $n = 2$ respectively. The following lemma assumes symmetric fees. The case with asymmetric fees is included in Appendix B (Lemma B1). We have:

Lemma 2 *For $c \leq c_l$ the issuer purchases both ratings, for $c_l < c \leq c_h$ the issuer purchases only one rating, and for $c > c_h$ no rating is purchased.*

The threshold values defined in (6) have a very clear economic interpretation. c_h represents the ex-ante marginal value of purchasing the first rating. It equals the reduction in the risk premium (the second term in Eq. (3)) the issuer expects from disclosing one rating over disclosing no rating.

[8]When indifferent between purchasing n or $n+1$ ratings, we assume the issuer purchases $n+1$ ratings. When indifferent between purchasing a rating from one RA or the other, we assume the issuer chooses one at random with equal probability. Both assumptions are immaterial in this context.

Similarly, c_l equals the marginal value of the second rating. It equals the reduction in the risk premium the issuer expects from disclosing two ratings over disclosing only one. Note that with full disclosure the marginal value of a second rating is independent of the realization of the first rating (so the solution is identical whether the preliminary ratings are provided to the issuer simultaneously or sequentially). Hence, conditional on having purchased a first rating, the issuer will always (never) purchase the second rating for $c \leq c_l$ ($c > c_l$). Ratings are imperfect substitutes in this framework, and the incremental reduction in uncertainty decreases with the number of ratings, implying $c_h > c_l$. Finally, the number of purchased ratings is naturally related to the value of information: the values c_h, c_l are increasing both in the variance of the fundamental and in risk aversion. In the risk neutral case, both c_l and c_h equal zero, so that costly ratings are never purchased.

3.2. Equilibrium in the fee-setting game

Next, we endogenize the fees as the result of competition between the two rating agencies. The following proposition describes the symmetric equilibrium in the transparent market.[9]

Proposition 1. *The equilibrium with endogenous fees in the transparent market is one of the following:*

1. *for $H \leq c_l$, RAs set $c_1 = c_2 = c_l$ and the issuer purchases both ratings;*
2. *for $c_l < H \leq c_h$, RAs set $c_1 = c_2 = H$ and the issuer purchases only one rating;*
3. *for $H > c_h$, RAs set $c_1 = c_2 = H$ and no rating is purchased.*

The proof of the proposition shows that the PC is never binding and that the value of information Ω is maximized. Hence, the equilibrium is efficient. Figure 1 provides a numerical illustration for different values of the ratings' production cost, H. The thick line is potential surplus, $\Omega + \Delta$. The gray and light gray areas represent issuer's profits Π from Eq. (5) and RAs profits, respectively. As the equilibrium is efficient, the sum of the two equals potential surplus. Moreover, RAs make positive profits only for $H < c_l$, when both ratings are purchased and RAs are therefore not competing. For $c_l < H \leq c_h$, only one rating is purchased and competition in fees drives profits to zero, so that in this region issuer's profits equal total potential surplus, $\Pi = \Delta + \Omega$. For $H > c_h$, the net value of

[9] In case 3 in the proposition, every $c \geq H$ is an equilibrium fee in which no ratings are purchased. In such case we assume that RAs set $c_i = H$ for $i = 1, 2$.

information, Ω from Eq. (4), is zero; the asset is sold to investors without acquiring any rating, and issuer's profits equal the gains from trade, $\Pi = \Delta$.

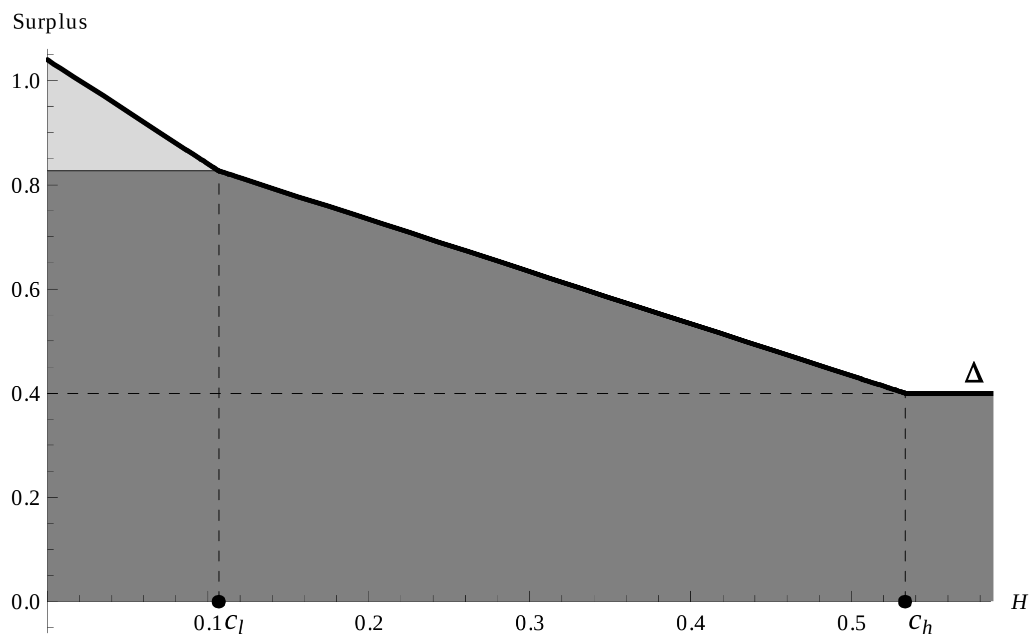

FIGURE 1. Parameter values: $\sigma_X^2 = 2$; $\sigma_\varepsilon^2 = 1$; $r = 0.4$; $V = 1.2$. Thick line: potential surplus, $\Omega + \Delta$. Dashed horizontal line: ex-ante gains from trade, Δ. Light gray area: RAs' profits. Gray area: issuer's profits Π from Eq. (5). Dashed horizontal line: ex-ante gains from trade, Δ.

4. Equilibrium in the opaque market

This section analyzes the case in which investors cannot observe the number of purchased ratings. In the opaque market, asymmetric information is more severe: the issuer has private information about which ratings are purchased. As consequence of this informational advantage, the issuer has an incentive to shop for ratings and disclose selectively. This section shows that the impact of these incentives on the issuer's strategy and the equilibrium are substantial.

4.1. *Issuer's strategy for exogenous fees*

First, consider the case in which the issuer sells the asset to investors without purchasing any rating. When investors anticipate the issuer following this strategy, the issuer sells the asset at price p_0 and realizes the gains from trade Δ. However, in an opaque market the issuer could deviate, purchase one rating, and then disclose it only if it were high enough. Suppose that investors react to such an off-equilibrium move assuming the issuer disclosed all purchased ratings. Then, the issuer would disclose if and only if

$$p(S_i) > p_0,$$

where p_0 is given in Eq.(1) and $p(\cdot)$ denotes the price function conditional on one rating under full disclosure,

$$p(S_i) = \mathrm{E}\left(X \right| S_i) - r\sigma^2_{X|S}. \tag{7}$$

For such a deviation not to be profitable, it has to be that its net expected benefits – the value for the option to disclose selectively – do not exceed the rating fee, that is,

$$\mathrm{E}\left[\max\left\{p(S_i) - p_0, 0\right\}\right] \leq c.$$

Solving the expectation, the above condition reads

$$c \geq \hat{c}_h := \left(\sigma^2_X - \sigma^2_{X|S}\right)^{1/2} g\left(r\sqrt{\sigma^2_X - \sigma^2_{X|S}}\right), \tag{8}$$

where the function $g(\cdot)$ is given by

$$g\left(t\right) := \Phi\left(t\right)t + f(t),$$

and $\Phi(\cdot)$ and $f(\cdot)$ denote the CDF and the PDF of a Standard Normal respectively. In other words, for the strategy of selling the asset without purchasing any rating to be an equilibrium, rating fees need to be at least the threshold $\hat{c}_h$.[10] Inspection of $g(\cdot)$ reveals that $g(t) > t$, which implies $\hat{c}_h > c_h$. Hence, for values of the rating's production cost such that the value of information is zero ($H \geq c_h$), the efficient outcome is not an equilibrium if RAs set fees below the threshold $\hat{c}_h$. In this case, an equilibrium with trading must involve the issuer purchasing some information, and RAs extracting some of the gains from trade. This example illustrates how the information friction induced by opaqueness influences the equilibrium in this context: the issuer cannot refrain from shopping for more ratings and disclosing selectively unless shopping costs (the fees) are high enough. As a result, in order to sell the asset to investors and realize the gains from trade, the issuer might purchase ratings at "unfair" prices. This allows RAs to extract rents, and the amount of information produced in equilibrium can be inefficiently high.

[10] The condition $c \geq \hat{c}_h$ is also sufficient, under the assumed beliefs, for this strategy to be an equilibrium (see Claim C5 in Appendix C).

Second, consider the case in which some information is acquired. We focus on strategies in which at least one rating is purchased with probability one. Unless both ratings are disclosed, investors do not observe the number of purchased ratings. In this case investors must form beliefs about the number of purchased ratings, and will assign probabilities to the unobservable event in which the issuer is disclosing information selectively. In an equilibrium, such probabilities need to be correct and, by Lemma 1, strictly less than one. How can a positive probability of selective disclosure be part of an equilibrium? Intuitively, the issuer could purchase a first rating and then decide to shop for the second rating only if the first rating is not good enough. If the second rating happens to be high, then the issuer might want to publish only this one and hide the first, less favorable, rating. For this option to be viable, it has to be that investors in equilibrium are uncertain as to whether the rating that is published is the result of selective disclosure or not. We remark that if the market was transparent, this would never be the case. The following definition illustrates the strategy upon which we will focus. We relabel ratings so that S_1 is the first rating that ends up being purchased.

Definition 1. *A constant $\bar{S}$, a function $s(\cdot)$ and a price function $p_{\bar{S}}(\cdot)$ constitute a* threshold strategy equilibrium *if:*

1. *for $c_1 = c_2 = c$, the following strategy is optimal for the issuer: purchase a first (randomly selected) rating, S_1, and then:*
   (a) *disclose S_1 and stop if $S_1 \geq \bar{S}$,*
   (b) *purchase the second rating if $S_1 < \bar{S}$, and then:*
      i. *disclose only S_2 if $S_2 \geq \bar{S}$ and $S_1 < s(S_2)$*
      ii. *disclose both ratings otherwise*
2. *$p_{\bar{S}}(\cdot)$ is consistent with the strategy of the issuer.*

We remark that the threshold strategy nests the cases of full disclosure of one rating for $\bar{S} \downarrow -\infty$, and full disclosure of both ratings for $\bar{S} \uparrow \infty$. For finite values of $\bar{S}$ the strategy involves discretionary disclosure with positive probability.

The task is to derive the functions $p_{\bar{S}}(\cdot)$, $s(\cdot)$ and identify conditions on $\bar{S}$ and c such that the conjectured strategy is indeed optimal and the price consistent with it. As a first step we take the threshold $\bar{S}$ and the function $s(\cdot)$ as given and derive the price function. When the issuer follows the strategy described above, two scenarios are possible: either investors observe both ratings, in which case all information is disclosed, or they observe a single rating, which we denote S_p, such that $S_p \geq \bar{S}$. In the latter case, investors' inference problem is as follows: S_p can be either the result of the first rating being high enough, $S_p = S_1 \geq \bar{S}$, or of the first rating being low and the second being high, that is, $S_p = S_2 \geq \bar{S}$ and $S_1 < s(S_2)$. Accordingly, investors use Bayes' Law to compute the posterior probability that the issuer disclosed information selectively, which we denote with $q_{(S_p)}$. Conditional on the information conveyed by S_p the distribution of the fundamental X is a mixture of two distributions:

$$\Pr\left(X|\, S_p\right) = (1 - q_{(S_p)}) \times \Pr\left(X|\, S_p = S_1\right) + q_{(S_p)} \times \Pr\left(X|\, S_p = S_2, S_1 < s(S_2)\right). \tag{9}$$

With CARA preferences, the price function $p_{\bar{S}}(\cdot)$ that is consistent with (9) is

$$p_{\bar{S}}(S_p) = p(S_p) - \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(\frac{s(S_p) - p(S_p)}{\sqrt{\sigma^2_{X|S} + \sigma^2_{\varepsilon}}}\right), \tag{10}$$

where $p(\cdot)$ is given in Eq. (7), and

$$\Gamma\left(t\right) := \frac{f\left(t\right)}{1 + \Phi(t)}. \tag{11}$$

Intuitively, the price in Eq. (10) is such that $p_{\bar{S}}(S_p) < p(S_p)$, as investors adjust pricing in response to the potential winners' curse implicit in only one rating being published.

Given $p_{\bar{S}}(\cdot)$, we now turn to the function $s(\cdot)$ that, for an exogenous threshold $\bar{S}$, makes the disclosure rule in (1.b) of Definition 1 optimal. Let t^* be the unique maximizer of $\Gamma\left(t\right)$, and denote

$$S^* := p_0 + t^* \frac{\sigma^2_X}{\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}} \tag{12}$$

$$\overrightarrow{S} := \frac{\sigma_X^{-2} + \sigma_{\varepsilon}^{-2}}{\sigma_{\varepsilon}^{-2}} \left(\bar{S} - S^*\right) + S^*. \tag{13}$$

Then, the function $s(\cdot)$ has the following form:

$$s(S_p) = \begin{cases} \bar{S}, & \text{for } \bar{S} \leq S^* \text{or } \left(\bar{S} > S^* \text{ and } S_p \geq \overrightarrow{S}\right) \\ \frac{\sigma_{\varepsilon}^{-2}}{\sigma_X^{-2} + \sigma_{\varepsilon}^{-2}} \left(S_p - S^*\right) + S^*, & \text{for } \bar{S} > S^* \text{ and } S_p \in \left[\bar{S}, \overrightarrow{S}\right) \end{cases} \tag{14}$$

The functional form in (14) implies that there are essentially two cases, depending on whether the threshold $\bar{S}$ is below or above the constant S^*. In the first case (i.e., $\bar{S} \leq S^*$), the issuer discloses S_2

alone whenever $S_2 \geq \bar{S}$ and both ratings otherwise. The second case is depicted in Figure 2: if the second rating is above $\bar{S}$ but not too high (i.e., below $\overrightarrow{S}$), the issuer will publish both ratings if the first rating is in the interval $\left[s(S_2), \bar{S}\right)$. Intuitively, it may be optimal to publish both ratings if they are sufficiently high and close to each other.

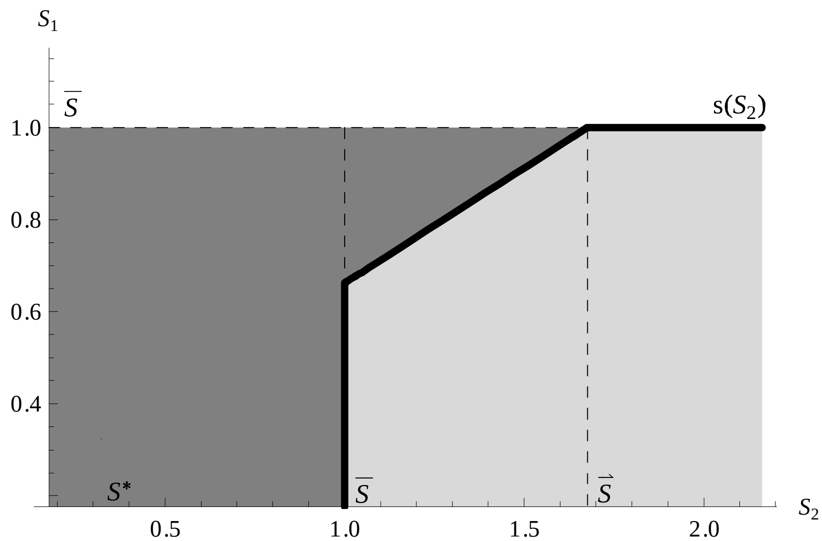

FIGURE 2. Disclosure rule conditional on both ratings being purchased and $S_1 < \bar{S}$. Exogenous parameter values: $\mu_X = 2$; $\sigma_X^2 = 1$; $\sigma_\varepsilon^2 = 1$; $r = 1$; $\bar{S} = 1$. Endogenous parameter values: $S^* = 0.32$, $\overrightarrow{S} = 1.68$. Thick line: $s(S_2)$ from Eq.(14). Light gray area: only S_2 is disclosed. Gray area: both ratings are disclosed.

Next, we consider what additional conditions guarantee the optimality of the issuer's conjectured strategy. To derive these conditions explicitly, we make the additional assumption that the noise in the ratings is nontrivial in the following sense:

$$\sigma_\varepsilon^2 \geq \bar{\sigma}\sigma_X^2,$$

where $\bar{\sigma}$ is a "small" constant.[11] Whenever $S_1 \geq \bar{S}$, the issuer could deviate by purchasing the second rating and then disclosing strategically depending on the realization of both ratings. While the cost of this deviation is just the fee, the expected benefit is the greatest for $S_1 = \bar{S}$. As a function of

[11] $\bar{\sigma} \approx 0.086$ is derived in Eq. (C9) in Appendix C as a sufficient condition for the proof of Eq. (15).

$\bar{S}$, this maximum expected benefit is strictly decreasing, unbounded above and bounded below by a positive constant, c_t, defined as

$$c_t := \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} g\left(-t^* + r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right). \tag{15}$$

This means that for each $c > c_t$ there exists a finite threshold, call it $\bar{S}_c$, such that if $\bar{S} \geq \bar{S}_c$ it is never optimal for the issuer to deviate by purchasing the second rating when $S_1 \geq \bar{S}$. As $\bar{S}_c$ is decreasing in c and $\lim_{c \downarrow c_t} \bar{S}_c = \infty$, for values of the fee $c \leq c_t$ the strategy from Definition 1 is an equilibrium only for $\bar{S}$ equal to plus infinity, which implies that both ratings are always purchased and disclosed. Any other deviation from the strategy outlined in Definition 1 necessarily results in one of the following out-of-equilibrium moves: either no rating is disclosed or only one rating is disclosed below the threshold $\bar{S}$. Worst case off-equilibrium beliefs about undisclosed ratings prevent the issuer from engaging in any such deviation. Summarizing, worst case beliefs support the equilibrium in which both ratings are disclosed for $c \leq c_t$ and any $\bar{S} \geq \bar{S}_c$ as an equilibrium threshold for $c > c_t$.

Note that for some values of the fees the equilibrium strategy we have described is not unique. To derive sharper results in such cases, off-equilibrium beliefs are refined to obtain a unique equilibrium. The refinement is described in Appendix C. In words, worst case off-equilibrium beliefs are softened whenever investors can interpret an off-equilibrium move as the issuer playing a different equilibrium strategy (among the set of strategies we considered here) for which she would obtain higher expected profits. As a result, the equilibrium that obtains is the one that entails the lowest potential for overproduction of information and breakdown of ex-ante efficient trade. This corresponds to the case in which no rating is purchased for $c \geq \hat{c}_h$ and to the lowest threshold (i.e., $\bar{S} = \bar{S}_c$) for $c \in (c_t, \hat{c}_h)$. We remark that as our focus is on the inefficiencies induced by ratings shopping, this approach is conservative.

The next proposition summarizes the analysis of this section. It describes the equilibrium strategy of the issuer for exogenously given symmetric fees. The case with asymmetric fees is presented in Appendix C (Lemma C2). We have:

Proposition 2. *Let $c_1 = c_2 = c$ and assume the PC is satisfied at the ex-ante stage. There exist off-equilibrium beliefs that support the following equilibrium:*

i) *If $c < \hat{c}_h$ the functions (10)-(14) constitute a threshold strategy equilibrium for some threshold $\bar{S}$. Whenever $c_t < \hat{c}_h$ the threshold $\bar{S}$ is finite and decreasing in c for all $c \in (c_t, \hat{c}_h)$; for $c \leq c_t$ the threshold $\bar{S} \uparrow \infty$ and both ratings are purchased and disclosed.*

ii) *If $c \geq \hat{c}_h$ no rating is purchased.*

The condition $c_t < \hat{c}_h$ depends on the primitives of the model and is satisfied for natural values of the parameters.[12] The proof of the proposition further shows that if the PC is satisfied ex-ante, then it is satisfied in all states along the equilibrium path.

4.2. Equilibrium in the fee-setting game

Fees are endogenized in a symmetric Nash equilibrium in the fee-setting game played by RAs in the initial stage. For any pair of fees such that $c_1 = c_2 = c$, each RA's expected profit is obtained anticipating the issuer's demand for ratings from Proposition 2. Whenever the fees are such that the demand for ratings is positive, the PC is satisfied if the value of the issuer's outside option at the ex-ante stage does not exceed the issuer's expected profits, that is, if

$$\mu_X - V \leq \mathrm{E}\left[p_{\bar{S}}(S_p)\mathbb{I}_{\{B\}} + p(S_1, S_2)\left(1 - \mathbb{I}_{\{B\}}\right)\right] - c\left[1 + \Phi\left(\frac{\bar{S} - \mu_X}{\sigma_s}\right)\right], \tag{16}$$

where $\mathbb{I}$ denotes the indicator function and B the event in which only one rating is published along the path of the threshold equilibrium. Let $v(H)$ be defined as the value of the holding cost such that the PC (16) binds when the fees equal H, and let c_2^* be defined as the value of the fees such that the PC (16) binds when both ratings are purchased and disclosed (i.e., for $\bar{S} \uparrow \infty$). Finally, let

$$\begin{aligned}\tilde{c} &:= \min\{c_t, \hat{c}_h\} \\ \hat{c}_l &:= \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} g\left(r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right). \end{aligned} \tag{17}$$

By the respective definitions and the fact that $t^* < 0$, it follows that $c_l < \hat{c}_l < \tilde{c}$. The next proposition describes the symmetric equilibrium in the opaque market.[13]

Proposition 3. *When the issuer's demand for ratings is as described in Proposition 2, the equilibrium with endogenous fees in the opaque market has the following form:*

1. *For $H \leq \tilde{c}$ such that $H \neq \hat{c}_h$ and $H \leq c_2^*$, then: either $c_2^* < \hat{c}_l$, and RAs set $c_1 = c_2 = c_2^*$; or $c_2^* \geq \hat{c}_l$ and any $c_1 = c_2 = c \in [\max\{H, \hat{c}_l\}, \min\{c_2^*, \tilde{c}\}] \setminus \hat{c}_h$ is an equilibrium fee. The issuer purchases and discloses both ratings.*

[12] Comparing (8) and (15) it is easy to verify that $c_t < \hat{c}_h$ for all $r \geq 0$ whenever $\sigma_\varepsilon^2 < \hat{\sigma}\sigma_X^2$, where $\hat{\sigma} \approx 2.186$.

[13] In case 3 and case 4 in the proposition, whenever every $c \geq H$ is an equilibrium fee in which no ratings are purchased, we assume that RAs set $c_i = H$ for $i = 1, 2$.

2. *For $c_t < \hat{c}_h$ and $H \in (c_t, \hat{c}_h)$ and $V \geq v(H)$, RAs set $c_1 = c_2 = H$ and the issuer follows the threshold strategy of Definition 1 for a finite threshold $\bar{S}$.*

3. *For $H \geq \hat{c}_h$, RAs set $c_1 = c_2 = H$ and no rating is purchased.*

4. *If neither of the above holds, RAs set $c_1 = c_2 = H$, the issuer retains the asset and there is no trade.*

Case 1 in the proposition implies that for some parameter values there are multiple equilibrium fees. This multiplicity affects the distribution of the surplus but not the equilibrium value of the threshold since both ratings are disclosed. Figure 3 shows the outcome of the model in the opaque market for values of the holding cost $V \in \left[r\sigma_X^2, \frac{3}{2}r\sigma_X^2\right]$, for values of the information production cost $H \in [0, \hat{c}_h]$, and fixed values of other parameters. From Proposition 3, for $H \geq \hat{c}_h$, the asset is sold without any rating being purchased in equilibrium. The gray area in the figure corresponds to parameter values for which both ratings are purchased and disclosed; the light gray area corresponds to parameter values for which ratings are disclosed selectively with positive probability. The empty area corresponds to the no-trade equilibrium: for these parameters the selling motive of the issuer is not sufficiently high to justify the purchase of ratings, but at the same time ratings are not sufficiently expensive for the issuer to refrain from shopping and disclosing selectively. In these cases, the asymmetric information problem induced by opacity and ratings shopping is so severe that the market breaks down, the issuer retains the asset and ex-ante valuable trading does not take place.

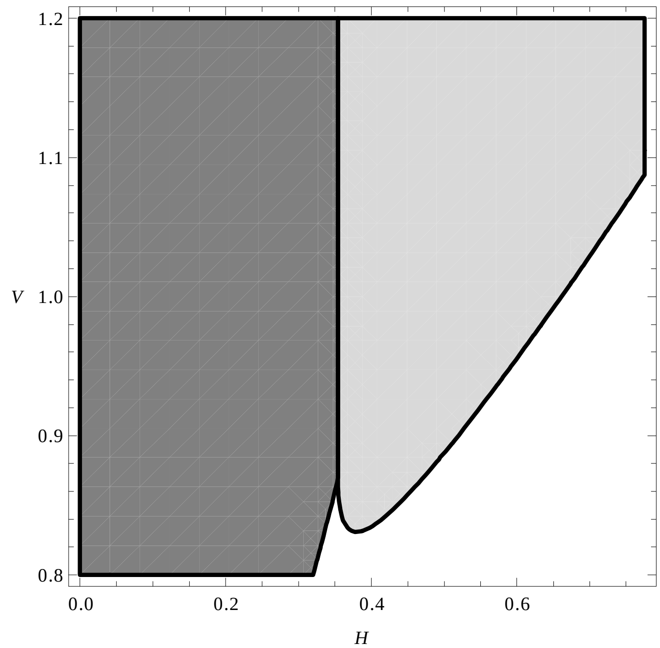

FIGURE 3. Parameter values: $\sigma_X^2 = 2$; $\sigma_\varepsilon^2 = 1$; $r = 0.4$. For these parameters, $\tilde{c} = 0.35$ and $\hat{c}_h = 0.78$. Gray area: full disclosure equilibrium with two ratings ($\bar{S} = \infty$). Light gray area: equilibrium with selective disclosure ($\bar{S} < \infty$). Empty areas: no-trade equilibrium.

Figure 4 sets the value of V such that trade always takes place and shows the resulting properties of the equilibrium for different values of H. Comparing Figure 4 with Figure 1 illustrates the impact of opaqueness on equilibrium in terms of efficiency and distribution of surplus. Inefficiency is measured by the vertical distance between the thick line, representing potential surplus, $\Omega + \Delta$, and the sum of RAs and issuer's profits. With opaqueness the equilibrium is inefficient for all $H \in (c_l, \hat{c}_h)$. In the figure, inefficiency is the result of inefficient overproduction of information. Also, the distribution of the surplus is greatly affected. For all $H \leq \tilde{c}$ both ratings are purchased and in this numerical example RAs charge $c = \tilde{c}$. As $\tilde{c} > c_h$, the costs the issuer is paying for the ratings exceed the value of information, and RAs extract rents from the gains from trade Δ. For H high enough ($H > \tilde{c}$), the equilibrium threshold is finite and competition in fees drives RAs profits to zero. However, the production of information is inefficient in equilibrium and issuer's profits are reduced below the ex-ante gains from trade, $\Pi < \Delta$. Issuer's profits equal Δ only if $H \geq \hat{c}_h$, in which case fees are high enough that the issuer can credibly commit not to purchase any rating and sell the asset.

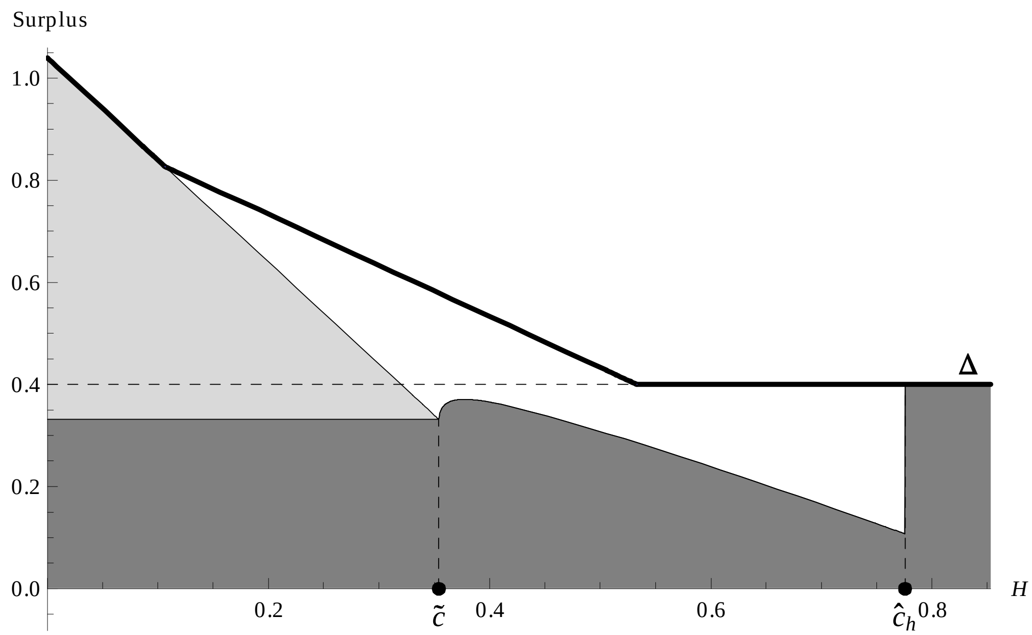

FIGURE 4. Parameter values: $\sigma_X^2 = 2$; $\sigma_\varepsilon^2 = 1$; $r = 0.4$; $V = 1.2$. Thick line: potential surplus, $\Omega + \Delta$. Dashed horizontal line: ex-ante gains from trade, Δ. Light gray area: RAs' profits. Gray area: issuer's profits Π from Eq. (5).

4.3. Threshold strategy and ratings inflation

In the opaque case, whenever equilibrium involves publication of fewer ratings than the number of ratings potentially available (i.e., only one rating), the distribution of these ratings is upward biased. One reason behind the bias is selective disclosure: if the first purchased rating is not sufficiently high, with positive probability it is not disclosed to investors. A second reason is that a single rating is disclosed only if above the threshold, which makes it more likely to select ratings for which the realization of the noise term is positive. As before, denote with B the set of states in which a single rating is disclosed along the path of play. The bias, b, is defined as

$$b := \mathrm{E}\left(\frac{S_p - X}{\sigma_X} \middle| B \right), \tag{18}$$

where S_p denotes the value of the rating that is disclosed, and the expected difference between S_p and the fundamental, X, is scaled by the standard deviation of X. Figure 5 plots rating bias (thick line) for values of the rating's production cost such that the resulting equilibrium involves publication of only one rating with positive probability. The figure also plots the probability that a single rating is published (dashed line), and the probability of selective disclosure conditional on a single rating being published (dotted line). The endogenous threshold above which the first rating is disclosed is decreasing in the cost (Proposition 2). As a result, the figure shows, the probability of a single rating being published increases with the cost, and the conditional probability of selective disclosure decreases with the cost. Since both the threshold and the conditional probability of selective disclosure decrease, rating bias decreases as well.

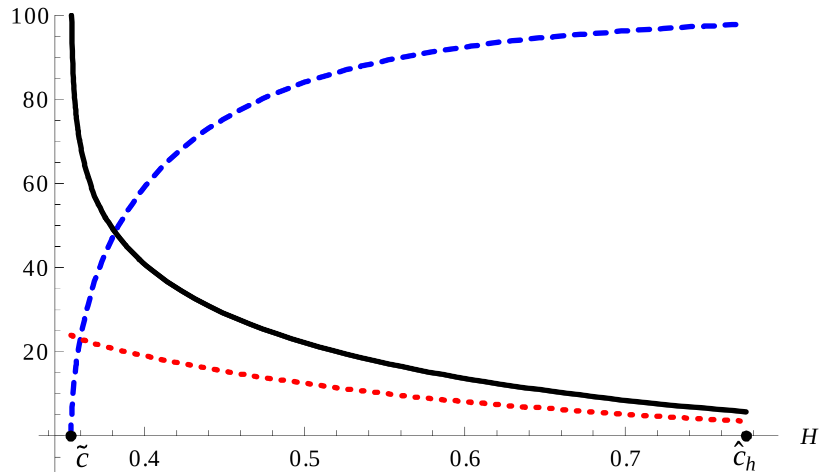

FIGURE 5. Parameter values: $\sigma_X^2 = 2$; $\sigma_\varepsilon^2 = 1$; $r = 0.4$. Thick line: rating bias (in %) from Eq. (18). Dashed line: probability (in %) that a single rating disclosed. Dotted line: probability (in %) of selective disclosure conditional on a single rating being published.

4.4. Endogenous opaqueness

So far we have taken the transparency regime of the rating process as given. A natural question is which regime would RAs choose if they could. More formally, assume each RA can independently choose the transparency regime as a strategic variable. The contract would therefore specify both the fee at which the rating is sold, and whether the RA would communicate to investors that the rating has been purchased. In such case, by Lemma 1, purchased ratings are eventually disclosed by the issuer. Therefore, choosing a transparent regime is in fact equivalent to RAs disclosing the purchased rating automatically. Fees and transparency regimes are then announced simultaneously by RAs at the beginning of the game and such announcement is observed by all players. The rest of the game is unchanged. The next proposition establishes whether the equilibrium outcomes described in Propositions 1 and 3 are robust to this extension.

Proposition 4. *When the degree of transparency is endogenously determined, then: i) the opaque market equilibrium of Proposition 3 is unaffected, and ii) there exists a value* $\bar{H}$ *such that in the transparent market equilibrium of Proposition 1 a RA can increase profits by offering opaque contacts for all* $H < \bar{H}$.

Given the previous discussion on the welfare properties of the different regimes, it is not surprising that the opaque regime emerges as an equilibrium, while the transparent does not. Enabling the issuer with the option to disclose the rating is beneficial to RAs, and allows them to extract rents in equilibrium.

4.5. Investors' welfare

Our definition of surplus in Section 2.1 abstracts from investors' welfare. To consider investors' welfare explicitly, investors' certainty equivalent of wealth is defined as the monetary value ϖ that solves $\mathbb{E}[u(w)] = u(\varpi)$, where w denotes investors' equilibrium end-of-period wealth.[14] Social Welfare is then defined as the sum of the equilibrium surplus, as defined in Section 2.1, and investors' certainty equivalent of wealth, that is,

$$\Pi + \text{RAs profits} + \varpi.$$

The following proposition establishes that the main normative implication of our paper is robust to considering investors' welfare explicitly. We have:

Proposition 5. *Social Welfare is greater in the transparent market equilibrium of Proposition 1 than in the opaque market equilibrium of Proposition 3.*

The intuition for this result is that more information reduces payoff uncertainty and results in a lower equilibrium risk premium. In turn, the reduced average profits from holding the asset have a negative effect on investors' welfare. As a consequence, the overproduction of information arising from opacity will generally make investors worse off.

[14] Beginning-of-period wealth is normalized to zero.

4.6. Other equilibria, rating bias and overproduction of information

If the rating process is opaque, issuers' incentives to shop for ratings and disclose selectively can result in inefficient production of information and biased ratings, even in the absence of any other friction. Our model portrays these effects with a specific equilibrium, but the insights of our model carry over to equilibria other than the one we describe here, as we illustrate in the following examples.

Consider the following variation of the threshold equilibrium of Definition 1 in which the issuer purchases a specific rating first, say, rating i, and then: discloses S_i and stops if S_i is in some open set $\Theta \subseteq \mathbb{R}$; purchases also rating $-i$ and discloses both ratings if $S_i \notin \Theta$. Our analysis implies that the issuer cannot commit not to shop for the second rating and disclose selectively when $S_i \in \Theta$ unless $c_{-i} \geq \hat{c}_l$.[15] Hence, as long as RA$_{-i}$ can profitably undercut to $H \leq c_{-i} < \hat{c}_l$, such equilibria can be sustained only for $H \geq \hat{c}_l$. As $\hat{c}_l > c_l$, these equilibria feature inefficient overproduction of information.[16,17]

Alternatively, consider a mixed strategy equilibrium in which the issuer purchases no rating with probability $q \in (0,1)$ and purchases only one rating with probability $1-q$, which is expected to be the cheaper rating if $c_1 \neq c_2$ and one at random if $c_1 = c_2$. When the rating is purchased it is disclosed if and only if above some threshold $\bar{S}$. Such an equilibrium can be shown to exist only for $H > \hat{c}_h$. As $\hat{c}_h > c_h$, information production is inefficiently high. As ratings are disclosed only if above a threshold, this equilibrium also features rating bias.

5. Extension: rating shopping with rating "adjustments"

A controversial assumption of our basic setup is that rating agencies are forced to provide an unbiased noisy signal of the asset payoff. This section extends the basic model by relaxing this assumption and providing a framework in which rating agencies may offer biased ratings to attract business from the issuer and the issuer may shop for ratings from multiple rating agencies in an opaque market. There are two potential sources of ratings bias in this context: issuers shopping for ratings and disclosing them selectively as well as rating agencies inflating ratings directly. Our analysis provides novel predictions with respect to the way in which these are interrelated.

[15] This result is derived in the proof of Lemma C2 in Appendix C.

[16] More specifically, $H > \hat{c}_l$ implies that any such equilibrium is inefficient for $\Theta \subset \mathbb{R}$, while for $\Theta = \mathbb{R}$ the equilibrium is inefficient whenever $\hat{c}_l > c_h$ (which is satisfied if risk aversion is low enough).

[17] Note that these equilibria require the issuer and the investors to coordinate on exactly one of the two (ex-ante identical) rating agencies, and with symmetric fees this coordination may be difficult to achieve, making this set of equilibria less plausible.

5.1. Setup

Assets and preferences

We modify our basic setup as follows. An issuer has one unit of an asset with random payoff

$$X = S_1 + S_2, \tag{19}$$

where, for $i = 1, 2$,

$$S_i \sim IIN(\mu, \sigma^2). \tag{20}$$

The remaining assumptions on preferences and assets traded are unchanged. Here we denote investors' risk aversion coefficient with γ and the rating that is published by RA_i with r_i.

Rating Agencies

We assume that RA_i faces a production cost equal to H to produce S_i, for $i = 1, 2$. The signal S_i, which we call a preliminary rating, is assumed to be verifiable by the issuer and contractible upon. However, and here we depart from our basic setup, the information produced by RAs is not immediately verifiable by investors. As a consequence, RA_i can report to investors any rating $r_i \in \mathbb{R}$ such that $r_i \neq S_i$. The terms of the rating contract specify an ex-ante fee c_i to be paid for RA_i to produce S_i. If the issuer pays c_i, she is entitled to the option to publish a rating equal to S_i. After S_i has been produced, RA_i and the issuer engage in "discussions" over the rating. We give all bargaining power to RAs by assuming RAs can make a take-it-or-leave-it offer for an "adjustment" in the rating.[18] Specifically, right after S_i is produced, RA_i offers to the issuer the option to disclose a different rating, $r_i \neq S_i$, for an additional fee $\tilde{c}_i$. Neither the act of purchasing a rating nor the outcome of the discussion stage is observed by investors, that only observe the ratings that the issuer decides to publish. This setup is meant to capture in a simplified way the practice, documented by Griffin and Tang (2012), by which RAs make adjustments to their direct model outputs. Here S_i reflects the model output and $r_i - S_i$ the rating adjustment. When r_i is disclosed to investors, RA_i faces an ex-post reputation cost. For tractability, we specify this cost exogenously and equal to

$$\kappa(S_i, r_i) = \frac{b}{2}(S_i - r_i)^2, \tag{21}$$

for some constant $b > 0$.

Timing

[18] The allocation of the bargaining power is irrelevant for the results in this section.

The timing of the benchmark model is modified to reflect the additional discussion stage and the decision of the issuer to accept the adjustments proposed by the RAs. Discussions over rating adjustments take place in stage 2 while the issuer is shopping for ratings: right after S_i has been produced, RA$_i$ offers the option to disclose the adjusted rating r_i for additional fee $\tilde{c}_i$. The issuer then decides whether to accept the adjustments proposed by the RAs or not and which ratings to disclose (if any) in stage 3. The remaining stages, as well the issuer's outside option, are unchanged.

Comments on the setup

Although the setup with independent signals described in Eqs. (19)-(20) and the setup from the previous sections are non-nested, the impact of this alternative signal structure on the resulting equilibrium is very minor.[19] The main qualitative difference of this alternative setup comes from the assumption that ratings are not verifiable by investors, which gives RAs an incentive to offer inflated ratings that optimally trade-off the additional fee an issuer is willing to pay for the rating adjustment and its ex-post reputation cost. The advantage of the simplified signal structure is to provide enough analytical tractability for an explicit analysis of this trade-off.

5.2. *Equilibrium analysis with opacity and non-verifiability*

Equilibrium rating adjustments and signal-jamming

Given the initial fees for preliminary ratings, a RA's strategy is described by a *rating function* and a *fee function*, that is, mappings $S_i \mapsto r(S_i)$ and $S_i \mapsto \tilde{c}(S_i)$. We focus on equilibria in which the function r is differentiable and invertible. Hence, when a rating $\tilde{r}_i$ is disclosed and investors expect the issuer to have purchased the rating adjustment, investors will price the asset according to the belief that $S_i = r^{-1}(\tilde{r}_i)$. Two properties of the equilibrium immediately obtain as a consequence of this framework. First, it has to be that investors expect published ratings to be the adjusted ratings, for otherwise a RA would profitably deviate by offering an inflated rating.[20] Second, ratings are always adjusted upward, for otherwise the issuer would get a better price disclosing the preliminary

[19] For instance, following the same steps that lead to Eq. (8) in Section 4.1, the value for the option to disclose selectively a first rating can be computed as $\underline{\hat{c}_h} := \sigma g(\gamma\sigma)$. As the marginal value of the first rating equals $\gamma\sigma^2$, there would be overproduction of information for $\overline{H} \in (\gamma\sigma^2, \underline{\hat{c}_h})$. Similarly, propositions equivalent to Proposition 2 and Proposition 3 can be shown to hold in this setup with minor modifications.

[20] More formally, assume in equilibrium investors expect the issuer not to have purchased a rating adjustment whenever a rating is disclosed that is in an open set (r_l, r_h). Then, for all realizations of $S_i \in (r_l, r_h)$, RA$_i$ could offer to disclose $\tilde{r}_i \in (S_i, r_h)$. Since investors believe $S_i = \tilde{r}_i$, the increase in the rating would translate into a higher price one-to-one, meaning the issuer would be willing to pay for the adjustment up to $\tilde{r}_i - S_i$. As $\tilde{r}_i - S_i - \frac{b}{2}(S_i - \tilde{r}_i)^2$ is increasing in $\tilde{r}_i$ for $\tilde{r}_i = S_i$, the deviation is profitable.

rating without purchasing the rating adjustment. The assumptions in Eqs. (19)-(21) then imply that a RA will not deviate from the strategy r only if, for all S_i, $r(S_i) \in \arg\max_{\tilde{r}} r^{-1}(\tilde{r}) - \frac{b}{2}(S_i - \tilde{r})^2$. The first order condition is

$$\left(r^{-1}(\tilde{r})\right)' + b\,(S_i - \tilde{r}) = 0,$$

which, after substituting $\tilde{r}$ with $r(S_i)$ and rearranging, gives the differential equation

$$r'(S_i) = \frac{1}{b\,(r(S_i) - S_i)}. \tag{22}$$

This equation has a linear solution, which is given

$$r(S_i) = S_i + b^{-1}. \tag{23}$$

Eq. (23) is the solution to the (linear) equilibrium in which RAs offer to disclose a constant rating adjustment. Furthermore, it is the only solution that is consistent with an equilibrium in which a rating is purchased and always disclosed.[21] As such, it provides the benchmark for the following analysis, which is instead focused on how the level of rating adjustments is affected in an equilibrium with rating shopping and selective disclosure.

Rating adjustments and the threshold equilibrium

We introduce the following definition of threshold equilibrium for the setup described in Eqs. (19)-(21), with the additional feature that RAs offer to disclose inflated ratings.[22]

Definition 2. *A constant $\bar{S}$ and a set of functions $r(\cdot)$, $\tilde{c}(\cdot)$ and $p_{\bar{S}}(\cdot)$, constitute a* threshold strategy equilibrium with rating adjustments *if the following strategies are optimal for the issuer and the RAs:*

1. *For $c_1 = c_2 = c$, the issuer pays for a first (randomly selected) preliminary rating, and then:*
    (a) *discloses r_1 and stops if $S_1 \geq \bar{S}$,*
    (b) *pays for the second preliminary rating if $S_1 < \bar{S}$, and then:*
        i. *discloses only r_2 if $S_2 \geq \bar{S}$*

[21] It is immediate to verify that for any other solution to Eq. (22) such that $r(S_i) \geq S_i$ for all S_i, the rating adjustment $r(S_i) - S_i$ diverges to infinity for $S_i \downarrow -\infty$. As the fee function must satisfy $\tilde{c}(S_i) \geq \kappa(S_i, r_i)$, the option of holding the asset would surely dominate the option of purchasing the rating adjustment for S_i sufficiently low.

[22] Aside from the rating adjustments, a qualitative difference with Definition 1 is that, in the eqilibrium from Definition 2, the issuer does not disclose any rating if both are below the threshold. This is now possible because, as information is not verifiable in this setup, it does not necessarily unravel and absence of disclosure can be consistent with equilibrium.

ii. *discloses no ratings otherwise.*

2. *After $S_i \geq \bar{S}$ is produced, RA_i offers to disclose $r_i = r(S_i)$ for additional fee $\tilde{c}(S_i)$ and the issuer accepts the offer.*

3. *The price function $p_{\bar{S}}(\cdot)$ is consistent with the strategy of the issuer and the RAs.*

To provide some intuition for why rating shopping and selective disclosure may affect the level of rating adjustments, notice that, in the conjectured equilibrium, RA$_i$'s profits from rating adjustments are foregone if its "model output," S_i, happens to be lower than the threshold value $\bar{S}$. This creates a tension, because now RA$_i$ might be tempted to recover some business by inflating its rating up to $r(\bar{S})$, which is the threshold above which investors expect ratings to be disclosed. For this not to happen, it must be that, for all $S_i < \bar{S}$, the issuer's willingness to pay for a rating equal or greater than $r(\bar{S})$ is more than offset by the ex-post reputation cost borne by the RA from inflating its rating up to $r(\bar{S})$. In turn, this requires equilibrium profits from rating adjustments to be zero at the threshold, a requirement that translates into a boundary condition for the differential equation (22) when $S_i = \bar{S}$. Whenever this boundary condition imposes the rating adjustment at the threshold to be greater than the benchmark adjustment, b^{-1}, then Eq. (22) implies that rating adjustments in the threshold equilibrium exceed the benchmark level for all $S_i \geq \bar{S}$ and monotonically decrease toward such benchmark level for larger values of S_i.[23]

This discussion suggests that the very nature of an equilibrium with rating shopping and selective disclosure may exacerbate RAs' incentives to adjust ratings above model outputs, further implying these rating adjustments to be more pronounced for published ratings that are closer to the threshold. The next proposition confirms that these features are in fact an equilibrium outcome of the model. We have:

Proposition 6. *Let $c_1 = c_2 = c$ be given. Then:*

i) *There exist off-equilibrium beliefs and a constant $\tilde{V}$ for which a threshold strategy equilibrium with rating adjustments exists for all $V \geq \tilde{V}$.*

ii) *There exist parameters for which, in any such equilibrium, rating adjustments exceed the benchmark value b^{-1} for all $S_i \geq \bar{S}$, decrease monotonically in S_i and approach b^{-1} as $S_i \uparrow \infty$.*

[23] To see this, rewrite Eq. (22) in terms of the rating adjustment, $y(S_i) = r(S_i) - S_i$, and note that b^{-1} is the steady state value of y.

Appendix E provides further details on the functions $r(\cdot)$, $\tilde{c}(\cdot)$ and $p_{\bar{S}}(\cdot)$, as well as sufficient conditions on the parameters for part ii) in Proposition 6.

6. Empirical implications

The framework developed in the paper offers a range of interesting empirical implications about the nature of credit ratings in the presence of potential ratings shopping. Empirical evidence about credit ratings has explored ratings in both the context of traditional bonds (such as corporate bonds) and mortgage-backed securitization tranching. To a degree these are very different contexts–the mortgage-backed securities setting is a relatively newer context with considerable uncertainty (especially due to the import of tranching) as compared to the traditional corporate (or municipal) bonds, for which there is much greater homogeneity in perspective. However, we do see evidence of the import of selective disclosure in both the corporate bond and tranching contexts. For example, the evidence in Kronlund highlights that even in the corporate bond context (where the potential for selective disclosure is more limited) that the market's pricing reflects the extent of the potential for ratings shopping and the anticipation of shopping. This evidence suggests the advantage for empirical analysis of utilizing a rational framework, rather than one based upon naïve or myopic pricing (see Bolton, Freixas and Shapiro (2012), Sangiorgi, Sokobin, and Spatt (2009) and Skreta and Veldkamp (2009)). More broadly, the detailed empirical evidence in Kronlund (2011) points to many dimensions in which corporate bond data is consistent with ratings shopping. For example, rating shopping is particularly prevalent for complex instruments (such as more junior and longer maturity instruments) in which rating agencies are likely to disagree. Furthermore, given the persistence in the views of particular rating agencies about specific types of issues, issuers would be more likely to solicit an agency that had viewed the corresponding issue favorably in the past. Kronlund (2011) also shows that defaults are more likely (fixing the rating) when the instrument is rated by an agency that rated it highly in the past and that market yields adjust accordingly.

The tranches underlying structured financing are considerably more complex than standard corporate bonds, so the potential for rating shopping is particularly great within the structured finance market (e.g., Skreta and Veldkamp (2009)). Benmelech and Dlugosz (2009, 2010) highlight the collapse of structured finance credit ratings during the financial crisis. They examine the impact of the number of rating agencies that rate an instrument on the subsequent likelihood of a downgrade. Tranches that are rated only by a single agency are most likely to be downgraded and have relatively larger ratings declines (suggesting greater inflation of those ratings compared to multiple ratings at a level).[24] Our analysis highlights (see Section 4.3, Figure 5 and Section 5.2 as well as related discussion in the Introduction) that in our opaque equilibrium that when a single rating is published it is upward biased.

The type of analysis we undertake in this paper also is particularly relevant for understanding multiple and split ratings. The information content in ratings reflects not only the ratings selected for publication and disclosure, but also indicative ratings (even though unobservable) that are not selected (also discussed in Sangiorgi, Sokobin and Spatt (2009)). For example, our analysis suggests that at the highest rating obtained, the larger the number of these ratings the more favorable the information content as it implies the ability to obtain several relatively favorable ratings as well as the presence of fewer unobservable ratings at lower levels. Analogously, a similar analysis can apply to split ratings. For example, in our opaque equilibrium the observation of a single rating would be less favorable than two ratings at the same level. This also suggests the adverse nature of the absence of ratings (e.g., unrated securities), especially when the costs of ratings are very low.[25] Unless the costs are especially high, unrated instruments are likely to reflect especially adverse information.[26] More generally, these types of models highlight the information content of published ratings at various levels.

The precise empirical implication involved between observing one and two ratings at the same level can be considered formally from our threshold equilibrium. If we compare situations in which *only* one rating is published with value S_p vs. both ratings being published with an average value equal to S_p, then, under conditions derived in Appendix F, we obtain

$$\mathrm{E}\left(X \middle| S_p \right) < \mathrm{E}\left(X \middle| \frac{S_1 + S_2}{2} = S_p \right).$$

This condition implies that, controlling for the level of ratings, a lower number of ratings should predict higher default probabilities and/or future downgrades. This is similar to the result in Benmelech and Dlugosz (2010), except that they do not control for the level of the rating.

Our formal analysis does highlight two reasons why issuers might want to publish multiple ratings,

[24] Recent evidence for CDOs that leads to differing conclusions is provided by Griffin, Nickerson and Tang (2012). They find that CDOs that are rated by a larger set of rating agencies are more likely to default, which is not consistent with our rating shopping perspective. Their evidence does highlight that the extent of rating agency model failures is enhanced by competitive ratings pressures.

[25] There is considerable evidence with respect to both multiple ratings and split ratings (e.g., see Bongaerts, Cremers and Goetzmann (2012), Livingston, Naranjo and Zhou (2005), Mattarocci (2005) and Livingston, Wei and Zhou (2010)).

[26] The extent of adverse inference due to the absence of a rating would reflect the cost of a rating and/or the extent of unrated issues.

even absent regulatory requirements to obtain multiple ratings. Because investors are risk averse, additional ratings reduce the required risk premium, offering more precision about the underlying signal.[27] Additionally, to the extent that investors expect the issuer to solicit multiple ratings, absence of publication suggests adverse information, and implies an information discount relative to the issues with public ratings. In fact, the second motive for multiple ratings is valid even in a risk-neutral setting. These motives tie closely to the "information production hypothesis" and "shopping hypothesis" in Bongaerts, Cremers and Goetzmann (2012).

At the heart of our equilibrium in the opaque formulation is the incentive for many parameter values to acquire excess ratings because of the potential benefits from selective disclosure for some realizations. For some parameters the ratings are fully disclosed in equilibrium, so in such situations the potential benefits of ratings shopping would not manifest itself in actual selective disclosure. Indeed, the prevalence of situations in which two and even three ratings are obtained in practice from the three major rating agencies is a central empirical fact, but it does not suggest that ratings shopping is unimportant. Indeed, our model highlights the inefficiency and overproduction of ratings, coupled with the potential considerable profitability of the rating agencies (Figure 4 depicts profitability in our model) when issuers derive valuable benefits from the possibility of selective disclosure. Somewhat surprisingly, our formulation demonstrates that the publication of many ratings is compatible to some degree with ratings shopping.

Finally, the analysis from Section 5 provides novel predictions with respect to the interaction between rating shopping and rating adjustments.[28] Our model predicts a negative relation between model-implied ratings and rating adjustments for ratings that are eventually disclosed. This prediction is consistent with the pattern documented by Griffin and Tang (2012) in the context of CDOs. Griffin and Tang (2012) define a rating adjustment as the difference between the proportion of a CDO-rated AAA in practice and the proportion implied by the RA main quantitative model output, and show how over half of the cross-sectional variation in adjustments is explained by (and negatively related to) the initial model-implied AAA proportion. As the amount of adjustment is shown to be positively related to future downgrades, their evidence is consistent with RAs inflating ratings to a larger extent for CDOs with lower model-implied AAA proportions.

[27] Also, see Skreta and Veldkamp (2009).

[28] In the context of our model, a rating adjustment is defined as the difference between the rating that is eventually assigned to a financial product and the rating that is initially implied by the RA model output.

7. Concluding comments

Our paper uses a model based upon rational expectations to examine conditions under which selective disclosure and ratings bias emerge in equilibrium. We highlight the role of the structure of equilibrium and regulatory policy about disclosure of contacts with rating agencies to purchase ratings. For example, under some conditions requiring the disclosure of the existence of indicative ratings may be equivalent to requiring disclosure of the indicative ratings themselves, eliminating ratings bias in equilibrium. In the absence of requiring disclosure of the contacts about indicative ratings (the opaque analysis), ratings bias can emerge even under rational expectations.

The focus in our paper on opacity and the structure of equilibrium also is relevant for a broader range of applications beyond our focus on credit rating agencies. For example, consider the situation in which "test takers" can request that only their highest score on a particular subject or exam be reported to outsiders. This would lead to considerable "excess" testing and the ability of the testing body or institution to extract additional rents from that process.[29] More generally, the endogenous structure of economic activity is an important potential source of information asymmetry that influences the analysis of institutional arrangements.

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Appendices

Appendix A: Notation and definitions

Conditional moments. Let $X \sim N(\mu_X, \sigma_X^2)$ and $S_i = X + \varepsilon_i$, with $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ for $i = 1, 2$. Let X, ε_1 and ε_2 be uncorrelated. Define

$$\mu(y) := \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + \sigma_\varepsilon^{-2}}(y - \mu_X); \quad \mu(y,x) := \mu_X + \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}}\left[(y - \mu_X) + (x - \mu_X)\right],$$

and let

$$\sigma_S^2 := \mathrm{Var}\left(S_i\right); \quad \sigma_{X|S}^2 := \mathrm{Var}\left(X|\, S_i\right); \quad \sigma_{X|2S}^2 := \mathrm{Var}\left(X|\, S_1, S_2\right); \quad \sigma_{S|S}^2 := \mathrm{Var}\left(S_i|\, S_{-i}\right).$$

Then, we have the following standard results

$$\begin{aligned} \mathrm{E}\left(X|\, S_i\right) &= \mu(S_i); & \sigma_{X|S}^2 &= \left(\sigma_X^{-2} + \sigma_\varepsilon^{-2}\right)^{-1}, \\ \mathrm{E}\left(X|\, S_1, S_2\right) &= \mu(S_1, S_2); & \sigma_{X|2S}^2 &= \left(\sigma_X^{-2} + 2\sigma_\varepsilon^{-2}\right)^{-1}. \\ \mathrm{E}\left(S_i|\, S_{-i}\right) &= \mu(S_{-i}); & \sigma_{S|S}^2 &= \sigma_{X|S}^2 + \sigma_\varepsilon^2. \end{aligned}$$

Equilibrium prices under full disclosure. In this CARA-normal framework, the (full disclosure) asset price conditional on, respectively, zero, one and two ratings being published is

$$p_0 = \mu_X - r\sigma_X^2 \tag{A1}$$
$$p(S_i) = \mu(S_i) - r\sigma_{X|S}^2 \tag{A2}$$
$$p(S_1, S_2) = \mu(S_1, S_2) - r\sigma_{X|2S}^2 \tag{A3}$$

Issuer's strategy profiles and further notation. In the proofs for Section 4 (Appendix C and Appendix D), the strategy profiles for the issuer upon which we focus are denoted as follows. We denote with γ_0 the strategy of selling the asset without purchasing any rating and with $\gamma_{\bar{S}}$ the threshold strategy of Definition 1 for a given threshold $\bar{S}$, and let $\gamma_\infty := \lim_{\bar{S}\uparrow\infty} \gamma_{\bar{S}}$. For the case of asymmetric fees, we denote with γ_i^Θ the strategy in which the issuer is purchases rating i first, and then: if S_i is in some open set $\Theta \subseteq \mathbb{R}$ the issuer discloses S_i and stops; if $S_i \notin \Theta$ the issuer purchases also rating $-i$ and discloses both. Note that this definition nests the cases in which both ratings are purchased and disclosed (γ_i^Θ for $\Theta = \emptyset$), and in which only rating i is purchased and disclosed (γ_i^Θ for $\Theta = \mathbb{R}$). Finally, we denote with $\Pi(\gamma)$ the issuer's ex-ante expected profits under γ net of the ex-ante value of the outside option, $\mu_X - V$.

Appendix B: Proofs for Section 3.

Proof of Lemma 2. From Lemma 1, purchased ratings are always disclosed in the transparent market. Anticipating full disclosure of purchased ratings, we solve backwards for the decision of how many ratings to purchase. If the issuer has purchased two ratings, final payoffs equal $p(S_1, S_2) - 2c$. One step back, if the issuer has purchased one rating, say, S_1, then she can either disclose the rating and stop, in which case payoffs equal $p(S_1) - c$, or purchase the second rating. Expected payoffs from purchasing the second rating, conditional on S_1 equal

$$\begin{aligned} \mathrm{E}\left(p(S_1, S_2) - 2c \mid S_1\right) &= \mathrm{E}\left(\mathrm{E}\left(X \mid S_1, S_2\right) \mid S_1\right) - r\sigma^2_{X|2S} - 2c \\ &= \mathrm{E}\left(X \mid S_1\right) - r\sigma^2_{X|2S} - 2c, \end{aligned}$$

The issuer will purchase the second rating if and only if

$$\mathrm{E}\left(X \mid S_1\right) - r\sigma^2_{X|2S} - 2c \geq \mathrm{E}\left(X \mid S_1\right) - r\sigma^2_{X|S} - c \Leftrightarrow c \leq r\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right) = c_l.$$

In other words, conditional on having purchased a first rating, the issuer will purchase the second rating iff $c \leq c_l$, regardless of the value of the first rating. Anticipating this, and going back to the decision of purchasing the first rating, the issuer will compare payoffs from selling the asset without having purchased any rating, p_0, to the expected payoffs from selling the asset having purchased and disclosed one rating (if $c > c_l$), or two ratings (if $c \leq c_l$). Therefore, if $c > c_l$, the issuer will purchase the first rating iff

$$p_0 \leq \mathrm{E}\left[p(S_i) - c\right] \Leftrightarrow \mu_X - r\sigma^2_X \leq \mu_X - r\sigma^2_{X|S} - c \Leftrightarrow c \leq r\left(\sigma^2_X - \sigma^2_{X|S}\right) = c_h,$$

and if $c \leq c_l$, the issuer will purchase the first rating iff

$$p_0 \leq \mathrm{E}\left[p(S_1, S_2) - 2c\right] \Leftrightarrow \mu_X - r\sigma^2_X \leq \mu_X - r\sigma^2_{X|2S} - 2c \Leftrightarrow c \leq \frac{1}{2}r\left(\sigma^2_X - \sigma^2_{X|2S}\right).$$

Simple algebra shows

$$\sigma^2_{X|S} - \sigma^2_{X|2S} < \frac{1}{2}\left(\sigma^2_X - \sigma^2_{X|2S}\right) < \sigma^2_X - \sigma^2_{X|S}, \tag{B1}$$

so the issuer will purchase zero ratings if $c > c_h$, one (and only one) rating if $c_l < c \leq c_h$, and both ratings if $c \leq c_l$.

The exact same procedure as above can be used to derive the issuer's demand for ratings in the case of asymmetric fees, which is described in the following lemma.

Lemma B1. *Let* $c_1 \neq c_2$. *For* $\max\{c_1, c_2\} \leq c_l$ *the issuer purchases both ratings; for* $\min\{c_1, c_2\} \leq c_h$ *and* $\max\{c_1, c_2\} > c_l$ *the issuer purchases only the cheaper rating, and for* $\min\{c_1, c_2\} > c_h$ *zero ratings are purchased.* ■

Proof of Proposition 1. We look for the symmetric Nash equilibrium in the fee-setting game. For any pair of fees such that $c_1 = c_2 = c$, each RA's expected profits are obtained anticipating the issuer's demand for ratings from Lemma 2. If a RA deviates to $c' \neq c$ such that $c' \geq H$, its expected profits are are obtained anticipating the issuer's demand with asymmetric fees as described in Lemma B1. We distinguish three cases depending on the value of the ratings' production cost H.

1. Let $H < c_l$. Any $c \in [H, c_l)$ is not an equilibrium as a RA can increase profits by deviating to $c + \varepsilon < c_l$. Any $c \in (c_l, c_h)$ is not an equilibrium either: only one rating is purchased and each RA expects $c/2$. A RA

can increase profits by undercutting to $c - \varepsilon$ with ε sufficiently small. A similar argument holds for $c \geq c_h$. Instead, at $c = c_l$ both ratings are purchased and no deviation can be profitable as a more expensive rating is not purchased.

2. Let $c_l \leq H < c_h$. Any $c \in (H, c_h]$ cannot be an equilibrium: only one rating is purchased and each RA expects $c/2$. A RA can increase profits by undercutting to $c - \varepsilon$ with $\varepsilon > 0$ sufficiently small. A similar argument holds for $c \geq c_h$. For $c = H$, RAs have no incentive to deviate as more expensive ratings would not be purchased.

3. Let $H \geq c_h$. Whenever $c_i \geq c_h$ for $i = 1, 2$, the issuer sells the asset without purchasing any rating. Hence, we assume that RAs set $c = H$.

With full disclosure $\mathcal{I}^{inv} = \mathcal{I}^I$, so that

$$\mathrm{E}\left(X|\mathcal{I}^I\right) - V = \mathrm{E}\left(X|\mathcal{I}^{inv}\right) - \Delta - r\sigma_X^2 < \mathrm{E}\left(X|\mathcal{I}^{inv}\right) - r\mathrm{Var}\left(X|\mathcal{I}^{inv}\right),$$

which implies the PC is satisfied in all states conditional on ratings being purchased. By construction, the issuer's ex-ante expected profits with endogenous fees are greater or equal than p_0, implying that the PC is always satisfied as a strict inequality ex-ante. Finally, by the definition of Ω in Eq. (4) and (B1), it is immediate to verify that for all $r \geq 0$,

$$\underset{n \in \{0,1,2\}}{\arg\max}\; r\left(\sigma_X^2 - \mathrm{Var}\left(X|\{S_i\}_{i=0}^n\right)\right) - n \times H = \begin{cases} 0 & \text{for } H \geq c_h \\ 1 & \text{for } H \in [c_l, c_h] \\ 2 & \text{for } H \leq c_l \end{cases}.$$

Hence, in the equilibrium of Proposition 1 the value of information Ω is maximized. ■

Appendix C: Proofs for Section 4.1.

Proof of Eq. (8). We are looking for the value of the fee c such that

$$\mathrm{E}\left[\max\{p(S_i), p_0\}\right] - c \leq p_0.$$

From Eq. (A1)-(A2), $p(S_i) > p_0 \Leftrightarrow S_i > p_0$. Therefore, c has to satisfy

$$c \geq \int_{p_0}^{\infty} \left[p(S_i) - p_0\right] \frac{1}{\sigma_S} f\left(\frac{S_i - \mu}{\sigma_S}\right) dS_i,$$

which by direct computation gives

$$c \geq \left(\sigma_X^2 - \sigma_{X|S}^2\right)^{1/2} g\left(r\sqrt{\sigma_X^2 - \sigma_{X|S}^2}\right) =: \hat{c}_h. \quad ■$$

Proof of Proposition 2. Outline. The proof is divided in seven steps that follow closely the derivation in Section 4.1 by giving details omitted in the text. Throughout this proof, intermediate results are stated and proved in Claim C1 to Claim C6, Lemma C1 and Lemma C2. We start from the threshold strategy equilibrium

of Definition 1. As a first step, we derive the price function $p_{\bar{S}}(\cdot)$ taking the strategy of the issuer as given. As in the main text, in the following we relabel ratings so that S_1 is the first rating that ends up being purchased.
Proof of Eqs. (9)-(11). Let A denote the event that is associated with selective disclosure, that is, $S_1 < s(S_2) \cap S_2 \geq \bar{S}$. Let B denote the event in which only one rating is published with value greater than $\bar{S}$, so that $\Pr(B)$ $=\Pr(A \cup S_1 \geq \bar{S})$ and $\Pr(B \cap A)$ $=\Pr(A)$. Defining $q_{(S_p)} :=\Pr(A|\, B \cap S_p)$, we compute

$$q_{(S_p)} = \frac{\Pr(A) \times \Pr(B|\, A) \times f_{S_p}(S_p|\, A \cap B)}{\Pr(B \cap A) \times f_{S_p}(S_p|\, B \cap A) + \Pr(B \cap A^C) \times f_{S_p}(S_p|\, B \cap A^C)},$$

where A^C denotes the complement of A, $\Pr(B|\, A) = 1$ and

$$\begin{aligned}
f_{S_p}(S_p|\, B \cap A) &= \frac{1}{\Pr(B \cap A)} \frac{1}{\sigma_S} f\left(\frac{S_p - \mu}{\sigma_S}\right) \Phi\left(\frac{s(S_p) - \mu(S_p)}{\sigma_{S|S}}\right) \mathbb{I}_{\{S_p \geq \bar{S}\}} \\
f_{S_p}\left(S_p|\, B \cap A^C\right) &= \frac{1}{\Pr(B \cap A^C)} \frac{1}{\sigma_S} f\left(\frac{S_p - \mu}{\sigma_S}\right) \mathbb{I}_{\{S_p \geq \bar{S}\}},
\end{aligned}$$

where $\mathbb{I}$ denotes the indicator function. Therefore, $q_{(S_p)}$ simplifies to

$$q_{(S_p)} = \frac{\Phi\left(\frac{s(S_p)-\mu(S_p)}{\sigma_{S|S}}\right)}{\left[\Phi\left(\frac{s(S_p)-\mu(S_p)}{\sigma_{S|S}}\right) + 1\right]} \mathbb{I}_{\{S_p \geq \bar{S}\}},$$

and the conditional density of X given $S_p \geq \bar{S}$ is given by

$$f_X(X|\, B \cap S_p) = q_{(S_p)} \times f_X(X|\, S_1 \leq s(S_p) \cap S_2 = S_p) + \left(1 - q_{(S_p)}\right) \times f_X(X|\, S_1 = S_p), \tag{C1}$$

where

$$\begin{aligned}
f_X(X|\, S_1 \leq s(S_p) \cap S_2 = S_p) &= \frac{1}{\sigma_{X|S}} f\left(\frac{X - \mu(S_p)}{\sigma_{X|S}}\right) \frac{\Phi\left(\frac{s(S_p)-X}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{s(S_p)-\mu(S_p)}{\sigma_{S|S}}\right)} \\
f_X(X|\, S_1 = S_p) &= \frac{1}{\sigma_{X|S}} f\left(\frac{X - \mu(S_p)}{\sigma_{X|S}}\right).
\end{aligned}$$

Normalizing initial wealth to zero, the expected utility of an agent with CARA preferences facing price P conditional on observing S_p and demanding θ units of the asset is given by

$$\mathrm{E}\left[u(w)|\, S_p, P\right] = -\int_{-\infty}^{\infty} \exp\left[-r\theta(X - P)\right] \times f_X(X|\, B \cap S_p)\, dX \tag{C2}$$

Solving the integral in (C2) using (C1), we find

$$\mathrm{E}\left[u(w)|\, S_p, P\right] = -\exp\left[-r\theta\left(\mu(S_p) - P - \theta\frac{r\sigma^2_{X|S}}{2}\right)\right] \frac{1 + \Phi\left(\frac{s(S_p)-\mu(S_p)+r\theta\sigma^2_{X|S}}{\sigma_{S|S}}\right)}{1 + \Phi\left(\frac{s(S_p)-\mu(S_p)}{\sigma_{S|S}}\right)}. \tag{C3}$$

Maximizing (C3) with respect to θ, imposing $\theta = 1$, solving for the price P and rearranging we find

$$P = p(S_p) - \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(\frac{s(S_p) - p(S_p)}{\sigma_{S|S}}\right),$$

where $\Gamma(\cdot)$ is given in (11). ■

As a second step we derive the function $s(\cdot)$ that makes the disclosure rule in part (1.b) optimal for an exogenous threshold $\bar{S}$. The analysis that follows makes use of the next claim.

Claim C1. There exists a unique value $t^* < 0$ such that $t^* + \Gamma(t^*) = 0$. Moreover, $\Gamma(t^*)$ is the unique maximizer of $\Gamma(t)$ and $\Gamma'(t) \geq 0 \Leftrightarrow t \leq t^*$

Proof of Claim C1. By the definition of $g(\cdot)$ we have $t^* + \Gamma(t^*) = 0 \Leftrightarrow t^* + g(t^*) = 0$. Existence of t^* follows from continuity of $g(\cdot)$ and $\lim_{t\uparrow\infty} g(t) + t = -\lim_{t\downarrow-\infty} g(t) + t = \infty$; uniqueness follows from $\frac{d}{dt}(t + g(t)) = 1 + \Phi(t) > 0$. As $g(0) > 0$, then $t^* < 0$. As $\Gamma(\cdot)$ is continuous, and $\lim_{t\uparrow\infty} \Gamma(t) = \lim_{t\downarrow-\infty} \Gamma(t) = 0$, and $\Gamma(t) > 0$ for all finite t, and $\Gamma'(t) = -\Gamma(t)[t + \Gamma(t)]$, the second part of the claim follows from the first part. ■

Proof of Eqs. (12)-(14). Let $S_1 < \bar{S}$ and $S_2 \geq \bar{S}$. First we derive S^* and then we verify the conjecture that for all $\bar{S} \leq S^*$ disclosing only S_2 dominates disclosing both ratings. Define $\psi_{\bar{S}}(S_1, S_2)$ as the price difference between disclosing only S_2 over disclosing both S_1 and S_2 when $s(S_2) = \bar{S}$, that is

$$\psi_{\bar{S}}(S_1, S_2) := p(S_2) - \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(\frac{\bar{S} - p(S_2)}{\sigma_{S|S}}\right) - p(S_1, S_2),$$

and define further $t(x, y) := \frac{x - p(y)}{\sigma_{S|S}}$. Some algebra and the definitions from Appendix A show that

$$t(\bar{S}, \bar{S}) = \left(\bar{S} - p_0\right) \frac{\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}}{\sigma^2_X} = \frac{p(\bar{S}, \bar{S}) - p(\bar{S})}{\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}}. \tag{C4}$$

Using the second equality in (C4) we rewrite $\psi_{\bar{S}}(\bar{S}, \bar{S})$ as

$$\psi_{\bar{S}}(\bar{S}, \bar{S}) = -\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \left[t(\bar{S}, \bar{S}) + \Gamma\left(t(\bar{S}, \bar{S})\right)\right].$$

Given $t(\bar{S}, \bar{S})$ is linear and strictly increasing in $\bar{S}$, by Claim C1 there exists a unique value of $\bar{S}$, call it S^*, such that $\psi_{S^*}(S^*, S^*) = 0$. Using the first equality in (C4) to solve $t^* = t(S^*, S^*)$ for S^* gives Eq. (12) in the text. As $\lim_{t\uparrow\infty} t + \Gamma(t) = -\lim_{t\downarrow-\infty} t + \Gamma(t) = \infty$, Claim C1 implies $t + \Gamma(t) \geq 0 \Leftrightarrow t \geq t^*$, and therefore

$$\psi_{\bar{S}}(\bar{S}, \bar{S}) \geq 0 \Leftrightarrow \bar{S} \leq S^*. \tag{C5}$$

Simple algebra yields

$$\frac{\partial}{\partial S_2} \psi_{\bar{S}}(S_1, S_2) = \frac{\sigma^2_{X|S} - \sigma^2_{X|2S}}{\sigma^2_{\varepsilon}} \left[1 + \Gamma'\left(t(\bar{S}, S_2)\right)\right] > 0, \tag{C6}$$

where the inequality follows from $\Gamma'(t) \geq 0$ for all $t \leq t^*$ and the fact that, for all $S_2 \geq \bar{S}$ and $\bar{S} \leq S^*$ we have

$$t^* = t(S^*, S^*) \geq t(\bar{S}, \bar{S}) \geq t(\bar{S}, S_2).$$

Together, (C5) and (C6) imply that if $s(S_2) = \bar{S}$ and $\bar{S} \leq S^*$, then we have $p_{\bar{S}}(S_2) \geq p(S_2, S_1)$ for all $S_2 \geq \bar{S}$ and $S_1 \leq \bar{S}$, which verifies the conjecture.

Next, we turn to the case $\bar{S} > S^*$. We will prove that for $S_2 \geq \overrightarrow{S}$ from Eq. (13) disclosing S_2 alone dominates disclosing both ratings for all $S_1 \leq \bar{S}$. Using again the conjecture $s(S_2) = \bar{S}$ for $S_2 \geq \overrightarrow{S}$, let $\overrightarrow{S}$ solve $\psi_{\bar{S}}(\bar{S}, \overrightarrow{S}) = 0$, that is,

$$p(\overrightarrow{S}) - \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(\frac{\bar{S} - p(\overrightarrow{S})}{\sigma_{S|S}}\right) = p(\bar{S}, \overrightarrow{S}). \tag{C7}$$

The last equation is equivalent to

$$\frac{\bar{S} - p(\overrightarrow{S})}{\sigma_{S|S}} + \Gamma\left(\frac{\bar{S} - p(\overrightarrow{S})}{\sigma_{S|S}}\right) = 0,$$

so that, by definition of t^*,

$$\frac{\bar{S} - p(\overrightarrow{S})}{\sigma_{S|S}} = \frac{S^* - p(S^*)}{\sigma_{S|S}}.$$

Solving the last equation for $\overrightarrow{S}$ and rearranging gives Eq. (13) in the text. The last equation also implies that for all $S_2 \geq \overrightarrow{S}$ we have $t(S^*, S^*) \geq t(\bar{S}, S_2)$. Therefore (C6) implies $\psi_{\bar{S}}(\bar{S}, S_2) > 0$ for all $S_2 > \overrightarrow{S}$, which verifies the conjecture.

What is left to prove is the conjecture for $s(\cdot)$ given $\bar{S} > S^*$ and $S_2 \in \left[\bar{S}, \overrightarrow{S}\right)$. We start from equating $p_{\bar{S}}(S_2)$ and $p(S_1, S_2)$ when $S_1 = s(S_2)$, that is,

$$p(S_2) - \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(\frac{s(S_2) - p(S_2)}{\sigma_{S|S}}\right) = p(s(S_2), S_2). \tag{C8}$$

Then, given the l.h.s. of Eq. (C8) is not a function of S_1 and the r.h.s. is strictly increasing in S_1, it is optimal to disclose S_2 alone if $S_1 < s(S_2)$, and both ratings otherwise. The last equality can be rearranged as

$$\frac{s(S_2) - p(S_2)}{\sigma_{S|S}} + \Gamma\left(\frac{s(S_2) - p(S_2)}{\sigma_{S|S}}\right) = 0,$$

from which we conclude, using the definition of t^*,

$$\frac{s(S_2) - p(S_2)}{\sigma_{S|S}} = \frac{S^* - p(S^*)}{\sigma_{S|S}}.$$

Solving the last equality for $s(S_2)$ and rearranging gives

$$s(S_2) = \frac{\sigma_\varepsilon^{-2}}{\sigma_X^{-2} + \sigma_\varepsilon^{-2}} (S_2 - S^*) + S^*.$$

Finally, notice that for $\bar{S} > S^*$ the function $s(\cdot)$ is continuous as $\lim_{S_p \uparrow \overrightarrow{S}} s(S_p) = \bar{S}$. This implies the price function $p_{\bar{S}}(\cdot)$ is continuous. ■

Remark C1. For $\bar{S} > S^*$ and $S_p \in \left[\bar{S}, \overrightarrow{S}\right]$ the term $s(S_p) - p(S_p)$ is constant and equal to $S^* - p(S^*)$. In this case the price discount is maximized, that is, $\Gamma\left(\frac{s(S_p)-p(S_p)}{\sigma_{S|S}}\right) = \Gamma(t^*) = \max_t \Gamma(t)$.

Remark C2. As $\Gamma'(t) \geq 0$ for all $t \leq t^*$, $\frac{d}{dS_p} p_{\bar{S}}(S_p) \geq \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}$. In particular,

$$\frac{d}{dS_p} p_{\bar{S}}(S_p) = \begin{cases} \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} + \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon}\Gamma'\left(\frac{\bar{S}-p(S_p)}{\sigma_{S|S}}\right), & \text{for } \bar{S} < S^* \text{or } \left(\bar{S} \geq S^* \text{ and } S_p > \overrightarrow{S}\right) \\ \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}, & \text{for } \bar{S} \geq S^* \text{ and } S_p \in \left[\bar{S}, \overrightarrow{S}\right]. \end{cases}$$

In the following claim we derive a condition on the parameters which will be used in the analysis that follows. Let $\tilde{t} := \arg\max_{t \leq t^*} \Gamma'(t)$ and define

$$\bar{\sigma} := \sqrt{1 + \Gamma'\left(\tilde{t}\right)} - 1 \approx 0.086. \tag{C9}$$

Claim C2: If $\sigma^2_\varepsilon \geq \bar{\sigma}\sigma^2_X$, then $\frac{d}{dS_p} p_{\bar{S}}(S_p) \leq 1$.

Proof of Claim C2. As $\Gamma'\left(\tilde{t}\right) \geq \Gamma'(t)$ for all $t \leq t^*$,

$$\frac{d}{dS_p} p_{\bar{S}}(S_p) \leq \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} + \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon}\Gamma'\left(\tilde{t}\right),$$

and therefore simple algebra shows

$$\frac{d}{dS_p} p_{\bar{S}}(S_p) \leq 1 \Leftrightarrow \sqrt{1 + \Gamma'\left(\tilde{t}\right)} - 1 \leq \frac{\sigma^2_\varepsilon}{\sigma^2_X}. \quad ■$$

As a third step, we derive conditions on the fee c and threshold $\bar{S}$ such that, whenever the realization of the first purchased rating is $S_1 \geq \bar{S}$, the issuer has no incentive to deviate from the conjectured strategy by purchasing the second rating. If the issuer deviates by purchasing the second rating, by disclosing selectively her payoff would be $\max\left\{p_{\bar{S}}(S_1), p_{\bar{S}}(S_2), p(S_1, S_2)\right\}$. Define $\nu(S_1, S_2)$ as the payoff from such deviation in excess of $p_{\bar{S}}(S_1)$, that is,

$$\nu(S_1, S_2) := \max\left\{0, p_{\bar{S}}(S_2) - p_{\bar{S}}(S_1), p(S_1, S_2) - p_{\bar{S}}(S_1)\right\}.$$

Define further $\Pi_d(\bar{S})$ as the issuer's expectation of $\nu(S_1, S_2)$ conditional on S_1 being exactly equal to $\bar{S}$, that is,

$$\Pi_d(\bar{S}) := \mathrm{E}\left(\nu(S_1, S_2) \middle| S_1 = \bar{S}\right).$$

Next lemma shows that $\Pi_d(\bar{S})$ is the upper bound to the issuer's expectation of $\nu(S_1, S_2)$ for all $S_1 \geq \bar{S}$, and that such upper bound is decreasing in the threshold $\bar{S}$.

Lemma C1 *For all $S_1 \geq \bar{S}$, we have*

$$\frac{\partial}{\partial S_1}\mathrm{E}\left(\nu(S_1, S_2) \middle| S_1\right) < 0 \tag{C10}$$

$$\frac{d}{d\bar{S}}\Pi_d(\bar{S}) < 0. \tag{C11}$$

Proof of Lemma C1. We first prove (C10). Let $\nu_n(\cdot,\cdot)$ denote the partial derivative of $\nu(\cdot,\cdot)$ with respect to the n-th argument. As $\nu(\cdot,\cdot)$ and $\nu_1(\cdot,\cdot)$ are continuous (over the relevant intervals), we have

$$\frac{\partial}{\partial S_1}\mathrm{E}\left(\nu(S_1,S_2)|\,S_1\right) = \frac{1}{\sigma_{S|S}}\int_{-\infty}^{\infty}\left[\nu_1(S_1,S_2)f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right)+\nu(S_1,S_2)\frac{\partial}{\partial S_1}f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right)\right]dS_2;$$

using $\frac{\partial}{\partial S_1}f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right) = -\frac{\partial}{\partial S_2}\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right)$ in the last expression, integrating by parts and rearranging gives

$$\frac{\partial}{\partial S_1}\mathrm{E}\left(\nu(S_1,S_2)|\,S_1\right) = \frac{1}{\sigma_{S|S}}\int_{-\infty}^{\infty}\left[\nu_1(S_1,S_2)+\nu_2(S_1,S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\right]f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right)dS_2. \tag{C12}$$

We will show that the term in the square brackets in Eq. (C12) is non positive and strictly negative for an open set of values of S_2. First, consider the case $\nu(S_1,S_2) = p(S_1,S_2) - p_{\bar{S}}(S_1)$. Then, we have

$$\begin{aligned}
\nu_1(S_1,S_2)+\nu_2(S_1,S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} &= \frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon} - p'_{\bar{S}}(S_1) + \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon} \\
&\le \frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon} - \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} + \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon} = 0,
\end{aligned}$$

where the inequality follows by $p'_{\bar{S}}(S_1) \ge \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}$ and the last equality from $\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon} = \frac{\sigma^2_{X|S}-\sigma^2_{X|2S}}{\sigma^2_\varepsilon}$. Second, consider $\nu(S_1,S_2) = p_{\bar{S}}(S_2) - p_{\bar{S}}(S_1)$, which is the case for S_2 high enough. We have

$$\begin{aligned}
\nu_1(S_1,S_2)+\nu_2(S_1,S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} &= -p'_{\bar{S}}(S_1) + p'_{\bar{S}}(S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} \\
&\le \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\left[p'_{\bar{S}}(S_2)-1\right] \le 0,
\end{aligned}$$

where the first inequality follows from $p'_{\bar{S}}(S_1) \ge \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}$ and the second from $p'_{\bar{S}}(S_2) \le 1$ (see Claim C2). As $p'_{\bar{S}}(S_2) < 1$ for $\frac{\bar{S}-p(S_2)}{\sigma_{S|S}} \ne \tilde{t}$, the second inequality is strict for an open set of values of S_2. Next, we prove (C11). The definitions of $s(\cdot)$ and $\overrightarrow{S}$ imply

$$\begin{aligned}
\Pi_d(\bar{S}) &= \int_{s(\bar{S})}^{s^{-1}(\bar{S})}\left[p(\bar{S},S_2)) - p_{\bar{S}}(\bar{S})\right]\frac{1}{\sigma_{S|S}}f\left(\frac{S_2-\mu(\bar{S})}{\sigma_{S|S}}\right)dS_2 + \\
&\quad + \int_{s^{-1}(\bar{S})}^{\infty}\left[p_{\bar{S}}(S_2) - p_{\bar{S}}(\bar{S})\right]\frac{1}{\sigma_{S|S}}f\left(\frac{S_2-\mu(\bar{S})}{\sigma_{S|S}}\right)dS_2,
\end{aligned} \tag{C13}$$

with $s^{-1}(\bar{S}) = \bar{S}$ for $\bar{S} \le S^*$ and $s^{-1}(\bar{S}) = \overrightarrow{S}$ for $\bar{S} > S^*$. Following the same steps that lead to Eq. (C12) we obtain

$$\frac{d}{d\bar{S}}\Pi_d(\bar{S}) = \frac{1}{\sigma_{S|S}}\int_{-\infty}^{\infty}\left[\frac{\partial}{\partial\bar{S}}\nu(\bar{S},S_2)+\nu_2(\bar{S},S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\right] f\left(\frac{S_2-\mu(\bar{S})}{\sigma_{S|S}}\right) dS_2. \tag{C14}$$

We will show that the term in the square brackets in Eq. (C14) is non positive and strictly negative for an open set of values of S_2. First, consider the case in which $\nu(\bar{S},S_2) = p(\bar{S},S_2) - p_{\bar{S}}(\bar{S})$. From Eq. (C13), this implies $s^{-1}(\bar{S}) > s(\bar{S})$ and hence $\bar{S} > S^*$, implying that $s(\bar{S}) - p(\bar{S})$ is not a function of $\bar{S}$ (see Remark C1). Therefore, the corresponding term in the square brackets in Eq. (C14) is

$$\frac{\partial}{\partial\bar{S}}\nu(\bar{S},S_2)+\nu_2(\bar{S},S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} = \frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon} - \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} + \frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon}\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} = 0.$$

Second, consider $\nu(\bar{S},S_2) = p_{\bar{S}}(S_2) - p_{\bar{S}}(\bar{S})$. For this to be the case we must have $p_{\bar{S}}(S_2) > p(\bar{S},S_2)$, therefore $S_2 \geq \overrightarrow{S}$ and hence $s(S_2) = \bar{S}$. Then,

$$\frac{\partial}{\partial\bar{S}}\nu(\bar{S},S_2)+\nu_2(\bar{S},S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} = \frac{\partial}{\partial\bar{S}}p_{\bar{S}}(S_2) - \frac{d}{d\bar{S}}p_{\bar{S}}(\bar{S}) + \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\frac{\partial}{\partial S_2}p_{\bar{S}}(S_2). \tag{C15}$$

It is straightforward to verify that

$$\frac{\partial}{\partial S_2}p_{\bar{S}}(S_2) = \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\left[1-\frac{\partial}{\partial\bar{S}}p_{\bar{S}}(S_2)\right],$$

which we use to substitute for $\frac{\partial}{\partial S_2}p_{\bar{S}}(S_2)$ in (C15) and obtain, after simple manipulations,

$$\begin{aligned}\frac{\partial}{\partial\bar{S}}\nu(\bar{S},S_2)+\nu_2(\bar{S},S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} &= \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\left(\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}-1\right) + \left(\sigma^2_{X|S}-\sigma^2_{X|2S}\right)^{1/2}\frac{d}{d\bar{S}}\Gamma\left(\frac{\bar{S}-p(\bar{S})}{\sigma_{S|S}}\right) + \\ &\quad -\frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon}\Gamma'\left(\frac{\bar{S}-p(S_2)}{\sigma_{S|S}}\right)\left[1-\left(\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\right)^2\right].\end{aligned} \tag{C16}$$

When evaluating Eq. (C16) we have to consider two cases. If $\bar{S} > S^*$, then $s(\bar{S}) - p(\bar{S})$ is a constant function of $\bar{S}$ and therefore

$$\frac{\partial}{\partial\bar{S}}\nu(\bar{S},S_2)+\nu_2(\bar{S},S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} = \frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\left(\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}-1\right) - \frac{\sigma^2_{X|2S}}{\sigma^2_\varepsilon}\Gamma'\left(\frac{\bar{S}-p(S_2)}{\sigma_{S|S}}\right)\left[1-\left(\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\right)^2\right] < 0.$$

If $\bar{S} \leq S^*$, then $s(\bar{S}) - p(\bar{S}) = \bar{S} - p(\bar{S})$ and therefore, after some manipulations, Eq. (C16) simplifies to

$$\frac{\partial}{\partial\bar{S}}\nu(\bar{S},S_2)+\nu_2(\bar{S},S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon} \leq \frac{\sigma^4_{X|S}}{\sigma^2_\varepsilon\sigma^2_X}\left[\frac{\sigma^2_{X|2S}}{\sigma^2_{X|S}}\Gamma'\left(\tilde{t}\right)-1\right] < 0,$$

where the last inequality follows from $\frac{\sigma^2_{X|2S}}{\sigma^2_{X|S}} < 1$ and $\Gamma'\left(\tilde{t}\right) < 1$. ■

Proof of Eq. (15). Lemma C1 implies that for the issuer not to purchase the second rating when $S_1 \geq \bar{S}$ it has to be that the cost of the deviation (the fee for the second rating) does not fall below the expected payoff from the deviation in any state (i.e., for any realization of S_1 such that $S_1 \geq \bar{S}$). Hence, c and $\bar{S}$ need to satisfy

$$c \geq \Pi_d(\bar{S}), \tag{C17}$$

where $\Pi_d(\bar{S})$ is given in Eq. (C13). As $\lim_{\bar{S}\uparrow\infty} \overrightarrow{S} = \infty$, it is immediate to verify that the second term in Eq. (C13) converges to zero as $\bar{S} \uparrow \infty$. Hence, the lower bound to $\Pi_d(\bar{S})$ is obtained as

$$\lim_{\bar{S}\uparrow\infty} \Pi_d(\bar{S}) = \lim_{\bar{S}\uparrow\infty} \int_{s(\bar{S})}^{\overrightarrow{S}} \left[p(S_2, \bar{S}) - p_{\bar{S}}(\bar{S})\right] \frac{1}{\sigma_{S|S}} f\left(\frac{S_2 - \mu(\bar{S})}{\sigma_{S|S}}\right) dS_2.$$

Solving the integral, taking the limit and rearranging gives the lower bound of $\Pi_d(\bar{S})$ as denoted by c_t in Eq. (15),

$$\lim_{\bar{S}\uparrow\infty} \Pi_d(\bar{S}) = \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \; g\left(-t^* + r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) =: c_t. \quad \blacksquare \tag{C18}$$

Claim C3. For each $c > c_t$, there exists a finite threshold, call it $\bar{S}_c$, such that $\Pi_d(\bar{S}_c) = c$. Moreover, S_c is decreasing in c.

Proof of Claim C3 As $\bar{S} \leq S^*$ the interval of integration in the first term in Eq. (C13) is zero, while as $\bar{S} \downarrow -\infty$ the second term in (C13) grows without bound:

$$\lim_{\bar{S}\downarrow-\infty} \Pi_d(\bar{S}) = \lim_{\bar{S}\downarrow-\infty} \mathrm{E}\left(p_{\bar{S}}(S_2)\middle|\, S_1 = \bar{S}\right) - p_{\bar{S}}(\bar{S}) = \lim_{\bar{S}\downarrow-\infty} \frac{\sigma^2_{X|S}}{\sigma^2_{\varepsilon}} \left(\frac{\sigma^2_{X|S}}{\sigma^2_{\varepsilon}} - 1\right) \bar{S} = \infty. \tag{C19}$$

Therefore, (C18)-(C19) and (C11) imply that $\Pi_d(\bar{S})$ is strictly decreasing in $\bar{S}$, unbounded above and bounded below by c_t. As $\Pi_d(\bar{S})$ is continuous, for all $c > c_t$ there exists a unique value of the threshold, which we denote S_c, such that

$$c = \Pi_d(S_c). \tag{C20}$$

Applying the implicit function theorem to Eq. (C20) and using (C11) shows that S_c decreases in c. $\blacksquare$

Remark C3. The price function $p_{\bar{S}}(\cdot)$ was derived assuming the issuer randomizes over the two ratings with equal probability. Given symmetry in costs and precision, it is indeed optimal for the issuer to do so.

As a fourth step, we rank issuer's ex-ante expected profits corresponding to $\gamma_{\bar{S}}$ and γ_0 for different values of the fee and prove conditions under which γ_0 is an equilibrium strategy.

Claim C4. *i)* $\frac{d\Pi(\gamma_{\bar{S}})}{d\bar{S}} < 0$ for all $c \geq c_t$; *ii)* $\Pi(\gamma_{\bar{S}}) < \Pi(\gamma_0)$ for all $c \geq \hat{c}_h$.

Proof of Claim C4 Part *i)*. Some manipulations show that the expression for the issuer's expected profits in the threshold equilibrium, $\Pi(\gamma_{\bar{S}})$, (the r.h.s. of the inequality 16 in the text net of $\mu_X - V$) simplifies to

$$\begin{aligned}\Pi(\gamma_{\bar{S}}) \;&=\; V - r\sigma^2_{X|2S} - c\left[1 + \Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right] + \\ &\quad + \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \int_{\bar{S}}^{\infty} \rho\left(\frac{s(S_p) - p(S_p)}{\sigma_{S|S}}, r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) \frac{1}{\sigma_s} f\left(\frac{S_p-\mu_X}{\sigma_s}\right) dS_p,\end{aligned}$$

where the function ρ is defined for $t \leq t^*$ and $\varepsilon \geq 0$ as

$$\rho(t,\varepsilon) := t - \varepsilon + g(t-\varepsilon) - [t + g(t)]\,\frac{1+\Phi\left(t-\varepsilon\right)}{1+\Phi\left(t\right)}.$$

Differentiating and simplifying, we obtain

$$\begin{aligned}\frac{\partial \Pi(\gamma_{\bar{S}})}{\partial \bar{S}} &= -\frac{1}{\sigma_S} f\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\left[c + \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \rho\left(\frac{\bar{S}-p(\bar{S})}{\sigma_{S|S}}, r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right)\right] + \\ &\quad + \sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}} \int_{\bar{S}}^{\infty} \frac{1}{\sigma_{S|S}} \frac{\partial}{\partial t}\rho\left(\frac{\bar{S}-p(S_p)}{\sigma_{S|S}}, r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right)\frac{1}{\sigma_S} f\left(\frac{S_p-\mu_X}{\sigma_s}\right) dS_p.\end{aligned}$$

Some simple manipulations and the definition of $\Gamma(t)$ show that

$$\frac{\partial}{\partial t}\rho(t,\varepsilon) = \left(\Gamma(t) + t\right)\left(1+\Phi\left(t-\varepsilon\right)\right)\left(\Gamma(t) - \Gamma(t-\varepsilon)\right) \leq 0,$$

where the inequality follows from $\Gamma'(t) \geq 0$ and $\Gamma(t) + t \leq 0$ for all $t \leq t^*$ (Claim C1). Therefore,

$$\rho(t,\varepsilon) \geq \rho(t^*,\varepsilon) = t^* - \varepsilon + g(t^* - \varepsilon). \tag{C21}$$

Using (C21) and the fact that $c \geq c_t$ as defined in (C18), we have

$$\begin{aligned}\frac{\partial \Pi(\gamma_{\bar{S}})}{\partial \bar{S}} &\leq -\frac{1}{\sigma_S} f\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}\left[g\left(-t^* + r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) + \right. \\ &\quad \left. -\left(-t^* + r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) + g\left(t^* - r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right)\right] < 0,\end{aligned}$$

where the last inequality follows by $g(x) > x$ and $g(x) > 0$.

Part *ii)*. We distinguish two cases. If $c \geq \max\{\hat{c}_h, c_t\}$, the statement follows from

$$\Pi(\gamma_{\bar{S}}) \leq \lim_{\bar{S}\downarrow -\infty} \Pi(\gamma_{\bar{S}}) = V - r\sigma^2_{X|S} - c < V - r\sigma^2_X = \Pi(\gamma_0),$$

where the first inequality is implied by part i) and the second inequality by $c \geq \hat{c}_h > c_h$. If $c_t > \hat{c}_h$ and $c \in [\hat{c}_h, c_t]$ we have $\bar{S} \uparrow \infty$ and therefore,

$$\Pi(\gamma_{\bar{S}}) = V - r\sigma^2_{X|2S} - 2c < V - r\sigma^2_X = \Pi(\gamma_0),$$

where the inequality follows by $c \geq \hat{c}_h > c_h$. ■

Claim C5. γ_0 is an equilibrium if and only if $c \geq \hat{c}_h$. This equilibrium is supported by full-disclosure off-equilibrium beliefs (i.e., that if only one rating is disclosed, only one rating was purchased).

Proof of Claim C5. The necessity part of the claim follows by construction of $\hat{c}_h$. Here we prove sufficiency by showing that, conditional on having deviated from γ_0 by purchasing a first rating, the issuer would never purchase a second rating if $c \geq \hat{c}_h$. We start by showing that the net expected profits of purchasing the second rating are maximized when the value of the first purchased rating, S_1, equals p_0. Note that if only one rating is published, under the assumed off-equilibrium beliefs, the asset is priced assuming full disclosure. Hence, having purchased S_1, the issuer will not purchase the second rating if

$$\mathrm{E}\left(\max\left\{p_0, p(S_1), p(S_2), p(S_1,S_2)\right\}\middle| S_1\right) - c \leq \max\left\{p_0, p(S_1)\right\}. \tag{C22}$$

If $S_1 \geq p_0$, then $p(S_1) \geq p_0$ and therefore (C22) can be rearranged as

$$c \geq \mathrm{E}\left(\varpi_H(S_1,S_2)\middle| S_1\right), \tag{C23}$$

where

$$\varpi_H(S_1,S_2) = \max\left\{0, p(S_2) - p(S_1), p(S_1,S_2) - p(S_1)\right\}.$$

Then, following similar steps as in the proof of Lemma C1, we obtain

$$\frac{\partial}{\partial S_1}\mathrm{E}\left(\varpi_H(S_1,S_2)\middle| S_1\right) = \int_{-\infty}^{\infty}\left[\frac{\partial}{\partial S_1}\varpi_H(S_1,S_2) + \frac{\partial}{\partial S_2}\varpi_H(S_1,S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\right]\frac{1}{\sigma_{S|S}}f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right)dS_2,$$

and simple algebra shows that the term in the square brackets in the last expression is negative, implying that the r.h.s. of (C23) is maximized for $S_1 = p_0$. If $S_1 \leq p_0$, then $p(S_1) \leq p_0$ and therefore (C22) can be rearranged as

$$c \geq \mathrm{E}\left(\varpi_L(S_1,S_2)\middle| S_1\right), \tag{C24}$$

where

$$\varpi_L(S_1,S_2) = \max\left\{0, p(S_2) - p_0, p(S_1,S_2) - p0\right\}.$$

Then, we obtain

$$\frac{\partial}{\partial S_1}\mathrm{E}\left(\varpi_L(S_1,S_2)\middle| S_1\right) = \int_{-\infty}^{\infty}\left[\frac{\partial}{\partial S_1}\varpi_L(S_1,S_2) + \frac{\partial}{\partial S_2}\varpi_L(S_1,S_2)\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}\right]\frac{1}{\sigma_{S|S}}f\left(\frac{S_2-\mu(S_1)}{\sigma_{S|S}}\right)dS_2,$$

and simple algebra shows that the term in the square brackets in the last expression is positive, implying that the r.h.s. of (C24) is maximized for $S_1 = p_0$. When $S_1 = p_0$, by comparing p_0, $p(S_1,S_2)$ and $p(S_2)$ in Eq. (A1)-(A3), it is immediate to verify that

$$\varpi_H(p_0,S_2) = \varpi_L(p_0,S_2) = \max\left\{0, p(S_2) - p_0\right\}.$$

Hence, the maximum expected benefit from purchasing the second rating can be computed as

$$\begin{aligned}\mathrm{E}\left(\max\left\{0, p(S_2) - p_0\right\}\middle| S_1 = p_0\right) &= \int_{p_0}^{\infty}\left[p(S_2) - p_0\right]\frac{1}{\sigma_{S|S}}f\left(\frac{S_2-\mu(p_0)}{\sigma_{S|S}}\right)dS_2 \\ &= \sigma_{S|S}\frac{\sigma^2_{X|S}}{\sigma^2_\varepsilon}g\left(r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) =: c^*,\end{aligned}$$

so that the issuer will never purchase the second rating if $c \geq c^*$. It is immediate to verify that $c^* < \hat{c}_h$. Hence, for $c \geq \hat{c}_h$ the issuer would never purchase the second rating if a first rating is purchased. Anticipating this, from the derivation of $\hat{c}_h$ it follows that γ_0 is an equilibrium strategy if $c \geq \hat{c}_h$. ■

As a fifth step, we present a refinement of off-equilibrium beliefs that supports the equilibrium of Proposition 2.

Off-equilibrium beliefs and equilibrium selection. Our analysis of the opaque regime with symmetric fees uncovers the following candidate equilibrium strategies. Worst case beliefs on potentially undisclosed ratings support any $\gamma_{\bar{S}}$ such that $\bar{S} \geq \bar{S}_c$ for $c > c_t$ and support γ_∞ for $c \leq c_t$. Full-disclosure off-equilibrium beliefs support γ_0 for $c \geq \hat{c}_h$ (Claim C5). For each such γ, denote with Ψ_γ the set of investors' information sets that are reached with positive probability and with β_γ the corresponding off-equilibrium beliefs. Denote with Υ_c the subset of these strategies that can be supported as an equilibrium when the fees equal c, i.e., γ_∞ for all c, $\gamma_{\bar{S}}$ such that $\infty > \bar{S} \geq \bar{S}_c$ for $c > c_t$ and γ_0 for $c \geq \hat{c}_h$. The refinement of off-equilibrium beliefs, $\hat{\beta}_\gamma$, is specified as follows.

For a given strategy $\gamma \in \Upsilon_c$, assume an off-equilibrium information set $\mathcal{I}^{inv} \notin \Psi_\gamma$ is reached, and consider the set of strategies $\Upsilon'_c \subset \Upsilon_c$ such that i) $\mathcal{I}^{inv} \in \Psi_{\gamma'}$ for all $\gamma' \in \Upsilon'_c$ and ii) $\Pi(\gamma') > \Pi(\gamma)$ for all $\gamma' \in \Upsilon'_c$. Then, either: *a*) Υ'_c is empty, in which case $\hat{\beta}_\gamma = \beta_\gamma$, or *b*) Υ'_c is non-empty, in which case investors interpret $\mathcal{I}^{inv}$ as the issuer playing γ^* such that $\gamma^* = \arg\max_{\gamma' \in \Upsilon'_c} \Pi(\gamma)$. Under $\hat{\beta}_\gamma$, part ii) in Claim C4 implies that the issuer would deviate from any $\gamma_{\bar{S}}$ by not purchasing any rating for $c \geq \hat{c}_h$. This pins down γ_0 as the only equilibrium strategy for $c \geq \hat{c}_h$, supported by full-disclosure off-equilibrium beliefs. Under $\hat{\beta}_\gamma$, part i) in Claim C4 implies for all $\bar{S} > \bar{S}_c$ there are states in which the issuer would deviate from $\gamma_{\bar{S}}$ (for instance, for $S_1 \in [\bar{S}_c, \bar{S})$ and S_2 low enough the issuer would disclose only S_1). This pins down $\bar{S}_c$ as the unique equilibrium threshold for $c_t < \hat{c}_h$ and $c \in (c_t, \hat{c}_h)$, supported by worst case off-equilibrium beliefs. If $c \leq c_t < \hat{c}_h$ or $c < \hat{c}_h \leq c_t$, the set Υ_c is only comprised of γ_∞, which is supported by worst case off-equilibrium beliefs.

As a sixth step, we prove that the PC is always satisfied if it is satisfied ex-ante.

Claim C6. If the PC is satisfied ex-ante whenever the issuer follows the threshold strategy in Proposition 2, then it is satisfied in all states along the equilibrium path.

Proof of Claim C6. Ex-ante, the PC reads $\Pi(\gamma_{\bar{S}}) \geq 0$, that is,

$$\mu_X - V \leq \mathrm{E}\left[p_{\bar{S}}(S_p)\mathbb{I}_{\{B\}} + p(S_1,S_2)\left(1 - \mathbb{I}_{\{B\}}\right)\right] - c\left[1 + \Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right] \tag{C25}$$

where B denotes the set of states in which only one rating is published. We note that the price $p_{\bar{S}}(\cdot)$ can be written as

$$p_{\bar{S}}(S_p) = \mathrm{E}\left[X \mid B \cap S_p\right] - r\sigma^2_{X|S} - \Lambda(S_p),$$

where $\Lambda(S_p)$ is given by

$$\Lambda(S_p) = \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}\left[\Gamma\left(\frac{s(S_p) - p(S_p)}{\sigma_{S|S}}\right) - \Gamma\left(\frac{s(S_p) - \mu(S_p)}{\sigma_{S|S}}\right)\right]$$

and $\Gamma'(t) \geq 0$ for $t \leq t^*$ implies $\Lambda(S_p) \geq 0$. Therefore, the PC (C25) can be written as

$$V \geq r\left[\sigma^2_{X|S}\Pr(B) + \sigma^2_{X|2S}(1 - \Pr(B))\right] + c\left[1 + \Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right] + \mathrm{E}\left[\Lambda(S_p)\mathbb{I}_{\{\omega \in B\}}\right]. \tag{C26}$$

When the issuer has purchased a first rating, we need to distinguish two cases depending on S_1 being above or below $\bar{S}$. In the latter case, the PC is satisfied if the outside option is not greater than the continuation payoff, that is

$$\mu(S_1) - V \leq -c + \mathrm{E}\left[\max\{p_{\bar{S}}(S_2), p(S_1,S_2)\} \mid S_1\right]$$

$$\therefore \qquad V \geq c + \mu(S_1) - \mathrm{E}\left[\max\{p_{\bar{S}}(S_2), p(S_1, S_2)\}|\, S_1\right] \tag{C27}$$

Notice that

$$\mathrm{E}\left[\max\{p_{\bar{S}}(S_2), p(S_1, S_2)\}|\, S_1\right] \geq \mathrm{E}\left[p(S_1, S_2)|\, S_1\right] = \mu(S_1) - r\sigma^2_{X|2S},$$

and therefore (C27) is satisfied for all $S_1 < \bar{S}$ if

$$V \geq c + r\sigma^2_{X|2S}. \tag{C28}$$

It is immediate to verify that if V is such that the ex-ante PC in (C26) holds, then (C28) also holds. If, instead, $S_1 \geq \bar{S}$, the PC is satisfied if

$$\mu(S_1) - V \leq p_{\bar{S}}(S_1). \tag{C29}$$

Rearranging, the PC is satisfied for all $S_1 \geq \bar{S}$ if

$$V \geq \max_{S_1 \geq \bar{S}} r\sigma^2_{X|S} + \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(\frac{s(S_1) - p(S_1)}{\sigma_{\,S|S}}\right) = r\sigma^2_{X|S} + \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(t^*\right) \tag{C30}$$

If the ex-ante PC in (C26) holds, then

$$\begin{aligned} V \quad &\geq \quad r\left[\sigma^2_{X|S}\Pr(B) + \sigma^2_{X|2S}(1 - \Pr(B))\right] + c\left[1 + \Phi\left(\frac{\bar{S} - \mu_X}{\sigma_s}\right)\right] \\ &= \quad c_l(\Pr(B) - 1) + r\sigma^2_{X|S} + c\left[1 + \Phi\left(\frac{\bar{S} - \mu_X}{\sigma_s}\right)\right]; \end{aligned} \tag{C31}$$

given $(1 - \Pr(B)) \leq \Phi\left(\frac{\bar{S} - \mu_X}{\sigma_s}\right)$ and $c > c_t > c_l$, then (C31) implies

$$V > r\sigma^2_{X|S} + \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} g\left(-t^* + r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right),$$

and therefore the ex-ante PC in (C26) implies (C30) if

$$g\left(-t^* + r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) > \Gamma\left(t^*\right).$$

Given that $\Gamma\left(t^*\right) + t^* = 0$, we are left with

$$g\left(-t^* + r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) > -t^*,$$

which is satisfied as g is increasing and $g(y) > y$. Finally, if the issuer has purchased both ratings, from Eq. (A3) and $V > r\sigma^2_X$, we have that for all S_1, S_2

$$p(S_1, S_2) > \mu(S_1, S_2) - V,$$

which implies that the PC is satisfied in all states. ∎

As a seventh and last step we describe the equilibrium with asymmetric fees. This is done in the following lemma, which assumes the PC is satisfied ex-ante.

Lemma C2. *Let $c_1 \neq c_2$. There exist off-equilibrium beliefs and a value $\hat{c}_l < \hat{c}_h$ such that: i) for $\max\{c_1, c_2\} < \hat{c}_l$, the issuer purchases both ratings; ii) for $\min\{c_1, c_2\} < \hat{c}_h$ and $\max\{c_1, c_2\} \geq \hat{c}_l$ the issuer purchases only the cheaper rating, and iii) for $\min\{c_1, c_2\} \geq \hat{c}_h$ no rating is purchased. All purchased ratings are disclosed.*

Proof. First, note that Claim C5 implies that γ_0 is an equilibrium if $\min\{c_1, c_2\} \geq \hat{c}_h$ (supported by full-disclosure off-equilibrium beliefs). Next, in line with the case of symmetric fees, consider equilibria in which at least one rating is purchased. Note that if $c_1 \neq c_2$ the threshold strategy of Definition 1 fails to be an equilibrium for all $\bar{S} < \infty$ as the issuer is not indifferent between the two ratings and would purchase the cheaper rating first. Consider instead the set of strategies denoted with γ_i^Θ for $i = 1, 2$. Under worst case off-equilibrium beliefs on ratings that are not disclosed, if investors expect the issuer to follow γ_i^Θ then it is never optimal for the issuer to deviate and disclose either only rating $-i$, or only rating $S_i \notin \Theta$, or no rating. Hence, worst case beliefs support γ_i^Θ for $\Theta = \emptyset$. If Θ is non-empty, however, upon observing $S_i \in \Theta$, the issuer could deviate, purchase rating $-i$ and disclose selectively. Hence, for the deviation not to be profitable, we must have

$$\mathrm{E}\left[\max\{p(S_i, S_{-i}), p(S_i)\} \middle| S_i\right] - c_{-i} \leq p(S_i), \text{ for all } S_i \in \Theta.$$

From Eqs. (A2)-(A3) we have $p(S_i, S_{-i}) > p(S_i) \Leftrightarrow S_{-i} > p(S_i)$. Therefore, c_{-i} has to satisfy

$$c_{-i} \geq \int_{p(S_i)}^{\infty} [p(S_i, S_{-i}) - p(S_i)] \frac{1}{\sigma_{S|S}} f\left(\frac{S_{-i} - \mu(S_i)}{\sigma_{S|S}}\right) dS_{-i}, \text{ for all } S_i \in \Theta.$$

Note that the value of this integral does not depend on S_i, and hence is independent of the particular Θ. Computing the integral and rearranging gives

$$c_{-i} \geq \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} g\left(r\sqrt{\sigma^2_{X|S} - \sigma^2_{X|2S}}\right) =: \hat{c}_l,$$

where it is immediate to verify $c_l < \hat{c}_l < \hat{c}_h$. When the issuer follows γ_i^Θ, her ex-ante expected profits equal

$$\begin{aligned}
\Pi(\gamma_i^\Theta) &= \mathrm{E}\left[(p(S_i) - c_i)\,\mathbb{I}_{\{S_i \in \Theta\}} + (p(S_i, S_{-i}) - c_i - c_{-i})\left(1 - \mathbb{I}_{\{S_i \in \Theta\}}\right)\right] - (\mu_X - V) \\
&= V - \left[\Pr(S_i \in \Theta)\left(c_i + r\sigma^2_{X|S}\right) + (1 - \Pr(S_i \in \Theta))\left(c_i + c_{-i} + r\sigma^2_{X|2S}\right)\right]. \qquad \text{(C32)}
\end{aligned}$$

Clearly, for a given non-empty Θ, $\Pi(\gamma_i^\Theta) > \Pi(\gamma_{-i}^\Theta)$ if and only if $c_i < c_{-i}$, that is, issuer's expected profits are higher if the cheaper rating is purchased first. Also, it is immediate to verify that $\Pi(\gamma_i^\Theta)$ is increasing in $\Pr(S_i \in \Theta)$ if and only if $c_{-i} > c_l$. This implies that if $c_1 < c_2$ and $c_2 \geq \hat{c}_l$ then $\Pi(\gamma_i^\Theta)$ is maximized for $i = 1$ and $\Theta = \mathbb{R}$, i.e., when the issuer purchases and discloses only the cheaper rating.

Given $c_1 \neq c_2$, denote with Υ_{c_1,c_2} the subset of these strategies that can be supported as an equilibrium when fees equal c_1, c_2 (i.e., $\gamma_i^\emptyset$ for all $c_i \neq c_{-i}$ for $i = 1, 2$; γ_i^Θ: $\Theta \subseteq \mathbb{R}$ and $\Theta \neq \emptyset$ for $c_{-i} \geq \hat{c}_l$ for $i = 1, 2$; γ_0 for $\min\{c_1, c_2\} \geq \hat{c}_h$). Given the properties of $\Pi(\gamma_i^\Theta)$ and the inequalities in (B1), it is easily verified that the same refinement of off-equilibrium beliefs presented for the case of symmetric fees can be adapted to Υ_{c_1,c_2} with almost no modifications to support the equilibrium stated in the lemma for $c_1 \neq c_2$. This completes the proof of Proposition 2. ■

Appendix D: Proofs for Section 4.2, Section 4.4 and Section 4.5.

The following claims are used in the proof of Proposition 3.

Claim D1. If $c < \hat{c}_h$ is such that the PC is satisfied ex-ante when the issuer follows the strategy of Proposition 2, then the PC would be satisfied were the issuer to purchase and disclose exactly one rating with fee equal to c.

Proof of Claim D1. When purchasing and disclosing exactly one rating, the issuer's PC is satisfied ex-ante if

$$V \geq r\sigma^2_{X|S} + c. \tag{D1}$$

Let c_1^* denote the value of the fee such that (D1) binds, that is,

$$c_1^* := \Delta + c_h. \tag{D2}$$

Let c_2^* be the value of the fees such that the ex-ante PC binds when both ratings are purchased and disclosed. As in this case the issuer's ex-ante profits (net of the outside option) equal $V - r\sigma^2_{X|2S} - 2c$, we have

$$c_2^* := \frac{1}{2}\left(\Delta + c_l + c_h\right).$$

It is immediate to verify that $c_1^* > c_2^*$, which implies that (D1) is satisfied if the PC is satisfied when both ratings are always purchased and disclosed. In the case $c \in (c_t, \hat{c}_h)$, if the PC (C26) holds then (D1) is implied by part i) of Claim C4. ■

Claim D2. Under the belief refinement specified in the proof of Proposition 2, no-trade is an equilibrium for $c_1 = c_2 = H$ if and only if $H < \hat{c}_h$ and either: i) $H < c_t$ and $H \geq c_2^*$ or ii) $H \in (c_t, \hat{c}_h)$ and $V \leq v(H)$. For these parameters no-trade is supported by worst case off-equilibrium beliefs if the issuer sells the asset and less than two ratings are disclosed.

Proof of Claim D2. Assume first that if the issuer deviates from no-trade by selling the asset and disclosing less than two ratings investors have worst case off-equilibrium beliefs. In this case the issuer has no incentive to deviate (and disclose both ratings) if and only if $c \geq c_2^*$. Next, consider the belief refinement of Proposition 2. The refinement applies only to two cases (in which the issuer would then profitably deviate from no-trade). These are: i) the off-equilibrium move of selling without disclosing any rating is interpreted as the issuer playing γ_0 for $c \geq \hat{c}_h$ (since $\Pi(\gamma_0) > 0$) and ii) the off-equilibrium move of selling and disclosing only one rating above $\bar{S}_c$ is interpreted as the issuer playing $\gamma_{\bar{S}_c}$ for $c \in (c_t, \hat{c}_h)$ if $\Pi(\gamma_{\bar{S}_c}) > 0$. For $c = H$, the latter conditions is satisfied if $V > v(H)$, where $v(H)$ is defined in the text. It follows, then, that for $c = H$ the issuer cannot profitably deviate from the no-trade equilibrium under the conditions given in the claim. ■

Proof of Proposition 3. We look for symmetric Nash equilibria in the fee-setting game. For any pair of fees such that $c_1 = c_2 = c$, each RA's expected profits are obtained anticipating the issuer's demand for ratings from Proposition 2 under the condition that the PC is satisfied ex-ante. If a RA deviates to $c' \neq c$ such that $c' \geq H$, its expected profits are are obtained anticipating the issuer's demand with asymmetric fees as described in Lemma C2 under the condition that the PC is satisfied ex-ante. We distinguish four cases depending on the value of the ratings' production cost H and the holding cost V. In the analysis below let $\varepsilon > 0$.

1. For $H \leq \tilde{c}$ we distinguish two cases, depending on whether $c_t \geq \hat{c}_h$ or $c_t < \hat{c}_h$.

(i) Let $H \leq c_t < \hat{c}_h$ and $H \leq c_2^*$. Then, the equilibrium at the fee-setting stage has the following form.

a) Let $c_2^* < \hat{c}_l$. Then $c = c_2^*$ is an equilibrium. In fact, for $c = c_2^*$ both ratings are purchased and the PC binds, so if a RA deviates to a higher rating it cannot be that both ratings are purchased, as this would violate the PC. For all $c > c_2^*$ the equilibrium involves either no ratings being purchased, or each rating being purchased with probability $\hat{p} \in (1/2, 1)$ if the issuer follows $\gamma_{\bar{S}}$ for $\bar{S} < \infty$. In the former case a RA can make positive profits by undercutting to $c - \varepsilon \in (H, \min\{c_1^*, \hat{c}_h\})$. In the latter case, if a RA undercuts to $c - \varepsilon < \hat{c}_l$ its rating is purchased with probability 1 (Claim D1 implies the PC is satisfied). Given $\hat{p} < 1$ there is always an $\varepsilon > 0$ small enough that $(c - \varepsilon) > \hat{p}c$, making the deviation profitable. For all $c < c_2^*$, either $c < H$ (if $H = c_2^*$), or a RA can increase profits by deviating to $c + \varepsilon < \hat{c}_l$ such that $2c + \varepsilon \leq 2c_2^*$, as both ratings are then purchased.

b) Let $c_2^* \geq \hat{c}_l$ and denote the set of fees

$$\mathcal{C} = \{c : c \geq \max\{H, \hat{c}_l\},\ c \leq \min\{c_2^*, c_t\}\}.$$

Note $\mathcal{C}$ is non-empty given the parameters under consideration. Any $c \in \mathcal{C}$ is an equilibrium fee. In fact, for all $c \in \mathcal{C}$ both ratings are purchased and any deviation to a higher fee implies $c + \varepsilon > \hat{c}_l$, so that the more expensive rating is not purchased. For all $c < \min \mathcal{C}$, either $c < H$ (if $H \geq \hat{c}_l$), or a RA can increase profits by deviating to $c + \varepsilon < \hat{c}_l$ such that $2c + \varepsilon \leq 2c_2^*$, as both ratings are then purchased. For all $c > \max \mathcal{C}$ the equilibrium involves either no ratings being purchased, or each rating being purchased with probability $\hat{p} \in (1/2, 1)$ if the issuer follows $\gamma_{\bar{S}}$ for $\bar{S} < \infty$. In the former case a RA can undercut to $c - \varepsilon \in (H, \min\{c_1^*, \hat{c}_h\})$ and make positive profits. In the latter case, a RA can profitably undercut to $c - \varepsilon$ such that $(c - \varepsilon) > \hat{p}c$ (Claim D1 implies the PC is satisfied).

(ii) Let $H < \hat{c}_h \leq c_t$ and $H \leq c_2^*$. The proof from case (i) goes through with the only modification that the set $\mathcal{C}$ is now defined as

$$\mathcal{C} = \{c : c \geq \max\{H, \hat{c}_l\},\ c \leq \min\{c_2^*, \hat{c}_h\}, c \neq \hat{c}_h\}.$$

2. Let $H \in (c_t, \hat{c}_h)$ and $V \geq v(H)$. Then $c = H$ is an equilibrium: any deviation to a higher fee is such that $c + \varepsilon > \hat{c}_l$ and results in the rating not being purchased. Any $c > H$ is not an equilibrium fee: either the PC (C25) is satisfied, in which case a RA can undercut to $c - \varepsilon$ such that $(c - \varepsilon) > \hat{p}c$ (Claim D1 implies the PC is satisfied), or the PC (C25) is violated, in which case no rating is purchased and a RA can undercut to $c - \varepsilon \in (H, \min\{c_1^*, \hat{c}_h\})$ (note $V \geq v(H)$ implies $H < c_1^*$).
3. Let $H \geq \hat{c}_h$. Whenever $c_i \geq \hat{c}_h$ for $i = 1, 2$, the issuer sells the asset without purchasing any rating. Hence, we assume that RAs set $c = H$.
4. Let V and H be such that neither of 1-3 are met. Let $c = H$. For these parameters, either $c_2^* < H$ implies $\Pi(\gamma_\infty) < 0$ for $H \leq c_t$, or $V < v(H)$ implies $\Pi(\gamma_{\bar{S}_c}) < 0$ for $H \in (c_t, \hat{c}_h)$, meaning the PC is violated under the strategy predicted by Proposition 2. For these parameters no-trade is an equilibrium (Claim D2), so the issuer retains the asset and RAs make zero profits. Any deviation of a RA to $c + \varepsilon$ is such that either $2c + \varepsilon > c_2^*$ or $c + \varepsilon > \hat{c}_l$, implying that if a RA deviates to a more expensive rating, this would not be purchased. Hence $c = H$ is an equilibrium. For all $c > H$ we have two cases. If $c_1^* > H$, then a RA can undercut to $c - \varepsilon \in (H, \min\{c_1^*, \hat{c}_h\})$ and make positive profits. If $c_1^* \leq H$, then no undercutting can be profitable and every $c > H$ is an equilibrium in which RAs make zero profits. Hence, we assume that RAs set $c = H$. ■

Proof of Proposition 4. Consider a RA that deviates from the equilibrium of Proposition 1 (Proposition 3) by offering an opaque (transparent) contact and fee $c' \geq H$. If $c' \neq c$, its expected profits are obtained anticipating that the issuer will follow a strategy $\gamma \in \Upsilon_{c_1, c_2}$ under the condition that the PC is satisfied under

γ. The set Υ_{c_1,c_2} is as defined in the proof of Lemma C2, with the difference that if rating i is transparent and rating $-i$ is opaque, then $\gamma_0 \in \Upsilon_{c_1,c_2}$ for $c_{-i} \geq \hat{c}_h$ and $\gamma^{\Theta}_{-i} \in \Upsilon_{c_1,c_2}$ for all c_{-i} and Θ.

We start by showing that if RAs can make the contact with the issuer opaque, the transparent outcome described in Proposition 1 cannot be an equilibrium for $H < \bar{H} := \min\{c^*_1, \hat{c}_l\}$ (note that the respective definitions imply $c_l < c^*_1$ and $c_l < \hat{c}_l$). Consider RA$_i$ that deviates from the equilibrium of Proposition 1 by offering an opaque contact and fee $c_i \in \left(\max\{c_l, H\}, \bar{H}\right)$. Note $c_i < \hat{c}_h$ implies $\gamma_0 \notin \Upsilon_{c_1,c_2}$ and $c_i < \hat{c}_l$ implies $\gamma^{\Theta}_{-i} \notin \Upsilon_{c_1,c_2}$ for all $\Theta \neq \emptyset$. Hence, $\Upsilon_{c_1,c_2} = \{\gamma^{\Theta}_i\colon\, \Theta \subseteq \mathbb{R},\, \gamma^{\emptyset}_{-i}\}$. For $H \leq c_l$, in the equilibrium of Proposition 1, $c_{-i} = c_l$ and therefore Eq. (C32) implies $\Pi(\gamma^{\emptyset}_{-i}) = \Pi(\gamma^{\Theta}_i)$ for all $\Theta \subseteq \mathbb{R}$. For $H > c_l$, in the equilibrium of Proposition 1, $c_{-i} > c_l$ and therefore Eq. (C32) implies $\Pi(\gamma^{\mathbb{R}}_i) > \Pi(\gamma^{\Theta}_i) \geq \Pi(\gamma^{\emptyset}_{-i})$ for all $\Theta \neq \mathbb{R}$. Note that for $H > c_l$ the issuer can profitably deviate from $\gamma^{\emptyset}_{-i}$ to $\gamma^{\mathbb{R}}_i$, as investors observe that rating $-i$ was not purchased. As $c_i < c^*_1$, the PC is satisfied as a strict inequality for $\gamma^{\mathbb{R}}_i$. Therefore, rating i is purchased with probability one. As $c_i > c$, the deviation is profitable.

In the opaque case, consider RA$_i$ that deviates from the outcome described in Proposition 3 by simultaneously switching to a transparent contact and fee c_i. For the deviation to be profitable it must be $c_i > c_{-i}$. As in the equilibrium of Proposition 3 we have $c_{-i} > c_l$, Eq. (C32) implies that, for any Θ, γ^{Θ}_i is dominated by $\gamma^{\mathbb{R}}_{-i}$. Since only rating $-i$ is opaque, the issuer can profitably deviate from any γ^{Θ}_i to $\gamma^{\mathbb{R}}_{-i}$, as investors observe that rating i was not purchased. Also, as $c_i > c_l$, Eq. (C32) implies that $\gamma^{\mathbb{R}}_{-i}$ dominates any γ^{Θ}_{-i} such that $\Theta \neq \mathbb{R}$. This implies that any $\gamma \in \Upsilon_{c_1,c_2}$ for which rating i is purchased with positive probability is dominated by $\gamma^{\mathbb{R}}_{-i}$. Therefore we conclude that rating i is not purchased and the deviation is not profitable. ■

Proof of Proposition 5. As the issuer's profits are defined net of the outside option of retaining the asset, Social Welfare is normalized to zero in the absence of trade. When trading occurs in equilibrium, $w = X - p$ and investor's ex-ante indirect utility can be written as

$$\mathrm{E}\left[u(w)\right] = \mathrm{E}\left[\mathrm{E}\left[u(-r\left(X-p\right))\middle|\, \mathcal{I}^{inv}\right]\right],$$

where $\mathcal{I}^{inv}$ denotes the investors' information set at the "interim stage" when purchasing the asset. Consider first an equilibrium in which n ratings are purchased and disclosed with probability one. In such equilibrium we have that $X|\,\mathcal{I}^{inv} \sim N(\mathrm{E}(\,X|\,\{S_i\}^n_{i=0})\,, \mathrm{Var}\,(\,X|\,\{S_i\}^n_{i=0}))$ and that $p = E\,(\,X|\,\{S_i\}^n_{i=0}) - r\mathrm{Var}\,(\,X|\,\{S_i\}^n_{i=0})$. Therefore, the interim indirect utility simplifies to a constant that only depends on the number of ratings

$$\mathrm{E}\left[u(-r\left(X-p\right))\middle|\, \mathcal{I}^{inv}\right] = -\exp\left(-\frac{r^2}{2}\mathrm{Var}\,(\,X|\,\{S_i\}^n_{i=0})\right).$$

Denoting with ϖ_n the corresponding certainty equivalent, it follows immediately that

$$\varpi_n = \frac{r}{2}\mathrm{Var}\,(\,X|\,\{S_i\}^n_{i=0})\,. \tag{D3}$$

Similarly, denoting with $\mathcal{S}_n$ the sum of the issuer's profits and RAs profits when n ratings are purchased and disclosed in equilibrium, we have

$$\mathcal{S}_n = V - r\mathrm{Var}\,(\,X|\,\{S_i\}^n_{i=0}) - nH$$

and therefore

$$\mathcal{S}_n + \varpi_n = V - \frac{r}{2}\mathrm{Var}\,(\,X|\,\{S_i\}^n_{i=0}) - nH.$$

Denoting with $\mathcal{W}^T$ Social Welfare in the transparent market equilibrium described in Proposition 1, we have

$$\mathcal{W}^T = \begin{cases} \mathcal{S}_2 + \varpi_2 & for\ H \leq c_l \\ \mathcal{S}_1 + \varpi_1 & for\ H \in (c_l, c_h] \\ \mathcal{S}_0 + \varpi_0 & for\ H > c_h. \end{cases} \tag{D4}$$

By the assumption $V > r\sigma_X^2$ and the definitions of c_l and c_h it is immediate to verify that $\mathcal{W}^T > 0$. Denoting with $\varpi(\gamma_{\bar{S}})$ the investors' certainty equivalent in the threshold equilibrium and with $\mathcal{W}^O$ the certainty equivalent in the opaque market equilibrium described in Proposition 3, we have

$$\mathcal{W}^O = \begin{cases} \mathcal{S}_2 + \varpi_2 & \textit{for } H \leq \tilde{c} \textit{ such that } H \neq \hat{c}_h \textit{ and } H \leq c_2^* \\ \Pi(\gamma_{\bar{S}}) + \varpi(\gamma_{\bar{S}}) & \textit{for } c_t < \hat{c}_h \textit{ and } H \in (c_t, \hat{c}_h) \textit{ and } V \geq v(H) \\ \mathcal{S}_0 + \varpi_0 & \textit{for } H \geq \hat{c}_h \\ 0 & \textit{if neither of the above holds.} \end{cases} \tag{D5}$$

Given $\tilde{c} = \min\{c_t, \hat{c}_h\} > c_l$, comparing (D4) and (D5) and using the definition of c_l it is immediate to verify that $\mathcal{W}^T \geq \mathcal{W}^O$ for $H \leq \tilde{c}$ and that $\mathcal{W}^T = \mathcal{W}^O$ for $H \geq \hat{c}_h$. Given $\mathcal{W}^T > 0$, it remains to show that $\mathcal{W}^T \geq \mathcal{W}^O$ for $c_t < \hat{c}_h$ and $H \in (c_t, \hat{c}_h)$. Form (D4) and the definitions of c_l and c_h it is immediate to verify that for $H > c_l$ we have $\mathcal{W}^T \geq \alpha\left(\mathcal{S}_1 + \varpi_1\right) + (1-\alpha)\left(\mathcal{S}_2 + \varpi_2\right)$ for all $\alpha \in [0,1]$. Hence, it is sufficient to show that for $H \in (c_t, \hat{c}_h)$ we have $\mathcal{W}^O \leq \beta\left(\mathcal{S}_1 + \varpi_1\right) + (1-\beta)\left(\mathcal{S}_2 + \varpi_2\right)$ for some $\beta \in [0,1]$. For this purpose, let us define the interim certainty equivalent as the value $\varpi\left(\mathcal{I}^{inv}\right)$ that solves $\mathrm{E}\left[u(w)|\,\mathcal{I}^{inv}\right] = u(\varpi\left(\mathcal{I}^{inv}\right))$. Then, the CARA assumption and Jensen's inequality imply

$$\varpi = -\frac{1}{r}\log\left(\mathrm{E}\left[-\mathrm{E}\left[u(w)|\,\mathcal{I}^{inv}\right]\right]\right) \leq \mathrm{E}\left[\varpi\left(\mathcal{I}^{inv}\right)\right].$$

The interim indirect utility in the threshold equilibrium can be written as

$$\mathrm{E}\left[u(w)|\,\mathcal{I}^{inv}\right] = \mathrm{E}\left[u(w)|\,S_p\right]\mathbb{I}_{\{B\}} + \mathrm{E}\left[u(w)|\,S_1, S_2\right]\left(1 - \mathbb{I}_{\{B\}}\right),$$

and therefore, using (C2) and (D3),

$$\varpi\left(\mathcal{I}^{inv}\right) = \left(-p_{\bar{S}}(S_p) - \varpi_1 + \mathrm{E}\left[X|\,B \cap S_p\right] + \frac{1}{r}\xi(r, S_p)\right)\mathbb{I}_{\{B\}} + \varpi_2\left(1 - \mathbb{I}_{\{B\}}\right)$$

where

$$\xi(r, S_p) := r\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}\Gamma\left(\frac{s(S_p) - \mu(S_p)}{\sigma_{S|S}}\right) - \log\left[\frac{1 + \Phi\left(\frac{s(S_p) - p(S_p)}{\sigma_{S|S}}\right)}{1 + \Phi\left(\frac{s(S_p) - \mu(S_p)}{\sigma_{S|S}}\right)}\right].$$

Using l'Hôpital's rule and the fact that $\Gamma(t)$ is increasing in t for $t < t^*$, we have

$$\begin{aligned} \lim_{r \downarrow 0}\xi(r, S_p) &= 0 \\ \frac{\partial}{\partial r}\xi(r, S_p) &= \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}\left[\Gamma\left(\frac{s(S_p) - \mu(S_p)}{\sigma_{S|S}}\right) - \Gamma\left(\frac{s(S_p) - p(S_p)}{\sigma_{S|S}}\right)\right] < 0, \end{aligned}$$

which imply $\xi(r, S_p) \leq 0$. It follows that $\mathcal{W}^O$ in the threshold equilibrium can be written as

$$\begin{aligned}
\mathcal{W}^O &\leq V-\mu_X+\mathrm{E}\left[p_{\bar{S}}(S_p)\mathbb{I}_{\{B\}}+p(S_1,S_2)\left(1-\mathbb{I}_{\{B\}}\right)+\varpi\left(\mathcal{I}^{inv}\right)\right]-H\left[1+\Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right] \\
&\leq V-\mu_X+\mathrm{E}\left[\mathrm{E}\left[X|\mathcal{I}^{inv}\right]-\left[\varpi_1\mathbb{I}_{\{B\}}+\varpi_2\left(1-\mathbb{I}_{\{B\}}\right)\right]\right]-H\left[1+\Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right] \\
&\leq V-\left[\varpi_1\mathrm{Pr}(B)+\varpi_2\left(1-\mathrm{Pr}(B)\right)\right]-H\left[1+\Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)\right].
\end{aligned}$$

As $(1-\mathrm{Pr}(B)) \leq \Phi\left(\frac{\bar{S}-\mu_X}{\sigma_s}\right)$, we can write

$$\mathcal{W}^O \leq \mathrm{Pr}(B)\left(\mathcal{S}_1+\varpi_1\right)+\left(1-\mathrm{Pr}(B)\right)\left(\mathcal{S}_2+\varpi_2\right).$$

As $\mathrm{Pr}(B) \in [0,1]$, this completes the proof. ■

Appendix E: Proofs for Section 6.

Equilibrium prices in the threshold equilibrium (Definition 2, part 3). Denote $\bar{r} := r(\bar{S})$. For the case in which only one rating is disclosed with value $r_p \geq \bar{r}$, the derivation of the pricing function follows closely the derivation of Eqs. (9)-(11) in Appendix C. Under the assumptions in Eqs. (19)-(21), the probability of selective disclosure given one rating is disclosed with value r_p, denoted $q(r_p)$, is constant and equal to

$$q(r_p)=\frac{\Phi\left(\frac{\bar{S}-\mu}{\sigma}\right)}{1+\Phi\left(\frac{\bar{S}-\mu}{\sigma}\right)}.$$

Hence, the resulting pricing function is given by

$$p_{\bar{S}}(r_p)=r^{-1}(r_p)+\mu-\gamma\sigma^2-\sigma\Gamma\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right). \tag{E1}$$

If no ratings are disclosed, the price, $p_{\bar{S}}(\emptyset)$, must reflect the belief that both preliminary ratings are below the threshold $\bar{S}$. Hence, the equilibrium price must satisfy

$$p_{\bar{S}}(\emptyset)=\frac{\mathrm{E}\left[X\times\exp\left(-\gamma X\right)\middle|\, S_1<\bar{S},S_2<\bar{S}\right]}{\mathrm{E}\left[\exp\left(-\gamma X\right)\middle|\, S_1<\bar{S},S_2<\bar{S}\right]}.$$

Since S_1,S_2 are IID, the previous expression simplifies to

$$\begin{aligned}
p_{\bar{S}}(\emptyset) &= 2\frac{\mathrm{E}\left[S_i\times\exp\left(-\gamma S_i\right)\middle|\, S_i<\bar{S}\right]}{\mathrm{E}\left[\exp\left(-\gamma S_i\right)\middle|\, S_i<\bar{S}\right]} \\
&= 2\mu-2\gamma\sigma^2-2\sigma h\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right),
\end{aligned} \tag{E2}$$

where $h(x) := f(x)/\Phi(x)$ and the second equality follows by direct computation.

Off-Equilibrium Beliefs. Taking the threshold $\bar{S}$ and the function $r(\cdot)$ as given, off-equilibrium beliefs are specified as follows. If both ratings are disclosed that are above $\bar{r}$, then investors believe that $S_i = r^{-1}(\tilde{r}_i)$ for $i = 1, 2$. For any rating $\tilde{r}_i$ that is disclosed below $\bar{r}$, investors believe that $S_i < \min\{\bar{S}, \tilde{r}_i\}$. Under these off-equilibrium beliefs it is immediate to verify that

$$p_{\bar{S}}(\tilde{r}_i, \tilde{r}_{-i}) = r^{-1}(\tilde{r}_i) + r^{-1}(\tilde{r}_{-i}) \text{ for } \tilde{r}_i, \tilde{r}_{-i} \geq \bar{r}$$

and that following inequalities hold:

$$p_{\bar{S}}(\tilde{r}_i, \tilde{r}_{-i}) < p_{\bar{S}}(\tilde{r}_i) \text{ for } \tilde{r}_i \geq \bar{r}, \tilde{r}_{-i} < \bar{r} \tag{E3}$$

$$p_{\bar{S}}(\tilde{r}_i, \tilde{r}_{-i}) \leq p_{\bar{S}}(\emptyset) \text{ for } \tilde{r}_i, \tilde{r}_{-i} < \bar{r} \tag{E4}$$

$$p_{\bar{S}}(\tilde{r}_i) \leq p_{\bar{S}}(\emptyset) \text{ for } \tilde{r}_i < \bar{r} \tag{E5}$$

Temporary assumptions. In the following analysis we will make use of the following two temporary assumptions. In the proof of Proposition 6 we will provide conditions that guarantee these assumptions to be verified.
H0: The issuer's outside option of holding the asset is always dominated on the equilibrium path.
H1: The issuer's expected continuation payoff when the first rating is below the threshold ($S_1 < \bar{S}$) equals $p_{\bar{S}}(\emptyset)$, that is,

$$p_{\bar{S}}(\emptyset) = -c + \mathrm{E}\left[p_{\bar{S}}(\emptyset)\mathbb{I}_{\{S_2<\bar{S}\}} + \left[p_{\bar{S}}(r(S_2)) - \tilde{c}(S_2)\right]\mathbb{I}_{\{S_2\geq\bar{S}\}} \,\middle|\, S_1 \right].$$

RA's optimal strategy (Definition 2, part 2). Here we take the initial fees $c_1 = c_2 = c$, the threshold $\bar{S}$, the price function $p_{\bar{S}}(\cdot)$ from Eqs. (E1) and (E2) and the issuer's strategy from Definition 2 and the previously described off-equilibrium beliefs as given and we derive the RAs' strategy for $S_i \geq \bar{S}$, that is, the functions $r(\cdot)$ and $\tilde{c}(\cdot)$. Then, we will derive the condition under which RAs do not offer any rating adjustment whenever $S_i < \bar{S}$.
<u>The rating function, $r(\cdot)$</u> Consider the function $r(\cdot)$ that is implicitly defined by

$$r(S_i) = S_i + b^{-1} + (\bar{y} - b^{-1})e^{b[\bar{r} - r(S_i)]}. \tag{E6}$$

Implicit differentiation of Eq. (E6) shows that this function is a solution to the initial value problem given by Eq. (22) and the boundary condition $\bar{r} = \bar{S} + \bar{y}$. Note that Eq. (E6) satisfies $r(S_i) \geq S_i$ for all $S_i \geq \bar{S}$ as long as $\bar{y} \geq 0$.
<u>The fee function, $\tilde{c}(\cdot)$</u> Under the assumption that RAs make a take-it-or-leave it offer to the issuer, the fee $\tilde{c}$ is set equal to the issuer's willingness to pay for the rating adjustment, and must satisfy $\tilde{c}(S_i) \geq \kappa(S_i, r_i)$ for RAs' profits to be non-negative. The issuer's willingness to pay for $r(S_i)$ depends on S_i and we must distinguish two cases, depending on whether $S_i \in [\bar{S}, \bar{r})$ or $S_i \geq \bar{r}$. *Case 1:* $S_i \geq \bar{r}$. Whether S_i is the first or the second preliminary rating, the issuer can disclose S_i and sell the asset for a price $p_{\bar{S}}(S_i)$ (in case S_i is the second rating, the inequality in (E3) implies that disclosing S_i dominates disclosing both S_i and S_{-i}). Hence, RA$_i$ sets $\tilde{c}(S_i) = p_{\bar{S}}(r(S_i)) - p_{\bar{S}}(S_i) = S_i - r^{-1}(S_i)$. Eq. (E6) implies that $r^{-1}(S_i) = S_i - b^{-1} - (\bar{y} - b^{-1})e^{b(\bar{r} - S_i)}$. Using this expression for $r^{-1}(S_i)$ and Eqs. (E6) and (22) it is easy to verify that i) $\lim_{S_i \uparrow \infty} \tilde{c}(S_i) - \kappa(S_i, r_i) = b^{-1}/2$ and ii) $\frac{d}{dS_i}\left[\tilde{c}(S_i) - \kappa(S_i, r_i)\right] = b\left[(\bar{y} - b^{-1})e^{b(\bar{r} - S_i)}\left(e^{b[S_i - r(S_i)]} - 1\right)\right]$. Given $S_i < r(S_i)$, the last expression has the sign of $\left(b^{-1} - \bar{y}\right)$. Then, for $\bar{y} \geq b^{-1}$ ii) is negative and i)-ii) imply $\tilde{c}(S_i) - \kappa(S_i, r_i) > 0$. For $\bar{y} < b^{-1}$, ii)

is positive and we must show $\tilde{c}(\bar{r})-\kappa(\bar{r},r(\bar{r}))\geq 0$ for all $\bar{y}\in[0,b^{-1})$. We have: iii) $\left[\tilde{c}(\bar{r})-\kappa(\bar{r},r(\bar{r}))\right]_{\bar{y}=0}=0$ and iv) $\frac{d}{d\bar{y}}\left[\tilde{c}(\bar{r})-\kappa(\bar{r},r(\bar{r}))\right]=1-b\left(\bar{r}-r(\bar{r})\right)\frac{d}{d\bar{y}}\left[\bar{r}-r(\bar{r})\right]>0$, where the inequality is implied by $\frac{d}{d\bar{y}}\left[\bar{r}-r(\bar{r})\right]>0$, which in turn is easily shown to be true by implicit differentiation of Eq. (E6) when valued at $\bar{r}$. Hence, iii) and iv) imply $\tilde{c}(\bar{r})-\kappa(\bar{r},r(\bar{r}))\geq 0$ for all $\bar{y}\in[0,b^{-1})$. *Case 2:* $S_i\in[\bar{S},\bar{r})$. In this case, the issuer's outside option is worth $p_{\bar{S}}(\emptyset)$. (This is the price she can sell the asset for if no ratings are disclosed or if the ratings that are disclosed are below $\bar{r}$.) Then, RA$_i$ sets $\tilde{c}(S_i)=p_{\bar{S}}(r(S_i))-p_{\bar{S}}(\emptyset)=S_i-\bar{S}+\sigma z\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right)$, the last equality follows by Eqs. (E1) and (E2) and $z(x):=x+2h(x)-\Gamma(x)$. Using Eqs. (E6) and (22) it is easy to verify that $\frac{d}{dS_i}\left[\tilde{c}(S_i)-\kappa(S_i,r_i)\right]=r(S_i)-S_i>0$. Hence, in order to have $\tilde{c}(S_i)\geq\kappa(S_i,r_i)$ for all $S_i\in[\bar{S},\bar{r})$ it is necessary and sufficient that $\tilde{c}(\bar{S})\geq\kappa\left(\bar{S},\bar{r}\right)$, which obtains if

$$\bar{y}\leq\sqrt{2b^{-1}\sigma z\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right)}. \tag{E7}$$

RA's optimal strategy for $S_i<\bar{S}$. We are left to show that RAs have no incentive to offer any rating adjustment whenever $S_i<\bar{S}$. For this to be the case, it is necessary and sufficient that, whenever RA$_i$ produces a preliminary rating $S_i<\bar{S}$, the issuer's willingness to pay for a rating that is as high as $\bar{r}$ is not greater than $\kappa(S_i,\bar{r})$, for all $S_i<\bar{S}$. Under H1, $p_{\bar{S}}(\emptyset)$ equals the issuer's continuation payoff regardless of whether $S_i<\bar{S}$ is the first or the second preliminary rating that is produced. Hence, whenever $S_i<\bar{S}$, the issuer would be willing to pay for $\bar{r}$ up to $p_{\bar{S}}(\bar{r})-p_{\bar{S}}(\emptyset)=\sigma z\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right)$. Hence, we must have $\sigma z\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right)\leq\kappa(S_i,\bar{r})$ for all $S_i<\bar{S}$, which requires

$$\bar{y}\geq\sqrt{2b^{-1}\sigma z\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right)}. \tag{E8}$$

Together, (E7) and (E8) pin down the value for the boundary condition $r(\bar{S})=\bar{S}+\bar{y}$, where $\bar{y}$ must satisfy $\bar{y}=\sqrt{2b^{-1}\sigma z\left(\frac{\bar{S}-\mu}{\sigma}+\gamma\sigma\right)}$.

Issuer's optimal strategy (Definition 2, part 1). Here we take c, $\bar{S}$ the previously described off-equilibrium beliefs, $p_{\bar{S}}(\cdot)$ and RAs' strategy from part 3 and part 2 above as given and verify that the issuer's strategy from part 1 in Definition 2 is indeed optimal. We start from part 1b of Definition 2, assuming $S_1<\bar{S}$ and the second preliminary rating (S_2) has been obtained. If $S_2<\bar{S}$, equilibrium continuation payoffs equal $p_{\bar{S}}(\emptyset)$ and (E4)-(E5) imply that the issuer cannot do better by disclosing any rating. If $S_2\geq\bar{S}$, note that $\tilde{c}(\cdot)$ is, by construction, such that the issuer cannot do better than purchasing and disclosing the adjusted rating. Next, consider part 1a of Definition 2, assuming $S_1\geq\bar{S}$. Again, by construction of $\tilde{c}(\cdot)$, the issuer cannot do better than the equilibrium payoff by disclosing S_1 or selling the asset without disclosing. However, we must check that the issuer has no incentives to deviate and purchase S_2. Here we must distinguish two cases, depending on whether $S_1\in[\bar{S},\bar{r})$ or $S_1\geq\bar{r}$. *Case 1:* $S_1\in[\bar{S},\bar{r})$. In this case the issuer's equilibrium continuation payoff equal $p_{\bar{S}}(\emptyset)$. It is easy to verify that, if the issuer purchases S_2, she cannot do better than $p_{\bar{S}}(\emptyset)$ for all realizations of $S_2<\bar{r}$, while for $S_2\geq\bar{r}$ the best option is to disclose only the second rating. By disclosing only the second rating, the issuer's payoff equals $p_{\bar{S}}(S_2)$ (regardless of whether she discloses the preliminary rating or the adjusted rating, by construction of $\tilde{c}(\cdot)$). Hence, the expected payoff from the deviation equals $-c+\mathrm{E}\left[p_{\bar{S}}(\emptyset)\mathbb{I}_{\{S_2<\bar{r}\}}+p_{\bar{S}}(S_2)\mathbb{I}_{\{S_2\geq\bar{r}\}}\middle|\,S_1\right]$. By construction of $\tilde{c}(\cdot)$ and independence of S_1 and S_2, the last expression equals the issuer's expected continuation payoff when $S_1<\bar{S}$. Under

H1, this equals exactly $p_{\bar{S}}(\emptyset)$, so the deviation is not profitable. *Case 2:* $S_2 \geq \bar{r}$. In this case the issuer's equilibrium continuation payoff equal $p_{\bar{S}}(S_1)$. By purchasing S_2, it is easy to verify that the issuer cannot do better than $p_{\bar{S}}(S_1)$ for all realizations of $S_2 < \bar{r}$, while for $S_2 \geq \bar{r}$ the best option is to disclose both adjusted ratings, which gives payoff $r^{-1}(S_1) + r^{-1}(S_2)$. Hence, the expected payoff from the deviation is $-c+\mathrm{E}\big[p_{\bar{S}}(S_1)\mathbb{I}_{\{S_2<\bar{r}\}} + \left[r^{-1}(S_1) + r^{-1}(S_2)\right]\mathbb{I}_{\{S_2\geq\bar{r}\}}\big|\, S_1\big]$. Using H1 and manipulating, this expression can be written as $p_{\bar{S}}(\emptyset)\left[1 - \Phi\left(\frac{\bar{S}-\mu}{\sigma}\right)\right] + r^{-1}(S_1) + \left[\mu - \gamma\sigma^2 - \sigma\Gamma\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right)\right]\left[2\Phi\left(\frac{\bar{S}-\mu}{\sigma}\right) - 1\right]$. Comparing this expression with equilibrium payoff $p_{\bar{S}}(S_1)$ and using Eqs. (E1) and (E2), it is immediate to verify that the issuer has no incentive to deviate. Finally, at the ex-ante stage, given symmetry the issuer is indifferent between the two RAs and will purchase a first rating if H0 holds.

Proof of Proposition 6. Part i) Given initial fees $c_1 = c_2 = c$ and the stated off-equilibrium beliefs, the results derived in this appendix imply that a threshold $\bar{S}$, the rating function in Eq. (E6) with $\bar{y} = \sqrt{2b^{-1}\sigma z\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right)}$ and the fee function given by $\tilde{c}(S_i) = S_i - \left(\bar{S} - \sigma z\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right)\right)\mathbb{I}_{\left\{S_i\in[\bar{S},\bar{r})\right\}} - r^{-1}(S_i)\mathbb{I}_{\{S_i>\bar{r})\}}$, constitute an equilibrium that conforms to Definition 2 if the threshold $\bar{S}$ and the holding cost V are such that H0 and H1 are satisfied. Using Eqs. (E1) and (E2) and the expression for $\tilde{c}(S_i)$ we can rearrange H1 as $c = -p_{\bar{S}}(\emptyset)+\mathrm{E}\big[p_{\bar{S}}(\emptyset)\mathbb{I}_{\{S_2<\bar{r}\}} + p_{\bar{S}}(S_2)\mathbb{I}_{\{S_2\geq\bar{r}\}}\big|\, S_1\big]$. Using the expression for $r^{-1}(\cdot)$, integrating and simplifying, the last equality can be expressed as $c = \Omega(\bar{S})$, where

$$\Omega(\bar{S}) := \sigma z\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right)\Phi\left(\frac{\mu-\bar{r}}{\sigma}\right) + \sigma g\left(\frac{\mu-\bar{r}}{\sigma}\right) + (\bar{y}-b^{-1})\left[\Phi\left(\frac{\mu-\bar{r}}{\sigma}\right) - e^{b\left(\bar{r}-\mu+b\sigma^2/2\right)}\Phi\left(\frac{\mu-\bar{r}}{\sigma} - b\sigma\right)\right].$$

The properties of the normal distribution imply that $\lim_{x\uparrow\infty}\frac{z(x)}{x} = \lim_{x\downarrow-\infty}\frac{z(x)}{|x|} = 1$. This result, the fact that $\bar{y} = \sqrt{2b^{-1}\sigma z\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right)}$ and l'Hôpital's rule can be used to show that $\lim_{\bar{S}\downarrow-\infty}\Omega(\bar{S}) = \infty$ and $\lim_{\bar{S}\uparrow\infty}\Omega(\bar{S}) = 0$. Hence, by continuity, there exists a finite value for the threshold $\bar{S}$ for which $\Omega(\bar{S}) = c$, implying that H1 is satisfied for such value. Next, let $\bar{S}$ be such that H1 is satisfied. Then, it is easy to verify that: the PC is satisfied at the ex ante stage if $V \geq V_0 := 2\left[\gamma\sigma^2 + \sigma h\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right)\right]$; the PC is satisfied for all values of the first purchased rating if $V \geq V_1 := V_0 + \bar{S} - \mu + \bar{y}$; the PC is satisfied for all values of the second purchased rating if $V \geq V_2 := V_0 + 2\left(\bar{S} - \mu\right) + \bar{y}$. Let $\tilde{V} := \max\{V_0, V_1, V_2\}$. Then, H0 is satisfied for $V \geq \tilde{V}$. Part ii) Given $\bar{y} = \sqrt{2b^{-1}\sigma z\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right)}$, we have $\bar{y} > b^{-1} \Leftrightarrow 2z\left(\frac{\bar{S}-\mu}{\sigma} + \gamma\sigma\right) > (b\sigma)^{-1}$. Inspection of the function $z(\cdot)$ shows that it is bounded below by $\underline{z}\approx 1.3$. Hence, for any $\bar{S} \in \mathbb{R}$, $\bar{y} > b^{-1}$ if $b\sigma > \underline{b\sigma} \approx 0.385$. For the remaining part of the statement, rewrite Eq. (22) in terms of the rating adjustment function, $y(S_i) := r(S_i) - S_i$. This gives $y'(S_i) = \left(b^{-1} - y(S_i)\right)/y(S_i)$. Then, it is immediate that $y'(S_i) < 0$ for $y(S_i) > b^{-1}$ and that $y'(S_i)$ approaches zero as $y(S_i)$ approaches b^{-1}. ■

Appendix F: Proofs for Section 5.

Here we prove conditions under which in the threshold equilibrium, the following condition holds:

$$\mathrm{E}\left(X|\,S_p\right) < \mathrm{E}\left(X|\,\frac{S_1+S_2}{2} = S_p\right). \tag{F1}$$

For this condition to be established, we need to compare cases in which in equilibrium either only one rating is published with value S_p or both ratings are published with average value equal to S_p. Note that for $\bar{S} \leq S^*$ both ratings are published only if both are below $\bar{S}$, so that the required comparison cannot be made. Also, for $\bar{S} > S^*$, it is never the case that both ratings are published with average value above $\overrightarrow{S}$. Therefore, we must restrict to parameters and states in which $\bar{S} > S^*$ and $S_p \in \left(\bar{S}, \overrightarrow{S}\right)$. In what follows we assume S_p and $\bar{S}$ satisfy this restriction. Given

$$\mathrm{E}\left(X|\,S_p\right) = \mu(S_p) - \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(\frac{s(S_p) - \mu(S_p)}{\sigma_{S|S}}\right)$$

and the fact that $\Gamma\left(\frac{s(S_p)-p(S_p)}{\sigma_{S|S}}\right) = \Gamma\left(t^*\right)$, (see Remark C1) condition (F1) is equivalent to

$$\mu(S_p, S_p) > \mu(S_p) - \left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2} \Gamma\left(t^* - r\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}\right) \Leftrightarrow$$

$$\frac{S_p - \mu_X}{\sigma_S} > -\left(1 + 2\frac{\sigma^2_X}{\sigma^2_\varepsilon}\right)^{1/2} \Gamma\left(t^* - r\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}\right) =: -T(\sigma_X, \sigma_\varepsilon, r). \tag{F2}$$

Furthermore, the definition of S^* implies

$$\frac{S^* - \mu_X}{\sigma_S} = \left(1 + 2\frac{\sigma^2_X}{\sigma^2_\varepsilon}\right)^{1/2} \left(t^* - r\left(\sigma^2_{X|S} - \sigma^2_{X|2S}\right)^{1/2}\right). \tag{F3}$$

Since $\Gamma\left(t^*\right) = -t^*$ (see Claim C1) then Eq. (F3) implies that condition (F2) is satisfied for all S_p if $r = 0$. For $r > 0$, condition (F2) is satisfied for $S_p \geq \mu_X - T(\sigma_X, \sigma_\varepsilon, r)\sigma_S$, where $T(\sigma_X, \sigma_\varepsilon, r) > 0$. For the parameters used in Figure 3 to Figure 5, $T(\sqrt{2}, 1, 0.4) \approx 0.604$.