Asset Markets and Monetary Policy

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Introduction

- financial market and economic activity strongly interact
- **appropriate monetary policy ?**
- Taylor rule   Taylor (1993)

  respond to inflation and output gap

  Svensson (1997, 1999)
  Woodford (2003)

- **Impact of asset price movements ?**

- Bernanke, Gertler & Gilchrist (1999)
  net worth moves procyclically
  can magnify disturbances

- Cecchetti et al. (2000)
  beneficial effects when responding to asset prices
  exogenous bubbles or non-fundamental asset price

- Dupor (2005)
  capital valuations are impacted by asset price bubbles
  distortions in aggregate demand

- Beaudry & Portier (2004)
  asset price bubbles resulting from asset price booms

- Christiano, Motto & Rostagno (2005)
  bubbles arise due to misperceived technology shocks

- Coenen & Wieland (2004)

  Eggertsson & Woodford (2003)

  Japanese deflationary experience:

  asset prices and product prices decline
  and inflation rates approach zero

- **quantitative easing**

  provide private economy with liquidity

  purchasing “bad assets”

  reviving the economy through government spending

- **current crises**
  triggered by subprime crisis
  global financial market meltdown
  banking crisis
  downturn in real economic activity
  partial deflation

  monetary policy and asset markets:
  U.S.: zero interest rates
  quantitative easing

- **dynamic portfolio approach**

  reveals transmission mechanism
  and limitations of monetary policy

  static: Tobin (1969, 1980)
  Frankel (1995)

  dynamic: Campbell & Viceira (2002)

  constant variances
  constant expected risk premia
  constant consumption-wealth ratio
  risk neutral assumptions

Benchmark Approach

Pl. & Heath (2006)

distinction between nominal and real assets

real world martingales

general stochastic processes

$\Longrightarrow$

interest rate rule

inflation rate rule

- **interest rate rule**

$$i_t = \left(a_t - \gamma\,\sigma_t^2\,(1 - |1 - \alpha_t|)\right)^+$$

a_t expected return

γ risk aversion

α_t fraction of wealth invested in the equity market

σ_t volatility of the equity index

- **inflation rate rule**

$$\pi_t = a_t - c_t + \frac{\gamma\,\sigma_t^2}{2}\,\alpha_t\,(\alpha_t - 1)$$

c_t consumption rate

Asset Market Dynamics

Merton (1992)
Cochrane (2001)
Campbell & Viceira (2002)

- savings account

$$\beta_t = \exp\left\{\int_0^t i_s\, ds\right\}$$

i_t nominal interest rate

- risky asset

$$dP_t = P_t\,(a_t\, dt + \sigma_t\, dz_t)$$

z_t standard Wiener process
$\sigma_t > 0$

- consumer price index

$$I_t = \exp\left\{\int_0^t \pi_s \, ds\right\}$$

π_t inflation rate

- consumption rate

$$c_t = c_0 \, \exp\left\{\int_0^t e_s \, ds\right\}$$

$c_0 > 0$

e_t growth rate of consumption rate

Budget equation

total wealth

$$dW_t = W_t\,((1-\alpha_t)i_t\,dt - c_t\,dt + \alpha_t(a_t\,dt + \sigma_t\,dz_t))$$

α_t fraction in the risky asset index P_t

$W_0 > 0$.

Maximization of aggregate consumed real wealth per unit of time:

$$\frac{c_s W_s}{I_s}$$

- **objective**

$$\max_{W} E_t \left(U \left(\frac{c_s W_s}{I_s} \right) \right)$$

for all $0 \leq t < s < \infty$

real world expectation

$U'(\cdot) > 0 \quad U''(\cdot) < 0$

Example:

- **power utility**

$$U(x) = \frac{x^{(1-\gamma)}}{1-\gamma}$$

$\gamma > 0,\ \gamma \neq 1,$

$\gamma \to 1$ logarithmic utility

no time horizon

consumption rate c_t given

Market Dynamics

- **martingale approach**

  assumes risk neutral martingales
  Cox & Huang (1989)
  Campbell & Viceira (2002)
  trends in real world dynamics are ignored

- **benchmark approach**

  assumes real world martingales
  Pl. & Heath (2006)

  no equivalent risk neutral probability measure needed
  trends in real world dynamics are taken into account

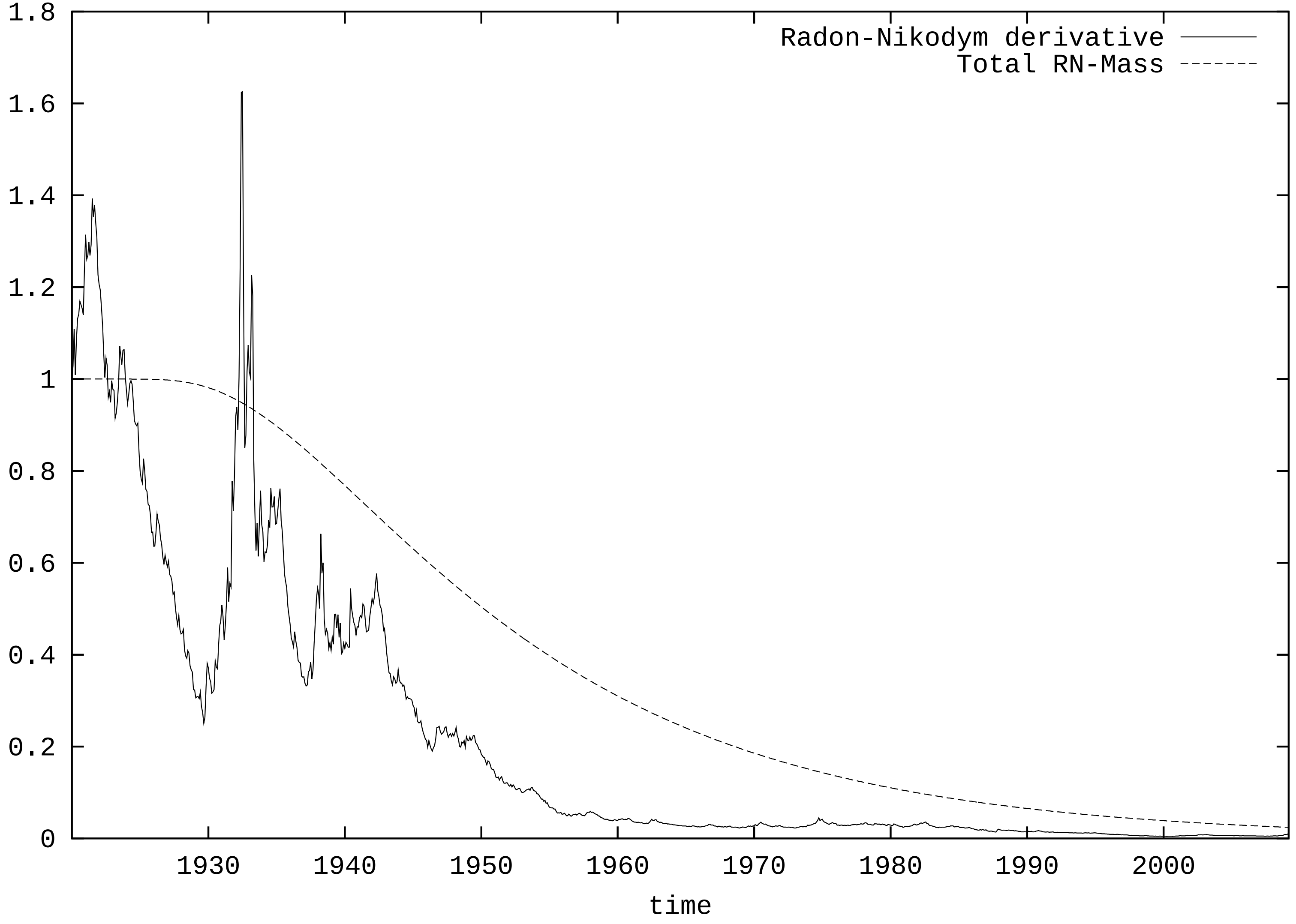

Figure 1: Radon-Nikodym derivative and total mass of putative risk neutral measure.

Optimal Wealth Dynamics

- **benchmark as best performing strictly positive portfolio**

  numeraire portfolio S_t^*

$$dS_t^* = S_t^*(i_t\,dt + \theta_t\,(\theta_t\,dt + dz_t))$$

market price of risk

$$\theta_t = \frac{a_t - i_t}{\sigma_t}$$

Long (1990), Becherer (2001), Pl. & Heath (2006)

growth optimal portfolio
Kelly (1956), Merton (1973)

$\Longrightarrow$ stochastic discount factor

$$M_t = \frac{1}{S_t^*}$$

similar as Cochrane (2001) but more general

$$dM_t = -\theta_t \, M_t \, dz_t - i_t \, M_t \, dt$$

$M_0 = 1$

- **accumulated total wealth** $W_t G_t$

$$G_t = \exp\left\{\int_0^t c_s \, ds\right\}$$

benchmarked accumulated total wealth

$$\tilde{W}_t = \frac{W_t \, G_t}{S_t^*}$$

$$d\tilde{W}_t = \tilde{W}_t \, (\alpha_t \, \sigma_t - \theta_t) \, dz_t$$

local martingale, supermartingale

Pl. & Heath (2006)

- **Law of the minimal price**

  Pl. (2008)

  $\Longrightarrow$

  real world pricing formula

$$U_t = S_t^* \, E_t\left(\frac{H_s}{S_s^*}\right)$$

- assume martingale property

$$\boldsymbol{E_t(W_s M_s G_s) = M_t\, W_t\, G_t}$$

for all $\boldsymbol{0 \leq t \leq s < \infty}$

martingale is most cost efficient

nonnegative supermartingale

- constraint optimization problem

$$
\begin{aligned}
v_t &= \max_{W} E_t \left(U\left(\frac{c_s W_s}{I_s} \right) \right) - \ell_t E_t \left(\tilde{W}_s - \tilde{W}_t \right) \\
&= \max_{W} E_t \left(U\left(\frac{c_s W_s}{I_s} \right) - \ell_t \left(\frac{W_s\, G_s}{S^*_s} - \frac{W_t\, G_t}{S^*_t} \right) \right)
\end{aligned}
$$

for $0 \leq t < s < \infty$

ℓ_t Lagrange multiplier

$\tilde{W}_t$ martingale

only real world probability used

- candidate for optimal wealth process

$$\left[U\left(\frac{c_s W_s}{I_s}\right) - \ell_t\left(\frac{W_s\,G_s}{S_s^*} - \frac{W_t\,G_t}{S_t^*}\right)\right] \to \max$$

$$U'\left(\frac{c_s W_s}{I_s}\right)\frac{c_s}{I_s} - \ell_t\,\frac{G_s}{S_s^*} = 0$$

$$\frac{c_s W_s}{I_s} = U'^{-1}\left(U'\left(\frac{c_s W_s}{I_s}\right)\right) = U'^{-1}\left(\ell_t\,\frac{G_s}{S_s^*}\,\frac{I_s}{c_s}\right)$$

$\Longrightarrow$

$$W_s = \frac{I_s}{c_s} U'^{-1}(\ell_t \phi_s)$$

with

$$\phi_s = \frac{G_s I_s}{S_s^* c_s}$$

Lagrange multiplier $\ell_t = \ell_0 = c_0 U'(c_0 W_0)$

$0 \leq t < s < \infty$

generally W maximizes

$$E_t \left(U \left(\frac{c_s W_s}{I_s} \right) \right)$$

- **benchmarked accumulated total wealth**

$$\frac{W_t\,G_t}{S_t^*} = M_t\,W_t\,G_t = F(\phi_t)$$

with

$$F(\phi) = \phi\,U'^{-1}(\ell_0\phi)$$

$$d\phi_t = \phi_t([c_t + \pi_t - e_t - i_t]\,dt - \theta_t\,dz_t)$$

$\Longrightarrow$ Itô formula

$$d\tilde{W}_t = \frac{\partial}{\partial \phi} F(\phi_t)\phi_t([c_t + \pi_t - e_t - i_t]\, dt - \theta_t\, dz_t)$$

$$+ \frac{1}{2}\frac{\partial^2}{\partial \phi^2} F(\phi_t)\phi_t^2\, \theta_t^2\, dt$$

Comparison of the drift coefficients

$\Longrightarrow$

$$0 = c_t + \pi_t - e_t - i_t - \frac{\theta_t^2}{2\,\gamma}$$

Example:

power utility

$$U'^{-1}(y) = y^{-\frac{1}{\gamma}}$$

$$F(\phi_s) = \ell_0^{\frac{1}{\gamma}} \, \phi_s^{1-\frac{1}{\gamma}}$$

$\Longrightarrow$

$\gamma_t = \gamma$

comparison of diffusion coefficients $\Longrightarrow$

$$\tilde{W}_t(\alpha_t\,\sigma_t - \theta_t) = -\frac{\partial}{\partial \phi}\,F(\phi_t)\phi_t\,\theta_t$$

$\Longrightarrow$

$$\alpha_t = \frac{\theta_t}{\bar{\gamma}_t\,\sigma_t}$$

with

$$\frac{1}{\bar{\gamma}_t} = 1 - \phi_t\,\frac{\frac{\partial}{\partial \phi}\,F(\phi_t)}{F(\phi_t)}$$

Example: power utility

$\Longrightarrow \quad \bar{\gamma}_t = \gamma$

Optimal Interest Rate

assume power utility

$\Longrightarrow$ **optimal interest rate:**

$$\tilde{i}_t = a_t - \gamma\, \alpha_t\, \sigma_t^2$$

Campbell & Viceira (2002), Pl. (1999)

cannot become negative $\Longrightarrow$

adjusted interest rate

$$i_t = (\hat{i}_t)^+$$

$\hat{i}_t$ - theoretical interest rate

- **negative optimal interest rates**

  Japanese stagnation
  great depression
  current financial crisis
  “quantitative easing”

- **change in consumption rate**

$$\frac{dc_t}{dt} = c_t\, e_t$$
$$= c_t \left(c_t + \pi_t - i_t - \frac{(a_t - i_t)^2}{2\gamma\, \sigma_t^2} \right)$$
$$= c_t \left(\pi_t - \tilde{\pi}_t - \frac{(i_t - \phi_t)^2}{2\gamma\, \sigma_t^2} \right)$$

with **critical inflation rate**

$$\tilde{\pi}_t = a_t - c_t - \frac{\gamma\, \sigma_t^2}{2}$$

and **intrinsic interest rate**

$$\phi_t = a_t - \gamma\, \sigma_t^2$$

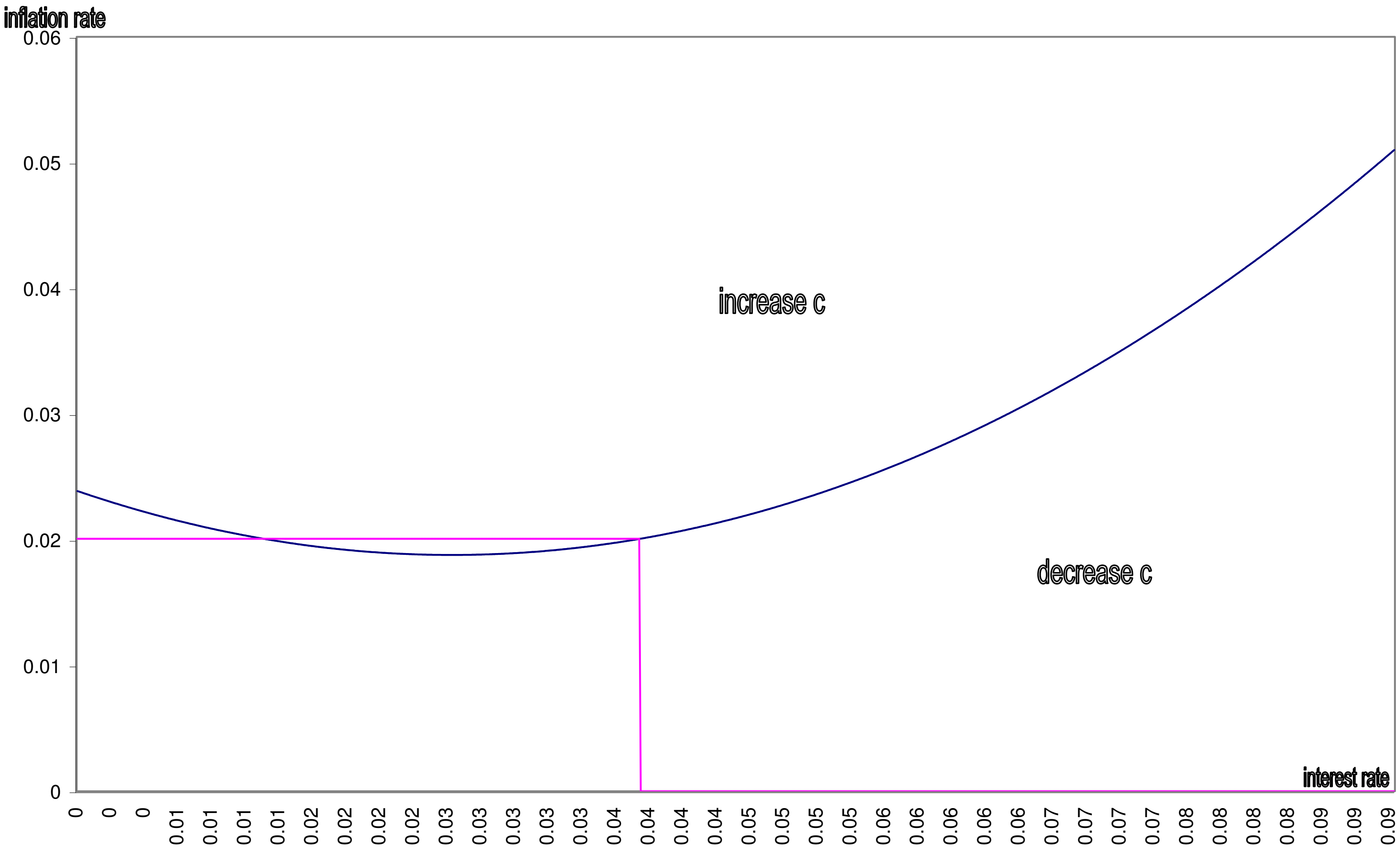

Figure 2: Inflation versus interest.

applying optimal interest rate

$\Longrightarrow$

$$\frac{dc_t}{dt} = c_t \left(\pi_t - \tilde{\pi}_t - \frac{\gamma\, \sigma_t^2}{2} (1 - \alpha_t)^2 \right)$$

$\Longrightarrow$ **Inflation Rate Rule:**

$$\pi_t = \tilde{\pi}_t + \frac{\gamma\, \sigma_t^2}{2} (1 - \alpha_t)^2$$

1. **Subcritical Inflation**

   $\pi_t < \tilde{\pi}_t$

   $\Longrightarrow \quad c_t$ decreases

   Interest rate does not matter !

   Nothing can stop downward trend !

   a_t may decrease

   $\Longrightarrow$ recession

   Subcritical inflation dangerous!

   $\Longrightarrow$ Low inflation rate targeting questionable !

2. **Supercritical Inflation**

$\pi_t \geq \tilde{\pi}_t$

- **without sufficient credit clearing**

$$\pi_t - \tilde{\pi}_t < \frac{\gamma\, \sigma_t^2}{2} \, (1 - \alpha_t)^2$$

$\Longrightarrow \quad c_t$ decreasing

not as strong as for $\pi_t < \tilde{\pi}_t$

Increased borrowing may further decrease c_t !

- **reversal:**

Making inflation sufficiently high

and balancing credit market !

- **Approximating Optimal Interest Rate**

  set interest rate i_t slightly above or below some interest rate

$$\bar{i}_t = \phi_t + \sqrt{2\,\gamma\,\sigma_t^2\,(\pi_t - \tilde{\pi}_t)}$$

- $i_t > \bar{i}_t$

  $\Longrightarrow$

$$(i_t - \phi_t)^2 \geq 2\,\gamma\,\sigma_t^2(\pi_t - \tilde{\pi}_t)$$

  $\Longrightarrow$ c_t **decreasing**

- $\phi_t < i_t < \bar{i}_t$

  $\Longrightarrow$ c_t **increasing**

  convenient mechanism

- for $\alpha_t < 1$ and inflation rate rule

  $\Longrightarrow \quad \bar{i}_t = \tilde{i}_t$ optimal interest rate

  and

$$\pi_t - \tilde{\pi}_t = \frac{\gamma\,\sigma_t^2}{2}\,(\alpha_t - 1)^2$$

- For $\alpha_t > 1$

  $\bar{i}_t$ not close to $\tilde{i}_t$

  Keep the inflation rather small !

  less optimal

Interest Rate Rule:

$$i_t = (\bar{i}_t)^+ = \left(a_t - \gamma\, \sigma_t^2\, (1 - |1 - \alpha_t|)\right)^+$$

assuming optimal inflation

- Create an economic environment where

$$a_t > \gamma \, \alpha_t \, \sigma_t^2 \quad \Longrightarrow \quad \tilde{i}_t > 0$$

$\Longrightarrow$ Avoid extended long boom with subsequent crash !

No cheap credit !

otherwise economic trap with zero interest

- **target inflation rate level**

$$\pi_t = a_t - c_t + \frac{\gamma\,\sigma_t^2}{2}\,\alpha_t\,(\alpha_t - 1)$$

after crash may be deflationary period if $\alpha_t \in (0, 1)$

when π_t on target, $\tilde{i}_t > 0$, $\alpha_t < 1$

$\Longrightarrow$ can steer the economy:

$$i_t > \tilde{i}_t \;\Rightarrow\; c_t \downarrow$$

$$i_t < \tilde{i}_t \;\Rightarrow\; c_t \uparrow$$

strongest impact on upward trend for

$$i_t \approx \phi_t = a_t - \gamma\, \sigma_t^2$$

optimal wealth evolution

when π_t on target, $\tilde{i}_t > 0$, $\alpha_t > 1$

$\Longrightarrow$ can steer the economy:

$$i_t > (\bar{i}_t)^+ \Rightarrow c_t \downarrow$$

$$i_t < (\bar{i}_t)^+ \Rightarrow c_t \uparrow$$

non-optimal wealth evolution

Set inflation π_t slightly above $\tilde{\pi}_t$!

Dangerous, could become subcritical

- when credit market clears

$$\alpha_t = 1$$

$\Longrightarrow$ target inflation rate minimal

least likely to have economic trap

For $c_t = c$, minimal inflation, $\alpha_t = 1 \Longrightarrow$

$$U\left(\frac{c\,W_t}{I_t}\right) = E_t\left(U\left(\frac{c\,W_s}{I_s}\right)\right)$$

fair monetary policy

- **benchmarked accumulated wealth**

  trendless

$$\frac{W_t \, G_t}{S_t^*} = E_t \left(\frac{W_s \, G_s}{S_s^*} \right)$$

  achieved in the least expensive manner

Country	a	i	π	σ	d	θ	$\gamma\alpha$	c	π_{min}	$\pi - \pi_{min}$
Australia	0.119	0.045	0.041	0.177	0.048	0.418	2.362	0.0910	0.034	0.007
Belgium	0.082	0.052	0.055	0.228	0.028	0.132	0.577	0.0056	0.039	0.016
Canada	0.097	0.049	0.031	0.168	0.041	0.286	1.701	0.0590	0.032	-0.001
Denmark	0.089	0.070	0.041	0.201	0.044	0.095	0.470	0.0334	0.035	0.006
France	0.121	0.043	0.079	0.231	0.036	0.338	1.462	0.0210	0.046	0.033
Germany	0.097	0.046	0.051	0.323	0.036	0.158	0.489	0.0075	0.035	0.016
Ireland	0.095	0.058	0.045	0.222	0.033	0.167	0.751	0.0027	0.044	0.001
Italy	0.120	0.047	0.091	0.294	0.038	0.248	0.845	-0.013	0.045	0.046
Japan	0.125	0.054	0.076	0.303	0.057	0.234	0.773	0.0055	0.033	0.043
Netherlands	0.090	0.037	0.03	0.210	0.049	0.252	1.202	0.0390	0.015	0.015
S. Africa	0.120	0.057	0.048	0.228	0.025	0.276	1.212	0.0470	0.064	-0.016
Spain	0.100	0.065	0.061	0.220	0.047	0.159	0.723	0.0300	0.036	0.025
Sweden	0.116	0.058	0.037	0.228	0.035	0.254	1.116	0.0530	0.052	-0.015
Switzerland	0.076	0.033	0.022	0.204	0.029	0.211	1.033	0.0330	0.026	-0.004
UK	0.101	0.051	0.041	0.200	0.038	0.250	1.250	0.0410	0.030	0.003
US	0.101	0.041	0.032	0.202	0.041	0.297	1.470	0.0530	0.030	0.002
Average	0.103	0.050	0.049	0.227	0.039	0.236	1.090	0.0330	0.038	0.011

Table 1: Estimates from sixteen markets

- market price of risk

$$\theta \approx \frac{a - i}{\sigma} \approx 0.236$$

- fraction times risk aversion

$$\gamma \, \alpha \approx \frac{a - i}{\sigma^2} = \frac{\theta}{\sigma} \approx 1.09$$

- consumption rate for $\alpha = 1$

$$c \approx i - \pi + \frac{(a-i)^2}{2\,\gamma\,\sigma^2} \approx 0.033 \approx 0.039 = d$$

- critical inflation

$$\tilde{\pi} \approx a - c - \frac{\gamma\,\sigma^2}{2} \approx 0.038 \leq 0.049 \approx \pi$$

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