

Bond Liquidity Premia

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Abstract

Recent models of limits to arbitrage imply that the tightness of funding conditions faced by financial intermediaries is a component of the pricing kernel. In the US, the repo market is the key funding market for traders and arbitrageurs implying in turn that the on-the-run premium shares a common component with the risk premia observed in other markets. This observation leads to the following identification strategy. We measure the value of liquidity from the cross-section of on-the-run premia by adding a liquidity factor to an arbitrage-free term structure model. As predicted, we find that liquidity value affects the cross-section of risk premia at quarterly and annual horizons. An increase in the value of liquidity predicts lower risk premia for on-the-run *and* off-the-run bonds but higher risk premia on Libor loans, swap contracts and corporate bonds. Moreover, the measured impact is pervasive through crisis and normal times. Finally, we find that liquidity value varies with changes in aggregate uncertainty, measured from S&P500 options, and with changes in monetary stance, measured from bank reserves and monetary aggregates. These linkages are consistent with the theory and suggest that different securities serve, in part, and to varying degrees, to fulfill investors' uncertain future needs for cash.

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“... a part of the interest paid, at least on long-term securities, is to be attributed to uncertainty of the future course of interest rates.”

(p.163)

“... the imperfect 'moneyness' of those bills which are not money [...] causes the trouble of investing in them and [causes them] to stand at a discount.”

(p.166)

“... In practice, there is no rate so short that it may not be affected by speculative elements; there is no rate so long that it may not be affected by the alternative use of funds in holding cash.”

(p.166)

John R. Hicks, *Value and Capital*, 2nd edition, 1948.

Introduction

Bond traders know very well that liquidity affects asset prices. One prominent case is the on-the-run premium, whereby the most recently issued (on-the-run) bonds sell at a premium relative to seasoned (off-the-run) bonds with similar coupons and maturities. Moreover, systematic variations in liquidity sometimes drive interest rates across several markets. A case in point occurred around the Federal Open Market Committee [FOMC] decision, on October 15, 1998, to lower the Federal Reserve funds rate by 25 basis points. In the meeting's opening, Vice-Chairman McDonough, of the New York district bank, noted increases in the spread between the on-the-run and the most recent off-the-run 30-year Treasury bonds (0.05% to 0.27%), the spreads between the rate on the fixed leg of swaps and Treasury notes with two years and ten years to maturity (0.35% to 0.70%, and 0.50% to 0.95% respectively), the spreads between Treasuries and investment-grade corporate securities (0.75% to 1.24%), and finally between Treasuries and mortgage-backed securities (1.10% to 1.70%). He concluded that we were seeing a run to quality and a serious drying up of liquidity¹. These events attest to the sometimes dramatic impact of liquidity seizures².

A common explanation for that and the more recent market turmoil is based on a common wealth shock to capital-constrained intermediaries (Shleifer and Vishny (1997), Kyle and Xiong (2001), Gromb and Vayanos (2002)). Intuitively, lower wealth hinders the speculators ability to pursue arbitrage opportunities across markets. Then, the risk premia for each market intermediated by a common set of intermediaries share a component measuring the tightness in the funding market

¹Minutes of the Federal Open Market Committee, October 15, 1998 conference call. See <http://www.federalreserve.gov/Fomc/transcripts/1998/981015confcall.pdf>.

²The liquidity crisis of 2007-2008 provides another example. Facing sharp increases of interest rate spreads in most markets, the Board approved reduction in discount rate, target Federal Funds rate as well as novel policy instruments (e.g. Term Auction Facility, Term Securities Lending Facility) to deal with the ongoing liquidity crisis.

(Brunnermeier and Pedersen (2008), Krishnamurthy and He (2008)). In the U.S. the repo market is the key market where investment banks, hedge funds and other speculators obtain the marginal funds for their activities (Adrian and Shin (2008)). We test the implication that the tightness of funding conditions in the repo market should be reflected in the risk premia across markets.

We introduce liquidity as an additional factor in an otherwise standard term structure model. Indeed, the modern term structure literature has not recognized the importance of aggregate liquidity for government bonds. We extend the no-arbitrage dynamic term structure model of Christensen et al. (2007) [CDR, hereafter] allowing for liquidity³ and we extract a common factor driving on-the-run premia across maturities. Identification of the liquidity factor is obtained by estimating the model from a panel of pairs of U.S. Treasury securities where each pair has similar cash flows but different ages. This sidesteps credit risk issues and delivers direct estimates of the liquidity value: it isolates price differences that can be attributed to liquidity. Of course, a recent empirical literature suggests that liquidity is priced on bond markets,⁴ but these empirical investigations are limited to a single market and do not measure liquidity value within a no-arbitrage framework.

Our main contribution is precisely to show that an aggregate liquidity risk component drives a substantial share of risk premia across different interest rate markets. By construction, an increase in the liquidity factor is associated with lower expected excess returns for on-the-run bonds. More interestingly, an increase in the liquidity factor has a large negative impact on risk premia for off-the-run government bonds. On the other hand, it raises the risk premium implicit in LIBOR, swap rates and corporate bond yields. Empirically, the pattern is consistent with accounts of flight-to-quality but the relationship is pervasive even in normal times. This adds considerably to the existing evidence pointing toward the importance of marketwide liquidity for risk premia. Commonality between the valuation of liquidity obtained from on-the-run premia and variations in risk premia across different markets suggests that different securities serve, in part and to varying degrees, to fulfill investors uncertain future needs for cash. That is, the aggregate demand for liquidity is a risk factor relevant for asset prices.

Our empirical findings can be summarized as follows. Figure 1 shows that the liquidity factor exhibits large variations through normal and crisis period. Nevertheless, it remains positive and implies an average premium of \$0.38 for a just-issued 10-year bond. Second, Figure 3a shows that an increase in the value of liquidity predicts lower expected excess returns and, thus, higher current valuations, for seasoned bonds. For a seasoned bond with two years to maturity, a one-standard deviation shock to liquidity predicts a decrease in excess returns of 182 basis points compared to an average excess returns of 214 basis points. This suggests that off-the-run government bonds are substitutes, albeit imperfect, to on-the-run issues. Like its more liquid counterpart, a seasoned

³This model captures parsimoniously the usual level, slope and curvature factors, while delivering good in-sample fit and forecasting power. Moreover, the smooth shape of Nelson-Siegel curves identifies small deviations, relative to an idealized curve, which may be caused by variations in market liquidity.

⁴See Longstaff (2000) for evidence that liquidity is priced for short-term U.S. Treasury security and Longstaff (2004) for U.S. Treasury bonds of longer maturities. See Collin-Dufresne et al. (2001), Longstaff et al. (2005), Ericsson and Renault (2006), Nashikkar and Subrahmanyam (2006) for corporate bonds.

bond offers low transaction costs and can be quickly converted into cash via the repo market. Then, all government bonds share a common, negative, liquidity component that leads to higher bond prices when the aggregate demand for liquidity increases. These results are robust to including term structure information among the regressors. Indeed, we confirm the presence in coupon-bond data of a forward rate factor summarizing term structure information about the risk premium (Cochrane and Piazzesi (2005)).

Next, we consider the predictive power of liquidity for the risk premium on short-term Eurodollar loans. Figure 3b shows that a substantial share of LIBOR spread variability is linked to variations of the liquidity factor, as much as 28% in univariate regressions. The relationship is significant, both statistically and economically. Consider a 1-year loan, whose spread averaged 45 basis points in our sample. Contemporaneously, and controlling for term structure information, a one-standard deviation shock to liquidity is associated with a LIBOR spread increase of 7 basis points. We also consider the spread of swap rates over par bond yields. Figure 3c shows that the information content of liquidity is important. For the 7-year swap contract, whose spread averaged 53 basis points, a shock to liquidity is associated with a spread increase of 9 basis points after controlling for term structure information. In univariate regressions, the R^2 ranges from 7 to 12%.

Finally, we consider risk premia on corporate bonds. We find that liquidity is a predictor for the spread of Moody's Aaa and Baa indices of corporate bond yields (see Figure 3d). In a panel of corporate bond spreads, we find that the impact of liquidity follows a flight-to-quality pattern. Corporate bonds of the highest credit quality appears to be liquid substitutes to government securities while the risk premia on bonds with lower ratings carry a positive liquidity component.

These results raise the all important issue of identifying macroeconomic drivers of the liquidity factor. Can we characterize the aggregate liquidity premium in terms of economic state variables? First, we consider principal components extracted by Ludvigson and Ng (2005) from a data set of 132 macroeconomic variables⁵. Consistent with theory, we find that the liquidity factor is associated with factors measuring monetary conditions in the economy and in the banking system. In particular, the evidence supports a strong link with measures of bank reserves at the Federal Reserve and measures of monetary aggregates. Second, we use implied volatility from S&P 500 index options as a proxy for aggregate uncertainty and find a significant positive link with liquidity valuation. Again, this is consistent with the view higher volatility tightens the capital position of arbitrageurs. Finally, consistent with the prediction of Brunnermeier and Pedersen (2008), our liquidity factor varies with measures of transaction costs on the bond market. These findings provide a third important empirical contribution.

Related Literature

The on-the-run liquidity premium was first documented⁶ by Warga (1992). Amihud and Mendelson (1991) and, more recently, Goldreich et al. (2005) confirm the link between the premium and

⁵We thank Sydney Ludvigson and Serena Ng for making their principal component data available.

⁶The U.S Treasury recognizes and takes advantages of this price differential: "In addition, although it is not a primary reason for conducting buy-backs, we may be able to reduce the government's interest expense by purchasing

expected transaction costs. More important for our purposes, Duffie (1996) provides a theoretical channel between on-the-run premia and lower financing costs on the repo market. Vayanos and Weill (2006) extend this view and model search frictions in both the repo and the cash markets explicitly.⁷ The link between the repo market and the on-the-run premium has been confirmed empirically. (See Jordan and Jordan (1997), Krishnamurthy (2002), Buraschi and Menini (2002) and Cheria et al. (2004).)

We differ from the modern term structure literature in two significant ways. First, the latter focuses almost exclusively on bootstrapped zero-coupon yields⁸. This approach is convenient because a large family of models delivers zero-coupon yields which are linear in the state variables (see Dai and Singleton (2000)). However, we argue that pre-processing the data wipes out the most accessible evidence on liquidity, that is the on-the-run premium. Therefore, we use coupon bond prices directly. However, the state space is no longer linear and we handle non-linearities with the Unscented Kalman Filter [UKF], an extension of the Kalman Filter for non-linear state-space systems (Julier et al. (1995) and Julier and Uhlmann (1996)). We first estimate a model without liquidity and, notwithstanding the differences in data and filtering methodology, our results are consistent with CDR. However, pricing errors in this standard term structure model reveals systematic differences, correlated with ages. Estimation of the model with liquidity by quasi-maximum likelihood produces a persistent factor capturing differences between prices of recently issued bonds and prices of older bonds. The on-the-run premium increases with maturity but decays exponentially with the age of a bond. These new features complete our contributions to the modeling of the term structure of interest rates in presence of a liquidity factor.

We also differ from the recent literature using a reduced-form approach to model convenience yields in interest rate markets (Duffie and Singleton (1997)). The cross-section of on-the-run premia cannot be modeled as a convenience yields affecting the instantaneous discount rate. Instead, it varies with the age of a bonds, a property that cannot be captured in a frictionless arbitrage-free model. Still, in a paper related to ours, Grinblatt (2001) argues that the convenience yields of U.S. Treasury bills can explain the U.S. Dollar swap spread. Recently, Liu et al. (2006) and Fedlhütter and Lando (2007) evaluate the relative importance of credit and liquidity risks in swap spreads. Other empirical investigations are related to our work. Jump risk (Tauchen and Zhou (2006)) or the debt-gdp ratio (Krishnamurthy and Vissing-Jorgensen (2007)) have been proposed to explain the non-default component of corporate spreads. Finally, Pastor and Stambaugh (2003) and Amihud (2002) provide evidence of an aggregate liquidity risk factor in expected stock returns.

The link between interest rates and aggregate liquidity is supported elsewhere in the theoretical literature. Svensson (1985) uses a cash-in-advance constraint in a monetary economy. Bansal and

older, “off-the-run” debt and replacing it with lower-yield “on-the-run” debt.” [Treasury Assistant Secretary for financial markets Lewis A. Sachs, Testimony before the House Committee on Ways and Means].

⁷Kiyotaki and Wright (1989) introduced search frictions in monetary theory and Shi (2005) extends this framework to include bonds. See Shi (2006) for a review. Search frictions can also rationalize the spreads between bid and ask prices offered by market intermediaries (Duffie et al. (2005)).

⁸The CRSP data set of zero-coupon yields is the most commonly used. It is based on the bootstrap method of Fama and Bliss (1987) [FB].

Coleman (1996) allow government-issued bonds to back checkable accounts and reduced transaction costs in a monetary economy. Luttmer (1996) investigates asset pricing in economies with frictions and shows that with transaction costs (bid-ask spreads) there is in general little evidence against the consumption-based power utility model with low risk-aversion parameters. Holmström and Tirole (1998) introduce a link between the liquidity demand of financially constrained firms and asset prices. Acharya and Pedersen (2004) propose a liquidity-adjusted CAPM model where transaction costs are time-varying. Alternatively, Vayanos (2004) takes transactions costs as fixed but introduces the risk of having to liquidate a portfolio. Lagos (2006) extends the search friction argument to multiple assets: in a decentralized exchange, agents with uncertain future hedging demand prefer assets with lower search costs.

The rest of the paper is organized as follows. The next section presents the model and its state-space representation. Section II describes the data and Section III introduces the estimation method based on the UKF. We report estimation results for models with and without liquidity in Section IV. Section V evaluates the information content of liquidity for excess returns and interest rate spreads while Section VI identifies economic determinants of liquidity. Section VII concludes.

I A Term Structure Model With Liquidity

We base our model on the Arbitrage-Free Extended Nelson-Siegel [AFENS] model introduced in CDR. This model belongs to the affine family (Duffie and Kan (1996)). The latent state variables relevant for the evolution of interest rates are grouped within a vector F_t of dimension $k = 3$. Its dynamics under the risk-neutral measure Q is described by the stochastic differential equation

$$dF_t^Q = K^Q(\theta^Q - F_t) + \Sigma dW_t^Q, \quad (1)$$

where dW_t is a standard Brownian motion process. Combined with the assumption that the short rate is affine in all three factors, the model then leads to the usual affine solution for discount bond yields.

In this context, CDR show that if the short rate is defined as $r_t = F_{1,t} + F_{2,t}$ and if the mean-reversion matrix K^Q is restricted to

$$K^Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix}, \quad (2)$$

then the absence of arbitrage opportunity implies the discount yield function,

$$y(F_t, m) = a(m) + F_{1,t}b_1(m) + F_{2,t}b_2(m) + F_{3,t}b_3(m), \quad (3)$$

with loadings given by

$$\begin{aligned} b_1(m) &= 1, \\ b_2(m) &= \left(\frac{1 - \exp(-m\lambda)}{m\lambda} \right), \\ b_3(m) &= \left(\frac{1 - \exp(-m\lambda)}{m\lambda} - \exp(-m\lambda) \right), \end{aligned} \tag{4}$$

where $m \geq 0$ is the length of time until maturity (see appendix for the $a(m)$ term).

These loadings are consistent with the static Nelson-Siegel representation of forward rates (Nelson and Siegel (1987), NS hereafter). Their shapes across maturities lead to the usual interpretations of factors in terms of level, slope and curvature. Moreover, the NS representation is parsimonious and imposes a smooth shape to the forward rate curve. Empirically, this approach is robust to over-fitting and delivers performance in line with, or better than, other methods for pricing out-of-sample bonds in the cross-section of maturities⁹. Conversely, its smooth shape is useful to identify deviations of observed yields from an idealized curve.

A dynamic extension of the NS model, the Extended Nelson-Siegel model [ENS], was first proposed by Diebold and Li (2006) and Diebold et al. (2006). Diebold and Li (2006) show large improvements in long-horizon interest rate forecasting. They argue that the ENS model performs better than the best essentially affine model of Duffee (2002) and point toward the model's parsimony to explain its successes. One concern is that the ENS model does not enforce the absence of arbitrage. This is precisely the contribution of CDR. They derive the class of arbitrage-free affine dynamic term structure models with loadings that correspond to the NS representation. Intuitively, an AFENS model corresponds to a canonical affine model in Dai and Singleton (2000) where the loading shapes have been restricted through over-identifying assumptions on the parameters governing the risk-neutral dynamics of latent factors. CDR compare the ENS and AFENS models and show that implementing these restrictions improves forecasting performances further.

Interestingly, CDR show that we are free to choose the drift and variance term for the dynamics under the physical measure

$$dF_t^P = K^P(\theta^P - F_t) + \Sigma dW_t^P. \tag{5}$$

Further, we impose that Σ is lower triangular and that K^P is diagonal¹⁰. We can then cast the model within a discretized state-space representation. The state equation becomes

$$(F_t - \bar{F}) = \Phi(F_{t-1} - \bar{F}) + \Gamma\epsilon_t, \tag{6}$$

⁹See Bliss (1997) and Anderson et al. (1996) for an evaluation of yield curve estimation methods.

¹⁰Formally, the assumption on Σ is required for identification purposes. In practice, the presence of the off-diagonal elements in the K^P matrix does not change our results. Moreover, CDR show that allowing for an unrestricted matrix K^P deteriorates out-of-sample performance.

where the innovation ϵ_t is standard Gaussian, the autoregressive matrix Φ is

$$\Phi = \exp\left(-K \frac{1}{12}\right) \quad (7)$$

and the covariance matrix Γ can be computed from

$$\Gamma = \int_0^{\frac{1}{12}} e^{-Ks} \Sigma \Sigma^T e^{-Ks} ds. \quad (8)$$

Finally, we define a new latent state variable, L_t , that will be driving the liquidity premium. Its transition equation is

$$(L_t - \bar{L}) = \phi^l (L_{t-1} - \bar{L}) + \sigma^l \epsilon_t^l, \quad (9)$$

where the innovation ϵ_t^l is standard Gaussian and uncorrelated with the innovation in term structure factors.

Typically, term structure models are not estimated from observed prices. Rather, coupon bond prices are converted to forward rates using the bootstrap method. This is convenient as affine term structure models deliver forward rates that are linear in state variables. It is also thought to be innocuous because bootstrapped forward rates achieve near-exact pricing of the original sample of bonds. Unfortunately, this extreme fit means that a naive application of the bootstrap pushes any liquidity effects and other price idiosyncracies into forward rates. Fama and Bliss (1987) handle this sensitivity to over-fitting by excluding bonds with “large” price differences relative to their neighbors. This approach is certainly justified for many of the questions addressed in the literature, but it removes any evidence of large liquidity effects¹¹. Moreover, the FB data set focuses on discount bond prices at annual maturity intervals. This smooths away evidence of small liquidity effects being passed through to forward rates. These effects would be apparent from reversals in the forward rate function at short maturity interval. Consider three quotes for bonds with successive maturities $M_1 < M_2 < M_3$. A relatively expensive quote at maturity M_2 induces a relatively small forward rate from M_1 to M_2 . However, the following normal quote with maturity M_3 requires a relatively large forward rate from M_2 to M_3 . This is needed to compensate the previous low rate and to achieve exact pricing as required by the bootstrap. However, the reversal cancels itself as we sum intra-period forward rates to compute annual rates.

Instead of using zero-coupon bond data, we proceed from observed coupon bonds with maturity,

¹¹See also the CRSP documentation for a description of this procedure. Briefly, a first filter includes a quote if its yield to maturity falls within a range of 20 basis points from one of the moving averages on the 3 longer or the 3 shorter maturity instruments *or* if its yield to maturity falls between the two moving averages. When computing averages, precedence is given to bills when available and this is explicitly designed to exclude the impact of liquidity on notes and bonds with maturity of less than one year. Amihud and Mendelson (1991) document that yield differences between notes and adjacent bills is 43 basis point on average, a figure much larger than the 20 basis point cutoff. The second filter excludes observations that cause reversals of 20 basis points in the bootstrapped discount yield function. The impact of these filters has not been studied in the literature.

say, M and intermediate coupon payoffs at maturities $m = m_1, \dots, M$. The price $D_t(m)$ of a discount bond with maturity m , used to price intermediate payoffs, is given by

$$D_t(m) = \exp(-m(a(m) + b(m)^T F_t)) \quad m \geq 0,$$

which follows directly from equation (3) but where we use vector notation for factors F_t and factor loadings $b(m)$. In a frictionless economy, the absence of arbitrage implies that the price of a coupon bond equals the sum of discounted coupons and principal. That is, the frictionless price is

$$P^*(F_t, Z_t) = \sum_{m=m_1}^M D_t(m) \times C_t(m), \quad (10)$$

where Z_t includes all characteristics relevant for pricing the bond. In this case, it includes the maturity M and the schedule of future coupons $C_t(m)$.

With a short-sale constraint on government bonds and a collateral constraint on the repo market, Luttmer (1996) shows that the set of stochastic discount factors consistent with the absence of arbitrage satisfies $P \geq P^*$. Moreover, Duffie (1996) and Vayanos and Weill (2006) show that the combination of these constraints with search frictions on the repo market induces differences in funding costs that favor recently issued bonds. Intuitively, the repo market provides the required heterogeneity between assets with identical payoffs. An investor cannot choose which bond to deliver to unwind a repo position; she must find and deliver the same security she had originally borrowed. Because of search frictions, then, investors are better off in the aggregate if they can coordinate around one security and reduce search costs. In practice, the repo rate is lower for this special issue to provide an incentive to bond holders to bring their bond to the repo market. Typically, recently issued, on-the-run, bonds benefit from these lower financing costs, leading to the on-the-run premium. Alternatively, differences in transaction costs drive a wedge between asset prices (Amihud and Mendelson (1986)). Empirically, both channels seem to be at work although the effect of direct transaction costs appears weaker (Amihud and Mendelson (1991) and Goldreich et al. (2005)) than the effect of repo rates (Jordan and Jordan (1997), Krishnamurthy (2002) and Cheria et al. (2004) as well as Buraschi and Menini (2002) for the German bonds market).

Then, we model the price, $P(F_t, L_t, Z_t)$, of a coupon bond with characteristics Z_t as the sum of discounted coupons to which we add a liquidity term,

$$P(F_t, L_t, Z_{n,t}) = \sum_{m=1}^{M_n} D_t(m) \times C_{n,t}(m) + \zeta(L_t, Z_{n,t}).$$

Here Z_t is a short hand for the maturity, coupon and age of the bond. Note that the liquidity term should be positive to be consistent with a model based on transaction costs. Grouping observations together, and adding a pricing error term, we obtain our measurement equation

$$P(F_t, L_t, Z_t) = C_t D_t + \zeta(L_t, Z_t) + \Omega \nu_t, \quad (11)$$

where C_t is the $(N \times M_{max})$ payoffs matrix obtained from stacking the N row vectors of individual bond payoffs and M_{max} is the longest maturity group in the sample. Shorter payoff vectors are completed with zeros. Similarly, $\zeta(L_t, Z_t)$ is a $N \times 1$ vector obtained by stacking the individual liquidity premium. D_t is a $(M_{max} \times 1)$ vector of discount bond prices and the measurement error, ν_t , is a $(N \times 1)$ gaussian white noise uncorrelated with innovations in state variables. The matrix Ω is assumed diagonal and its elements are a linear function of maturity,

$$\omega_n = \omega_0 + \omega_1 M_n,$$

which reduce substantially the dimension of the estimation problem. However, leaving the diagonal elements of Ω unrestricted does not affect our results¹².

Our specification of the liquidity premium is based on a latent factor common to all bonds but with loadings that vary with a bond's maturity and age. Warga (1992) documented the links of the average on-the-run premium with bond ages and maturities. The on-the-run premium is given by

$$\zeta(L_t, Z_{n,t}) = L_t \times \beta_{M_n} \exp\left(-\frac{1}{\kappa} \text{age}_{n,t}\right) \quad (12)$$

where age_t is the age, in years, of the bond at time t . The parameter β_M controls the average of the liquidity premium within maturity group M . Below, we fix $\beta_{max} = 1$ to identify the level of the liquidity factor. The parameter κ controls the liquidity premium's decay as a bond becomes older. The gradual decay of the premium with age has been documented by Goldreich et al. (2005). For instance, immediately following its issuance (i.e.: $\text{age} = 0$), the loading on the liquidity factor is $\beta_M \times 1$. Taking $\kappa = 0.5$, the loading decreases by half within any maturity group after a little more than 4 months following issuance : $\zeta(L_t, 4) \approx \frac{1}{2}\zeta(L_t, 0)$.

Equation (11) shows that omitting the liquidity term will push the liquidity effect into pricing errors, possibly leading to biased estimators and large filtering errors. Alternatively, adding a liquidity term amounts to filtering a latent factor present in pricing errors. However, this factor captures that part of pricing errors correlated with bond ages. Our maintained hypothesis is that any such factor can be interpreted as a liquidity effect. While the specification above reflects our priors about the impact of age and maturity, the scale parameters are left unrestricted at estimation and we allow for a continuum of shapes for the decay of liquidity.

As discussed above, a positive liquidity term is consistent with the absence of arbitrage given the frictions observed on the bond market. Our specification delivers a discount rate function consistent with off-the-run valuations. However, a structural specification of the liquidity premium raises important challenges. The on-the-run premium is a real arbitrage opportunity unless we explicitly consider the cost of shorting the more expensive bond or, alternatively, the benefits accruing to

¹²This can be explained by noting that the level factor explains most of yields variability. Its impact on bond prices is linear in duration and duration is approximately linear in maturity, at least for maturities up to 10 years. Bid-ask spreads increase with maturity and may also contribute to an increase in measurement errors with maturity.

the bondholder from a lower repo rate. These features are absent from the current crop of term structure models. Moreover, a joint model of the term structure of repo rates and of government yields is not itself free of arbitrage unless we simultaneously model the convenience yields of holding short-term government securities. This follows from the observation that a Treasury bill typically offers a lower yield than a repo contract with the same maturity. This is beyond the scope of this paper. Our strategy bypasses these challenging considerations but still allows us to identify the on-the-run liquidity premia and uncover an aggregate liquidity factor. We now turn to a description of the data.

II Data

We use end-of-month prices of U.S. Treasury securities from the CRSP data set. We exclude callable bonds, flower bonds and other bonds with tax privileges, issues with no publicly outstanding securities, bonds and bills with less than 2 months to maturity and observations with either bid or ask prices missing¹³. We focus on the period from 12/1985 to 12/2007 because the tax premium and the on-the-run premium are hard to untangle in the prior period. Before 1986, interest income had a favorable tax treatment compared to capital gains and investors favored high-coupon bonds. In a period of rising interest rates, recently issued bonds had relatively high coupons and were priced at a premium both for their liquidity and for their tax benefits. Green and Ødegaard (1997) document that the high-coupon tax premium mostly disappeared when the asymmetric treatment of interest income and capital gains was eliminated in the 1986 tax reform.

At each observation date, the CRSP data set¹⁴ provides quotes on all outstanding U.S. Treasury securities. We construct bins around maturities of 3, 6, 9, 12, 18, 24, 36, 48, 60, 84 and 120 months to provide a sufficient coverage of the term structure. Then, for each date, and within each bin, we choose a pair of securities to identify any on-the-run premium. First, we want to pick the on-the-run security, if available, and, second, a security with matching maturity. Unfortunately, on-the-run bonds are not directly identified in the CRSP database. Instead, we use time since issuance as a proxy and pick the most recently issued security in each maturity bin. To complete the pair, we pick the off-the-run bond with time remaining to maturity closest to the center of the maturity bin. By construction, then, securities within each pair have the same credit quality, similar times to maturity and similar coupons. Also, we minimize maturity differences between on-the-run bonds and their off-the-run companions throughout the sample. Finally, pinning the off-the-run security in at the center of its bin ensures a stable coverage of the term structure of interest rates.

The most recent issue at a given bin and date is not always an on-the-run security. This may be due to the absence of new issuance in some maturity bins throughout the whole sample (e.g.

¹³Along the way, we exclude some suspicious quotes. CRSP ID #19920815.107250 on August 31st 1987, #20041031.202120 on November 29th 2002, #20070731.203870 on May 31st 2006 and #20080531.204870 on November 30th 2007 were removed because yields to maturity were suspicious. CRSP ID #20040304.400000 has a maturity date preceding its issuance date, as dated by the U.S. Treasury.

¹⁴See Elton and Green (1998) and Piazzesi (2005) for discussion of the CRSP data set.

18 months to maturity) or within some sub-periods (e.g. 84 months to maturity). Alternatively, the on-the-run bond may be a few months old, due to the quarterly issuance pattern observed in some maturity categories. In any case, this introduces variability in age differences which, in turn, identifies how the liquidity premium varies with age. Still, the most important aspect of our sample is that whenever an on-the-run security is available, any large price difference cannot be rationalized from small coupon and maturity differences under the no-arbitrage restriction. Instead, age differences are magnified in the sample and any price difference common across maturities and correlated with age will be attributed to the liquidity effect.

We now investigate some features of our sample of $265 \times 22 = 5830$ observations. The first columns of Table I present means and standard deviations of age for each liquidity-maturity category. The average off-the-run security is always older than the corresponding on-the-run security. Typically, the off-the-run security has been in circulation for more than a year. In contrast, the on-the-run security is typically a few months old and, even, a few weeks old in the 6 and 24-month categories. While a relatively low mean age for the recent issue indicates a regular issuance pattern, the relatively high standard deviation in the 36 and 84-month categories reflects the decision by the U.S. Treasury to stop the issuance cycles at these maturities.

[Table I about here.]

The middle columns of Table I presents means and standard deviations of duration¹⁵. Note that average duration is almost linear in maturity. Also, as expected, duration is similar within pairs implying that the averages of cash flow maturities are very close. By construction, categories with regular issuance patterns exhibit pairs with smaller duration differences on average. This helps identify price differences due to the on-the-run premium. Finally, the last columns of Table I indicates that the average term structure of coupons is upward sloping. Standard deviations indicate important variations in coupon rates. This is in part due to the general decline of interest rates throughout the sample. Notwithstanding this important variation, coupon differences are typically small. This was another of our objectives. To summarize our strategy, differences in duration and coupon rates are kept small within each pair to highlight the effect of age, and thus liquidity, on prices.

III Estimation Methodology

Equations (6), (9) and (11) can be summarized as a state-space system

$$(X_t - \bar{X}) = \Phi_X(X_{t-1} - \bar{X}) + \Sigma_X \epsilon_t \quad (13)$$

$$P_t = \Psi(X_t, C_t, Z_t) + \Omega \nu_t, \quad (14)$$

where $X_t \equiv [F_t^T \ L_t]^T$ and Ψ is the (non-linear) mapping of cash flows C_t , bond characteristics, Z_t , and current states, X_t , into prices.

¹⁵Duration is the relevant measure to compare maturities of bonds with different coupons.

Estimation of this system is challenging: we do not know the joint density of factors and prices. Various strategies to deal with non-linear state-space systems have been proposed in the filtering literature: the Extended Kalman Filter (EKF), the Particle Filter (PF) and more recently the Unscented Kalman Filter¹⁶ (UKF). The UKF is based on a method for calculating statistics of a random variable which undergoes a nonlinear transformation. It starts with a well-chosen set of points with given sample mean and covariance. The nonlinear function is then applied to each point and moments are computed from transformed points. This approach has a Monte Carlo flavor but the sample is drawn according to a specific deterministic algorithm. It delivers second-order accuracy but with no increase in computing cost relative to the EKF. Moreover, analytical derivatives are not required. The UKF has been introduced in the term structure literature by Leippold and Wu (2003) and in the foreign exchange literature by Bakshi et al. (2005). Recently, Christoffersen et al. (2007) compared the EKF and the UKF for the estimation of term structure models. They conclude that the UKF improves filtering and reduces estimation bias.

To set up notation, we state the standard Kalman filter algorithm as applied to our model. We then explain how the unscented approximation helps overcome the challenge posed by a non-linear state-space system. Consider the case where Ψ is linear in X whereas state variables and bond prices are jointly Gaussian. In this case, the Kalman recursion provides optimal estimates of current state variables given past and current prices. The recursion works off estimates of state variables and their associated MSE from the previous step,

$$\begin{aligned}\hat{X}_{t+1|t} &\equiv E[X_{t+1}|\mathfrak{S}_t], \\ Q_{t+1|t} &\equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{X}_{t+1|t} - X_{t+1})^T\right],\end{aligned}$$

where \mathfrak{S}_t belongs to the natural filtration generated by bond prices. The associated prediction of bond prices, and its MSE, are given by

$$\begin{aligned}\hat{P}_{t+1|t} &\equiv E[P_{t+1}|\mathfrak{S}_t] \\ &= \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}),\end{aligned}\tag{15}$$

$$\begin{aligned}R_{t+1|t} &\equiv E\left[(\hat{P}_{t+1|t} - P_{t+1})(\hat{P}_{t+1|t} - P_{t+1})^T\right] \\ &= \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1})^T \hat{Q}_{t+1|t} \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}) + \Omega,\end{aligned}\tag{16}$$

using the linearity of Ψ . The next steps compare predicted to observed bond prices and update

¹⁶See Julier et al. (1995), Julier and Uhlmann (1996) and Wan and der Merwe (2001) for a textbook treatment. Another popular bypasses filtering altogether. It assumes that some prices are observed without errors and obtains factors by inverting the pricing equation. In our context, the choice of maturities and liquidity types that are not affected by measurement errors is not innocuous and impacts estimates of the liquidity factor.

state variables and their MSE,

$$\hat{X}_{t+1|t+1} = \hat{X}_{t+1|t} + K_{t+1}(P_{t+1} - \hat{P}_{t+1|t}), \quad (17)$$

$$Q_{t+1|t+1} = Q_{t+1|t} + K_{t+1}^T (R_{t+1|t})^{-1} K_{t+1}, \quad (18)$$

where

$$\begin{aligned} K_{t+1} &\equiv E \left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{P}_{t+1|t} - P_{t+1})^T \right], \\ &= Q_{t+1|t} \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}), \end{aligned} \quad (19)$$

which measure co-movements between pricing and filtering errors. Finally, the transition equation gives us a conditional forecast of X_{t+2} ,

$$\hat{X}_{t+2|t+1} = \Phi_X \hat{X}_{t+1|t+1}, \quad (20)$$

$$Q_{t+2|t+1} = \Phi_X^T Q_{t+1|t+1} \Phi_X + \Sigma_X \Sigma_X^T. \quad (21)$$

The recursion delivers series $\hat{P}_{t|t-1}$ and $R_{t|t-1}$ for $t = 1, \dots, T$. Treating $\hat{X}_{1|0}$ as a parameter, and using for $R_{t|t-1}$ the unconditional variance of prices, the sample log-likelihood is

$$L(\theta) = \sum_{t=1}^T l(P_t, \theta) = \sum_{t=1}^T \left[\log \phi(\hat{P}_{t+1|t}, R_{t+1|t}) \right], \quad (22)$$

where $\phi(\cdot, \cdot)$ is the multivariate Gaussian density.

However, because $\Psi(X, C)$ is not linear, equations (15) and (16) do not correspond to the conditional expectations of prices and the associated MSE. Also, (19) does not correspond to the conditional covariance between pricing and filtering errors. Still, the updating equations (17) and (18) remain linear: these are justified as linear projections. Then we can recover the Kalman recursion provided we obtain approximations of the relevant conditional moments. This is precisely what the unscented transformation achieves, using a small deterministic sample from the conditional distribution of factors while maintaining a higher order approximation than linearization¹⁷. We can then use the likelihood given in (22), but in a QML context. Using standard results, we have $\hat{\theta} \approx N(\theta_0, T^{-1}\Omega)$ where $\hat{\theta}$ is the QML estimator of θ_0 and the covariance matrix is

$$\Omega = E \left[(\zeta_H \zeta_{OP}^{-1} \zeta_H)^{-1} \right], \quad (23)$$

where ζ_H and ζ_{OP} are the alternative representations of the information matrix, in the Gaussian

¹⁷See Appendix A.

case. These can be consistently estimated via their sample counterparts. We have

$$\hat{\zeta}_H = -T^{-1} \left[\frac{\partial^2 L(\hat{\theta})}{\partial \theta \partial \theta'} \right] \quad (24)$$

and

$$\hat{\zeta}_{OP} = T^{-1} \sum_{t=1}^T \left[\left(\frac{\partial l(t, \hat{\theta})}{\partial \theta} \right) \left(\frac{\partial l(t, \hat{\theta})}{\partial \theta} \right)^T \right]. \quad (25)$$

Finally, the model implies some restrictions on the parameter space. In particular, ϕ_l and diagonal elements of Φ must lie in $(-1, 1)$ while κ and λ must remain positive. In practice, large values of κ or λ lead to identification problems and are excluded. Finally, we maintain the second covariance contour of state variables inside the parameter space associated with positive interest rates. The filtering algorithm often fails outside this parameter space. None of these constraints binds around the optimum and estimates remain unchanged when the constraints are relaxed.

IV Estimation Results

We first estimate a restricted version of our model, excluding liquidity. Filtered factors and parameter estimates are consistent with results obtained by CDR from zero-coupon bonds. More interestingly, the on-the-run premium reveals itself in the residuals from the benchmark model. This provides a direct justification for linking the premium with the age and maturity of each bond. We then estimate the unrestricted liquidity model. The null of no liquidity is easily rejected and the liquidity factor captures the systematic differences between on-the-run and off-the-run bonds. Finally, estimates imply that the on-the-run premium increases with maturity but decreases with the age of a bond.

A Results For The Benchmark Model Without Liquidity

Estimation¹⁸ of the benchmark model put the curvature parameter at $\hat{\lambda} = 0.6834$ when time periods are measured in years. The standard error is 0.0234 when using the QMLE covariance matrix and 0.0042 when using the MLE covariance matrix. This estimate lies between the two values obtained by CDR and pins the maximum curvature loading just above 31 months to maturity. Estimates for the transition equation are given in Table (IIa) and filtered values of latent term structure factors are presented in Figure 3.

[Table II about here.]

The results imply average short and long term discount rates of 3.74% and 5.45% respectively. Looking at the dynamics, we see that the level factor is very persistent, perhaps a unit root. This

¹⁸Estimation is implemented in MATLAB via the *fmincon* routine with the medium-scale (active-set) algorithm. Using different starting values, a maximum was reached at 2492.1. To compute standard errors, we use the final Hessian update (BFGS formula) and each observation gradient is obtained through a centered finite difference approximation evaluated at the optimum.

is a standard result: it reflects the gradual decline of interest rates in our sample. The slope factor is slightly less persistent and exhibits the usual association with business cycles. It enters positive territory before the recessions of 1990 and 2001. The other period of inversion starting in 2006 is the so-called “conundrum” episode. The curvature factor is closely related to the slope factor.

Standard deviations of pricing errors are given by

$$\sigma(M_n) = 0.0226 + 0.0284 \times M_n,$$

(0.022) (0.019)

implying standard deviations of 0.051 and 0.307 dollars for maturities of 1 and 10 years respectively. This translates in yield pricing errors of 5.1 and 4.4 basis points using durations of 1 and 7 years. Table (IIIa) gives more information on the fit of the benchmark model. Root Mean Squared Errors (RMSE) increase from \$0.045 and \$0.048 for 3-month on-the-run and off-the-run securities respectively, to \$0.366 and \$0.392 at 10-year maturity. As discussed above, the monotonous increase of RMSE with maturity may reflect the higher sensitivity of longer maturity bonds to interest rates. It may also be due to higher uncertainty regarding the true price, as signaled by wider bid-ask spreads. For the entire sample, the RMSE is \$0.19.

Notwithstanding the differences between estimation approaches, our results are consistent with CDR. Estimation from coupon bonds or from bootstrapped data seem to provide similar pictures of the underlying term structure of interest rates and the approximation introduced when dealing with nonlinearities appears innocuous. However, preliminary estimation of forward rate curves with the bootstrap smooths away evidence of the on-the-run premia. In contrast, our sample comprises on-the-run and off-the-run bonds. Any systematic price differences not due to cash flow differences will be revealed in the pricing errors.

[Table III about here.]

Table IIIa confirms that Mean Pricing Errors (MPE) are higher for on-the-run securities than for off-the-run securities. Residuals of the former are systematically higher than residuals of the latter. For a recent 12-month T-Bill, the average difference is close to \$0.08, controlling for cash flow differences. Similarly, a recently issued 5-year bond is \$0.25 more expensive on average¹⁹. To get a clearer picture of the link between age and price differences, consider Figure 4. The top panels plot residual differences within the 60-month and 84-month category, respectively. The bottom panels plot the ages of each bond in these categories. Figure 4c shows that there has been regular issuance of 5-year bonds over the sample. As expected, the difference between residuals is almost always positive. The on-the-run (i.e. low age) bond appears overpriced compared to the off-the-run (i.e. high age) bond. The 84-month category provides an interesting example. Figure 4d shows that the U.S. Treasury stopped issuing 7-year bonds in 1993. The liquidity premium was positive while the Treasury proceeded with regular issuance but stopped when issuance ceased. Afterwards, each

¹⁹Note that the price impact of liquidity increases with maturity. This is consistent with the results of Amihud and Mendelson (1991).

pair is made of two old 10-year bonds, aged 2 and 3 years on average, and evidence of a premium disappears from the residuals. This correspondence between issuance patterns and systematic pricing errors can be observed in every maturity category. The evidence is consistent with an on-the-run premium and with our specification: the premium increases with maturity but decreases with age.

Bonds with 24 months to maturity seem to carry a smaller liquidity premium than what would be expected given the regular monthly issuance in this category. Note that a formal test rejects the null hypothesis of zero-mean residual differences. Interestingly, Jordan and Jordan (1997) could not find evidence of a liquidity or specialness effect at that maturity²⁰. A smaller price premium for 2-year notes is intriguing and we can only conjecture as to its causes. Recall that the magnitude of the premium depends on the benefits of higher liquidity, both in terms of lower transaction costs and repo rates. However, it also depends on the expected length of time a bond will offer these benefits. Results in Jordan and Jordan (1997) suggests that 2-year notes remain “special” for shorter periods of time (see Table I, p.2057). Similarly, Goldreich et al. (2005) find that the on-the-run premium on 2-year notes goes to zero faster than other maturities, on average. This is consistent with its short issuance cycle. Alternatively, holders of long-term bonds may re-allocate funds from their now short maturity bonds into newly issued longer term securities. If the two-year mark serves as a focus point for buyers and sellers, this may cause a larger volume of transactions around this key maturity, increasing the liquidity value of surrounding assets.

B Results For The Liquidity Model

Estimation of the unrestricted model leads to a substantial increase in the log-likelihood. The benchmark model is nested with 15 parameter restrictions and the improvement in likelihood is such that comparing the LR test-statistic to its asymptotic distribution leads to a p-value that is essentially zero²¹. The estimate for the curvature parameter is now $\hat{\lambda} = 0.7138$ with QMLE and MLE standard errors of 0.0315 and 0.0035. Results for the transition equations are given in Table (IIb). These imply average short and long term discount rates of 3.91% and 5.55% respectively. The standard deviations of measurement errors are given by

$$\sigma(M_n) = \begin{matrix} 0.0309+ & 0.0278 \times M_n, \\ (0.057) & (0.041) \end{matrix}$$

which implies standard deviations of \$0.059 and \$0.309 for bonds with one and ten years to maturity respectively. Overall, parameter estimates and latent factors are relatively unchanged compared to the benchmark model.

²⁰See Jordan and Jordan (1997) p. 2061: “With the exception of the 2-year notes [...], the average price differences in Table II are noticeably larger when the issue examined is on special.”

²¹The benchmark model reached a maximum at 2492.1 while the liquidity model reached a maximum at 3691.3 leading to an LR test-statistic of 2398. The corresponding critical value for a test with a 0.1% level, taken from the $\chi^2(15)$ distribution, is 37.7

Consider now Figure 1 which traces the path of liquidity valuation. First, it exhibits important variations in the sample. Nonetheless, it remains positive at all dates although it was left unrestricted at estimation. This is consistent with the prediction that the on-the-run premium is due to frictions in the cash and the repo markets. The factor’s unconditional mean implies that a just-issued bond with 10 years to maturity is worth \$0.38 more, on average, than a similar off-the-run bond. This translates into a large negative impact on returns as the premium eventually goes to zero. Recall however that this predictable return differential corresponds to anticipations of lower transaction and funding costs.

We estimate the decay parameter at $\hat{\kappa} = 0.74$ with QMLE and MLE standard errors of 0.33 and 0.06 respectively. This implies a reduction by half of the liquidity premium after a little more than 6 months. Estimates of β are given in Table IV. Note that the level of the liquidity premium increases with maturity. The pattern accords with the observations made from the residuals of the model without liquidity. Table IIIb shows RMSE improvements for almost all maturities while the overall sample RMSE decreases from \$0.19 to \$0.14. Moreover, Table IIIa shows that the model eliminates most of the systematic differences between on-the-run and off-the-run bonds. There is still some liquidity effect in the 10-year category where the average error in this category decreases from \$0.295 to \$0.183. Part of the variation in the 10-year on-the-run premium is not common with variations in other maturity groups.

[Table IV about here.]

Finally, Figures 5a and 5b draw the residual differences within the 60-month and 84-month category, respectively. This is another way to see that the model removes systematic differences between residuals. Overall, the evidence points toward a single systematic factor pricing the liquidity benefits of on-the-run U.S. Treasury securities. We interpret this liquidity factor as a measure of the value of liquidity to investors. The results below show that its variations also explain a substantial share of the variation in the risk premia observed across interest rate markets.

V The Information Content Of Liquidity

In this section, we document a tight linkage between the liquidity factor and an aggregate liquidity premium prevalent across interest rate markets. Of course, increases in the liquidity factor necessarily leads to lower excess returns for on-the-run bonds. We show here that it also leads to lower risk premia for off-the-run bonds as well as higher risk premia on LIBOR loans, swap contracts and corporate bonds. Thus, although the payoffs of these other assets do not appear to be related to the convenience yield of on-the-run securities, a substantial share of the risk premium they carry is driven by a common liquidity factor. The behavior of the aggregate liquidity is similar to the often cited “flight-to-liquidity” phenomenon but remains pervasive in normal market conditions. This commonality across liquidity premia accords with a substantial theoretical literature supporting the existence of an economy-wide liquidity risk premium (Svensson (1985), Bansal and Coleman (1996), Holmtröm and Tirole (1998, 2001), Acharya and Pedersen (2004), Vayanos (2004), Lagos

(2006)). The following section presents our results. For consistency, and unless otherwise stated, we use the AFENS model with liquidity whenever we need to compute zero coupon yields or returns, par yields, forward rates and the risk-free rate.

A *Off-the-run Government Bonds*

We first document the negative relationship between liquidity and expected excess returns on off-the-run bonds. Figure 3a displays the liquidity factor along with excess returns on a 2-year off-the-run bond. Clearly, these variables move in opposite directions throughout the sample but note also the sharp rises in spreads occurring around the crash of October 1987, the LTCM crisis in August 1998 and at the end of the millennium. At first, this tight link between on-the-run premia and returns from government bonds may be surprising. Recall that on-the-run bonds trade at a premium due to their anticipated transaction costs and financing advantages on the cash and repo markets. However, although off-the-run bonds demand higher costs, they can be readily converted into cash via the repo market. This is especially true relative to other asset classes. Thus, we expect seasoned bonds to be substitutes to on-the-run bonds. Higher demand for liquidity raises the value of on-the-run bonds and their substitutes, decreasing the off-the-run bond risk premium. We test this hypothesis and perform regressions of off-the-run bond excess returns on the liquidity factor. We also include term structure factors as they span the information content of forward rates (Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005)) but do not suffer from their near-collinearity.

[Table V about here.]

Table V presents results from these regressions²². We consider excess returns for off-the-run bonds with maturities 2, 3, 4, 5, 7 and 10 years and for investment horizons of 6, 12, 18 and 24 months. First, Table Va presents the estimated average risk premia. These are large, ranging from 72 to 310 basis points at an horizon of 12 months, and consistent with a period of declining interest rates. Next, Table Vb presents estimates of the liquidity coefficients. The results are conclusive. Estimates of the liquidity coefficients are negative at all horizons and maturities. The predictive content of liquidity peaks at the annual horizon. There, the evidence of predictability from the liquidity factor is solid with t-statistics ranging from 2.93 up to 3.22 across maturities. Moreover, the impact of liquidity on excess returns is economically significant: a one-standard deviation shock to liquidity lowers excess returns by 47 and 350 basis points for maturities of two and ten years respectively. At this horizon, R^2 statistics range from 20% to 36%. Of course, these coefficients of variation pertain to the joint explanatory power of all regressors. Repeating these regressions but excluding liquidity leads to a reduction in R^2 of 10% or more.

The regressions above use excess returns and term structure factors computed from the model. One concern is that model misspecification leads to estimates of term structure factors that do not

²²In the following regressions, we standardize each variable by subtracting its mean and dividing by its standard deviation. This eases the interpretation of regression coefficients. In particular, for each risk premium regression, the constant corresponds to an estimate of the average risk premium. Also, the coefficient of the liquidity factor measures the impact on expected returns, in basis points, of a one-standard deviation shock to liquidity.

correctly capture the information content of forward rates. Another concern is that misspecification induces spurious correlations between excess returns and liquidity. As a robustness check against both possibilities, we re-examine the predictability regressions but using different samples of excess returns and forward rates obtained from the CRSP zero-coupon data set.

We perform regressions of annual excess returns on bonds with maturity from 2 to 5 years. As regressors, we include the liquidity factor along with annual forward rates at horizon from 1 to 5 years. Excess returns and forward rates are obtained from CRSP FB data set. Table VIa presents results. Compared to our previous results (Figure Vb) coefficients of the liquidity factor appear unchanged or, if anything, slightly higher. We conclude that the predictability power of the liquidity factor is robust to how we compute excess returns and forward rates.

[Table VI about here.]

Moreover, this confirms that the AFENS model captures important aspects of excess returns. Table VIb provides results for the regressions of CRSP excess returns on CRSP forward rates, excluding the liquidity factor. This is a replication of the unconstrained regressions in Cochrane and Piazzesi (2005) but for our shorter sample period and it confirms that their stylized predictability results hold in this sample. The predictive power of forward rates is substantial and we recover the tent-shaped pattern of coefficients across maturities. Table VIc provides results for regression of excess returns from the model on the CRSP forward rates. Comparing the last two panels, we see that average excess returns, forward rate coefficients, as well as R^2 are similar across data sets. This is striking given that excess returns were recovered using very different approaches. We conclude that the AFENS model captures the stylized facts of bond risk premia, which is an important measure of success for term structure models²³. We also conclude that the empirical facts highlighted by Cochrane and Piazzesi (2005) are not an artefact of the bootstrap method.

Taken as a whole, the results provide evidence that on-the-run premia and bond risk premia share a common component. Also, off-the-run bonds appear to be liquid substitutes to their recently issued counterparts. The theoretical and empirical literature point toward short-run fluctuations in transaction costs and financing advantages of new issues as the source of the on-the-run premium. The implication is that a component of bond risk premia common with the on-the-run premium is, in fact, linked to variation of liquidity risk. This leaves as unlikely any role for the traditional explanations of bond risk premia, such as inflation risk or interest rate risk. Similarly, Longstaff (2004) documents price differences between off-the-run U.S Treasury bonds and Refcorp bonds²⁴ with similar cash flows. He argues that discounts on Refcorp bond are due to "...varying preferences for the liquidity of Treasury bonds, especially in unsettled markets."

²³Fama (1984b) originally identified this modeling challenge but see also Dai and Singleton (2002). Other stylized facts are documented in Fama (1976), (1984a), and(1984b), as well as Startz (1982) for maturities below 1 year. See also Shiller (1979), Fama and Bliss (1987), Campbell and Shiller (1991). Our conclusions hold if we use Campbell and Shiller (1991) as a benchmark.

²⁴Refcorp is an agency of the U.S. government. Its liabilities have their principals backed with U.S. Treasury bonds and coupons explicitly guaranteed by the U.S. Treasury.

B LIBOR Spreads

In this section, we provide evidence of a liquidity component in the risk compensation from money market loans. Specifically, we find that the liquidity factor is related to expected excess returns on interbank loans, measured from London Inter Bank Offer Rate (LIBOR) spreads. However, in contrast with the government bond market, the relationship is positive: higher valuation of liquidity in on-the-run markets predicts higher LIBOR spreads. Figure 3b highlights the positive correlation between liquidity and the 12-month LIBOR spread. Beyond the events discussed in the previous section, note also the sharp rise in LIBOR spread and liquidity associated with the unraveling of mortgage-backed asset markets toward the end of 2007. Interbank loans appear to be poor substitutes to U.S. Treasury securities. Rather, LIBOR spreads in part reflect the opportunity cost, in terms of future liquidity, of an interbank loan. Indeed, in order to convert a loan back to cash, a bank must enter into a new bilateral contract to borrow money. The search costs of this transaction depend on the number of willing counterparties in the market, which in turn may be linked to anticipation of the bank's default risk. Thus, it may be difficult at critical times to convert a LIBOR position back to cash, which may explain why the liquidity factor is positively linked with LIBOR spreads²⁵.

We obtain LIBOR data from Datastream for the period from 01/1987 up to 12/2007. We use off-the-run zero-coupon yields²⁶ to compute LIBOR spreads on loans with maturities of 1, 3, 6, 9 and 12 months. We consider regressions of these spreads on the liquidity and term structure factors. First note from Table VIIa that the average risk premium increases with maturity, ranging from 34 to 47 basis points. Next, Table VIIb presents the liquidity coefficients at horizons of 0, 6, 12, 18 and 24 months. These are significant and negative at all maturities and horizons. Contemporaneously, a one-standard deviation shock to the value of liquidity raises the spread of an interbank loan by 7 basis points, after controlling for term structure information. The impact increases with the forecasting horizon, reaching 13 basis points. Table VIIc shows that the explanatory power is substantial across maturities, ranging from 40% to 54% in these multivariate regressions. Finally, using liquidity only in the regressions delivers higher coefficients and explanatory power that ranges from 20% to 27%. Again, the information content remains substantial at long horizons.

[Table VII about here.]

C Swap Spreads

The impact of liquidity on risk premia extends to the swap market. In this section we show that swap spreads vary positively with liquidity value. This link can be observed in Figure 3c. To the extent that swap rates are determined by anticipations of future LIBOR rates, our results support previous literature (Grinblatt (2001), Duffie and Singleton (1997), Liu et al. (2006) and

²⁵Note that this does not preclude that part of the LIBOR spread is due to the higher default risk of the average issuer compared to the U.S. government.

²⁶We use model-implied yields but results are similar using yields from the Svensson, Nelson and Siegel method (Gurkaynak et al. (2006)) available at (<http://www.federalreserve.gov/pubs/feds/2007>).

Fedlhütter and Lando (2007)) pointing toward LIBOR liquidity as an important driver of swap spreads. However, a liquidity premium common to U.S. Treasury securities and swap contracts may be also due to common priced variations of liquidity in these markets. We do not distinguish between these alternative channels here.

We obtain a sample of swap rates from DataStream, starting in 04/1987 and up to 12/2007. We focus on maturities of 2, 3, 4, 5 and 7 years and compute the spread of swap rates above the yield to maturity of the corresponding off-the-run par coupon bond. Table VIII shows the results from regressions of swap spreads on the liquidity and term structure factors. First, note from Table VIIIa that the average risk premium rises with maturity, ranging from 44 to 53 basis points, extending the pattern of LIBOR risk premia. Next, Table VIIIb presents estimates of the liquidity coefficients in forecasting regressions at horizons of 0, 6, 12, 18 and 24 months. The results are conclusive. Contemporaneously, and controlling for term structure factors, a one-standard deviation shock to the value of liquidity raises swap spreads from 7 to 9 basis points across maturities. The corresponding t-statistics are 2.06 and 2.91 and the explanatory power is substantial across maturities, ranging from 21% to 24%. Using only liquidity as a regressor delivers higher coefficients and the explanatory power ranges from 7% to 12%. Then, expected excess returns on swap contracts also include compensation for aggregate liquidity. Interestingly, aggregate liquidity affects swap spreads and LIBOR spreads similarly, even at maturity up to 84 months. This suggests that anticipation of liquidity compensation in the interbank loan market, rather than liquidity risk, is the main driver behind the aggregate liquidity component of swap risk premium.

[Table VIII about here.]

D Corporate Spreads

In this section, we show that corporate bond yield spreads also offer a compensation for aggregate liquidity. Figure 3d traces the liquidity factor along with the Moody's Baa corporate index spread. The close link between liquidity value and corporate spread is striking. Corporate spreads and liquidity were linked in October 1987, August 1998, December 2000 and in the end of 2008. Most interesting is the common variation in normal times throughout most of the sample. Empirically, we find that the impact of liquidity has a "flight-to-quality" pattern across credit ratings. Following an increase in the value of liquidity, the largest spread increases are observed for bonds with lowest credit quality. The impact then decreases as we consider higher ratings. The sign of the relationship even becomes positive in a different, shorter, sample of corporate spreads. Our results are consistent with the evidence of an aggregate liquidity effect documented by Collin-Dufresne et al. (2001). They find that most of the variations of non-default corporate spreads are driven by a single latent factor. In turn, this factor is related to the 30-year on-the-run premium. The evidence is also consistent with the differential impact of liquidity across ratings found by Ericsson and Renault (2006).

Our analysis begins with Moody's Aaa and Baa indices of corporate bond yields. In a comple-

mentary exercise, below, we use a sample of NAIC transaction data²⁷ with a better coverage of the credit spectrum. We obtain the indices for our entire sample from the Federal Reserve of New York. Spreads are computed as the differences with the 10-year off-the-run zero-coupon yield. Table IXa presents results from predictability regressions on the liquidity and term structure factors. The results are striking. Contemporaneously, a shock to aggregate liquidity raises Aaa and Baa spreads by 15 and 100 basis points, respectively, with t-statistics of 7.7 and 12.5. This compares with an average premium of 221 basis points for bonds with Aaa ratings and an average premium of 331 basis points for bonds with Baa ratings. Together, term structure and liquidity factors produce R^2 of 28% and 44% while liquidity alone produces R^2 of 19% and 26%. Moreover, liquidity coefficients are positive and decay only slowly at longer horizons. The presence of a liquidity component in corporate spreads, common with on-the-run premia, LIBOR spreads and swap spreads is strong evidence of a key role for aggregate liquidity in the determination of risk compensation offered by corporate bonds.

[Table IX about here.]

Note that the differential impact of liquidity for these two ratings suggests a flight-to-liquidity pattern across credit quality. Unfortunately, this sample does not provide a detailed picture of the credit spectrum. Another approach is to use a sample of NAIC transaction data for the period from 02/1996 to 12/2001. Once restricted to end-of-month observations, the sample includes 2,171 transactions over 71 months. To preserve parsimony, we group ratings in five categories. Group 1 includes ratings of lowest credit risk (e.g. Aaa) and group 5 includes ratings of highest credit risk (e.g. C-). We consider regression of NAIC corporate spreads on the liquidity and term structure factors but we also include the control variables used by Ericsson and Renault (2006). These are the VIX index, the returns on the S&P500 index, a measure of market-wide default risk premium and an on-the-run dummy signalling whether that particular bond was on-the-run at the time of the transaction²⁸.

The panel regressions of credit spreads for bond i at date t are given by

$$sprd_{i,t+h} = \alpha + \beta_{1,h}L_tI(G_i = 1) + \dots + \beta_{5,h}L_tI(G_i = 5) + \gamma_h^T X_{t+h} + \epsilon_{i,t+h} \quad (26)$$

for horizons $h = 0, 6, 12, 18, 24$ months. Also, L_t is the liquidity factor and $I(G_i = j)$ is an indicator function equals to one if the credit rating of bond i belongs in group $j = 1, \dots, 5$. Control variables are grouped in the vector X_{t+h} . Table IXb presents the results. The flight-to-quality pattern clearly emerges from the results. Consider the contemporaneous (i.e. $h = 0$) regression. For the highest rating category, an increase in liquidity value of one standard deviation decreases spreads by 39 and 25 basis points in groups 1 and 2 respectively. The effect is smaller and statistically undistinguishable from zero for group 3. Coefficients then become positive implying increases in

²⁷We thank Jan Ericsson for providing the NAIC transaction data and control variables. See Ericsson and Renault (2006) for a discussion of their data set.

²⁸We excluded control variables that represent term structure information. Moreover, we do not include individual bond fixed-effects as our sample is small relative to the number (998) of securities.

spreads of 23 and 30 basis points for groups 4 and 5, respectively. This is an average effect through time and across ratings within each group. Looking at predictability results, we see a similar flight-to-liquidity pattern at all horizons. Finally, explanatory power also rises with the horizon; from 6% up to 9% at an horizon of 24 months²⁹.

The results obtained with spreads computed from Moody's indices and NAIC transactions differ in one key aspect. Using Moody's data, estimates of liquidity coefficients imply that corporate spreads of bonds with Aaa ratings increase when the value of liquidity increases. Then, this sample suggests that corporate bonds of high credit quality are not considered by investors as liquid substitutes to government bonds. In contrast, estimates of liquidity coefficients obtained from NAIC data imply that a positive liquidity shock leads to lower corporate spreads in the highest rating group. Two important differences between samples may explain the results. First, Moody's indices cover a much longer time span. The pattern of liquidity premia across the quality spectrum may be time-varying. Second, the composition of the index is likely to be different from the composition of NAIC transaction data. The impact of liquidity on corporate spreads may not be homogenous across issues. For example, the maturity or the age of a bond, the industry of the issuer and security-specific option features may introduce heterogeneity.

E Discussion

The results above show that aggregate liquidity plays a major role in capital markets. When on-the-run premia vary, we observe systematic variations of risk premia across interest rate markets. That is, when the liquidity of on-the-run bonds becomes more valuable to investors, we observe changes of risk premia on the markets for off-the-run U.S. government bonds, eurodollar loans, swap contracts, and corporate bonds. Note that focusing on the common component of on-the-run premia helps filtering out local supply or idiosyncratic effects on prices. Empirically, the impact of aggregate liquidity on asset pricing appears strongly during crisis and the pattern is suggestive of a flight-to-quality behavior. Nevertheless, its impact is pervasive even in normal times, which points toward an important role for an aggregate liquidity risk premium. Also, the long-horizon predictive power of liquidity suggests a low frequency link, possibly due to an underlying general equilibrium relationship. Jointly, the evidence is hard to reconcile with theories based on variations of credit, inflation or interest rate risks. Instead, a plausible alternative is that some assets earn an aggregate liquidity premium if they can be used by investors to meet their uncertain future demand for cash while other assets offer a compensation for being exposed to variations in the aggregate demand for liquidity. In this context, it is interesting that the aggregate liquidity risk premium is negative for government bonds but positive in other markets. This confers a special status to government obligations, although the case of high-quality corporate bonds is ambiguous. The pattern of compensation for aggregate liquidity risk across markets is likely a reflection of the pattern of covariations between payoffs and aggregate liquidity states. The next section identifies

²⁹We do not report other coefficients. Briefly, the coefficients on the level factor are negative and significant, but only contemporaneously. All other coefficients are insignificant but these results are not directly comparable with Ericsson and Renault (2006) due to differences of models and sample frequencies.

candidate determinants of liquidity valuation and to characterize aggregate liquidity states in terms of known economic indicators.

VI Determinants Of Liquidity Value

Given the wide reach of aggregate liquidity risk across markets, it is important to identify the economic drivers of aggregate liquidity states. However, the liquidity factor aggregates very diverse economic information. Among other causes, the value of liquidity services depends on investors’s desire for liquidity. We then consider macroeconomic information summarizing the state of the economy. We find that liquidity valuation varies with measures of change in monetary aggregates and change in bank reserves at the Federal Reserve. Next, the value of liquidity may also depend on anticipations of the future path of liquidity demand. Either through changes in anticipated shocks to aggregate wealth, or through variations in risk aversion, measures of aggregate uncertainty may be related to the liquidity factor. We find that liquidity valuation increases with a measure of implied volatility from the stock market. Liquidity is also related to the amount of liquidity services offered by on-the-run bonds. We find that liquidity value rises when recently issued bonds offers relatively lower bid-ask spreads³⁰.

A Macroeconomic Variables

Ludvigson and Ng (2005) [LN hereafter] summarize 132 macroeconomic series into 8 principal components. They can then explore, in a parsimonious way, the predictive content of a large information set for bond returns. They find that a “real” and an “inflation” factor³¹ have substantial predictive power for bond excess returns. They also find that a “financial” factor is significant but that much of its information content is subsumed in the Cochrane-Piazzesi measure of bond risk premium. We find that the liquidity factor is also related to the “inflation” and “financial” factors but not to the “real” factor. Interestingly, we find, in addition, that the liquidity factor relates to measures of monetary aggregates and of bank reserves.³² These results are consistent with Longstaff (2004), who establishes a link between variations of RefCorp spreads and measures of flows into money market mutual funds, Longstaff et al. (2005), who document a similar link for the non-default component of corporate spreads and, finally, Chordia et al. (2005), who document that money flows and monetary surprises affect measures of bond market liquidity.

Table X displays results from a regression of liquidity on macroeconomic factors (Regression A). Macroeconomic components with significant coefficients are $F2$ and $F4$, the “financial” and

³⁰We also considered the Pastor-Stambaugh measure of aggregate stock market liquidity and found no link with the liquidity factor.

³¹Ludvigson and Ng (2005) use univariate regressions of individual series on each principal component to characterize its information content. For example, the “real” factor was labeled as such because it has high explanatory power for real quantities (e.g. Industrial Production).

³²A significant link between liquidity and one of the principal components of LN does not necessarily require that this component predicts excess returns. The liquidity factor is endogenous and its loadings on the underlying macroeconomic variables is unlikely to be constant through time.

“inflation” factor of LN, as well as $F5$, $F6$ and $F7$. Individual coefficients have similar magnitude and the R^2 is 28%. The “financial” factor relates to different interest rate spreads, which is consistent with the link between the liquidity factor and risk premia. Factor $F5$ is a “housing” factor. It contains substantial information on interest rate levels, housing starts and new building permits. Results below show that the significance of the “inflation” and “housing” factors is not robust.

[Table X about here.]

However, factors $F6$ and $F7$ share a similar and extremely interesting interpretation: these are “monetary conditions” factors. Both have highest explanatory power for the rate of change in reserves and non-borrowed reserves of depository institutions. Next, factor $F6$ has most information for the rate of change of the monetary base and the M1 measure of money stock. It also contains some information from the PCE indices. Beyond bank reserves, factor $F7$ is most informative for the spreads of commercial paper and three-month Treasury bills above the Federal Reserve funds rate. To a lesser extent, it is linked to the rate of change of the PCE, housing starts and new building permits. The importance of measures of monetary aggregates, reserves and short term interest rate spreads points toward a link between the value of liquidity and financial constraints in the banking system. Monetary policy and the credit channel appear to be linked with variations in the valuation of marketwide liquidity. The link between measures of activities in the residential construction sector may be linked to the availability of loanable funds in the banking system. A decrease of real estate prices leads to write-offs, increases in loss reserves by banks and reductions of the quantity of funds available for new loans.

B Aggregate Uncertainty

Higher aggregate liquidity valuation is associated with higher aggregate uncertainty. We use implied volatility from options on the S&P 500 stock index as proxy for aggregate uncertainty. The S&P500 index comprises a large share of aggregate wealth and its implied volatility can be interpreted as a forward looking indicator of wealth volatility. The sample comprises monthly observations of the CBOE VOX index from 01/1986 to 12/2007. Table X presents results from a regression of liquidity on aggregate uncertainty (Regression B). The coefficient is positive and significant and the R^2 is 14%. A one-standard deviation shock to implied volatility raises the liquidity factor by \$0.09. Figure 6b shows that volatility peaks are often associated with rises in liquidity valuation.

C Transaction Costs Variables

Coupon bond quotes from the CRSP data set include bid and ask prices. At each point in time, we consider the entire cross-section of bonds and compute the difference between the median and the minimum bid-ask spreads. This measures the difference in transaction costs between the most liquid bond and a typical bond. Table X presents the results from a regression of liquidity on this measure of relative transaction costs. The coefficient is positive and significant. This implies

that the liquidity factor increases when the median bid-ask spread moves further away from the minimum spread. That is, on-the-run bonds become more expensive when they offer relatively lower transaction costs. The explanatory power of bid-ask information is substantial, as measured by an R^2 of 28%. However, there is a sharp structural break in this relationship. All the statistical evidence is driven by observations preceding 1990 as made clear by Figure 6a. The first break in this process coincides with the advent of the GovPX platform while the second, around 1999, matches the introduction of the eSpeed electronic trading platform. Although transaction costs are an important determinant of the on-the-run premium, the lack of variability since these breaks implies a lesser role in the variations of liquidity valuation.

D Combining Regressors

Finally, Table X reports the results from a regression combining all the economic information considered above (Regression D). We experimented with different specifications and reached the following conclusions. First, coefficients on the “monetary conditions” factors remain significant and relatively unchanged. Next, the presence of the bid-ask spread measure makes the F8 factor significant. This factor contains S&P500 index information (i.e. index level, dividend yield, price-earning). This may explain why we find that including the macroeconomic factors makes the implied volatility coefficient insignificant. The information content of implied volatility may be subsumed by the information content of the valuation and valuation ratios on the stock market. On the other hand, information from Bid-Ask data subsumes the “inflation” and “housing” factors. Given the period of relevance of bid-ask spread information, this indicates that the relationship between these two factors and liquidity is not stable over time. Overall, the evidence from macroeconomic variables supports our interpretation of the liquidity factor as an indicator of the state of aggregate liquidity.

VII Conclusion

We augment the Arbitrage-Free Extended Nelson-Siegel term structure model of Christensen et al. (2007) by allowing for a liquidity factor driving the on-the-run premium. Estimation of the model proceeds directly from coupon bond prices, based on recent advances in non-linear filtering. We identify from a panel of bonds a common liquidity factor driving on-the-run premia at different maturities. Its effect increases with maturity and decreases with the age of a bond.

Our measure of the state of liquidity predicts a substantial share of the risk premium on off-the-run bonds. It also predicts LIBOR spreads, swap spreads and corporate bond spreads. The pattern across interest rate markets and credit ratings is consistent with accounts of flight-to-liquidity events. However, the effect is pervasive even in normal times. The evidence points toward the importance of aggregate liquidity and aggregate liquidity risk compensation in asset pricing. We find that measures of changes in the stock of money and measures of the availability of funds in the banking system are important determinants of our measure of aggregate liquidity. To a lesser

extent, the liquidity factor varies positively with transaction costs and aggregate uncertainty. Our results are robust to changes in data set and to the inclusion of term structure information.

A large aggregate liquidity risk premium is present across interest rate markets. This also suggests that financial assets are in part valued for the monetary services they render. Investors consider the relative ease of converting financial investments back to cash before deciding upon their portfolio allocation. It remains to be seen if the impact of aggregate liquidity extends to expected excess returns on stocks. In this context, our results suggest that on-the-run premia can be used to identify variations in the compensation for aggregate liquidity risk.

A Unscented Kalman Filter

The UKF is based on an approximation to any non-linear transformation of a probability distribution. It has been introduced in Julier et al. (1995) and Julier and Uhlmann (1996) (see Wan and der Merwe (2001) for textbook treatment) and was first imported in finance by Leippold and Wu (2003).

Given $\hat{X}_{t+1|t}$ a time- t forecast of state variable for period $t + 1$, and its associated MSE $\hat{Q}_{t+1|t}$ the unscented filter selects a set of Sigma points in the distribution of $X_{t+1|t}$ such that

$$\begin{aligned}\bar{x} &= \sum_i w^{(i)} x^{(i)} = \hat{X}_{t+1|t} \\ \mathbf{Q}_x &= \sum_i w^{(i)} (x^{(i)} - \bar{x})(x^{(i)} - \bar{x})' = \hat{Q}_{t+1|t}.\end{aligned}$$

Julier et al. (1995) proposed the following set of Sigma points,

$$x^{(i)} = \begin{cases} \bar{x} & i = 0 \\ \bar{x} + \left(\sqrt{\frac{N_x}{1-w^{(0)}} \Sigma_x} \right)_{(i)} & i = 1, \dots, K \\ \bar{x} - \left(\sqrt{\frac{N_x}{1-w^{(0)}} \Sigma_x} \right)_{(i-K)} & i = K + 1, \dots, 2K \end{cases}$$

with weights

$$w^{(i)} = \begin{cases} w^{(0)} & i = 0 \\ \frac{1-w^{(0)}}{2K} & i = 1, \dots, K \\ \frac{1-w^{(0)}}{2K} & i = K + 1, \dots, 2K \end{cases}$$

where $\left(\sqrt{\frac{N_x}{1-w^{(0)}} \Sigma_x} \right)_{(i)}$ is the i -th row or column of the matrix square root. Julier and Uhlmann (1996) use a Taylor expansion to evaluate the approximation's accuracy. The expansion of $y = g(x)$ around \bar{x} is

$$\begin{aligned}\bar{y} &= E[g(\bar{x} + \Delta x)] \\ &= g(\bar{x}) + E \left[D_{\Delta x}(g) + \frac{D_{\Delta x}^2(g)}{2!} + \frac{D_{\Delta x}^3(g)}{3!} + \dots \right]\end{aligned}$$

where the $D_{\Delta x}^i(g)$ operator evaluates the total differential of $g(\cdot)$ when perturbed by Δx , and evaluated at \bar{x} . A useful representation of this operator in our context is

$$\frac{D_{\Delta x}^i(g)}{i!} = \frac{1}{i!} \left(\sum_{j=1}^n \Delta x_j \frac{\partial}{\partial x_j} \right)^i g(x) \Big|_{x=\bar{x}}$$

Different approximation strategies for \bar{y} will differ by either the number of terms used in the expansion or the set of perturbations Δx . If the distribution of Δx is symmetric, all odd-ordered terms are zero. Moreover, we can re-write the second terms as a function of the covariance matrix P_{xx} of Δx ,

$$\bar{y} = g(\bar{x}) + (\nabla^T P_{xx} \nabla) g(\bar{x}) + E \left[\frac{D_{\Delta x}^4(g)}{4!} + \dots \right]$$

Linearisation leads to the approximation $\hat{y}_{lin} = g(\bar{x})$ while the unscented approximation is exact

up to the third-order term and the σ -points have the correct covariance matrix by construction. In the Gaussian case, Julier and Uhlmann (1996) show that same-variable fourth moments agree as well and that all other moments are lower than the true moments of Δx . Then, approximation errors of higher order terms are necessarily smaller for the UKF than for the EKF. Using a similar argument, but for approximation of the MSE, Julier and Uhlmann (1996) show that linearisation and the unscented transformation agree with the Taylor expansion up to the second-order term and that approximation errors in higher-order terms are smaller for the UKF.

B Arbitrage-Free Yield Adjustment Term

Christensen et al. (2007) show that the constant, $a(m)$ is given by

$$\begin{aligned}
a(m) = & -\sigma_{11}^2 \frac{m^2}{6} - (\sigma_{21}^2 + \sigma_{22}^2) \left[\frac{1}{2\lambda^2} - \frac{1 - e^{-m\lambda}}{m\lambda^3} + \frac{1 - e^{-2m\lambda}}{4m\lambda^3} \right] \\
& - (\sigma_{31}^2 + \sigma_{32}^2 + \sigma_{33}^2) \\
& \times \left[\frac{1}{2\lambda^2} + \frac{e^{-m\lambda}}{\lambda^2} - \frac{me^{-2m\lambda}}{4\lambda} - \frac{3e^{-2m\lambda}}{4\lambda^2} - \frac{2(1 - e^{-m\lambda})}{m\lambda^3} + \frac{5(1 - e^{-2m\lambda})}{8m\lambda^3} \right] \\
& - (\sigma_{11}\sigma_{21}) \left[\frac{m}{2\lambda} + \frac{e^{-m\lambda}}{\lambda^2} - \frac{1 - e^{-m\lambda}}{m\lambda^3} \right] \\
& - (\sigma_{11}\sigma_{31}) \left[\frac{3e^{-m\lambda}}{\lambda^2} + \frac{m}{-2\lambda} + \frac{me^{-m\lambda}}{\lambda} \right] \\
& - (\sigma_{21}\sigma_{31} + \sigma_{22}\sigma_{32}) \\
& \times \left[\frac{1}{\lambda^2} + \frac{e^{-m\lambda}}{\lambda^2} - \frac{e^{-2m\lambda}}{\lambda^2} - \frac{3(1 - e^{-m\lambda})}{m\lambda^3} + \frac{3(1 - e^{-2m\lambda})}{4m\lambda^3} \right], \tag{27}
\end{aligned}$$

References

- Acharya, V. V. and L. Pedersen (2004). Asset pricing with liquidity risk. *NBER* (W10814).
- Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5, 31–56.
- Amihud, Y. and H. Mendelson (1986). Asset pricing and the bid-ask spreads. *Journal of Financial Economics* 17, 223–249.
- Amihud, Y. and H. Mendelson (1991). Liquidity, maturity and the yields on u.s treasury securities. *The Journal of Finance* 46, 1411–1425.
- Anderson, N., F. Breedon, M. Deacon, A. Derry, and G. Murphy (1996). *Estimating and interpreting the yield curve*. Series in Financial Economics and Quantitative Analysis. John Wiley & Sons.
- Bakshi, G., P. Carr, and L. Wu (2005). Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies. *Journal of Financial Economics*. forthcoming.
- Bansal, R. and W. Coleman (1996). A monetary explanation of the term premium, equity premium and risk-free rate puzzle. *The Journal of Political Economy* 104, 396–409.
- Bliss, R. (1997). Testing term structure estimation methods. *Advances in Futures and Options Research* 9, 197–231.
- Brunnermeier, M. K. and L. Pedersen (2008). Market liquidity and funding liquidity. *Journal of Financial Economics*, forthcoming.
- Buraschi, A. and D. Menini (2002). Liquidity risk and specialness. *Journal of Financial Economics* 64, 243–284.
- Campbell, J. and R. Shiller (1991). Yield spreads and interest rate movements: A bird’s eye view. *Review of Economic Studies* 58, 495–514.
- Cheria, J., E. Jacquier, and R. Jarrow (2004). A model of the convenience yields in on-the-run treasuries. *Review of Derivatives Research* 7, 79–97.
- Chordia, T., A. Sarkar, and A. Subrahmanyam (2005). An empirical analysis of stock and bond market liquidity. *Review of Financial Studies* 18, 85–129.
- Christensen, J., F. Diebold, and G. Rudebusch (2007). The affine arbitrage-free class of nelson-siegel term structure models. Technical Report 20, FRBSF.
- Christoffersen, P., K. Jacobs, L. Karoui, and K. Mimouni (2007). Estimating term structure models using swap rates. McGill University.

- Cochrane, J. and M. Piazzesi (2005). Bond risk premia. *American Economic Review* 95, 138–160.
- Collin-Dufresne, P., R. Goldstein, and J. Spencer Martin (2001). The determinants of credit spread changes. *The Journal of Finance* 56, 2177–2207.
- Dai, Q. and K. Singleton (2000). Specification analysis of affine term structure models. *The Journal of Finance* 55, 1943–1978.
- Dai, Q. and K. Singleton (2002). Expectation puzzles, time-varying risk premia and affine model of the term structure. *Journal of Financial Economics* 63, 415–441.
- Diebold, F. and C. Li (2006). Forecasting the term structure of government bond yields. *Journal of Econometrics* 130, 337–364.
- Diebold, F., G. Rudebusch, and S. Aruoba (2006). The macroeconomy and the yield curve : A dynamic latent factor approach. *Journal of Econometrics* 131, 309–338.
- Duffee, G. (2002). Term premia and interest rate forecast in affine model. *The Journal of Finance* 57, 405–443.
- Duffie, D. (1996). Special repo rates. *The Journal of Finance* 51, 493–526.
- Duffie, D., N. Gârleanu, and L. Pedersen (2005). Over-the-counter markets. *Econometrica* 73, 1815–1847.
- Duffie, D. and R. Kan (1996). A yield-factor model of interest rates. *Mathematical Finance* 6, 379–406.
- Duffie, D. and K. Singleton (1997). An econometric model of the term structure of interest-rate swap yields. *The Journal of Finance* 52, 1287–1321.
- Elton, E. and T. Green (1998). Tax and liquidity effects in pricing government bonds. *The Journal of Finance* 53, 1533–1562.
- Ericsson, J. and O. Renault (2006). Liquidity and credit risk. *The Journal of Finance* 61, 2219–2250.
- Fama, E. (1976). Forward rates as predictors of future spot rates. *Journal of Financial Economics* 3, 361–377.
- Fama, E. (1984a). The information in the term structure. *Journal of Financial Economics* 13, 509–528.
- Fama, E. (1984b). Term premium in bond returns. *Journal of Financial Economics* 13, 529–546.
- Fama, E. and R. Bliss (1987). The information in long-maturity forward rates. *American Economic Review* 77, 680–692.

- Fedlhütter, P. and D. Lando (2007). Decomposing swap spreads. Copenhagen Business School.
- Goldreich, D., B. Hanke, and P. Nath (2005). The price of future liquidity : Time-varying liquidity in the u.s. treasury market. *Review of Finance* 9, 1–32.
- Green, R. and B. Ødegaard (1997). Are there tax effects in the relative pricing of the u.s. government bonds? *The Journal of Finance* 52, 609–633.
- Grinblatt, M. (2001). An analytic solution for interest rate swap spreads. *International Review of Finance* 2, 113–149.
- Gurkaynak, R., B. Sack, and J. Wright (2006). The u.s. treasury curve: 1961 to present. Technical Report 2006-28.
- Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *The Journal of Political Economy* 106, 1–40.
- Jordan, B. and S. Jordan (1997). Special repo rates: An empirical analysis. *The Journal of Finance* 52, 2051–2072.
- Julier, S. and J. Uhlmann (1996). A general method for approximating nonlinear transformations of probability distributions. Robotics Research Group, University of Oxford.
- Julier, S., J. Uhlmann, and H. Durrant-Whyte (1995). A new approach for filtering nonlinear systems. *The Proceedings of the American Control Conference* 3, 1628–1632. Seattle.
- Kiyotaki, N. and R. Wright (1989). On money as a medium of exchange. *The Journal of Political Economy* 97, 927–954.
- Krishnamurthy, A. (2002). The bond/old-bond spread. *Journal of Financial Economics* 66, 463–506.
- Krishnamurthy, A. and Z. He (2008). Intermediary asset prices. Technical Report W14517, NBER.
- Krishnamurthy, A. and J. Vissing-Jorgensen (2007). The demand for treasury debt. Technical Report W12881, NBER.
- Lagos, R. (2006). Asset prices and liquidity in an exchange economy. Technical Report 373, Federal Reserve Bank of Minneapolis.
- Leippold, M. and L. Wu (2003). Design and estimation of multi-currency quadratic models. *European Finance Review* 7, 47–73.
- Liu, J., F. Longstaff, and R. Mandell (2006). The market price of risk in interest rate swaps: The roles of default and liquidity risks. *Journal of Business* 79, 2337–2359.

- Longstaff, F. (2000). The term structure of very short-term rates : New evidence for the expectation hypothesis. *Journal of Financial Economics* 58, 397–415.
- Longstaff, F. (2004). The flight-to-liquidity premium in U.S. treasury bond prices. *The Journal of Business* 77, 511–526.
- Longstaff, F., S. Mithal, and E. Neis (2005). Corporate yield spreads: Default risk or liquidity? new evidence from the credit-default swap market. *The Journal of Finance* 60, 2213–2253.
- Ludvigson, S. and S. Ng (2005). Macro factors in bond risk premia. Technical Report 11703, NBER.
- Luttmer, E. (1996). Asset pricing in economies with frictions. *Econometrica* 64, 1439–1467.
- Nashikkar, A. and M. Subrahmanyam (2006). Latent liquidity and corporate bond yield spreads. Stern School of Business.
- Nelson, C. and A. Siegel (1987). Parsimonious modeling of yield curves. *The Journal of Business* 60, 473–489.
- Pastor, L. and R. Stambaugh (2003). Liquidity risk and expected stock returns. *The Journal of Political Economy* 111, 642–685.
- Piazzesi, M. (2005). *Affine term structure models*. in Handbook of Financial Econometrics. Elsevier.
- Shi, S. (2005). Nominal bonds and interest rates. *International Economic Review* 46, 579–612.
- Shi, S. (2006). Viewpoint: a microfoundation of monetary economics. *Canadian Journal of Economics* 39, 643–688.
- Shiller, R. (1979). The volatility of long-term interest rates and expectation models of the term structure. *The Journal of Political Economy* 87, 1190–1219.
- Startz, R. (1982). Do forecast errors of term premia really make a difference between long and short rates. *Journal of Financial Economics* 10, 323–329.
- Svensson, L. (1985). Money and asset prices in a cash-in-advance economy. *The Journal of Political Economy* 93, 919–944.
- Tauchen, G. and H. Zhou (2006). Realized jumps on financial markets and predicting credit spreads. Technical Report 2006-35, Federal Reserve Board.
- Vayanos, D. (2004). Flight to quality, flight to liquidity, and the pricing of risk. *NBER* (W10327).
- Vayanos, D. and P.-O. Weill (2006). A search-based theory of the on-the-run phenomenon. *NBER* (W12670).

Wan, E. and R. V. der Merwe (2001). *Kalman filtering and neural network*, Chapter The unscented Kalman filter. Wiley and Sons.

Warga, A. (1992). Bond returns, liquidity and missing data. *The Journal of Financial and Quantitative Analysis* 27, 605–617.

Table I: Summary statistics of bond characteristics. We present summary statistics of age (months), duration (months) and coupon (%) for each maturity and liquidity category. New refers to the on-the-run security and Old refers to the off-the-run security (see text for details). In each case, the first column gives the sample mean and the second column gives the sample standard deviation. Coupon statistics are not reported for maturity categories of 12 months and less as T-bills do not pay coupons. Monthly data from CRSP (1985:12-2007:12).

Maturity	Age		Duration		Coupon					
	Old	New	Old	New	Old	New				
3	12.04	9.33	1.64	0.09	3.01	0.03	4.38	0.10		
6	16.96	6.31	0.12	0.11	6.00	0.10	5.90	0.11		
9	14.47	6.12	4.41	4.87	8.89	0.11	10.02	0.36		
12	13.14	5.87	2.48	3.88	11.77	0.23	12.13	1.07		
18	7.22	5.99	3.49	0.63	17.36	0.42	19.4	0.80	6.91	3.05
24	22.75	13.48	0.33	0.52	22.54	0.58	22.66	0.71	7.19	2.82
36	24.65	10.32	4.28	6.45	32.51	1.42	32.58	2.47	7.58	2.96
48	18.61	9.64	4.35	3.01	41.87	2.28	43.99	4.34	7.47	3.02
60	29.31	21.33	2.35	3.89	50.27	3.02	51.2	2.93	7.86	2.73
84	34.37	8.73	12.11	11.77	65.61	4.92	68.35	8.31	7.84	2.52
120	15.2	18.78	4.08	7.67	84.08	8.20	85.12	8.93	7.24	2.21

Table II: Parameter estimates for transition equations. Panel (a) presents estimation results for the AFENS model without liquidity. Panel (b) presents estimation results for AFENS model with liquidity. For each parameter, the first standard error (in parenthesis) is computed from the QMLE covariance matrix (see Equation 23) while the second is computed from the sum of outer products of gradients (see Equation 25). Estimation is based on coupon bonds data from CRSP 12/1985-12/2007.

(a) Benchmark Model

	\bar{F}	K	$\Sigma (\times 10^2)$		
Level	0.0545	0.171	0.68		
	(0.0152)	(0.106)	(0.39)		
	(0.0134)	(0.072)	(0.02)		
Slope	-0.0171	0.182	0.76	0.84	
	(0.0203)	(0.195)	(0.74)	(0.55)	
	(0.0166)	(0.135)	(0.05)	(0.04)	
Curvature	-0.0129	0.885	-0.10	0.39	2.34
	(0.0083)	(1.29)	(1.88)	(1.92)	(1.66)
	(0.0072)	(0.288)	(0.17)	(0.20)	(0.13)

(b) Model with liquidity

	\bar{F}	K	$\Sigma (\times 10^2)$		
Level	0.0555	0.181	0.71		
	(0.0239)	(0.228)	(0.46)		
	(0.0132)	(0.079)	(0.02)		
Slope	-0.0164	0.186	-0.76	0.84	
	(0.0177)	(0.341)	(0.66)	(0.48)	
	(0.0186)	(0.138)	(0.05)	(0.04)	
Curvature	-0.0163	0.914	0.20	0.30	2.34
	(0.0087)	(1.178)	(2.10)	(1.93)	(1.52)
	(0.0076)	(0.301)	(0.04)	(0.19)	(0.11)
	\bar{L}	ϕ_l	σ_l		
Liquidity	0.375	0.965	0.068		
	(0.212)	(0.044)	(0.040)		
	(0.76)	(0.085)	(0.016)		

Table III: Panel (a) presents Mean Pricing Errors (MPE) from AFENS models with and without liquidity. Panel (b) presents Root Mean Squared Pricing Errors (RMSPE) from the AFENS model with and without liquidity. The columns correspond to liquidity category where New refers to on-the-run issues and Old refers to off-the-run issues. Coupon bonds data from CRSP 12/1985-12/2007.

(a) Mean Pricing Errors

Mean Pricing Errors				
Maturity	Benchmark Model		Liquidity Model	
	Old	New	Old	New
3	0.015	0.025	0.014	0.002
6	-0.003	0.019	0.022	-0.003
9	-0.032	0.023	-0.011	0.002
12	-0.042	0.034	-0.024	0.003
18	-0.059	-0.056	-0.010	-0.004
24	-0.027	0.001	-0.005	-0.009
36	0.008	0.071	0.030	-0.010
48	-0.005	0.082	0.014	$2.5e^{-5}$
60	0.008	0.253	0.038	0.003
84	-0.167	-0.043	0.037	-0.014
120	-0.237	0.057	-0.161	0.023
All	-0.049	0.042	-0.005	-0.001

(b) Root Mean Squared Errors

Mean Pricing Errors				
Maturity	Benchmark Model		Liquidity Model	
	Old	New	Old	New
3	0.048	0.045	0.048	0.038
6	0.035	0.040	0.041	0.034
9	0.055	0.061	0.038	0.047
12	0.073	0.079	0.059	0.057
18	0.084	0.085	0.052	0.056
24	0.061	0.083	0.054	0.082
36	0.104	0.139	0.098	0.088
48	0.173	0.185	0.156	0.121
60	0.228	0.320	0.201	0.130
84	0.326	0.299	0.208	0.211
120	-0.237	0.057	-0.161	0.023
All	-0.049	0.042	-0.005	-0.001

Table IV: On-the-run premium. Each line corresponds to a maturity category (months). The first two columns provide the average of residual differences in each maturity category for the AFENS model with and without maturity, respectively. The last three columns display estimates of the liquidity level, $\hat{\beta}$, followed by standard errors (in parenthesis). The first standard error is computed from the QMLE covariance matrix (see Equation 23) while the second is computed from the sum of outer products of gradients (see Equation 25). Estimation is based on coupon bonds data from CRSP 12/1985-12/2007.

Maturity	Residuals Differences		$\hat{\beta}$	Liquidity Level	
	Benchmark	Liquidity		QMLE s.e.	MLE s.e.
3	0.011	-0.013	0.148	(0.118)	(0.010)
6	0.022	-0.025	0.132	(0.118)	(0.014)
9	0.055	0.013	0.195	(0.129)	(0.017)
12	0.075	0.027	0.188	(0.114)	(0.019)
18	0.003	0.006	-0.074	(0.039)	(0.023)
24	0.028	-0.004	0.092	(0.035)	(0.019)
36	0.064	-0.040	0.407	(0.213)	(0.063)
48	0.089	-0.014	0.594	(0.359)	(0.065)
60	0.248	-0.035	0.965	(0.491)	(0.071)
84	0.124	-0.052	0.975	(0.379)	(0.084)
120	0.295	0.183	1		
All	0.092	0.004			

Table V: Results from regressions of excess returns from off-the-run bond on the liquidity and term structure factors. Each variable is obtained from the AFENS model, demeaned and divided by its standard deviation. The specification is $xr_{t+h}^{(m)} = \alpha_h^{(m)} L_t + \beta_h^{(m)T} F_t + \epsilon_{(t+h)}^{(m)}$ where $xr_{t+h}^{(m)}$ is the excess returns at horizon h (months) on a bond of maturity m (months), L_t is the liquidity factor and F_t is the vector of term structure factor. Panel (a) contains estimates of α and Panel (b) contains estimates of δ . Newey-West standard errors (h+3 lags) are included in parenthesis. Panel (c) presents R^2 . 24-months excess return on 2-year bonds (lower left corner of each panel is always zero) by construction.

(a) Average risk premia										
Bond Maturity										
Horizon	2	3	4	5	7	10				
6	1.11 (0.39)	1.70 (0.62)	2.15 (0.81)	2.49 (0.98)	2.98 (1.27)	3.50 (1.70)				
12	0.72 (0.28)	1.34 (0.51)	1.80 (0.68)	2.14 (0.82)	2.62 (1.05)	3.10 (1.37)				
18	0.37 (0.14)	1.02 (0.38)	1.53 (0.55)	1.91 (0.67)	2.44 (0.84)	2.96 (1.05)				
24		0.70 (0.24)	1.28 (0.41)	1.72 (0.53)	2.33 (0.67)	2.94 (0.81)				

(b) Liquidity Coefficients										
Bond Maturity										
Horizon	2	3	4	5	7	10				
6	-0.58 (0.32)	-0.99 (0.55)	-1.38 (0.75)	-1.74 (0.93)	-2.42 (1.25)	-3.37 (1.70)				
12	-0.47 (0.16)	-0.96 (0.31)	-1.42 (0.44)	-1.82 (0.57)	-2.54 (0.80)	-3.50 (1.15)				
18	-0.17 (0.08)	-0.50 (0.24)	-0.79 (0.38)	-1.05 (0.49)	-1.52 (0.69)	-2.16 (0.96)				
24		-0.26 (0.19)	-0.48 (0.34)	-0.68 (0.45)	-1.02 (0.62)	-1.49 (0.81)				

(c) R^2										
Bond Maturity										
Horizon	2	3	4	5	7	10				
6		15.8	16.7	17.5	18.6	19.2				
12	15.1	20.4	25.7	28.4	32.6	36.0				
18	20.7	19.4	20.5	22.5	27.5	33.6				
24		22.2	18.7	17.1	16.9	20.3				

Table VI: Results from regressions of excess returns from off-the-run bonds on forward rates and the liquidity factor. Variables are demeaned and divided by their standard deviations. The specification is $xr_{t+12}^{(m)} = \alpha^{(m)} + \delta^{(m)} L_t + \beta^{(m)T} f_t + \epsilon_{(t+12)}^{(m)}$ where $xr_{t+h}^{(m)}$ is the annual excess returns on a bond with maturity m (months), L_t is the liquidity factor and f_t is the vector of forward rates $f_t^{(h)}$ at horizon h . Panel (a) presents results using excess returns and forward rates computed from FB data with the liquidity factor from the AFENS model. Panel (b) excludes the liquidity factor. Panel (c) excludes the liquidity factor and uses excess returns computed from the model. Newey-West standard errors (in parenthesis) with 15 lags.

(a) Excess returns and forward rates from FB with the liquidity factor

Maturity	cst	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	L_t	R^2
2	0.72 (0.23)	0.05 (0.67)	-2.01 (1.27)	2.97 (1.49)	1.07 (0.96)	-1.46 (0.66)	-0.51 (0.18)	0.31
3	1.31 (0.43)	-0.29 (1.27)	-3.71 (2.36)	6.51 (2.79)	1.06 (1.73)	-2.50 (1.30)	-1.06 (0.36)	0.32
4	1.79 (0.57)	-1.08 (1.73)	-3.86 (3.20)	7.70 (3.73)	1.95 (2.24)	-3.27 (1.79)	-1.56 (0.50)	0.34
5	1.98 (0.69)	-2.15 (2.08)	-2.90 (3.98)	8.41 (4.50)	0.88 (2.61)	-2.54 (2.23)	-1.96 (0.63)	0.33

(b) Excess returns and forward rates from FB

Maturity	cst	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	L_t	R^2
2	0.72 (0.24)	-0.43 (0.75)	-1.34 (1.27)	2.66 (1.77)	0.99 (1.04)	-1.53 (0.72)		0.21
3	1.31 (0.45)	-1.27 (1.46)	-2.33 (2.24)	5.86 (3.30)	0.88 (1.92)	-2.64 (1.42)		0.19
4	1.79 (0.61)	-2.52 (2.00)	-1.83 (2.96)	6.74 (4.45)	1.70 (2.54)	-3.46 (1.96)		0.20
5	1.98 (0.73)	-3.96 (2.40)	-0.35 (3.55)	7.20 (5.32)	0.56 (3.01)	-2.79 (2.44)		0.18

(c) Excess returns from the model and forward rates from FB

Maturity	cst	$f_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$	L_t	R^2
2	0.70 (0.24)	-0.14 (0.75)	-1.89 (1.22)	2.96 (1.72)	0.92 (1.00)	-1.49 (0.71)		0.22
3	1.31 (0.45)	-1.16 (1.44)	-2.01 (2.23)	4.96 (3.28)	1.18 (1.88)	-2.42 (1.39)		0.19
4	1.77 (0.61)	-2.48 (1.99)	-1.24 (3.04)	6.12 (4.52)	1.16 (2.56)	-2.91 (1.98)		0.18
5	2.12 (0.74)	-3.88 (2.40)	-0.01 (3.71)	6.67 (5.49)	1.04 (3.08)	-3.11 (2.47)		0.17

Table VII: Results from regressions of LIBOR spreads on the liquidity and term structure factors from the AFENS model. Spreads are computed above the AFENS zero-coupon yield curve. Each variable is demeaned and divided by standard deviation. The specification is $spr d_{t+h}^{(m)} = \alpha_h^{(m)} L_t + \beta_h^{(m)T} F_t + \epsilon_{(t+h)}^{(m)}$ where $spr d_{t+h}^{(m)}$ is the spread at time $t+h$ (months) on a loan with maturity m (months), L_t is the liquidity factor and F_t is the vector of term structure factor. Panel (a) contains estimates of α_0 and Panel (b) contains estimates of $\delta_h^{(m)}$. Newey-West standard errors (h+3 lags) are include in parenthesis. Panel (c) presents R^2 .

(a) Average Risk Premium										
Horizon	Loan Maturity									
	1	3	6	9	12					
0	0.34	(0.03)	0.38	(0.02)	0.41	(0.02)	0.43	(0.2)	0.47	(0.02)

(b) Liquidity Coefficients										
Horizon	Loan Maturity									
	1	3	6	9	12					
0	0.073	(0.02)	0.079	(0.02)	0.068	(0.01)	0.070	(0.01)	0.069	(0.01)
6	0.045	(0.02)	0.062	(0.02)	0.067	(0.02)	0.078	(0.02)	0.081	(0.02)
12	0.091	(0.02)	0.105	(0.02)	0.103	(0.02)	0.120	(0.02)	0.131	(0.02)
18	0.088	(0.02)	0.094	(0.02)	0.094	(0.02)	0.110	(0.02)	0.118	(0.02)
24	0.109	(0.02)	0.112	(0.02)	0.113	(0.02)	0.124	(0.02)	0.130	(0.02)

(c) R^2						
Horizon	Loan Maturity					
	1	3	6	9	12	
0	39.3	45.4	47.7	51.7	53.6	
6	42.9	44.5	41.9	41.7	41.1	
12	49.0	46.6	37.9	36.6	36.4	
18	51.5	43.9	30.8	27.1	25.6	
24	58.7	54.0	40.7	33.7	30.1	

Table VIII: Results from regressions of swap spreads on the liquidity and term structure factors obtained from the AFENS model. Spreads are computed above the AFENS par yield curve. Each variable is demeaned and divided by its standard deviation. The specification is $spr d_{t+h}^{(m)} = \alpha_h^{(m)} L_t + \beta_h^{(m)T} F_t + \epsilon_{(t+h)}^{(m)}$ where $spr d_{t+h}^{(m)}$ is the spread at time $t+h$ (months) on a contract with maturity m (months), L_t is the liquidity factor and F_t is the vector of term structure factor. Panel (a) contains estimates of α_0 and Panel (b) contains estimates of $\delta_h^{(m)}$. Newey-West standard errors (h+3 lags) are included in parenthesis. Panel (c) presents R^2 .

(a) Average Risk Premium						
Horizon	Contract Maturity			Contract Maturity		
	24	36	48	48	60	84
0	0.44	0.47	0.48	0.48	0.49	0.53
	(0.03)	(0.04)	(0.03)	(0.03)	(0.3)	(0.03)

(b) Liquidity Coefficients						
Horizon	Contract Maturity			Contract Maturity		
	24	36	48	48	60	84
0	0.068	0.073	0.076	0.076	0.079	0.094
	(0.027)	(0.027)	(0.027)	(0.027)	(0.026)	(0.026)
6	0.072	0.079	0.086	0.086	0.090	0.111
	(0.028)	(0.029)	(0.028)	(0.028)	(0.027)	(0.026)
12	0.094	0.102	0.111	0.111	0.112	0.137
	(0.030)	(0.030)	(0.030)	(0.030)	(0.029)	(0.028)
18	0.140	0.149	0.152	0.152	0.149	0.166
	(0.029)	(0.030)	(0.029)	(0.029)	(0.028)	(0.027)
24	0.111	0.121	0.125	0.125	0.120	0.128
	(0.027)	(0.026)	(0.026)	(0.027)	(0.026)	(0.024)

(c) R^2						
Horizon	Contract Maturity			Contract Maturity		
	24	36	48	48	60	84
0	20.7	21.1	21.9	21.9	21.9	24.4
	(12.6)	(11.7)	(13.5)	(13.5)	(15.0)	(20.8)
12	14.2	14.8	17.0	17.0	18.2	23.2
	(22.9)	(24.5)	(25.7)	(25.7)	(26.0)	(29.6)
24	18.8	20.9	21.3	21.3	20.7	20.9

Table IX: Results from regressions of corporate spreads on the liquidity and term structure factors obtained from the AFENS model. Each variable is demeaned and divided by its standard deviation. Newey-West standard errors (h+3 lags) are in parenthesis. Panel (a) presents results using Moody's indices where spreads are computed above the model-implied yield curve. The specification is $spr d_{t+h}^{(m)} = \alpha_h^{(m)} + \delta_h^{(m)} L_t + \beta_h^{(m)T} F_t + \epsilon_{(t+h)}^{(m)}$ where $spr d_{t+h}^{(m)}$ is the spread at time $t + h$ (months) on a contract with maturity m (months), L_t is the liquidity factor and F_t is the vector of term structure factor. Panel (b) presents results using NAIC data. See text for the specification. Newey-West standard errors (h+3 lags) are included in parenthesis.

(a) Coefficient Estimates - Moody's Indices

Rating	Avg Premium	Horizon			R^2	
		0	6	12		18
Aaa	2.214 (0.041)	0.154 (0.032)	0.128 (0.045)	0.141 (0.049)	0.193 (0.048)	0.166 (0.054)
Baa	3.314 (0.056)	0.999 (0.140)	0.787 (0.192)	0.833 (0.164)	1.050 (0.142)	0.470 (0.207)
Aaa		27.8	20.7	21.3	36.2	39.7
Baa		43.9	24.5	25.2	43.3	54.7

(b) Coefficient Estimates - NAIC Transactions

Rating	Avg Premium	Horizon			R^2	
		0	6	12		18
G1	1.514 (0.186)	-0.389 (0.138)	-0.420 (0.148)	-0.255 (0.111)	-0.142 (0.112)	-0.137 (0.133)
G2	1.648 (0.205)	-0.251 (0.122)	-0.314 (0.118)	-0.158 (0.097)	-0.081 (0.090)	-0.050 (0.107)
G3	2.251 (0.299)	-0.076 (0.157)	-0.099 (0.168)	0.123 (0.136)	0.277 (0.141)	0.309 (0.176)
G4	3.383 (0.586)	0.234 (0.121)	0.293 (0.134)	0.420 (0.099)	0.529 (0.101)	0.566 (0.119)
G5	3.703 (0.536)	0.304 (0.121)	0.436 (0.143)	0.647 (0.154)	0.794 (0.144)	0.861 (0.149)
All		6.49	6.11	7.01	7.89	9.15

Table X: Results from regressions of the liquidity factor on selected economic variables. BA is the difference between the minimum and the median bid-ask spreads across all bond price observation on a given date. VXO is the implied volatility from S&P500 call options. $F1$ to $F8$ are principal components of macroeconomic series from Ludvigson and Ng. Newey-West standard errors (3 lags) are included in parenthesis.

Model	$ cst $	Regressors								$ R^2 $		
		$ BA $	$ VXO $	$ F1 $	$ F2 $	$ F3 $	$ F4 $	$ F5 $	$ F6 $		$ F7 $	$ F8 $
A	0.475 (0.033)			0.037 (0.027)	0.089 (0.021)	-0.008 (0.020)	0.055 (0.025)	0.043 (0.021)	-0.074 (0.025)	0.082 (0.026)	-0.019 (0.019)	30.4
B	0.432 (0.028)	0.140 (0.037)										31.7
C	0.431 (0.033)		0.094 (0.027)									14.2
D	0.474 (0.028)	0.102 (0.035)	0.031 (0.031)	0.013 (0.022)	0.060 (0.021)	-0.010 (0.016)	0.025 (0.019)	0.019 (0.019)	-0.056 (0.025)	0.070 (0.024)	-0.048 (0.016)	45.2

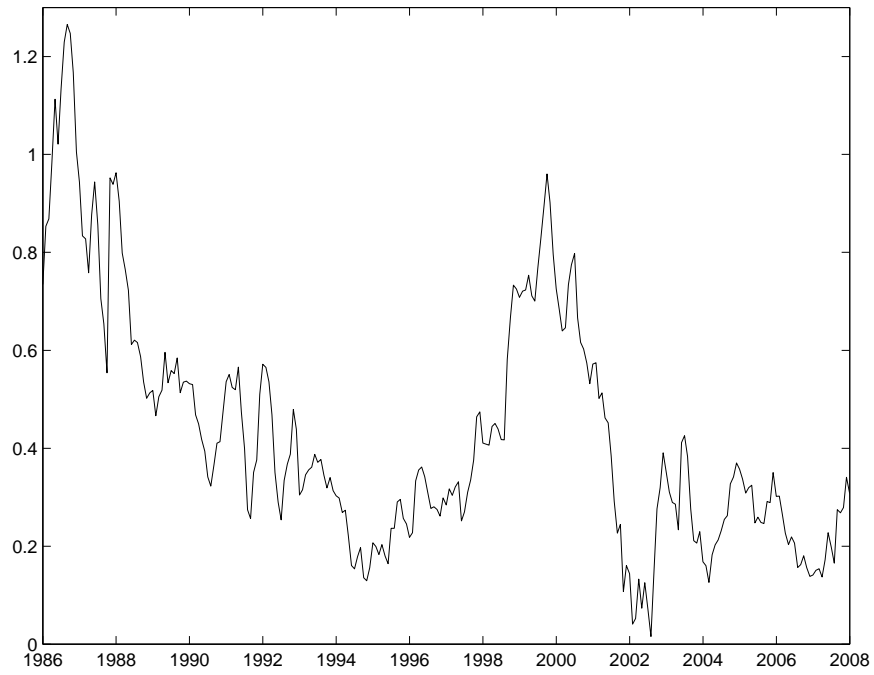
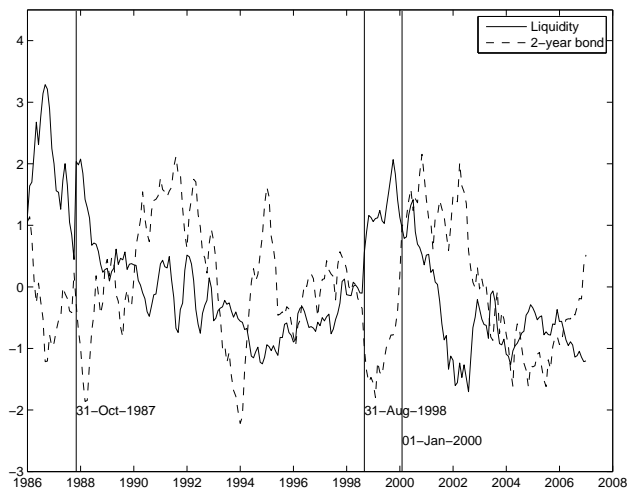
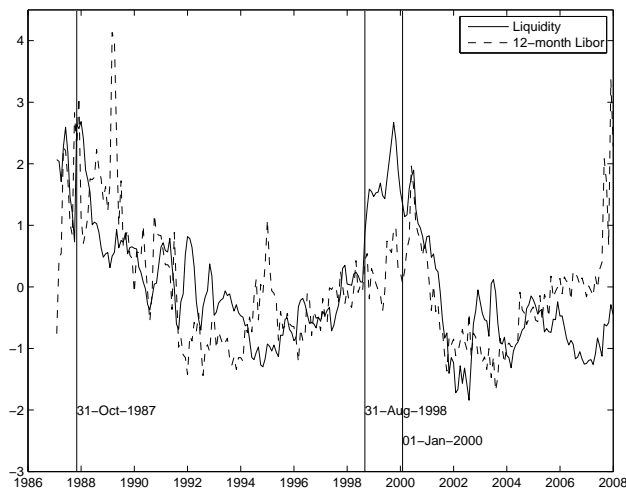


Figure 1: Liquidity factor with loading $\beta_{10} = 1$ with the scale in dollar. The factor corresponds to the on-the-run premium of a just-issued 10-year government coupon bond. Estimation of the AFENS model with liquidity using a sample of coupon bonds (CRSP 12/1985-12/2007).

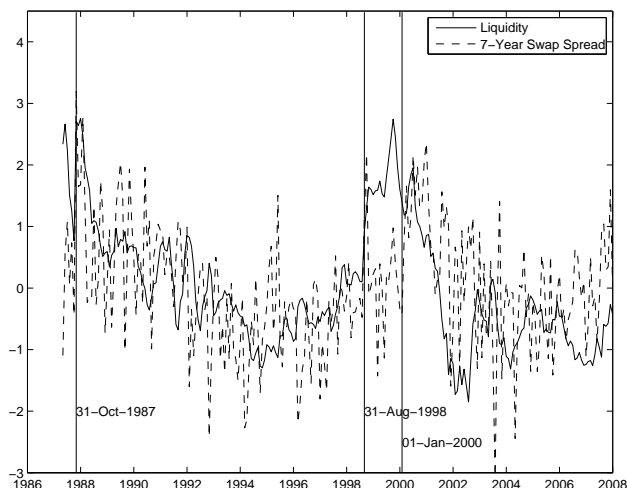
Figure 2: Comparison of the liquidity factor with expected risk premium in different markets. Panel (a) compares with the risk premium on 2-year off-the-run U.S. Treasury bond. Panel 3b compares with risk premium on a 12-month LIBOR loan. Panel 3c compares with risk premium on 7-year swap contract. Panel 3c compares with Moody's index of risk premium on Baa corporate bonds.



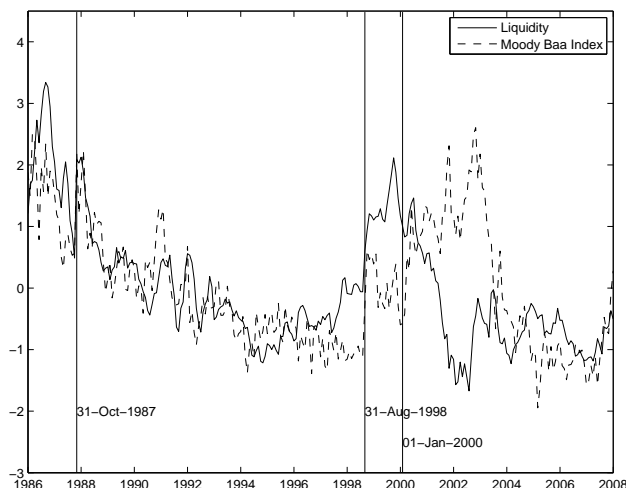
(a) Liquidity and off-the-run excess returns



(b) Liquidity and 12-month LIBOR spread



(c) Liquidity and 7-year swap spread



(d) Liquidity and Moody's corporate spread index

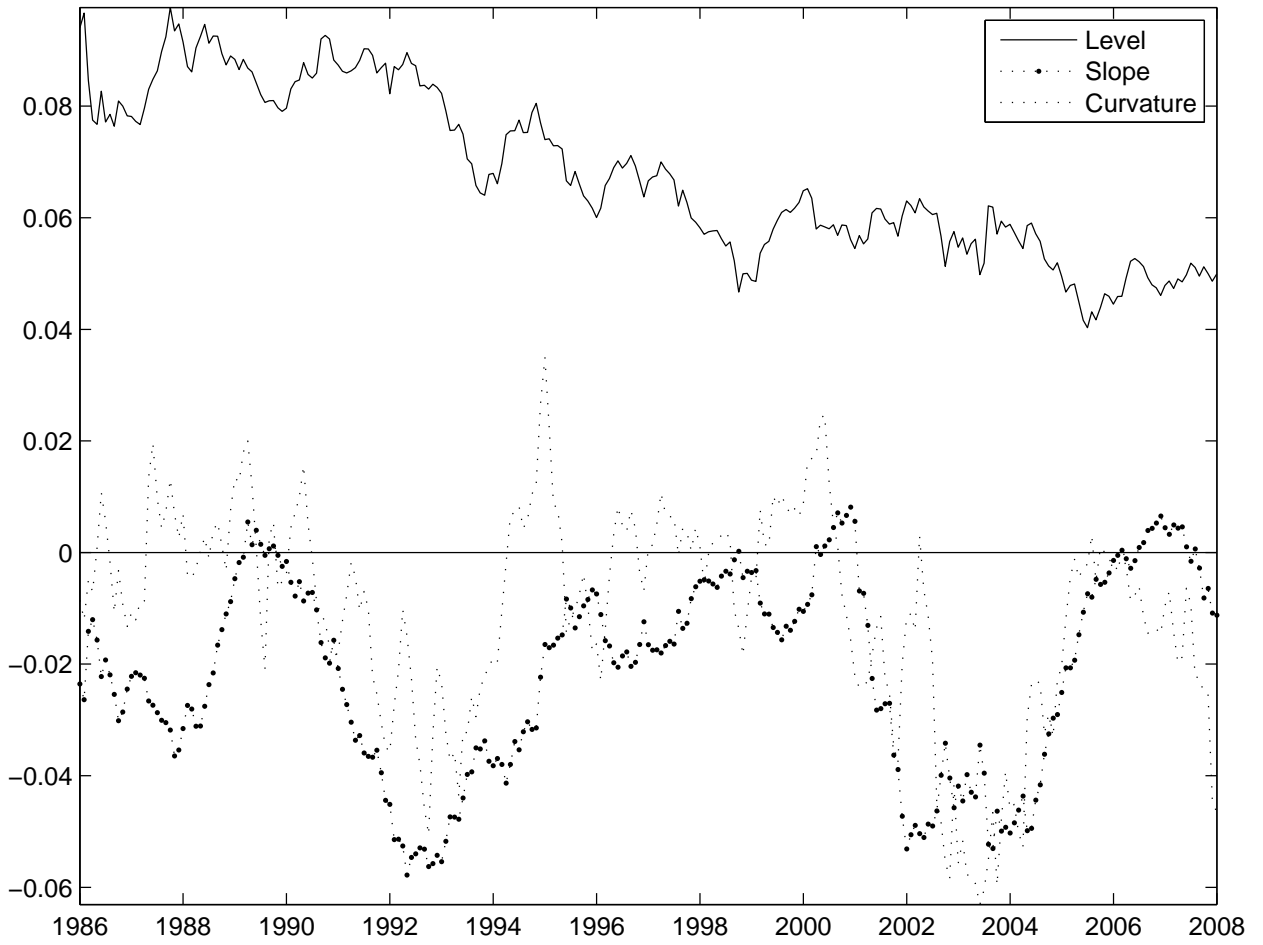
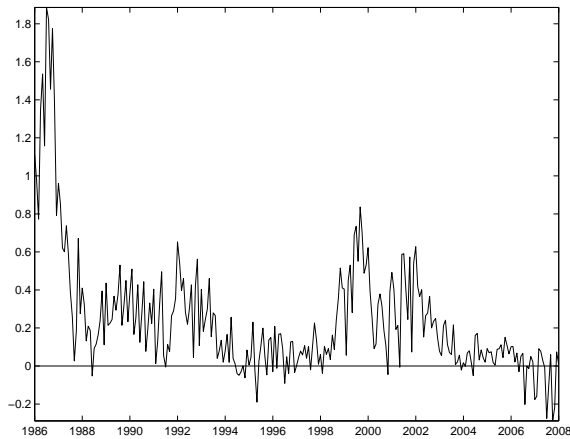
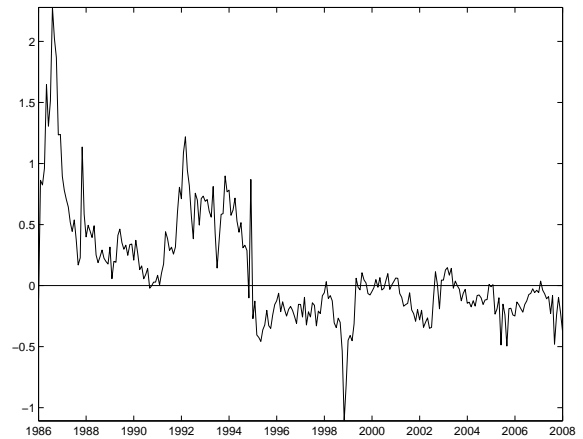


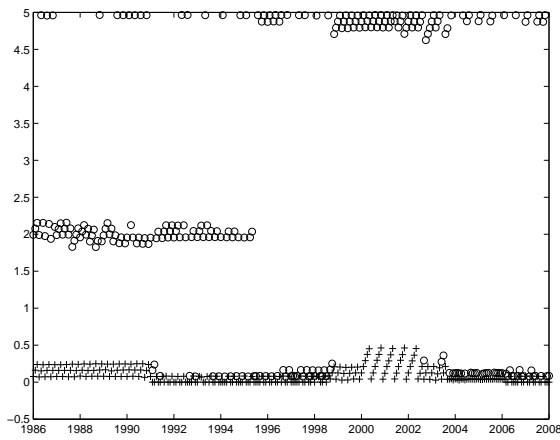
Figure 3: Term Structure factors from AFENS model without liquidity. The scale is in percentage. Coupon bonds data from CRSP 12/1985-12/2007.



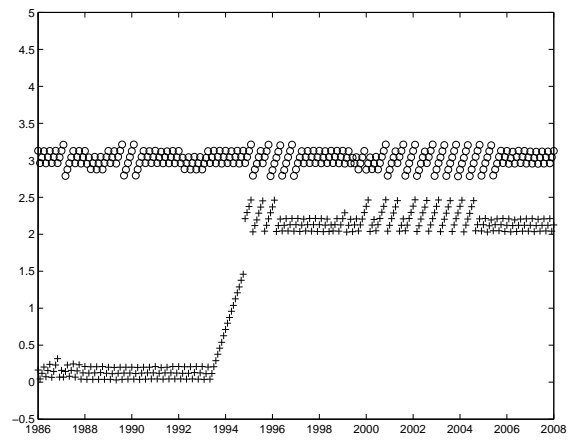
(a) 60-month residual difference



(b) 84-month residual difference

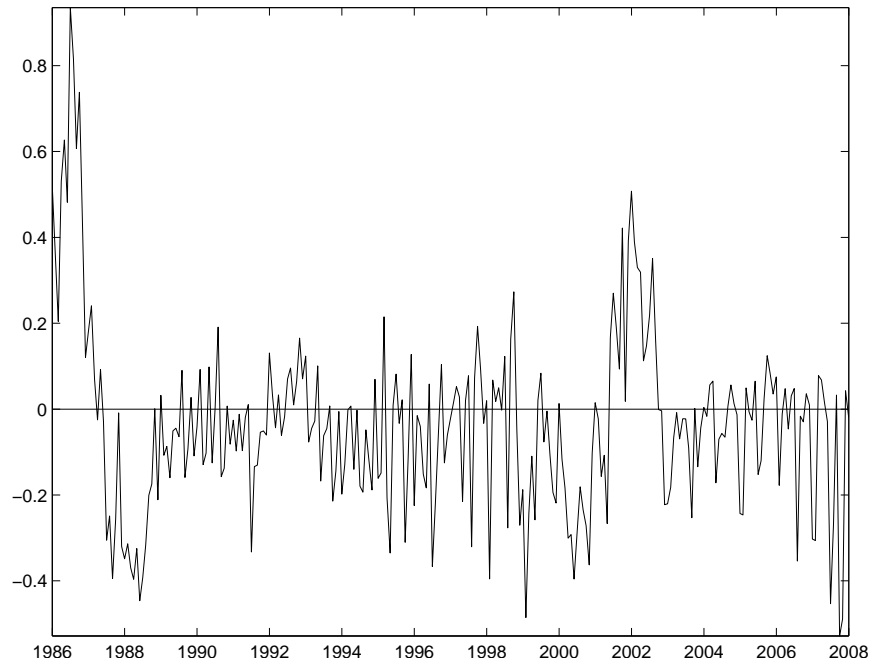


(c) 60-month ages

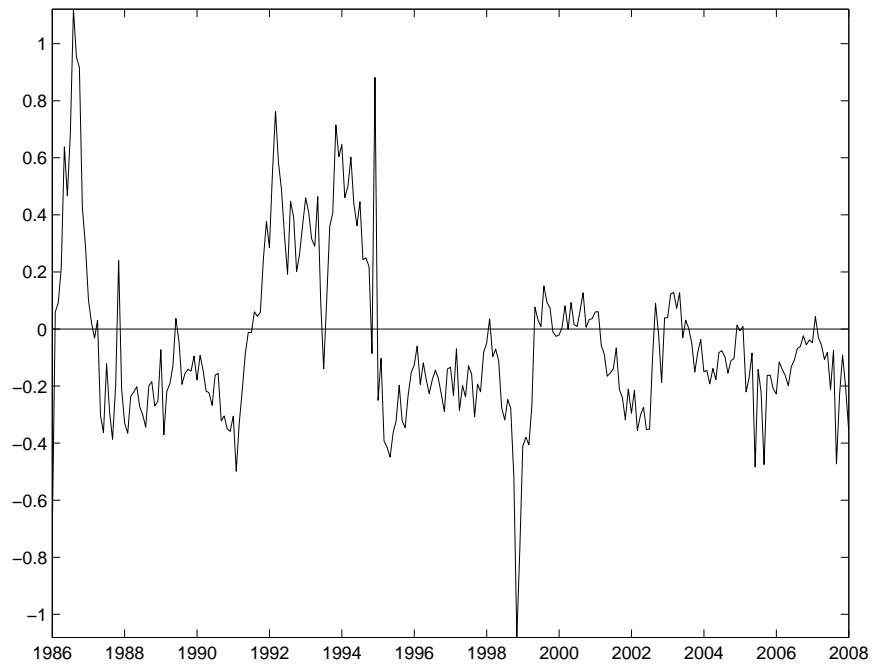


(d) 84-month ages

Figure 4: Comparison of residual differences and ages for the benchmark AFENS model without liquidity. Panel (a) presents differences between the residuals (dollars) of the on-the-run and off-the-run bonds in the 60-month category. Panel (b) presents the residuals 84-month category. Panel (c) and (d) present years from issuance for on-the-run (+) and off-the-run (o) bonds in each of the 60-month and the 84-month category, respectively. Coupon bonds data from CRSP 12/1985-12/2007.

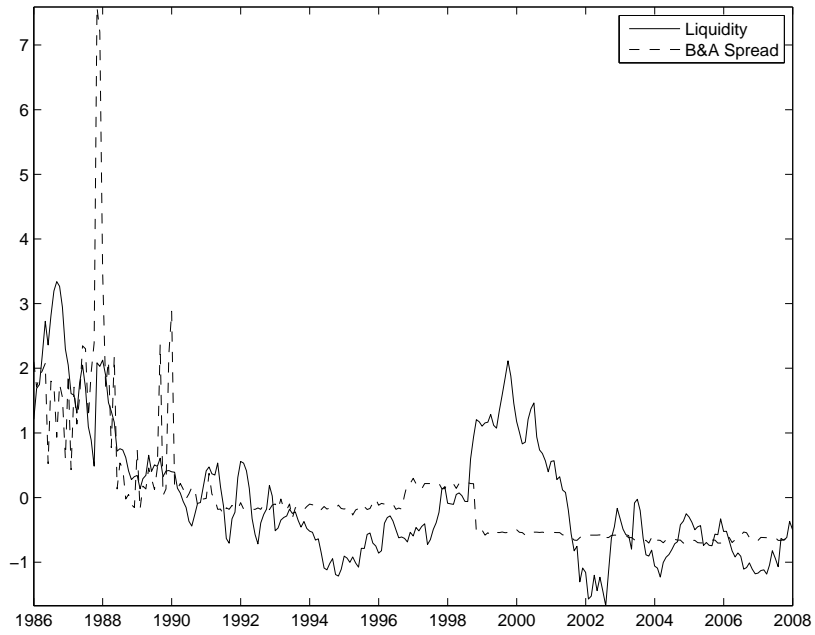


(a) 60-month residual differences

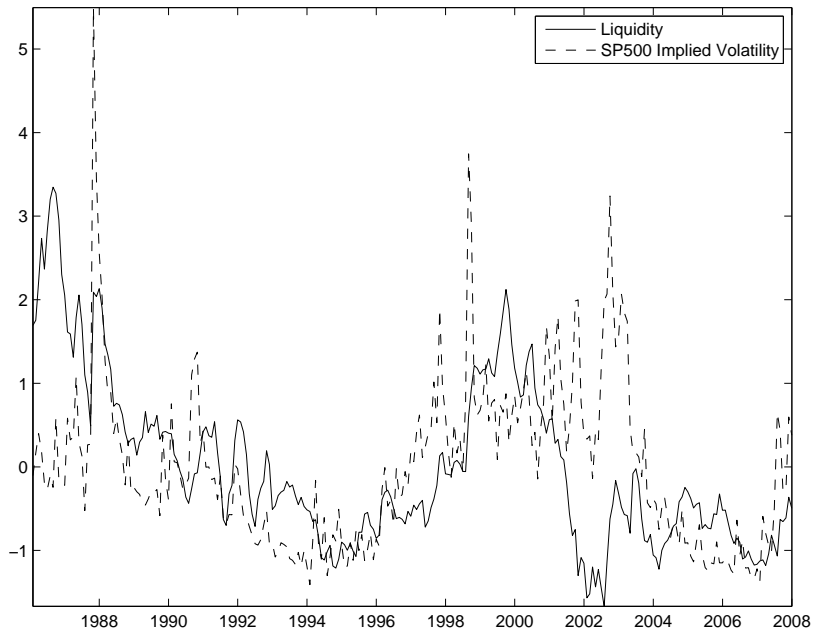


(b) 84-month residual differences

Figure 5: Residual analysis the AFENS model with liquidity. Panel (a) present differences between residuals (dollars) of the on-the-run and off-the-run bonds in the 60-month category. Panel (b) presents differences between residuals (dollars) in the 84-month category. Coupon bonds data from CRSP 12/1985-12/2007.



(a) Bid-Ask Spread and Liquidity



(b) Volatility and Liquidity

Figure 6: Panel (a) traces the liquidity factor and our measure of relative transactions costs. The latter is computed as the difference between the median and the minimum bid-ask spread at each observation date. Panel (b) traces the liquidity factor and implied volatility from S&P 500 call options. The liquidity factor is obtained from the AFENS model with liquidity. Estimation from CRSP coupon bond data 12/1985-12/2003.