

Limits of Limits of Arbitrage: Theory and Evidence*

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Abstract

We present a model where arbitrageurs operate on an asset market that can be hit by information shocks. Before entering the market, arbitrageurs are allowed to optimize their capital structure, in order to take advantage of potential underpricing. We find that, at equilibrium, some arbitrageurs always receive funding, even in low information environments. Other arbitrageurs only receive funding in high information environments. The model makes two easily testable predictions: first, arbitrageurs with stable funding should experience more mean reversion in returns, in particular following low performance. Second, this larger mean reversion should be lower, if many other funds have stable funding. We test these predictions on a sample of hedge funds, some of which impose impediments to withdrawal to their investors.

1 Introduction

In the literature on limits to arbitrage, a widening of the mispricing of an asset may lead arbitrageurs to unwind their positions, which further amplifies the initial mispricing (Shleifer and Vishny, 1997, Gromb and Vayanos, 2002). Such forced unwinding occurs because, as arbitrageurs lose money on their trades, their investors (brokers, banks, limited partners etc) demand early reimbursement of their claims. Thus, existing theories of the limits to arbitrage assume that arbitrageurs cannot design their capital structure *ex ante* (for instance, by taking on long term debt) in order to avoid such value destroying events.

This paper starts from the simple fact that this assumption does not always hold in reality, and investigates its theoretical and empirical consequences. In the hedge fund industry, investors often agree to limit their ability to withdraw their funds. About 20% of the hedge funds in our sample have lockup periods of typically one, or even two years, during which investors cannot redeem their shares (Aragon, 2007, has a similar proportion). Once they are able to do so, they must give the fund advance notice (typically a month) and then obtain

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redemption at fixed dates (typically a quarter). For the average hedge fund in our sample, we estimate the minimum duration of funds to be equal to 5 months, and 10 months for funds with lockup periods. Interestingly, such share restrictions can be found with hedge funds investing in illiquid securities (such as fixed income), but also with funds dealing with stocks (such as “long short equity” funds). They are what we call “limits of limits of arbitrage”: thanks to them, some market participants can afford to underperform in the short run while they hold on to ultimately profitable arbitrage opportunities.

Thus, at least some arbitrageurs choose the maturity of their investors’ claims. To understand the determinants and consequences of such a capital structure decision, we first build a model where arbitrageurs optimally design the securities that they issue, and then engage in arbitrage on the same market. Arbitrageurs differ in skill. We posit that arbitrageur skill affects long term asset payoffs in some states of nature only (“low information states”). We first find that, at equilibrium, prices in low information states are lower, because scarce arbitraging skills are needed to trade in these states. Furthermore, at equilibrium investors guarantee funding to skilled arbitrageurs in low information (low price) states, while unskilled arbitrageurs only receive funding in high information (high price) states. The intuition comes from the asset price equilibrium: if investors did not guarantee funds to some arbitrageurs in low information states, asset prices in these states would collapse, which would make investment attractive. Even when arbitrageur skill is not contractible upon, the equilibrium capital structure choice is separating: skilled arbitrageurs choose guaranteed funding, while unskilled arbitrageurs choose funding only contingent on high information (high prices). Thus, there is optimal differentiation at equilibrium.

Our model generates two easily testable predictions. First, conditional on past bad performance, funds with guaranteed funding outperform other funds. As argued above, in low information states, scarce skills are needed which lowers current prices. Thus, funds with guaranteed funding invest more often in states where the assets are underpriced, and thus outperform other funds who take less advantage of underpricing. Our second prediction is cross sectional: in industries where the fraction of arbitrageurs with guaranteed funding is large, these arbitrageurs overperform other funds less. The mechanism is deeply rooted in the model: if more arbitrageurs receive guaranteed funding, underpricing in the low information state will be reduced. Since these arbitrageurs benefit from underpricing, their overperformance will be reduced.

We then test these two predictions on hedge fund data. We use the fact that, in our data, funds with impediments to withdrawal (such as long redemption periods, or lockup periods) experience less outflows when they underperform (Ding et al., 2008, find similar evidence). Thus, these share restrictions, which we can observe from the data, are a good proxy for “guaranteed funding” in our model. We find that the first prediction of our model holds in the data: conditional on bad past performance, funds with impediments to withdrawal do “bounce back more”, i.e., have higher expected returns. We also find some support for the second prediction. To test it, we look at investment styles where impediments to withdrawal are prevalent. We find that, in these styles, funds with such share restrictions overperform

other funds relatively less than in styles where such impediments are relatively rare, although these results are somewhat less robust.

This paper contributes to two strands of literature. First, we extend Shleifer and Vishny’s model of limits of arbitrage by allowing arbitrageurs to optimally choose their capital structure in order to avoid inefficient liquidation. In this sense, our paper is closely related to independent work by Stein (2009), which is the only paper, to the best of our knowledge, that explicitly seeks to endogenize arbitrageurs’ capital structures. Compared to his model, our theory endogenizes the cost of external finance more explicitly and makes testable predictions on observed arbitrageur returns, that we can bring to the data. What is common to the two models is that underpricing in bad states of nature leads to more investment in these states, through the optimal structure choice. This feedback mechanism is not present in Shleifer and Vishny (1997) nor in Stein (2005), who does not endogenize prices. The rest of the limits to arbitrage literature considers the destabilizing feedback that goes through the wealth of arbitrageurs. Gromb and Vayanos (2002), Brunnermeier and Pedersen (2008) and Acharya and Viswanathan (2007) model intermediaries that need to unwind their positions when collateral prices decrease, which amplifies price drops. In all these models, even if mispricing can be very large, there is no contractual way to take advantage of this.¹

Second, we shed new light on existing evidence from the (mostly hedge) funds literature. First, our findings are related to a recent paper on open end mutual funds by Coval and Stafford (2008): they look at asset fire sales following massive redemptions at mutual funds, and find a significant price impact. Thus, their paper suggests (but does not test) that mutual fund performance should display some persistence, in particular conditional on past low performance. We propose a theory of why some funds may seek protection against massive redemptions, and what returns dynamics should look like in protected and unprotected funds. Second, some papers show that the presence of impediments to withdrawal is correlated with unconditional fund performance (Aragon, 2007, Agarwal et al., 2008): their explanation is that investors earn a premium for the illiquidity of their investment. Other papers have informally argued that hedge funds act as liquidity providers (see, e.g., Agarwal, Fung, Loon and Naik, 2008). The present article suggests a potential reason why illiquid funds can afford to issue illiquid shares in the first place: because illiquidity allows them to reap the gains of arbitrage, they can pay the illiquidity premium to their investors. Third, we develop and test a theory of the mean reversion of fund returns. Interestingly, some of the existing hedge funds literature has focused on the *positive* relation between autocorrelation and share

¹Also related to this paper is Lerner and Schoar (2004). They test a model where (private equity) fund managers make their shares illiquid in order to select “patient” investors. Their capital structure focus differs from ours in an important way: they look at the ability to sell shares to *other investors*, while we look at the ability to sell shares to the *fund*. In addition, if we were to transpose their mechanism in our setting where funds interact through buying and selling the same asset, funds would screen “patient investors” until mispricing disappears. There would be no prediction on the link between share restriction and fund returns dynamics.

Casamatta and Pouget (2008) solve a model where investors give fund managers incentives to search for information on assets. In their model, the optimal contract features short term performance pay. The cost of such contracts is that they reduce market efficiency by deterring information acquisition.

restrictions (see, e.g., Aragon, 2007) while we find clear evidence of such a *negative* relation. The difference between these studies and ours is the frequency at which autocorrelation is computed: we work at the annual level, while existing papers work at the monthly level. At the monthly frequency, the existing literature argues that reported returns of illiquid assets are smoothed. At the annual frequency, this paper argues that arbitrage induces a mean reversion in fund returns. To some extent, such evidence is reminiscent to insights from the strategic allocation literature (Campbell and Viceira, 2002) which argues that long term investors have a comparative advantage at investing in mean reverting assets.

The rest of the paper follows a simple structure. Section 2 describes, solves the model and derives predictions and comparative statics. Section 3 tests the model. Section 4 concludes.

2 Model

2.1 Set-Up

There are competitive, risk neutral, investors. Investors want to purchase an asset which is in unit supply, but cannot do so themselves. They delegate this task to a measure 1 of fund managers. Fund managers are risk neutral and limitedly liable; each of them starts with initial wealth A .

2.1.1 Sequence of Events

There are four periods $t = 0, 1, 2, 3$ and the discount rate is zero. At $t = 0$, investors contract with managers. The optimal contract will specify the amount of funds that the investor will entrust to the manager, both in $t = 1$ and 2, and conditional on the state of nature in $t = 2$.

At date $t = 1$, each fund manager *learns about the asset* he will be trading: the acquired knowledge will only be useful in period $t = 2$. Learning effort costs C to the manager. As explained below, we assume that learning effort is not contractible. With high learning effort, the manager becomes skilled with probability μ ; with low learning effort, with probability $\mu - \Delta\mu$. The manager does not know whether he is skilled until period 3. After the learning phase, managers use entrusted funds to purchase assets at unit price P . The market for assets clears.

At date $t = 2$, the market can be in one of three states. With probability λ_U , the market is in state U : in this state, knowledge acquired in period 1 is useless (think for instance of a bull market where everyone can generate high returns). It becomes public knowledge that the asset will generate $t = 3$ cash flows of $V > 0$. All fund managers liquidate their positions from $t = 1$, pay off their investors, and use newly entrusted funds, as specified in the optimal contract, to repurchase the same assets. The market clears again at price P_U .

With probability λ_M , the market is in state M . In this state, we assume that a second asset, which is a priori not distinguishable from the first asset, appears. Because they cannot be differentiated from each other, both assets trade at the same price, but we assume that the second asset has zero present value, while the first asset has, as in state U , a PV of V .

Furthermore, we assume that only a fraction μ of the managers picks the “right” asset. The important hypothesis is that, in state M , the asset PV does not depend on $t = 1$ effort. Thus, compared to state U , state M is a *bad* state, in the sense that there is less information than in state U , but the state is equally bad for all managers, irrespective of their $t = 1$ learning decisions. In this state, the market for assets clears at price P_M .

Last, with probability λ_D , the market is in state D . Exactly as in state M , a second asset appears that has a PV of zero, but this time skilled managers can differentiate between the two. Thus, an important difference between states M and D is that in state D , date 1 learning effort matters. In this sense, state D also is a *bad* state, but it is worse for managers who did not learn in $t = 1$. In this state, the “right” asset market clears at price P_D , which is also the price of the “wrong” asset whose market we do not model.

At date $t = 3$, assets held in portfolios mature and payoffs are realized. If the “right” asset is held, its payoff is V . In states M and D , we assume that only $V - B$ can be pledged to the investors. We think of B as the rent of an unmodeled agency conflict in period 3: for instance, the manager can sell the asset on a black market for price B and consume the proceeds. To simplify exposition, we assume that this agency conflict does not exist in state U , in which case the entire present value of the asset V can be pledged to investors. All intuitions of this model would carry through without this assumption.

All in all, states U , M and D vary along two key dimensions. First, in state U , expected cash flows from assets are higher than in states M and D . Expected present value in U is always V ; in state M it is only μV . In state D only skilled managers will be able to buy good assets, so the expected payoff is at most μV . This feature of the model ($\mu < 1$) is not entirely necessary for most intuitions to carry through; we will explain why later on. The second difference between the three states is that, in state D , managerial skill matters. Thus, a manager who is committed more funds in state D will have more incentive to learn. This second dimension of our model is essential.

2.1.2 Contracts

We assume that the financial contract specifies four amounts entrusted to the manager: I , in period 1, and (I_U, I_M, I_D) in period 2, conditional on states U , M and D . Thus, we make two implicit, and mostly simplifying, assumptions. First, we assume that date 1 learning effort cannot be contracted upon. As we explain later on, this assumption is not necessary to obtain our results. Second, we assume that the date 2 state of nature is contractible. This is the case for instance if period 2 returns are contractible.² It is precisely the goal of this paper to study the impact of the contingent financing of arbitrageurs.

The financial contract could also include a compensation for the manager in some states of nature, in order to induce the manager to put in high learning effort. To simplify the analysis, we will impose restrictions on parameter values such that the incentive compatibility

²Indeed, the asset prices in the three period 2 states of nature will be different from each others. Therefore, the rational expectations equilibrium we will find when the state of nature is contractible, persists when the state of nature is observable but not verifiable, and returns are contractible.

constraint is not binding. Therefore, adding an incentive fee on top of the private benefit B per unit of asset will not be part of an optimal contract.

In the empirical application, we will think of contracts where $I_D > 0$ as contracts with impediments to withdrawals. Such contracts guarantee $t = 2$ inflows even when the fund underperforms, i.e., when asset prices go down in state D .

2.1.3 Modeling Strategy

Before we solve the model, it is worthwhile to discuss our modelling strategy. With moral hazard, another possible strategy could have been to think of withdrawals as an ex ante optimal “punishment” strategy. Underperforming managers are punished for low effort provision, while well performing managers are rewarded through continuation. Such a model delivers similar comparative static properties as the one we study in this paper, but the implied contract has the important drawback of not being renegotiation-proof. Once low effort has been provided, assets are fairly priced in equilibrium, and their expected return is non negative. Thus, shutting down the fund is never an optimal decision ex post and the punishment is non credible.

One second alternative would have been to model limits of arbitrage as arising because investors learn about the fund manager’s skill, assuming such a skill is fixed from the beginning, i.e., not obtained through learning. If the fund underperforms, then it becomes optimal to withdraw investment because the chances that the manager is incompetent are high. In such a model, there would be no reason for an investor to lock his money up in the fund because there is no efficiency gain to do so.

To make impediments to withdrawals optimal from a contracting perspective, they must have an incentive property, so we choose a moral hazard setting: learning entails an “effort”. An alternative model would be to assume that managers have fixed types (skilled or unskilled) and that investors seek to design separating contracts. Such a model would be almost identical to the model we present in this paper, except that learning is exogenous. Such a model would generate identical predictions to the ones we derive and test here.

2.2 Solving the Model

We first derive the optimal contracts for given expected asset prices, and then solve for the rational expectations equilibrium of the asset market. This allows us to (1) characterize the equilibrium and (2) find a relationship between impediments to withdrawals ($I_D > 0$), and the equilibrium returns of the funds.

2.2.1 Optimal Contracts

In this section, we take the sequence of future prices P , P_U , P_M and P_D as given. The optimal contract solves the manager’s objective function, which is the project’s NPV, under the constraints that the profit pledgeable to investors is nonnegative and that the manager exerts the desired level of learning effort (Tirole, 2006).

We first focus on high learning effort funds:

$$\begin{aligned} \max_{I, I_U, I_M, I_D} \quad & I[\lambda_U P_U + \lambda_M P_M + \lambda_D P_D - P] + \lambda_U I_U [V - P_U] \\ & + \lambda_M I_M [\mu V - P_M] + \lambda_D I_D [\mu V - P_D], \end{aligned}$$

subject to

$$\begin{aligned} & I[\lambda_U P_U + \lambda_M P_M + \lambda_D P_D - P] + \lambda_U I_U [V - P_U] \\ & + \lambda_M I_M [\mu(V - B) - P_M] + \lambda_D I_D [\mu(V - B) - P_D] + A \geq 0, \end{aligned}$$

and

$$\lambda_D \Delta \mu B I_D > C.$$

The objective function is the overall NPV of the fund. The first term is the total profit made between period 1 and 2, which is equal to the expected price increase times the amount invested in $t = 1$. Given that this profit is free from any agency consideration, it can be pledged to the investor at 100%, which is why it also appears as such in the first (investor participation) constraint. The second term is the $t = 2$ NPV realized in state U , which can also be fully pledged. The third term is the expected NPV in state M . In this case, the manager will purchase the right asset with probability μ , since learning effort has been made, and, as appears in the first constraint, only $\mu(V - B)$ per asset purchased can be pledged at $t = 0$. In state D , the conditional expected payoff per asset is the same, because the manager puts in high effort. The second constraint is the manager's incentive compatibility constraint which ensures that, in period 1, high learning effort is always preferred. Given our parameters restrictions below, this constraint will never bind at equilibrium.

It is clear from the above problem that

$$P = \lambda_U P_U + \lambda_M P_M + \lambda_D P_D$$

will have to hold in equilibrium. If this is not the case, I will be equal to $+\infty$ or $-\infty$. Hence, markets in $t = 1$ are fully efficient in this model because there is no agency friction in $t = 1$: any profit from arbitrage is pledgeable, so that infinite amount of funds can be used to finance arbitrageurs. This reduces arbitrage opportunities to zero. For the same reason, the same happens in state U : $P_U = V$. Thus, all funds receive an indeterminate amount of funding in state U .

Moreover, it is easy to see that $P_M \leq \mu V$ and $P_D \leq \mu V$ have to hold in equilibrium, otherwise no fund would be willing to hold the asset in state M or in state D . At the same time, $P_M > \mu(V - B)$ and $P_D > \mu(V - B)$. This comes from the fact that the marginal pledgeable income of investment has to be strictly negative in equilibrium. If this is not the case, fund managers could raise money to invest more, as the NPV of doing so is strictly positive. This would contradict the equilibrium.

Given these properties and the convenient linearity of the problem, we obtain that

$$\begin{aligned} I_D &= \frac{1}{\lambda_D} \frac{A}{P_D - \mu(V - B)}, \\ I_M &= 0, \\ NPV &= \frac{A}{P_D - \mu(V - B)} (\mu V - P_D) - C, \end{aligned}$$

if $P_D < P_M$. Hence, if the price in state D is low enough compared to the price in state M , it is then optimal to allocate all pledgeable income in state D where the asset is relatively cheap. In contrast, as soon as $P_M < P_D$,

$$\begin{aligned} I_D &= 0, \\ I_M &= \frac{1}{\lambda_M} \frac{A}{P_M - \mu(V - B)}, \\ NPV &= \frac{A}{P_M - \mu(V - B)} (\mu V - P_M) - C. \end{aligned}$$

When the asset is cheap in state M , it is optimal to allocate all pledgeable income in state M .

Computations and expressions are very similar when the learning effort is low. In this case, the fund will invest in state D only when $P_M > \frac{\mu}{\mu - \Delta\mu} P_D$, and in state M only when the reverse inequality holds. This leads us to the following lemma:

Lemma 1. *For given asset prices $P_M \leq V$ and $P_D \leq \mu V$, there are five regimes:*

In all regimes, all funds receive funding in state U . In addition,

1. $P_M < P_D$: both high and low effort funds invest in state M only.
2. $P_M = P_D$: high effort funds are indifferent between investing in state M and D ; low effort funds invest in state M only.
3. $P_D < P_M < \frac{\mu}{\mu - \Delta\mu} P_D$: high effort funds invest in state D only; low effort funds invest in state M only.
4. $P_M = \frac{\mu}{\mu - \Delta\mu} P_D$: high effort funds invest in state D only; low effort funds are indifferent between investing in state M and D .
5. $\frac{\mu}{\mu - \Delta\mu} P_D < P_M$: both high and low effort funds invest in state D only.

The results of this lemma are intuitive: high P_M discourages funds to invest in state M . In addition, high effort funds have higher returns to investing in state D , since this is when learning effort pays off. Hence, high effort funds are ready to invest in state D for higher levels of P_D .

The above lemma also indicates that cases 1 and 5 cannot be equilibrium outcomes, since in these cases there is no demand for assets in either state D or M . Putting aside the knife-edge cases 2 and 4, this suggests that in equilibrium both levels of learning effort coexist: high

effort funds invest in state D only, while low effort funds invest in state M only.³ We now turn to the description of the equilibrium.

2.2.2 Equilibrium

Following the above discussion, we restrict ourselves to $P_D < P_M < \frac{\mu}{\mu - \Delta\mu} P_D$. Let α be the equilibrium fraction of high effort funds. In equilibrium, since both categories of funds coexist, funds have to be indifferent, ex ante, between putting in high effort (and buy in state D) or low effort (and buy in state M):

$$\frac{\mu V - P_D}{P_D - \mu(V - B)} - \frac{C}{A} = \frac{\mu V - P_M}{P_M - \mu(V - B)}. \quad (1)$$

We now need to compute equilibrium prices P_M and P_D . Aggregate asset demand by funds in state M and state D has to be equal to supply (assumed equal to 1). Hence

$$P_M = \mu(V - B) + \frac{\mu(1 - \alpha)A}{\lambda_M}, \quad (2)$$

$$P_D = \mu(V - B) + \frac{\mu\alpha A}{\lambda_D}. \quad (3)$$

The price in each state is higher, the higher the expected payoff, the higher the equity of managers, and the higher the number of funds operating in this state. Plugging back (2) and (3) into indifference condition (1), we obtain the following equation for the equilibrium α :

$$\frac{\lambda_D}{\alpha} - \frac{C}{B} = \frac{\lambda_M}{1 - \alpha}. \quad (4)$$

It is straightforward to see that $\alpha \in (0; 1)$. Moreover, α is increasing in λ_D , and decreasing in λ_M and C/B . When the cost of making effort (C) decreases, or when the gains of making effort (λ_D) are larger, there will be more high effort funds operating in equilibrium.

So far we have assumed that, once the contract is signed, the fund manager puts in the expected amount of effort. It is straightforward to see that a manager with $I_D = 0$ will make no learning effort, since it will never pay off. A fund manager with $I_D > 0$ puts in high effort if and only if his incentive constraint is satisfied

$$I_D = \frac{1}{\alpha\mu} > \frac{C}{\lambda_D B \Delta\mu}, \quad (5)$$

i.e., I_D is large enough to make the gain of learning $\lambda_D \Delta\mu B I_D$ larger than the effort cost C . Condition (5) ensures that providing the manager with an incentive fee on top of the private benefit B is not part of an optimal contract.

³Case 2 cannot actually arise in equilibrium, otherwise a high training effort fund would be indifferent in $t = 2$ between investing in state M or D . In $t = 1$, it would then be optimal to make low effort and invest in state M only, to save the training effort cost, hence there would be not demand for the asset in state M . By contrast, case 4 can be an equilibrium outcome for some parameter values. To clarify exposition, we rule them out in the following.

Finally, we need to ensure that asset prices in period 2 are below their fundamental value in equilibrium, else conditions (2) and (3) do not apply. This occurs if and only if

$$A < \frac{\lambda_M B}{1 - \alpha}. \quad (6)$$

Intuitively, if fund managers have little equity, their demand will be so low that prices do not reach their fundamental values, even in state M .

Hence, an equilibrium is defined by equation (4), under conditions (5) and (6). Equilibrium prices also have to satisfy $P_D < P_M < \frac{\mu}{\mu - \Delta\mu} P_D$. The following proposition characterises such an equilibrium, and provides a parameter condition for its existence.

Proposition 1. *There exist \bar{A} and $\underline{\Delta\mu} < \mu$ such that, if*

$$\Delta\mu > \underline{\Delta\mu} \quad \text{and} \quad A < \bar{A},$$

1. *The only equilibrium is an equilibrium where $\alpha \in (0; 1)$ funds make high learning effort and are only committed funds in states U and D , and $1 - \alpha$ funds make low learning effort and are only entrusted funds in states U and M .*
2. *α is defined by equation (4). Equilibrium prices are such that $P_D < P_M$.*
3. *The ex ante optimal contract is renegotiation-proof in equilibrium.*

Proof. Let α be the (unique) positive solution of equation (4). Let $\bar{A} = \lambda_M B / (1 - \alpha)$. The condition $A < \bar{A}$ is equivalent to condition (6) which is therefore satisfied. Let $\underline{\Delta\mu} = C\alpha\mu / \lambda_D B$. The condition $\Delta\mu > \underline{\Delta\mu}$ ensures that the incentive compatibility constraint for high effort funds holds. From (4), it is easy to see that $\underline{\Delta\mu} < \mu$.

From equilibrium prices (2) and (3), it is easy to obtain that

$$P_M - P_D = \left(\frac{1 - \alpha}{\lambda_M} - \frac{\alpha}{\lambda_D} \right) \mu A$$

is strictly positive by virtue of (4). Finally, straightforward manipulations show that $P_M < \frac{\mu}{\mu - \Delta\mu} P_D$ is equivalent to

$$\frac{\Delta\mu}{\mu} > \frac{1 - \frac{\alpha}{\lambda_D} \frac{\lambda_M}{1 - \alpha}}{1 + \frac{V - B}{A} \frac{\lambda_M}{1 - \alpha}} = \frac{\frac{\alpha}{\lambda_D} \frac{C}{B}}{1 + \frac{V - B}{A} \frac{\lambda_M}{1 - \alpha}}$$

by equation (4), which holds when $\Delta\mu > \underline{\Delta\mu}$. \square

The optimal contract is renegotiation-proof because, in equilibrium, the asset price is always above the marginal pledgeable payoff, but below the marginal NPV. As a result, the manager cannot raise new funds, since he can only promise a negative income $V - B - P_i$, $i = M, D$, nor is he willing to cut down investment, since he obtains utility B per asset invested. Put differently, the contract is renegotiation-proof because continuation is as optimal

ex post as it is ex ante: the size of the surplus does not increase nor decrease and there is thus no scope for renegotiation.

One can compute the relative underpricing in state D as the difference $P_M - P_D$. Given that the expected PV of the assets is $\mu(V - B)$ in both states, $P_M - P_D$ measures the price difference that is not due to a difference in expected payoff, but simply a lack of invested funds. This underpricing is given by

$$P_M - P_D = \left(\frac{1 - \alpha}{\lambda_M} - \frac{\alpha}{\lambda_D} \right) \mu A, \quad (7)$$

therefore it is an increasing function of C/B . It is equal to 0 if $C/B = 0$. As the cost of learning tends to zero, more and more funds are willing to raise money in state D . This brings prices in state D closer to fundamental value.

This remark extends the model of Shleifer and Vishny (1997) to a full contracting framework, where investors are also able to commit funding in the low state of nature, i.e., when the asset is underpriced. What we show is that, when assets are underpriced, there is an incentive for another class of funds to specialize in this state. Yet, because arbitrage in this state is costly (managers need to learn enough about the asset), state D prices cannot increase too much.

A related point is that learning is necessary in our model to obtain underpricing. Assume that C is so large that it is never optimal to learn in period 1. In this case, we are looking for an equilibrium where both funds investing in states M , and in state D , make no learning effort. It is easy to verify that there is no price distortion. The intuition is that entering state D is now costless as it entails no learning effort: free entry in the two states eliminates the relative underpricing in state D .

2.3 Predictions of the Model

Our first prediction is related to net of fee conditional returns. For both types of funds, expected returns conditional on good performance in period 2 (i.e., in state U) are equal to zero, since $P_U = V$. This comes from the fact that there is no agency friction B in this state. Including one would not change the results: both types of funds would still have the same expected returns in this state, since they would purchase the same asset at the same price.

Our model has more interesting predictions on returns conditional on low performance in period 2:

$$E(R_3 | R_2 \text{ is low}, I_D > 0) = \mu(V - B) - P_D = -\frac{\mu\alpha A}{\lambda_D}, \quad (8)$$

$$E(R_3 | R_2 \text{ is low}, I_D = 0) = \mu(V - B) - P_M = -\frac{\mu(1 - \alpha)A}{\lambda_M}. \quad (9)$$

It appears clearly that expected returns of high effort funds are larger than expected returns of low effort funds, since high effort funds invest in state D , where assets are significantly underpriced ($P_D < P_M$). What is interesting is that this prediction holds in equilibrium, even though “entry” in both states of nature is free at the contracting stage.

Implication 1. *High effort funds exhibit more mean reversion in returns, in particular when past returns are low:*

1. *Conditional on high past returns, both funds have similar expected returns*

$$E(R_3|R_2 \text{ is high}, I_D > 0) = E(R_3|R_2 \text{ is high}, I_D = 0).$$

2. *Conditional on low past returns, high effort funds overperform low effort ones*

$$E(R_3|R_2 \text{ is low}, I_D > 0) > E(R_3|R_2 \text{ is low}, I_D = 0).$$

Before we proceed, we note that the model would have exactly the same properties if learning effort was contractible. The only difference is that the incentive compatibility condition on $\Delta\mu$ would be replaced by another (slightly weaker) condition on $\Delta\mu$ to ensure that high learning effort is optimal for funds invested in state D . Thus, the differential mean reversion does not hinge on the contractibility or incontractability of learning effort.

The above prediction is related to Aragon (2007) and Agarwal et al. (2008), who find empirically that hedge funds with impediments to withdrawal tend to exhibit superior performance, even after controlling for usual risk factors. They interpret this correlation as evidence that investors demand a premium for holding illiquid (i.e., locked up) shares.⁴ And indeed, given the loss of (put) option value, the cost of illiquidity to investors can be quite sizeable (Ang and Bollen, 2008, perform a calibration using a real option model). Our model has the feature that high effort funds may exhibit higher performance under some circumstances. Using (8) and (9), we find easily that the excess unconditional performance of high effort funds is given by

$$E(R_3|I_D > 0) - E(R_3|I_D = 0) = (1 - 2\alpha)\mu A$$

High effort funds outperform in our model as long as $\alpha < 1/2$. If α is small enough, fewer funds invest in state D while more funds invest in state M . Thus, underpricing in state D is large enough to make high effort funds outperform low effort ones.

Our model does not only predict that high effort funds' returns mean revert more, but also makes predictions on the relative size of the mean reversion. Intuitively, as the share of high effort funds α increases, there is more investment in state D , which increases P_D and reduces P_M . Given that high effort funds invest in state D , their outperformance should therefore be reduced. Thus, across asset markets, α and the overperformance of high effort funds conditional on past low returns should be negatively correlated. Given that α and the extent of mean reversion are both endogenous, we need to verify that this intuition holds in the model. We do this in the proposition below.

Implication 2. *When C decreases:*

1. *α increases.*

⁴It is interesting to note that the presence of impediments to withdrawals does not necessarily mean that investment is illiquid. One possibility is that shares, even though not immediately redeemable, can be traded among investors on a secondary market. See for instance Ramadorai (2008) for a description of such a market.

2. The outperformance of high effort funds, conditional on bad performance,

$$\rho = E(R_3|R_2 \text{ is low}, I_D > 0) - E(R_3|R_2 \text{ is low}, I_D = 0),$$

decreases.

Proof. From (8) and (9),

$$\rho = \left(\frac{1 - \alpha}{\lambda_M} - \frac{\alpha}{\lambda_D} \right) \mu A$$

is a decreasing function of α . As C decreases, α increases, which reduces ρ . \square

We now turn to formal tests of our empirical predictions. We do not observe learning effort in the data, but we know from the model that learning effort is high for funds who still receive funding in state D , i.e., when past performance has been relatively poor. We use the fact that, in our data, funds with impediments to withdrawal face lower reductions in assets under management conditional on bad performance (see also Ding et al., 2008, for related evidence). Thus, we use the presence as strong impediments to withdrawal as our measure that $I_D > 0$, and test the two predictions derived here.

3 Empirical Evidence

3.1 Data Description

We start from a June 2008 download of EurekaHedge, a hedge fund data provider. The download provides us with monthly data from June 1987 until June 2008. 6,070 funds are initially present in the sample, with a total of 366,728 observations. Every month, each fund reports asset under management and net of fee returns. We delete from the data set all funds that have less than \$20m under management.

Our main results use annual data, but we also use higher frequency information on returns (monthly and quarterly, see below). Descriptive statistics on returns and AUM are provided at the annual frequency in Table 1, Panel A. Mean annual return is about 11% net of fees. Mean assets under management are 300 million dollars. Also available from the data are fund level characteristics that do not change over time, whose descriptive statistics are reported in Panel B. Using these information, we construct two dummy variables to capture the presence of “strong” impediments to withdrawal:

- **Lockup dummy:** In some cases, investors agree to lock their investment in the fund for a given period of time after their investment. Out of 5,154 funds for which share restrictions are known, the mean lockup period is about 2.6 months. This mean conceals a lumpy distribution: 21% of the funds have lockup periods, 15% have a lockup period of 12 months, and only 2% have a longer lockup period. The percentage of funds with lockup periods that we have in our dataset is similar to what Aragon (2007) has in his TASS extract.

- **Redemption dummy:** Once the lockup period has passed, investors can redeem their shares, but still face constraints. Redemption can only occur at fixed moments of the year. For 3,152 funds (53% of the total), redemption is monthly. It is quarterly for 25% of the funds (1,499), and annual in 151 cases. In addition, investors have to notify the fund of their withdrawal before the redemption period. This notice period is lower than 1 month in 30% of the cases, equal to 1 month in 30% of the cases, and is equal or above one quarter in 15% of the cases. We construct a dummy variable equal to one when the sum of the redemption and notice periods is equal or longer than a quarter (90 days). The mean value of this sum is equal to 92 days; for 38% of the funds, it is equal or larger than a quarter.

Table 1: Summary Statistics

	All funds with AUM>20m		
	Mean	Std	Obs
Panel A: Annual variables			
Return	10.5	13.5	7,041
AUM	301	554	7,041
Net Inflows	18.0	75.3	7,041
Net Inflows × (Net Inflows < 0)	-13.0	22.6	7,041
Panel B: Fixed Characteristics			
Long Short	0.44	-	5,310
Global Macro	0.06	-	5,310
Fixed Income	0.05	-	5,310
Lockup Period (months)	2.6	5.8	5,154
Lockup dummy	0.21	-	5,154
Notice + Redemption Periods	92	84	4,805
Quarterly Not.+Red. dummy	0.38	-	4,805

Data: EurekaHedge, 1994-2007. Annual data, excluding funds with AUM lower than 20 million USD.

Finally, the spearman correlation between the lockup dummy and the redemption dummy is 41% (using one data point per fund). Thus, even though this correlation is positive and statistically significant, which indicates some complementarity between the two forms of share restriction, it is far from being equal to 1. In particular, 23% of the funds without lockup have “redemption periods”.

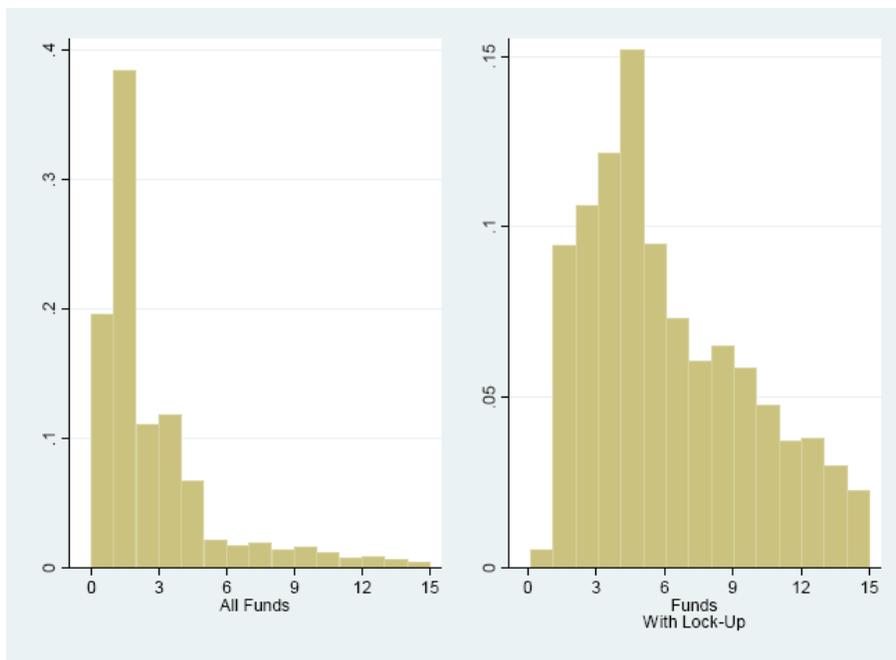
How effectively constrained are hedge fund investors? To answer this question, we compute the mean duration of capital, for each fund, separately for each year. We do this by including the effects of lockup periods, redemption date and advance notice. We use the following

formula:

$$\text{Duration}_{it} = \text{Notice}_i + \frac{\text{Redemption Period}_i}{2} + \frac{1}{\text{AUM}_{it}} \sum_{s=0}^{\text{Lockup Period}_i} \text{Net Inflow}_{it-s} \times 1_{\{\text{Net Inflow}_{it-s} > 0\}} \times (\text{Lockup Period}_i - s).$$

The first part of this formula accounts for the effect of notice and redemption periods. The implicit assumption behind this formula is that fund’s distance to the next redemption period is uniformly distributed. The second part accounts for the effect of lockup periods. For each past net inflow into the fund, it computes the remaining lockup duration (for instance, 5 month old inflows have a duration of 7 months if the lockup period is one year). We then normalize by current assets under management. We use monthly data. Following the literature on fund flows (Chevalier and Ellison, 1997, Sirri and Tufano, 1998), we compute net inflows by taking the difference between monthly AUM growth and monthly returns, and remove outliers. Overall, the above formula is an approximation. First, past inflows are computed net of outflows. This procedure leads us to underestimate gross inflows if they occur at the same time as gross outflows. Second, when shares are still locked up, the notice and redemption periods are in part ineffective. This leads the above formula to overestimate duration.

Figure 1: Duration of Fund Liabilities



Data: EurekaHedge, 1994-2007. Monthly data, excluding funds with AUM lower than 20 million USD.

We plot the sample distribution of estimated durations in Figure 1. Taking all fund-months

in the sample, the sample mean of this measure is 3 months. On average, the contributions of potential lockup periods and redemption and notice are of similar sizes. The 25th, 50th and 75th percentiles of the distribution are respectively 1, 1.5 and 3.5 months. The time series of the mean duration exhibits a clear downward trend, from 3.5 months in 1996 to about 2.6 months in 2007. If we focus on the subgroup of funds with lockup periods (21% of our sample), mean duration is, unsurprisingly, much larger: 8.2 months (median is 5.8). Thus even though most funds have relatively short duration of liabilities, there is a group of funds for which the mean dollar of AUM is secured for at least half a year.

As expected, such impediments to withdrawals do indeed prevent outflows from happening in the data. To check this, we run the following regression on annual data:

$$\text{Outflow}_{it} = \gamma_i + \beta.1_{\{r_{it-1} < r_{t-1}^{rf}\}} + \delta.1_{\{r_{it-1} < r_{t-1}^{rf}\}} \times \text{Impediment}_i + \varepsilon_{it}$$

where Outflow_{it} is a variable equal to 0 if the fund experiences net inflows in year t , and equal to net inflows if net inflows are negative. $1_{\{r_{it-1} < r_{t-1}^{rf}\}}$ is a dummy variable equal to 1 if past year's returns have been lower than the risk-free rate, as measured by the yield on 3-month Treasury Bill. Impediment_i is one of the two measures described above: a lockup period, or a redemption period of at least a quarter. We include a fund specific fixed effect γ_i and cluster error terms at the year level.

Table 2: Outflows and Impediments to Withdrawal

Dependent variable	Net inflows $_{it} \times (\text{Net inflows}_{it} < 0)$		
Impediment to withdrawal	None	Lockup	Quarterly Redemption
	(1)	(2)	(3)
$(r_{it-1} < r_{t-1}^{rf})$	-0.11*** (0.02)	-0.12*** (0.02)	-0.14*** (0.02)
$(r_{it-1} < r_{t-1}^{rf})$ $\times \text{Impediment}_i$	-	0.04** (0.02)	0.04*** (0.01)
Fund FE	Yes	Yes	Yes
Observations	4,825	4,690	4,162
<i>Adj. R</i> ²	0.54	0.54	0.55

Data: EurekaHedge, 1994-2007. Annual data, excluding funds with AUM lower than 20 million USD. The dependent variable is equal to annual net inflows if they are negative, and zero else. Net inflows are computed as the difference between the growth in AUM minus net-of-fee returns. All specifications include fund specific fixed effects. In column (1), the only regressor is a dummy equal to 1 if the past annual return was lower than the yield on the 3 month T-bill. In column (2), we interact with the fact that fund i has a lockup period of at least a year. In column (3) we interact with the fact that redemption + notice periods is at least 120 days. Error terms are clustered at the year level. *, **, and *** means statistically different from zero at 10, 5 and 1% levels of significance.

Regression results are reported in Table 2. As shown in the first column, if past perfor-

mance is below the safe rate of return, outflows increase by 11% of AUM on average. This is sizeable, compared to mean annual outflows of 13% in the data, and a cross sectional standard deviation of 22% (see Table 1). As shown in columns 2 and 3, such a large sensitivity is somewhat reduced, yet not totally erased, by the presence of impediments to withdrawal. Conditional on low performance, funds with such share restrictions experience outflows of 8% of AUM, compared to 12% without such restrictions. Hence, the sensitivity is reduced by about one third.

3.2 Evidence from Conditional Returns

We test here our prediction 1: illiquid funds overperform liquid funds relatively more after bad performance than after good performance. To do this, we first run the following regression:

$$r_{it} = \gamma_i + \beta \cdot 1_{\{r_{it-1} < r_{t-1}^{rf}\}} + \delta \cdot 1_{\{r_{it-1} < r_{t-1}^{rf}\}} \times \text{Impediment}_i + \varepsilon_{it} \quad (10)$$

where r_{it} is the annual return of fund i in year t . We use annual data because at the annual frequency returns are less likely to be polluted by asset illiquidity problems (Lo, 2008; more on this below). γ_i is a fund-specific fixed effect, designed to capture heterogeneity in risk exposure and alphas, across funds (but our results are unchanged in the absence of fixed effects). We cluster error terms at the year level. Our theory predicts that the extent of mean reversion in returns should be larger for illiquid funds, i.e., $\delta > 0$.

Table 3 reports the results. Consistently with the first prediction of our model, the mean reversion of returns significantly increases with impediments to withdrawal. After returns below the risk-free rate, annual returns increase by 2.6 points for funds with no lockup and by 7.7 points for funds with a lockup; the difference is strongly significant (column 1). When we look at redemption periods, we find that returns increase by 3.4 points after low performance when the notice + redemption period is shorter than a quarter, and by 6.7 when it is longer than a quarter (column 2). Again, the difference is strongly significant.

Individual fixed effects may introduce a mechanical tendency to generate mean reversion of returns in the regressions. Indeed, adding a fund specific fixed effect amounts to replacing every variable by its difference from the individual average. If a fund's return in year $t - 1$ is below its average return, then its returns in year t will tend to be above average. The bias may be more severe, the smaller the number of time periods. This is *a priori* not a problem for the regressions in columns 1 and 2, since we are interesting in the difference in mean reversion between funds with a lockup and funds with no lockup, not in the absolute level of mean reversion. In terms of equation (10), the estimate of β might be biased, but the estimate of δ should not. In particular, funds with lockups appear in the data during approximately the same length of time (4.5 years on average) as funds with no lockup (4.6 years on average). To confirm that intuition, we rerun our regressions without fixed effects. Consistently with a bias in the absolute level of mean reversion when fund specific fixed effects are used, the estimates of β are significantly lower without fixed effects (columns 3 and 4). Furthermore, the difference in mean reversion between funds with a lockup and funds with no

Table 3: Conditional Returns and Impediments to Withdrawal

Dependent variable	r_{it}				
	Impediment to withdrawal	Lockup	Quarterly Redemption	Lockup	Quarterly Redemption
	(1)	(2)	(3)	(4)	
Impediment _{<i>i</i>}	-	-	-0.7	1.6	
			(0.8)	(1.1)	
$(r_{it-1} < r_{t-1}^{rf})$	2.6	3.4	-3.4	-3.2*	
	(2.3)	(2.0)	(2.1)	(1.8)	
$(r_{it-1} < r_{t-1}^{rf})$ × Impediment _{<i>i</i>}	5.1***	3.3***	5.4***	5.4***	
	(1.4)	(1.0)	(1.8)	(1.3)	
Fund FE	Yes	Yes	No	No	
Observations	4,412	3,902	4,412	3,902	
Adj. R^2	0.48	0.49	0.01	0.02	

Data: EurekaHedge, 1994-2007. Annual data, excluding funds with AUM lower than 20 million USD. The dependent variable is the annual net-of-fee return. In columns (1) and (2) the specifications include fund specific fixed effects; in columns (3) and (4) they do not. In columns (1) and (3), the regressors are a dummy equal to 1 if the past annual return was lower than the yield on the 3-month T-bill, and that dummy interacted with the fact that fund i has a lockup. In columns (2) and (4), we interact with the fact that redemption + notice periods is at least 90 days. Error terms are clustered at the year level. *, **, and *** means statistically different from zero at 10, 5 and 1% levels of significance.

lockup is not modified and remains strongly significant. Results are similar when impediments to withdrawal are measured with the quarterly redemption dummy.

Note that each year an average of 6% of funds drop from the data. This could potentially bias our results if exit is correlated both with past returns and with current (at the time of exit) returns, and if these correlations are different for funds with a lockup and funds with no lockup. Funds can drop from the data either because they are liquidated, or because they voluntarily stop reporting their returns. The former is presumably associated with poor performance. The latter might occur when the fund is doing well. Indeed, funds report to data vendors for the purpose of indirect marketing to potential investors; when a fund performs well and has large capital flows, it may have no incentive to continue reporting. If, for instance, funds exit the database after bad performance, and also perform poorly the year of liquidation, but the econometrician does not observe that last poor performance, then our regressions overestimate the mean reversion of returns. However, there is *a priori* no reason to believe that this bias should be correlated with impediments to withdrawal, hence the test of our prediction 1 should not be biased. To check further that intuition, we construct a dummy variable equal to one the first year a fund disappears from the data. We then regress that exit dummy on our variable of bad performance interacted with our measures of impediment to withdrawal.

Table 4: Probability of Exit and Impediments to Withdrawal

Dependent variable	Exit _{it}				
	Impediment to withdrawal	Lockup	Quarterly Redemption	Lockup	Quarterly Redemption
	(1)	(2)	(3)	(4)	
Impediment _i	-	-	-0.01**	-0.01*	
			(0.00)	(0.00)	
($r_{it-1} < r_{t-1}^{rf}$)	0.06***	0.07***	0.08***	0.09***	
	(0.01)	(0.01)	(0.02)	(0.02)	
($r_{it-1} < r_{t-1}^{rf}$) × Impediment _i	-0.05**	-0.05*	-0.03	-0.04	
	(0.02)	(0.02)	(0.03)	(0.03)	
Fund FE	Yes	Yes	No	No	
Observations	4,707	4,171	4,707	4,171	
Adj. R ²	0.62	0.61	0.02	0.02	

Data: EurekaHedge, 1998-2007. Annual data, excluding funds with AUM lower than 20 million USD. The dependent variable is a dummy equal to one if the fund exits from the data in the current year. In columns (1) and (2) the specifications include fund specific fixed effects; in columns (3) and (4) they do not. In columns (1) and (3), the regressors are a dummy equal to 1 if the past annual return was lower than the yield on the 3-month T-bill, and that dummy interacted with the fact that fund i has a lockup. In columns (2) and (4), we interact with the fact that redemption + notice periods is at least 90 days. Error terms are clustered at the year level. *, **, and *** means statistically different from zero at 10, 5 and 1% levels of significance.

Results are reported in Table 4. In columns 1 and 2, we find that funds tend to exit after bad performance, and that the relation almost disappear for funds with a lockup and for funds with a long redemption + notice period. In columns 3 and 4, we find similar results without fund specific fixed effects, although the difference between funds with and without impediments to withdrawal is not significant any more. Overall, these results suggest that, if there is any bias due to exit, the bias tends to underestimate the relative mean reversion of illiquid funds. We check in unreported regressions that if a poor performance is assigned to funds that exit the database, then our results are reinforced.

Results from Table 3 show that there is a significantly larger tendency for returns of illiquid funds to mean revert, but it does not differentiate between mean reversion in bad states of nature and mean reversion in good states of nature. Our model, however, does predict such an asymmetry. Prediction 1 suggest that most of the mean reversion should be conditional on bad states of nature. This comes from the fact that, in the model, bad states of nature are states where assets are underpriced, while there is no mispricing in the good states of nature. Such a prediction holds even if $\mu = 1$, i.e., expected asset payoffs are similar in states U , M , and D .

To test it, we check if there is a difference in mean reversion between funds with past low returns, and events with past high returns. We define low returns as above, i.e., as cases

when returns are below the safe rate of return. This corresponds to approximately 21% of the observations in our dataset. Against this background, we define high returns as cases where past annual returns are above 20% net of fee: this threshold is chosen because it isolates about 21% of our fund-year observations, so the identifying power of the data should be similar for mean reversion conditional on high and low returns.

Table 5: Conditional Returns and Impediments to Withdrawal: Good States versus Bad States

Dependent variable	r_{it}				
	Impediment to withdrawal	Lockup	Quarterly Redemption	Lockup	Quarterly Redemption
	(1)	(2)	(3)	(4)	
Impediment _{<i>i</i>}	-	-	-0.3	1.6	
			(0.7)	(0.9)	
$(r_{it-1} < r_{t-1}^{rf})$	1.8	2.8	-1.7	-1.7	
	(1.9)	(1.7)	(2.0)	(1.7)	
$(r_{it-1} > 20\%)$	-2.9*	-2.1	6.3***	6.3**	
	(1.5)	(2.0)	(2.1)	(2.8)	
$(r_{it-1} < r_{t-1}^{rf})$ × Impediment _{<i>i</i>}	5.0***	2.8***	5.0***	5.4***	
	(1.4)	(0.8)	(1.5)	(1.0)	
$(r_{it-1} > 20\%)$ × Impediment _{<i>i</i>}	0.5	-1.0	-1.5	-1.1	
	(1.0)	(2.1)	(1.9)	(2.1)	
Fund FE	Yes	Yes	No	No	
Observations	4,412	3,902	4,412	3,902	
<i>Adj. R</i> ²	0.48	0.49	0.04	0.04	

Data: EurekaHedge, 1994-2007. Annual data, excluding funds with AUM lower than 20 million USD. The dependent variable is the annual net-of-fee return. In columns (1) and (2) the specifications include fund specific fixed effects; in columns (3) and (4) they do not. In columns (1) and (3), the regressors are a dummy equal to 1 if the past annual return was lower than the yield on the 3-month T-bill, a dummy equal to 1 if the past annual return was above 20%, and these dummies interacted with the fact that fund *i* has a lockup. In columns (2) and (4), we interact with the fact that redemption + notice periods is at least 90 days. Error terms are clustered at the year level. *, **, and *** means statistically different from zero at 10, 5 and 1% levels of significance.

We then run regressions similar in spirit to (10): we also introduce a dummy variable equal to one if past returns have been above 20%, in addition to the dummy that past returns are below the risk-free rate. Results are reported in Table 5. In column 1, we interact these two terms with the lockup dummy. The asymmetry appears clearly: conditional on bad performance, funds with lockups overperform funds without lockups by about 5 percentage points. Conditional on good past performance, both categories of funds experience the same decline in expected returns. In column 2, the same asymmetry is present with our second measure of impediment to withdrawal. In columns 3 and 4, we rerun these two regressions

without the fund specific fixed effects. The asymmetry between bad states of nature and good states of nature continue to appear significantly.

3.3 Mean Reversion Conditional on Other Funds Capital Structures

In this section, we test our prediction 2. This prediction states that the difference between high and low effort funds' expected returns, conditional on low past performance, is a decreasing function of α . When the fraction of funds with $I_D > 0$ decreases, the price difference between states D and M will expand, which will increase the scope for mean reversion for funds that invest in state D . In our model, α is determined in equilibrium but the proof of prediction 2 ensures that this negative covariation should be there as underlying parameters change.

Econometrically, prediction 2 rewrites, for fund i at date t operating on asset market s

$$r_{ist} = \gamma_i + \beta \cdot \mathbf{1}_{\{r_{ist-1} < r_{t-1}^{rf}\}} + \delta \cdot \mathbf{1}_{\{r_{ist-1} < r_{t-1}^{rf}\}} \times \text{Impediment}_i \\ + \zeta \cdot \mathbf{1}_{\{r_{ist-1} < r_{t-1}^{rf}\}} \times \alpha_s + \eta \cdot \mathbf{1}_{\{r_{ist-1} < r_{t-1}^{rf}\}} \times \text{Impediment}_i \times \alpha_s + \varepsilon_{it} \quad (11)$$

where α_s corresponds to α in the model; α_s is the fraction of other illiquid funds. Ideally, to compute α_s , we should focus on pure players on the same asset market where fund i operates. It is, however, not feasible, first because in general funds may operate on several assets markets, and also because the data does not provide us with the positions of funds on each market. Thus, we use the style classification of the database, implicitly assuming that funds operating within a given style buy the same assets. Thus, α_s will be the fraction, in the style where fund i is operating, of funds with a lockup, or the fraction of funds with at least a quarterly redemption. Our model unambiguously predicts that the difference in mean reversion should be lower in styles where there are more other funds with impediments to withdrawal, so that $\eta < 0$.

Table 6 reports estimates of equation (11) on our sample. Column 1 measures impediments as lockups. Conditional on past low performance, illiquid funds outperform by about 13 percentage points if there no other illiquid funds at all in the style. If the fraction of other funds with lockups is 20% (the sample average), the outperformance of illiquid funds goes down to 7 percentage points. The differential effect of α_s is statistically significant at the 5% level. In column 2, we use quarterly redemptions as our measure of impediments to withdrawals, both at the fund level and to compute α_s . With this specification, the result is less favorable to our theory: the estimate of η is insignificant and economically negligible.

3.4 Comparison with Existing Literature

Our main empirical result is thus that funds experience more mean reversion in returns when they have share restrictions. In this section, we show how our results complement the existing literature on hedge funds.

Getmansky, Lo and Makarov (2004) have designed a measure of returns “smoothing” by funds (named θ_0). This measure is econometrically complex to put in place, but the principle is to look at autocorrelation of monthly returns. If monthly returns are very autocorrelated,

Table 6: Mean Reversion: The Impact of Other Funds' Illiquidity

Dependent variable	r_{it}	
Impediment to withdrawal	Lockup	Quarterly Redemption
	(1)	(2)
$(r_{it-1} < r_{t-1}^{rf})$	0.1	4.5**
	(2.4)	(2.0)
$(r_{it-1} < r_{t-1}^{rf})$ \times Impediment $_i$	12.7***	3.8
	(3.3)	(5.5)
$(r_{it-1} < r_{t-1}^{rf})$ \times α_{st}	13.7**	-4.4
	(5.1)	(3.3)
$(r_{it-1} < r_{t-1}^{rf})$ \times Impediment $_i \times \alpha_{st}$	-31.1**	0.3
	(13.8)	(12.0)
Fund FE	Yes	Yes
Observations	4,412	3,902
$Adj. R^2$	0.49	0.49

Data: EurekaHedge, 1994-2007. Annual data, excluding funds with AUM lower than 20 million USD. The dependent variable is the annual net-of-fee return. All specifications include fund specific fixed effects. In column (1), we use as measure of impediment to withdrawal the fact that fund i has a lockup. In column (2) the impediment dummy is equal to 1 if redemption + notice periods is at least 90 days. Error terms are clustered at the year level. *, **, and *** means statistically different from zero at 10, 5 and 1% levels of significance.

then it is likely that funds smooth returns across months to minimize volatility. Such a strategy is easier to put in place for assets whose prices cannot easily marked to market, so θ_0 is also considered as a proxy for asset illiquidity. Consistent with the idea that impediments to withdrawals help funds to buy illiquid assets, Aragon (2007), Ding et al. (2008) and Liang and Park (2008) have shown that high θ_0 funds also tend to have share restrictions.

This contradiction with our results (we find *less* returns persistence for funds with share restrictions) is only apparent, because we focus on annual returns, which are less likely to be smoothed. Illiquidity and window dressing issues should therefore generate less autocorrelation at this frequency. In fact, as we show in Tables 3 and 4, column 1, annual returns are more likely to mean revert, for the average hedge fund in our sample.

This difference between existing results and ours does not come from the fact that we are using a different dataset (most papers use Lipper/TASS), but really from the fact that we work with annual data. To check this, we look at the correlation between impediments to withdrawal and the autocorrelation of monthly returns. First, we compute, for each fund, a measure of the first order autocorrelation in monthly returns: we find an average of 0.10. This figure is consistent with what can be found in papers using other datasets: for instance, Lo (2008) finds that the mean first order autocorrelation of fund returns is 0.08 using the

TASS/Lipper database (his Table 2.6). Consistently with Liang and Park (2008), we also find that autocorrelation is positively correlated with the presence of a lockup period: in our dataset, the correlation is 0.08 (compared to 0.09 in their study), statistically significant at the 1% level. Thus, our data generates the same pattern as existing papers: impediments to withdrawal are associated with more autocorrelated monthly returns.

Table 7: Conditional Returns and Impediments to Withdrawal: Higher Frequency Evidence

Dep. Variable	r_{it}		
Panel A: Monthly frequency			
	All	Long short equity	Fixed Income
	(1)	(2)	(3)
$(r_{it-1} < r_{t-1}^{rf})$	-0.38** (0.18)	-0.44** (0.21)	-0.45*** (0.14)
$(r_{it-1} < r_{t-1}^{rf})$ \times Lockup _{<i>i</i>}	-0.14* (0.08)	0.14 (0.13)	-0.49*** (0.14)
Fund FE	Yes	Yes	Yes
Observations	120,734	51,963	6,929
<i>Adj. R</i> ²	0.06	0.05	0.10
Panel B: Quarterly frequency			
	All	Long short equity	Fixed Income
	(1)	(2)	(3)
$(r_{it-1} < r_{t-1}^{rf})$	-0.10 (0.42)	-0.42 (0.58)	-0.49 (0.40)
$(r_{it-1} < r_{t-1}^{rf})$ \times Lockup _{<i>i</i>}	0.53** (0.26)	1.37*** (0.24)	-1.24** (0.55)
Fund FE	Yes	Yes	Yes
Observations	34,447	14,828	1,989
<i>Adj. R</i> ²	0.15	0.15	0.18

Data: EurekaHedge, 1994-2007. Annual data, excluding funds with AUM lower than 20 million USD. The dependent variable is the net-of-fee return. Panel A uses monthly returns, while Panel B uses quarterly returns. All specifications include fund specific fixed effects. In column (1), we look at all funds. In column (2), we restrict the sample to the long-short equity style. In column (3), we restrict the sample to funds operating in the fixed income style. Error terms are clustered at the month (Panel A) and quarterly (Panel B) level. *, **, and *** means statistically different from zero at 10, 5 and 1% levels of significance.

To further check this, we run regression (10) with *monthly*, instead of yearly, data, on all funds. We find, in the first column of Panel A of Table 7, that coefficient on low past performance is -0.38 and the interaction term is -0.14 , but significant at the 10% level only. Thus, at very high frequency, our data, like others, generate the positive autocorrelation pattern found in the literature.

This analysis suggests that the correlation between impediments to withdrawals and autocorrelation of returns is affected by two opposite forces: at the annual frequency, illiquid funds mean revert more because they take advantage of temporary mispricing, while at the monthly frequency, they exhibit more autocorrelation because they hold illiquid assets whose prices display a significant inertia. To disentangle these two effects, we run regression (10) on different groups of hedge funds, depending on the liquidity of the asset market they operate on. If we focus on funds managing liquid assets such as (long-short) equity funds, we find some evidence that persistence of monthly returns is weaker for funds with lockup periods (Panel A, column 2), although the relation is not statistically significant. So even with monthly data, the evidence from “liquid styles” is more in line with our theory. For fixed income funds, the pattern is reversed (Panel A, column 3): for this strategy, as the existing literature would predict, share restrictions means more smoothing, and therefore more autocorrelation. In unreported regressions, we find similar results when we measure impediments to withdrawal with redemption periods.

With quarterly data, our predictions start to have more bite. In Panel B of Table 7, we re-estimate equation (10), using as the LHS variable quarterly, instead of monthly, returns. On the whole panel of funds, we find evidence of more mean reversion for funds with lockup periods (Panel B, column 1), while there was less mean reversion at the monthly frequency (Panel A, column 1). For equity funds, the effect is even more spectacular (Panel B, column 2). For fixed income funds, the movement of persistence is still present at the quarterly frequency, but less significant (Panel B, column 3). In unreported regressions, we find similar results when we measure impediments to withdrawal with redemption periods.

4 Conclusion

In this paper, we have developed and tested a model of delegated fund management in equilibrium. The starting point was Shleifer and Vishny (1997): arbitrageur invest in a common market. Arbitrage opportunities may generate temporary underperformance. Our model explicitly models the contract that ties the investor and the fund manager. Because in some cases the underpricing can be so severe, we find that it is always optimal for the investor to commit not to redeem his shares, for some funds only. Hence, we predict that funds with share restrictions will outperform those without such restrictions after past bad performance. We find evidence consistent with this in the data.

5 References

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