Momentum and Mean-Reversion in Strategic Asset Allocation*

Ralph S.J. Koijen
Tilburg University,[†]
CentER and Netspar

Juan Carlos Rodriguez
Tilburg University[‡]
and CentER

Alessandro Sbuelz
University of Verona[§]

This version: August 2006

Abstract

We study a dynamic asset allocation problem in which expected stock returns are predictable, focusing on an investor with a medium-term horizon of up to five years. At these horizons, both return continuation (momentum) and mean-reversion are of central importance in the asset allocation problem. Researchers have extensively investigated the impact of mean-reversion on optimal portfolio choice, but its interplay with return continuation has not been explicitly addressed so far. We introduce a tractable continuous time model that captures these two predictability features of stock market returns. Our model predicts that hedging demands are negative for short to medium-term investors, and that the total allocation to stocks does not increase monotonically with the investor's horizon. Moreover, the value of hedging time-variation in investment opportunities, i.e. acting strategically, is substantially higher in the presence of return continuation. The utility gains from including momentum are preserved if we impose realistic borrowing and short-sales constraints and allow the investor trade on a monthly frequency, but disappear at an annual trading frequency.

*We would like to thank Theo Nijman, Anna Pavlova, Pieter Jelle van der Sluis, Jules van Binsbergen, and Bas Werker and seminar participants at the University of Salerno, University of Konstanz, VII Workshop on Quantitative Finance, University of Perugia, International Summer School in 'Risk Measurement and Control', University of Rome "La Sapienza" for useful discussions and suggestions.

†Department of Finance and CentER, Tilburg University, P.O. Box 90153, 5000 LE, Tilburg, The Netherlands. Phone: +31-13-466-3238, Fax: +31-13-466-2875, E-mail: R.S.J.Koijen@TilburgUniversity.nl.

‡Department of Finance and CentER, Tilburg University, P.O. Box 90153, 5000 LE, Tilburg, The Netherlands. Phone: +31-13-466-3262, Fax: +31-13-466-2875, E-mail: J.C.Rodriguez@TilburgUniversity.nl.

§Economics Department, SAFE Center, University of Verona, Via Giardino Giusti 2, 37129, Verona, Italy, Phone: +39-045-8054922, Fax: +39-045-8054935, E-mail: Alessandro.Sbuelz@Univr.it.

1 Introduction

Investors deciding strategically about their allocation of total capital to equities have to account for time-variation in investment opportunities offered by this asset class. Empirical evidence indicates that market returns tend to continue over short periods and reverse over longer periods (see Lo and Mackinlay (1988), Fama and French (1988), and Poterba and Summers (1988) for seminal contributions). Momentum or return continuation makes stocks riskier over short periods, inducing a decrease in the optimal allocation to stocks. In contrast, mean-reversion or return reversals reduce the long-term risk of stocks, which leads to an increase in the optimal allocation to stocks (see Campbell and Viceira (1999, 2005)).

Several recent papers acknowledge the importance of momentum on the optimal strategic allocation to stocks. Balvers and Mitchell (1997) show that the optimal allocation to stocks is decreasing in the investment horizon when returns are positively autocorrelated. Brandt (1999) finds an initial decline in the optimal allocation to stocks due to positive autocorrelation in short to medium-term stock returns using a non-parametric approach. This effect is stronger for more aggressive investors. Likewise, Aït-Sahalia and Brandt (2001) use a single index model to estimate the conditional Euler equation and find that the optimal hedging demands are weak and negative, consistent with the presence of momentum in returns.

On the other hand, mean-reversion is the dominant characteristic of equity index returns at longer horizons. Campbell and Viceira (1999), Wachter (2002), Campbell, Chan, and Viceira (2003) have illustrated that, as a consequence of this phenomenon, the optimal long-term allocation to equities is larger than the myopic allocation and increases monotonically with the investment horizon. This result is robust to parameter uncertainty (see Barberis (2000)), and transaction costs (see Balduzzi and Lynch (1999)).[1]

Although the impact of predictability in the form of continuation or reversal has been widely studied in the strategic asset allocation literature, the possible effects of their joint presence have not received similar attention.[2] Such issue is relevant, however, since there is substantial evidence that the investment horizons of a large portion of institutional investors have shortened in recent years (see Baker (1998)), a phenomenon that may be due to agency

[1]The predictive power of the dividend yield for equity returns has been debated recently in Ang and Bekaert (2006), Goyal and Welch (2003, 2006), Campbell and Yogo (2005), and Campbell and Thompson (2005), among others. The empirical support for the predictive power of the dividend yield has weakened during the nineteens.

[2]Recent contributions, like Campbell et al. (2003) do allow, by their use of a VAR model, for return continuation and reversal. However, their models are calibrated on a value-weighted index and estimated using quarterly data, a frequency and index for which continuation is barely noticeable.

problems that arise in delegated portfolio management.[3] As John Bogle, a veteran of the investment management business, put it at the beginning of the 2000s:[4]

> a dramatic decline in investment horizons has also changed the industry. ... Portfolio managers turn over at a rapid rate. The average manager lasts just six years, and then a new broom sweeps the portfolio clean. ... Fund shares – once held by long-term investors for an average of 12.5 years – are today held for an average of just over two years.

We study in this paper a dynamic asset allocation problem in which both momentum and mean-reversion are accommodated. In doing this, we emphasize the impact of stock return predictability on investors with medium-term investment horizons of up to 5 years. At these horizons, both return continuation and reversal play a vital role in determining the total allocation to stocks.

As an example of the simultaneous presence of continuation and reversals in broad market indices, consider the following predictive regressions on postwar data that runs from from January of 1946 to December of 2005. The first regression corresponds to the CRSP value-weighted (VW) index and the second regression to the CRSP equally-weighted (EW) index. We regress simple monthly returns on one month lagged (R_t) returns and the dividend yield (D_t). The dividend yield is constructed by summing the dividends over the last 12 months and dividing by the current price, following Campbell et al. (2003).

$$\begin{array}{lcccccccccc} \text{VW:} & R_{t+1} & = & 0.0398 & + & 0.0089D_t & + & 0.0561R_t & + & \epsilon_{t+1}, & R^2 = 1.07\%, \\ & & & (0.0130) & & (0.0038) & & (0.0371) & & & \\ \text{EW:} & R_{t+1} & = & 0.0313 & + & 0.0059D_t & + & 0.1982R_t & + & \epsilon_{t+1}, & R^2 = 4.07\%, \\ & & & (0.0153) & & (0.0041) & & (0.0366) & & & \end{array} \tag{1}$$

where the standard errors have been reported in parentheses. In both regressions, past returns predict future returns with a positive sign, which is indicative of momentum. As expected, the predictive power of past returns is stronger in the equally-weighted index than

[3] The literature has been mainly focusing on explaning why institutional investors do not have long investment horizons. Explanations range from the characteristic of funds, see for instance Baker (1998), to agency problems that arise naturally in delegated portfolio management, see Shleifer and Vishny (1997) and van Binsbergen, Brandt, and Koijen (2006).

[4] "How Mutual Funds Lost Their Way", remarks by John C. Bogle, Founder and Former Chairman, The Vanguard Group, *The Wall Street Journal*, June 20, 2000. Bogle has confirmed these views in "The Mutual Fund Industry in 2003: Back to the Future", Before the Harvard Club of Boston, the Harvard Business School Association of Boston, and the Boston Security Analysts Society Boston, Massachusetts, January 14, 2003.

in the value-weighted index. This evidence is consistent with known results in the empirical asset pricing literature (see Lo and Mackinlay (1988)). The dividend yield, instead, is a stronger predictor in the value-weighted index. The fact that continuation is weak in the value-weighted index may explain why the literature has tended to overlook momentum. The value-weighted index is the natural choice in strategic asset allocation studies, and it would be surprising to find that continuation affects the allocation to stocks of an investor having a 20-year horizon. However, continuation may play a role in the decisions of the 1 to 5-year investor, horizons whose practical relevance we have discussed above. Actually, this paper shows that even the small continuation exhibited by the CRSP value-weighted index has a noticeable effect on the optimal allocation to stocks of a short to medium-term investor.

The emphasis on return continuation also relates our paper to the momentum literature pioneered by Jegadeesh and Titman (1993). In this literature, profits of such momentum strategies can be generated along three lines: differences in (unconditional) expected returns (see Conrad and Kaul (1998)), positive autocorrelation in winning and losing portfolio returns (see Moskowitz and Grinblatt (1999)), or cross-serial correlations (see Lewellen (2002)). Given the fact that in our economy there is just a single stock index, we can only accommodate the second source of momentum profits. However, our model can be extended easily to the construction of optimal portfolios exploiting positive autocorrelation in momentum portfolios, taking at the same time mean-reversion into account.[5]

This paper provides two main contributions to the dynamic asset allocation literature. First, we introduce an intuitive and parsimonious continuous time model that accommodates the joint presence of momentum and mean-reversion. In this model expected index returns are governed by two state variables, a weighted average of past returns and the dividend yield. The model manages to incorporate the history of stock returns as a state variable while keeping the tractability of a Markovian framework. The model nests the well-known models of Campbell and Viceira (1999) and Wachter (2002), which therefore constitute a natural benchmark for our work. We consider these restricted versions to rigorously study the incremental effect of momentum on strategic asset allocation. Second, we explicitly focus on the case of an investor with an intermediate investment horizon, contributing in this way to bridge the gap between the long-term asset allocation literature that emphasizes the economic value of return reversal, and the tactical return continuation or momentum

[5]The model developed in this paper extends to multiple indices and can therefore be used to analyze optimal momentum strategies in the presence of mean-reversion as opposed to naively constructed winner and loser portfolios, see Koijen, Rodriguez, and Sbuelz (2006).

literature.

We calibrate our model to the CRSP value-weighted and equally-weighted indices[6] as they provide two polar cases for our study: one in which continuation is weak relative to reversal (the value-weighted index), and one in which continuation is stronger (the equally-weighted index).[7] We find, for both indices, that hedging demands are negative for short to medium-term investors. This happens because at short horizons momentum is the dominating force, and investors hedge the increased risk by reducing their market exposures. As a consequence of these negative hedging demands, we find that the total allocation to stocks does not increase monotonically with the investor's horizon: it starts at the myopic level, then goes down, reaches a minimum that depends on the index to which the model is calibrated, and finally starts going up as mean-reversion becomes dominating. The minimum allocation to stocks, which is lower than the myopic allocation, occurs at about 6 and 18 months when the model is calibrated to the value-weighted and the equally-weighted indices, respectively. These results imply that there exists a positive investment horizon (about one year for the value-weighted index and 5 years for the equally-weighted index) at which the investor behaves as she were myopic. Finally, we find that the value of hedging time-variation in investment opportunities is substantially higher in the presence of return continuation, although the pattern of these costs, as a function of time, depends strongly on the index. Interestingly, these costs are reduced, but do not vanish completely, when we discretize the model and impose trading frequency, short-sales and borrowing constraints on the investor.

This paper proceeds as follows. In Section 2, we introduce a new model of stock returns, in which the equity risk premium is driven by both a weighted average of past returns and a fundamental variable. In Section 3, the optimal portfolio for a CRRA investor who derives utility from terminal wealth is derived analytically. Section 4 calibrates the model of Section 2, and a restricted version, to the data. In Section 5, we determine the optimal investment strategies and the utility losses from suboptimal strategies. Section 6 finally concludes.

[6]There is evidence that investors actually implement return continuation strategies at an index level, see for instance Amin, Coval, and Seyhun (2004). Moreover, Balvers and Wu (2006) find that a combination of momentum-contrarian strategies, used to select from among several developed stock markets at a monthly frequency, outperform both pure momentum and pure contrarian strategies.

[7]Continuation varies widely across indices. The CRSP value and equally-weighted indices that we use in this paper have a first monthly autocorrelation of 6 and 20%, respectively (post II WW period). Although we only calibrate our model to the two CRSP indices, it is interesting to note that other broad indices have first autocorrelations that are in the middle of these two. For example, the MSCI Europe index has a first monthly autocorrelation of 10.32% for the period 1970-2005.

2 Financial market

In this section we introduce an intuitive and parsimonious model of stock returns that accommodates both return continuation over short horizons and return reversals over longer horizons. The model nests the works of Campbell and Viceira (1999) and Wachter (2002), which only accommodate mean-reversion in stock returns.

Momentum over short horizons implies that the recent development of the stock price has predictive power for future returns. To capture this idea, we introduce a state variable that summarizes the past performance of the stock. Mean-reversion in stock returns is usually modeled via persistent financial ratios as the dividend yield or the price earnings ratio predicting future stock returns (see Campbell and Viceira (1999) and Campbell et al. (2003)). In order to allow for both momentum and mean-reversion, we assume that the investor predicts the next return using a weighted average of fundamental and performance information.

Define S_t as the price level of a stock, or index, at time t. We then characterize the dynamics of stock returns as:

$$\frac{\mathrm{d}S_t}{S_t} = (\phi M_t + (1-\phi)\mu_t)\,\mathrm{d}t + \sigma_S' \mathrm{d}Z_t,\ 0 \leq \phi < 1, \tag{2}$$

with σ_S a two-dimensional volatility vector and Z_t a two-dimensional vector of independent Brownian motions. All correlations between the various processes will be captured by the volatility vectors. Note that the instantaneous expected return in (2) depends on two state variables, μ_t and M_t, which we discuss now in detail.

We construct the variable M_t in such a way that it reflects the past performance of the index and term it the *performance variable.* More precisely, we define M_t as a weighted sum of past returns:

$$M_t = \int_0^t e^{-(t-u)} \frac{\mathrm{d}S_u}{S_u}, \tag{3}$$

where $e^{-(t-u)}$ is the weighting scheme.[8] The dynamics of the performance variable follows from the dynamics of stock returns. To see this, totally differentiate (3) with respect to time

[8] We can in fact allow for more general weighting schemes. The empirical section illustrates that there is no need to consider more general structures since this model is capable to fit the short-term and long-term autocorrelations exactly.

to obtain:

$$\mathrm{d}M_t = \frac{\mathrm{d}S_t}{S_t} - M_t \mathrm{d}t, \tag{4}$$

and replace equation (2) in (4). This results in the following dynamics for the performance variable:

$$\mathrm{d}M_t = (1-\phi)\left(\mu_t - M_t\right)\mathrm{d}t + \sigma_S' \mathrm{d}Z_t. \tag{5}$$

It follows from (5) that the performance variable fluctuates around a stochastic mean, μ_t. We interpret μ_t as the expected return on the stock based only on fundamental information, that is, on information that is not related to the stock's past performance. We indicate the variable μ_t therefore as the *fundamental variable*. Furthermore, we assume that μ_t is a stationary variable, following an Ornstein-Uhlenbeck process:

$$\mathrm{d}\mu_t = \alpha\left(\mu_0 - \mu_t\right)\mathrm{d}t + \sigma_\mu' \mathrm{d}Z_t,\ \alpha > 0, \tag{6}$$

where μ_0 is the (constant) long-run expected rate of return, α is the rate at which μ_t converges to μ_0, and σ_μ a two-dimensional vector of instantaneous volatilities. From equation (2) the (instantaneous) expected return can be recasted as:

$$\mathbb{E}_t\left(\frac{\mathrm{d}S_t}{S_t}\right) = \left(\mu_t + \phi\left(M_t - \mu_t\right)\right)\mathrm{d}t. \tag{7}$$

This representation admits the following interpretation. If $\phi > 0$ and the performance of the stock has in the recent past been above (below) the current expected return conditional on fundamental information, the investor predicts a return that is above (below) μ_t. In contrast, $\phi = 0$ means that past performance has no predictive power for future stock returns. In this case we recover the classical continuous time models of drift-based predictability in which the stochastic mean μ_t contains the fundamental component needed to forecast returns.[9] Along these lines, the model admits both momentum and mean-reversion and nests the return models that have become the workhorse in the strategic asset allocation literature.

Apart from the stock (index), the investor's asset menu also contains a riskless cash account whose price at time t is indicated by B_t. The dynamics of the cash account is given by:

$$\frac{\mathrm{d}B_t}{B_t} = r\mathrm{d}t, \tag{8}$$

[9] Consider among others Campbell and Viceira (1999) and Wachter (2002).

with r the constant riskless interest rate.

Finally, note that although M_t depends on the whole history of the stock, the triplet (S_t, M_t, μ_t) forms a three-dimensional Markov process.

3 Optimal strategic asset allocation

In this section we solve the strategic asset allocation faced by a long-term investor who derives utility from terminal wealth.[10] We assume that the preferences of the investor can be represented by a CRRA utility index with a constant coefficient of relative risk aversion equal to γ. The investor dynamically allocates the capital available, W_t, to stocks and the cash account. Hence, the investment problem for an investor with an investment horizon equal to $T-t$ can be formalized as:

$$J(W, M, \mu, t, T) = \max_{(\pi_s)_{s\in[t,T]}} \mathbb{E}_t \left(\frac{1}{1-\gamma} W_T^{1-\gamma} \right), \tag{9}$$

subject to the dynamic budget constraint

$$\frac{\mathrm{d}W_t}{W_t} = \left(\pi_t \left(\mu_t + \phi \left(M_t - \mu_t\right) - r\right) + r\right) \mathrm{d}t + \pi_t \sigma_S' \mathrm{d}Z_t, \tag{10}$$

in which π_t denotes the fraction of wealth allocated to stocks at time t and $J(W, M, \mu, t, T)$ the value function corresponding to the optimal investment strategy. The problem is solved by means of dynamic programming and we refer to Appendix A for details on the derivation. The next proposition provides the main result for the value function, J, and the optimal dynamic strategic allocation.

Proposition 1 *For an investor with an investment horizon $\tau = T - t$ and constant coefficient of relative risk aversion γ, the value function is of the form:*

$$\begin{aligned} J(W, M, \mu, t, T) &= \frac{1}{1-\gamma} W^{1-\gamma} H(Y, \tau), \\ H(Y, \tau) &= \exp\left(\frac{1}{2} Y' A(\tau) Y + B(\tau)' Y + C(\tau)\right), \end{aligned}$$

with $Y = (M, \mu)'$, $A \in \mathbb{R}^{2\times 2}$, $B \in \mathbb{R}^{2\times 1}$, and $C \in \mathbb{R}$. A, B, and C are given in Appendix A as the solution to a set of ordinary differential equations. The optimal strategic allocation to

[10] Since our financial market model is (dynamically) incomplete, the investment problem with intermediate consumption is non-trivially harder than the terminal wealth problem, see Wachter (2002) and Liu (2006).

stocks is given by:

$$\pi_t^\star = \frac{1}{\gamma}\frac{\mu_0 - r}{\sigma_S'\sigma_S} + \frac{1}{\gamma}\frac{\mu_t - \mu_0}{\sigma_S'\sigma_S} \quad + \quad \frac{\phi}{\gamma}\frac{(M_t - \mu_t)}{\sigma_S'\sigma_S} + \ldots$$
$$\frac{1}{\gamma\sigma_S'\sigma_S}\sigma_S'\Sigma'\left(\frac{1}{2}\left(A(\tau) + A(\tau)'\right)Y + B(\tau)\right) \tag{11}$$

with $\Sigma = (\sigma_S, \sigma_\mu)' \in \mathbb{R}^{2\times 2}$. The remainder, $1 - \pi_t^\star$, is invested in the cash account.

This proposition states that the optimal fraction invested in stocks contains four components. The first component is the unconditional myopic demand for stocks. That is, when the performance and fundamental variable equal their unconditional expectations, this first component denotes the investment in equities when the investor disregards future changes in investment opportunities. The second and third components are timing portfolios that exploit the information contained in the fundamental and performance variables. Fundamentals above their unconditional expectations always lead to an increase in the allocation to equity (the second component). Regarding the third component, we find that when past performance (M_t) has been strong relative to μ_t, return continuation induces the investor to increase the allocation to equities. As the investor anticipates that changes in the state variables have an impact on future investment opportunities, the optimal strategic allocation contains hedging demands to hedge adverse changes in future investment opportunities. The final component measures the impact of shocks to the performance and the fundamental variable on the value function. The more sensitive the value function to changes in the performance or the fundamental variable, the larger the induced hedging demands will be.

4 Data and estimation

In this section, we assess empirically the impact of the joint presence of momentum and mean-reversion on strategic asset allocation using the model of Section 2. As argued in the introduction, we focus on investors with a realistic intermediate investment horizon up to five years. In order to highlight the role of momentum, we also estimate a restricted version of our model that allows only for mean-reversion, i.e. $\phi = 0$. This restricted version, in which the equity risk premium is only driven by fundamental information, has been studied extensively in the literature (see e.g. Campbell and Viceira (1999) and Wachter (2002)) and constitutes therefore a natural benchmark. We discuss the estimation procedure, the data used in estimation, and the resulting estimates of our model and of the restricted version.

In order to estimate the financial market model of Section 2, we require proxies for both the fundamental mean, μ_t, and the performance variable, M_t. In line with the literature mentioned above, the fundamental mean is modeled affine in the (log) dividend yield, i.e.:[11]

$$\mu_t = \mu_0 + \mu_1(D_t - \mu_D) = \mu_0 + \mu_1 X_t, \tag{12}$$

with D_t indicating the (log) dividend yield, $\mathbb{E}(D_t) = \mu_D$, and $X_t = D_t - \mu_D$ denoting the de-meaned dividend yield. The construction of the performance variable, M_t, is slightly more involved. The investor observes the stock's performance and uses this information to construct the weighted average of past returns. Given that stock returns are observed at discrete points in time, we approximate the integral in (3) using monthly stock returns and through a standard Euler discretization:

$$M_t \approx \sum_{i=1}^{t} e^{-i} \left(\frac{S_{t-i+1} - S_{t-i}}{S_{t-i}} \right). \tag{13}$$

To summarize, the financial market model, which acommodates both momentum and mean-reversion, is given by:

$$\frac{\mathrm{d}S_t}{S_t} = ((\mu_0 + \mu_1 X_t)(1-\phi) + \phi M_t)\,\mathrm{d}t + \sigma_S' \mathrm{d}Z_t, \tag{14}$$

$$\mathrm{d}M_t = (1-\phi)\,(\mu_0 + \mu_1 X_t - M_t)\,\mathrm{d}t + \sigma_S' \mathrm{d}Z_t, \tag{15}$$

$$\mathrm{d}X_t = -\alpha X_t \mathrm{d}t + \sigma_X' \mathrm{d}Z_t, \tag{16}$$

with $\sigma_X = \sigma_\mu / \mu_1$.

We use monthly US data over the period January 1946 up to December 2005 to estimate the model. The data is obtained from the Center for Research in Security Prices (CRSP). We use the return on both the value-weighted and equally-weighted index, including dividends, on the NYSE, NASDAQ, and AMEX markets. Lo and MacKinlay (1988) and Khil and Lee (2002) have shown that index momentum is more prominently present in the equally-weighted index, which renders a natural application of the model. To calculate the dividend yield, we first construct the dividend payout series using the value-weighted return including dividends, and the price index series associated with the value-weighted return excluding dividends. We take the dividend series to be the sum of dividend payments over the past

[11] The use of the dividend yield is not fully uncontroversial, see Ang and Bekaert (2006) and Goyal and Welch (2003, 2006). However, Lettau and van Nieuwerburgh (2006) and Boudoukh et al. (2006) have shown recently that the predictive power of the dividend yield is significantly enhanced once structural breaks are allowed or the payout yield is used, respectively.

year. The dividend yield is then the log of the ratio between the dividend and the current price index, in line with Campbell et al. (2003). We set the instantaneous short rate to $r = 4\%$ in annual terms.

In our model of the financial market as summarized in (14), (15), and (16), the uncertainty is driven by two independent Brownian motions. In order to ensure statistical identification of the volatility matrix, we normalize:

$$\begin{pmatrix} \sigma_S' \\ \sigma_X' \end{pmatrix} \tag{17}$$

to be lower triangular. Next, we discretize the continuous time model exactly at a monthly frequency, which results in a tri-variate Gaussian VAR-model for log returns, the performance variable, and the dividend yield, which are all observed. We employ maximum likelihood to estimate the structural parameters of the model. However, when the model is estimated using unconstrained maximum likelihood, we find that the implied autocorrelations fit the sample autocorrelations poorly. We therefore optimize the likelihood under constraints on the autocorrelation function. More specifically, we impose that the model-implied autocorrelations fit the first order sample autocorrelation of one month and 24 months returns exactly. For the value-weighted index, the first order autocorrelation of one month returns equals 6.2% and of 24 months returns (using non-overlapping returns) −6.9%. For the equally-weighted index, on the contrary, these numbers change to 19.9% and −7.7%, respectively. As such, the estimation method employed can be viewed as a GMM procedure in which both the efficient moments and economically motivated moments are combined. Finally, it is well-known that estimates of the unconditional expectation of persistent series, like the dividend yield, are inaccurate. Therefore, we de-mean the dividend yield using its sample average and use the de-meaned predictor variable, X_t, in estimation.

We are mainly interested in the interaction between momentum and mean-reversion in the strategic asset allocation problem of an investor who has a short to medium-term investment horizon. To draw a comparison with a model that only allows fundamental information to predict the equity risk premium, we also estimate the model under the condition that $\phi = 0$. Since this model is, by construction, not able to accommodate both positive and negative autocorrelations in returns, we only require the long-term autocorrelation to match its sample counterpart. By re-estimating the model, we consider an investor who, potentially erroneously, disregards the information contained in the performance variable.

The model estimates in monthly terms are portrayed in Table 1. Panel A portrays the

Panel A: Estimation results

	Momentum and mean-reversion		Mean-reversion only	
	VW	EW	VW	EW
	$\frac{\mathrm{d}S_t}{S_t} = (\phi M_t + (1-\phi)\mu_t)\,\mathrm{d}t + \sigma'_S \mathrm{d}Z_t,\ \mu_t = \mu_0 + \mu_1 X_t$			
ϕ	0.15	0.39	0	0
μ_0	0.92%	1.15%	0.94%	1.20%
μ_1	0.016	0.012	0.015	0.0059
$\sigma_{S(1)}$	5.24%	6.28%	5.37%	6.68%
	$\mathrm{d}X_t = -\alpha X_t \mathrm{d}t + \sigma'_X \mathrm{d}Z_t$			
α	0.011	0.0094	0.010	0.0076
$\sigma_{X(1)}$	-5.77%	-8.43%	-5.34%	-6.66%
$\sigma_{X(2)}$	1.35%	1.52%	1.35%	1.45%

Panel B: Autocorrelations

	Momentum and mean-reversion		Mean-reversion only	
	VW	EW	VW	EW
1 month				
Sample	6.2%	19.8%	6.2%	19.8%
Model implied	6.2%	19.8%	-0.3%	-0.4%
24 months				
Sample	-6.9%	-7.7%	-6.9%	-7.7%
Model implied	-6.9%	-7.7%	-6.9%	-7.7%

Table 1: Estimates of the financial market model in Section 2.
We use data over the period January 1946 up to December 2005 to estimate the financial market model in Section 2 ('Momentum and mean-reversion'). Panel A provides the estimates and Panel B portrays both the sample and model implied first order autocorrelations of one month and 24 months returns. The column with the heading 'Mean-reversion only' presents the estimates of a restricted version of the model in which only fundamental information affects the equity risk premium, i.e. $\phi = 0$. The model is estimated by means of maximum likelihood with constraints on the autocorrelation function. The abbreviation 'VW' refers to the value-weighted index and likewise 'EW' to the equally-weighted index. The main text provides further details.

estimation results for the model of Section 2 ('Momentum and mean-reversion') and the restricted model in which only fundamental information predicts the equity risk premium ('Mean-reversion only'), i.e. $\phi = 0$. Panel B compares the sample estimates of the one month and 24 months returns first order autocorrelation with the model implied estimates. The results are presented for both the value-weighted index ('VW') and the equally-weighted index ('EW'). For the stock price process, two results are worth mentioning. First, the parameter ϕ, which weighs the relative importance of fundamental and performance information in predicting equity returns, is almost three times higher in the equally-weighted than in the value-weighted index. This is consistent with the predictive regressions presented in the introduction: while past performance is a key driver of expected returns in the equally-weighted index, fundamental information plays a more prominent role in the value-weighted index. Second, the instantaneous volatility of stock returns, $\sigma_{S(1)}$, is lower in the case of the full model as opposed to the restricted model. This is a consequence of the enhanced predictive power of incorporating past performance information. Since the expected returns (μ_0) are comparable, this implies that the myopic allocation to stocks will be higher in the model that accommodates both momentum and mean-reversion. The estimates for the dividend yield reveal the well-documented features of financial ratios, namely high persistence and high (negative) correlation with stock return innovations.

Panel B illustrates that the model that accommodates both momentum and mean-reversion is able to fit two autocorrelations with different signs excellently. The restricted model, which has only one state variable, can produce autocorrelations that are either negative or positive, but not both. This potential source of misspecification turns out to be particularly serious in the estimation based on the equally-weighted index.

5 Empirical results

5.1 Optimal strategic allocation

In this section we study the implications of the estimates reported in Table 1 for strategic asset allocation. We first analyze the optimal investment strategy of an investor who accommodates both momentum and mean-reversion, and compare the results of this analysis to the case of an investor who restricts her attention to mean-reversion only. Next, we determine the economic loss, measured in utility terms, of not incorporating performance information in the investment policy. Finally, we assess the utility loss of behaving myopically with respect to momentum and mean-reversion.

The optimal allocations to stocks are portrayed in Figure 1 for an investor with a coefficient of relative risk aversion of $\gamma = 5$. The left panels provide the results for the value-weighted index and the right panels for the equally-weighted index. To emphasize the case of the medium term investor, the two top panels focus on investment horizons of up to five years. The two bottom panels extend the results to investment horizons of up to 15 years, i.e. the long-term investment problem that has been studied rigorously in the literature.

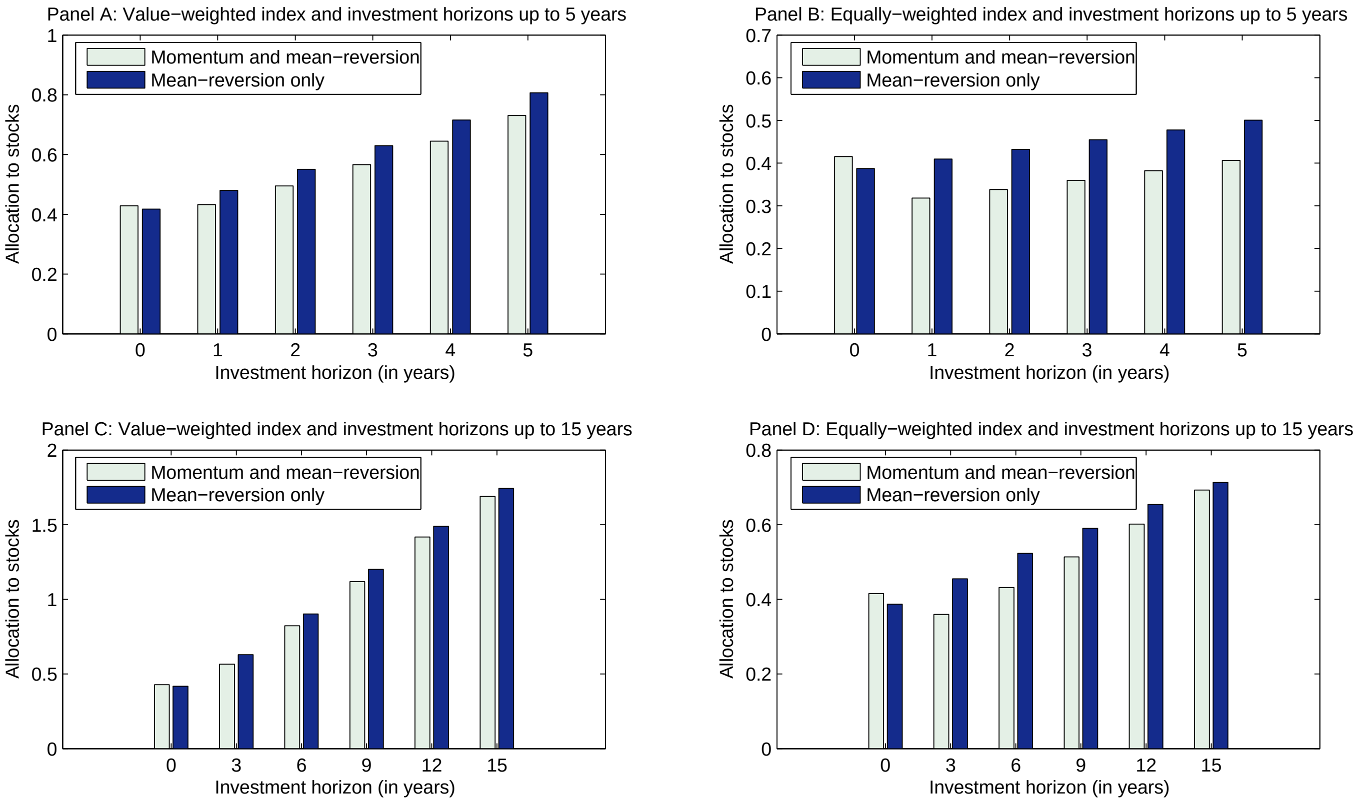

Figure 1: Optimal strategic allocation to stocks for different investment horizons. Panel A and B consider investment horizons up to 5 years, whereas Panel C and D investment horizons up to 15 years. Panel A and C portray the results for the value-weighted index, whereas Panel B and D are based on the equally-weighted index. In all figures, the light blue bars correspond to the optimal strategy which incorporates momentum and mean-reversion. The dark blue bars, on the contrary, only incorporate mean-reversion. The investor's coefficient of relative risk aversion equals $\gamma = 5$. In all panels, the horizontal axis indicates the investment horizon and the vertical axis the optimal fraction of wealth allocated to stocks.

Four aspects of Figure 1 are worth highlighting. First, the initial allocation to stocks, i.e. the myopic allocation, is always higher for the investor who uses performance information. This is because the enhanced predictive power sharpens the forecast of future returns, thereby reducing the conditional volatility. Second, investors who acknowledge the strategic nature of the investment problem hold significant hedging demands. The hedging demand of a long-term investor is positive and increases with the investment horizon. This phenomenon is due to long-term mean reversion in stock returns, and it has been widely documented in the literature. As a result of mean-reversion, long-term equity returns are less risky which induces an increase in the optimal allocation. On the other hand, the hedging demand of a short to medium-term investor is greatly affected by return continuation. Once the model is calibrated to the equally-weighted index, the hedging demand is negative for an investor with an investment horizon of up to six years. Moreover, the hedging demand is also negative for an investor with an investment horizon of up to one year once the model is calibrated to the value-weighted index, even when the evidence of return continuation in this index is rather weak. Momentum leads to an increase of the riskiness of short- to medium-term stock returns. This induces the investor to decrease the optimal allocation to stocks. These results suggest that ignoring return continuation may have nontrivial consequences for the investment policies of medium-term investors. As an example, consider in panel B the investor who has a 5-year investment horizon and who accounts for both momentum and mean-reversion. This investor allocates slightly less than 40% to stocks, while the investor that ignores the presence of return continuation allocates more than 50% to stocks, a relative increase of 25%.

Third, it turns out that at a long horizon of, say, 15 years, the differences between the two investment strategies are negligible. This, together with the negativity of the hedging demand at medium-term horizons discussed above, implies that the total allocation to stocks is not monotonic in the investment horizon. A striking consequence of the non-monotonicity of the total allocation to stocks is that there exists a positive investment horizon at which the hedging demands to hedge momentum and mean-reversion cancel out exactly, and so the investor behaves as if she were myopic. This investment horizon is about 5 years for the equally-weighted index, and about one year for the value-weighted index. These results are a direct consequence of the interplay between continuation and reversal, and they have not been explored in the literature so far.

Fourth, as suggested already by the predictive regressions presented in the introduction, we find that time-variation in expected returns of the value-weighted index is predominantly driven by changes in the fundamental variable, in our case the dividend yield. In contrast, for the equally-weighted index it is mainly performance information what matters for investors with investment horizons up to five years.

To highlight the different impact of momentum and mean-reversion on the strategic allocation, we decompose the total hedging demands into two components for the model, which allows for predictability by the performance and fundamental variable. The total hedging demands are given by:

$$\frac{1}{\gamma}\left(\sigma_S'\sigma_S\right)^{-1}\sigma_S'\Sigma' J_{WY}, \tag{18}$$

with J_{WY} the partial derivative with respect to wealth W, and the state variables $Y = (M, \mu)$. The hedging demand induced by momentum is therefore given by:

$$\frac{1}{\gamma}\left(\sigma_S'\sigma_S\right)^{-1}\sigma_S'\sigma_S J_{WM} = \frac{1}{\gamma} J_{WM}, \tag{19}$$

and likewise for mean-reversion by:

$$\frac{1}{\gamma}\left(\sigma_S'\sigma_S\right)^{-1}\sigma_S'\sigma_\mu J_{W\mu}. \tag{20}$$

Figure 2 portrays this decomposition of the total hedging demand for the same horizons and indices as in Figure 1. Note that, by definition, both components sum exactly to the total hedging demand. In all cases, the hedging demand of a myopic investor equals by default zero. For longer investment horizons, investors optimally hold significant hedging demands. Once calibrated to the value-weighted index, the hedging demand to hedge mean-reversion dominates the hedging demand induced by momentum after an investment horizon of one year. This is caused by the high persistence of the fundamental variable, and also by the relatively minor importance of the performance variable. In case of calibration to the equally-weighted index, we find that the hedging demand induced by momentum exceeds the one induced by mean-reversion, and the total demand is therefore negative. At an investment horizon of 5 years, the two hedging demands approximately cancel and the investor behaves myopically. For longer investment horizons, hedging time-variation in investment opportunities caused by the fundamental variable becomes more important, and the total hedging demand is positive.

5.2 Economic costs of sub-optimal strategic allocations

In this section we assess the economic importance of i) taking into account momentum for strategic asset allocation and ii) acting strategically as opposed to myopically. We construct various sub-optimal strategies that shed light on the aspects of the optimal strategic allocation that matter to investors.

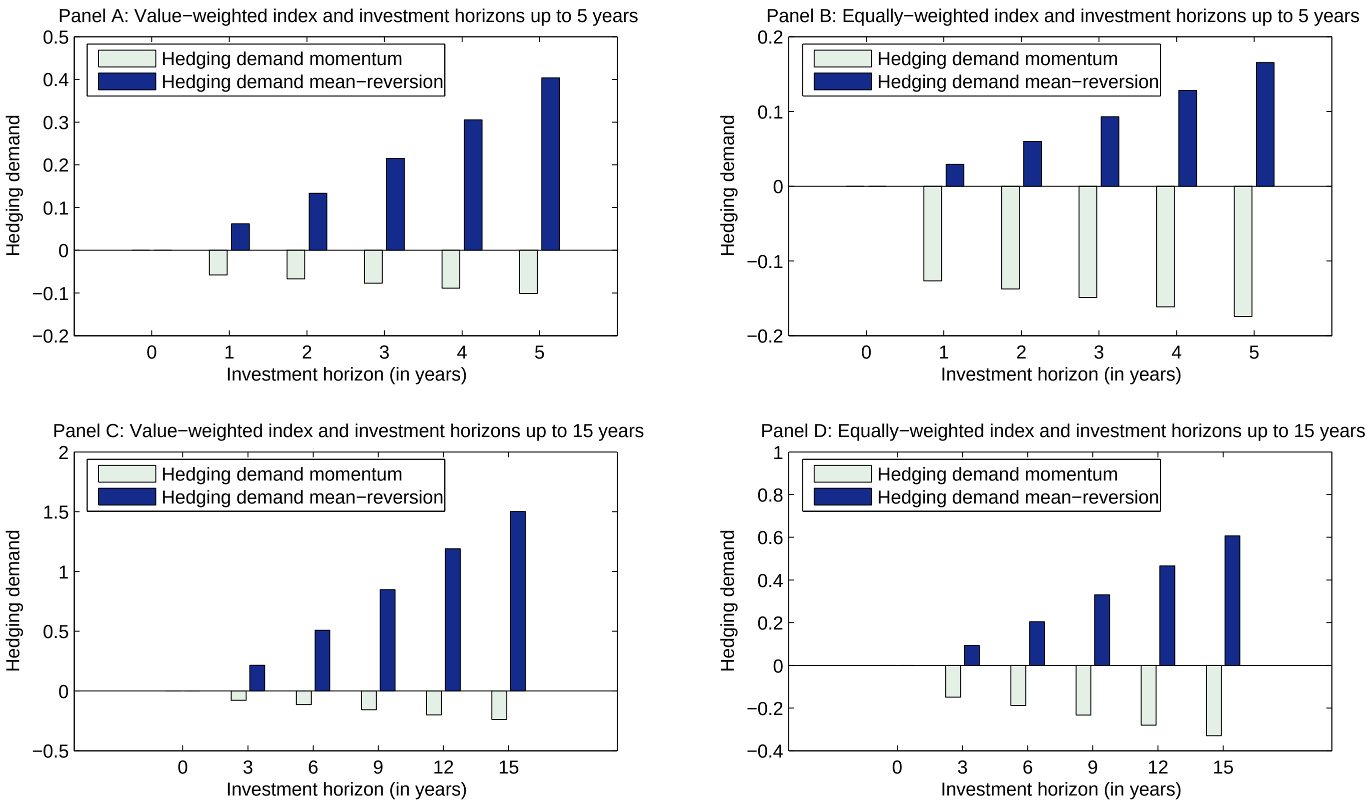

Figure 2: Decomposition of the total hedging demands.
Panel A and B consider investment horizons up to 5 years, whereas Panel C and D investment horizons up to 15 years. Panel A and C portray the results for the value-weighted index, whereas Panel B and D are based on the equally-weighted index. In all figures, the light blue bars correspond to the hedging demands induced by momentum. The dark blue bars, on the contrary, correspond to the hedging demands due to mean-reversion. The investor's coefficient of relative risk aversion equals $\gamma = 5$. In all panels, the horizontal axis indicates the investment horizon and the vertical axis the optimal fraction of wealth allocated to stocks.

We first address the question of how important momentum is, especially for investors with investment horizons up to five years. We therefore evaluate the strategy resulting from the model that ignores momentum, i.e. $\phi = 0$, in the unrestricted model. Recall that the restricted model and the unrestricted model are estimated separately, although the same data and estimation procedures are used in both cases. By estimating a separate model under the constraint, we mimic the behavior of an investor who considers momentum to be of minor importance for her strategic allocation. For both the optimal and sub-optimal investment strategies we determine the certainty equivalent return, and report the annualized loss in certainty equivalent wealth from following a sub-optimal strategic allocation. More specifically, if we denote the value function resulting from the optimal strategy by J_1 and from the sub-optimal by J_2, the annualized utility costs are given by:

$$\phi = \left(\frac{J_2}{J_1}\right)^{\frac{1}{T(1-\gamma)}} - 1, \tag{21}$$

with T the investment horizon expressed in years. The utility costs are portrayed in Figure 3.

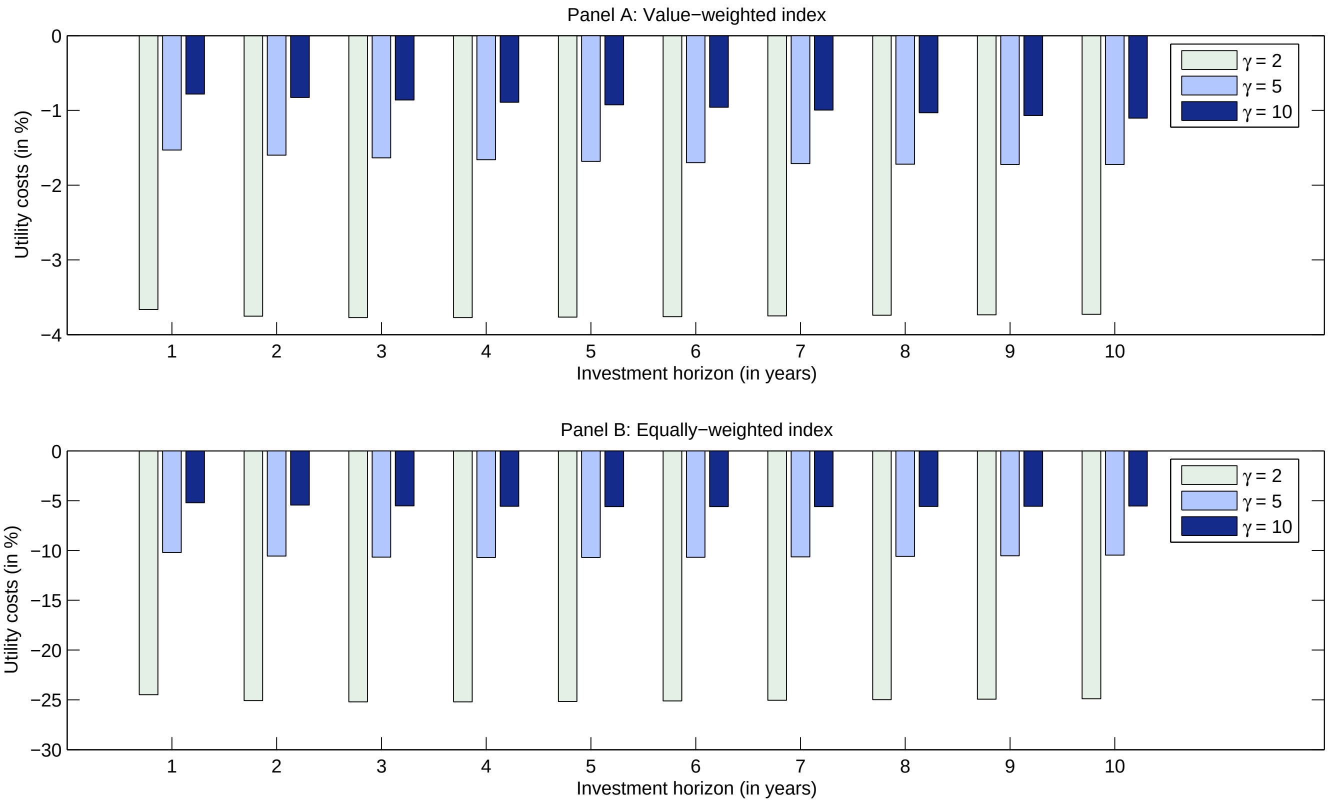

Figure 3: Annualized utility costs of ignoring momentum.
Panel A portrays the utility costs of ignoring momentum for the value-weighted index and Panel B for the equally-weighted index. In order to determine the utility costs, a model which imposes the constraint $\phi = 0$ is estimated. The horizontal axis depicts the investment horizon and the vertical axis the utility costs in percent per year. The results are portrayed for three levels of risk aversion, namely $\gamma = 2, 5, 10$. The main text provides further details.

Panel A portrays the utility costs for the value-weighted index and Panel B for the equally-weighted index. We find that the utility costs of ignoring momentum are substantial for both indices. This is remarkable, given that the evidence of return continuation in the value-weighted index is rather weak. Consider for instance an investor with a coefficient of relative risk aversion of $\gamma = 5$. This investor is willing to sacrifice 1.5% per year to be able to implement a momentum strategy on the basis of the value-weighted index and almost 10% in the case of the equally-weighted index, for which past returns have strong predictive power. Note that the annualized utility costs of ignoring momentum are invariant to the investment horizon. Despite the strong support for the importance of momentum in strategic problems, these numbers are to be interpreted with some caution. Once the investor incorporates performance information into the investment strategy, the investment policy becomes substantially more volatile. Even small degrees of drift predictability will be maximally exploited in our continuous time model. In practice, however, investors have to trade at discrete intervals and incur in transaction costs once implementing the strategy. Therefore, we will perform the same calculations at the end of this section for an investor who trades in discrete time and is subject to borrowing and short-sales constraints.

A large part of the utility costs described above arises from the improved tactical opportunities offered by return predictability via past performance. In what follows we also study the strategic aspects of momentum and mean-reversion. Towards this end, we compare a myopic strategy with the optimal strategy, which accounts for time-variation in investment opportunities. Both strategies are evaluated using the estimated parameters of our general model. The results are depicted in Figure 4.

Panel A corresponds again to the value-weighted index and Panel B to the equally-weighted index. The results for the two indices are dramatically different, which reveals the fundamental difference between momentum and mean-reversion from a strategic perspective. In the case of the value-weighted index, fundamental information is what matters most in capturing the time-variation of expected returns. When the model is calibrated to this index, the utility costs of ignoring the strategic aspects of the investment problem are gradually increasing in the investment horizon. Instead, what captures time-variation of investment opportunities in the equally-weighted index is mainly past performance, and mean-reversion tends to dominate only after investment horizons of 5 to 6 years. So, when the model is calibrated to the equally-weighted index, we find that the costs of acting myopically are substantially higher for short to medium-term investment horizons. Also, the utility costs are close to constant over the first 5 to 6 years, and then gradually increase. This implies that behaving strategically is also important for a stock index whose expected return is influenced mainly by past performance. Due to the low persistence of return information, the value of hedging is high even for short horizons, but remains constant afterwards. In contrast, for stock indices whose expected return is predicted by fundamental information, the value of hedging for the horizons we are interested in is rather modest. This is due to the high persistence of fundamental information (as proxied by the dividend yield) over time. For the

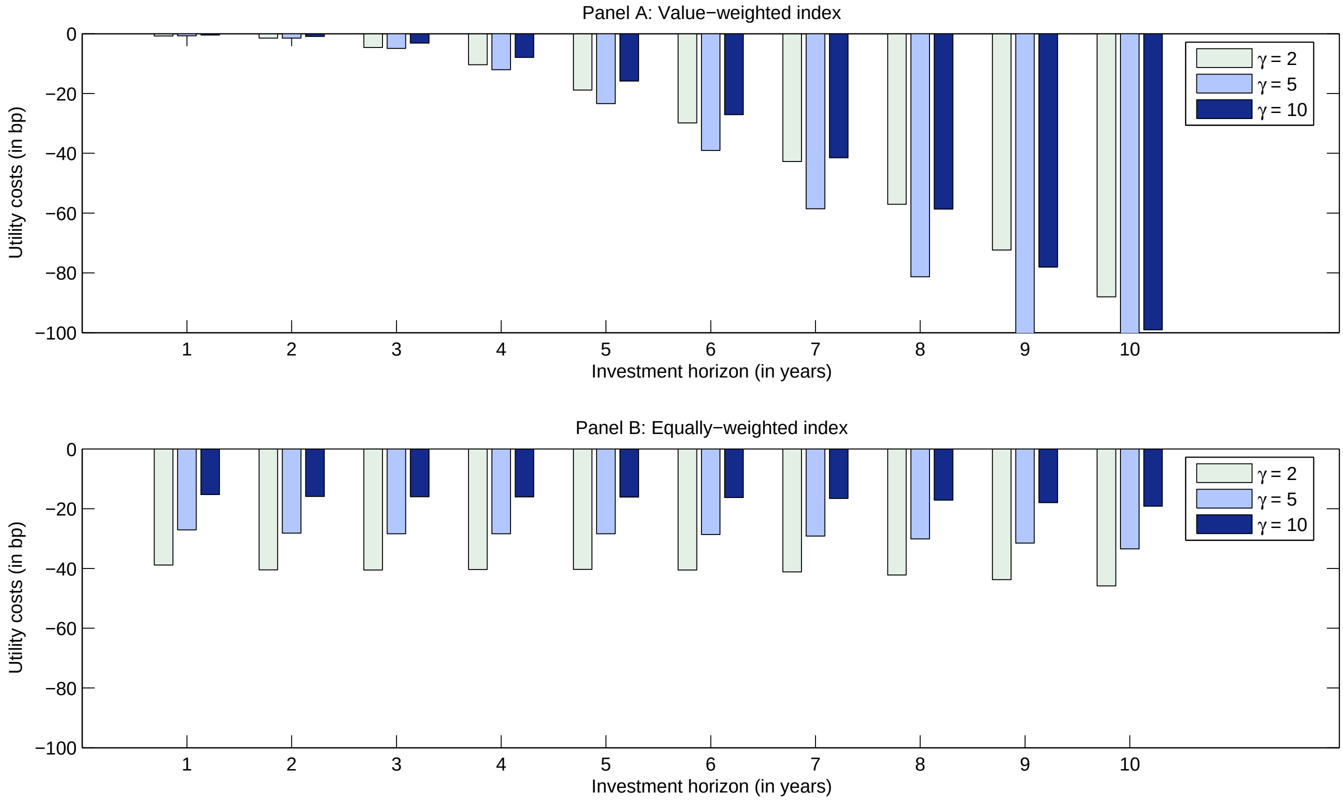

Figure 4: Annualized utility costs of behaving myopically.
Panel A portrays the utility costs of behaving myopically for the value-weighted index and Panel B for the equally-weighted index. The horizontal axis depicts the investment horizon and the vertical axis the utility costs in basis points per year. The results are portrayed for three levels of risk aversion, namely $\gamma = 2, 5, 10$. The main text provides further details.

very long-term investors, finally, like, for instance, investors saving for retirement, behaving strategically becomes increasingly important, no matter the index under consideration.

Staggering as they are, even for the model calibrated to the value-weighted index, the costs of ignoring momentum shown in Figure 3 have been derived under the assumptions that the investor can trade continuously and without any restrictions. Under these stylized assumptions the investor will fully exploit the information in the performance variable. In Table 2 we measure the costs of ignoring momentum for an investor who has been restricted to trade at higher frequencies and on whom borrowing and short-sales constraints have been imposed. The optimal investment strategy and value function have determined using the simulation-based approach developed in Brandt, Goyal, Santa-Clara, and Stroud (2005). We refer to Appendix B for further details on the numerical procedure. While less pronounced, Panel A: Value-weighted index

γ	2	4	6	8	10
Monthly	-0.3%	-0.4%	-0.4%	-0.3%	-0.3%
Quarterly	-0.1%	-0.1%	-0.1%	-0.1%	-0.1%
Annually	0.0%	0.0%	0.0%	0.0%	0.0%

Panel B: Equally-weighted index

γ	2	4	6	8	10
Monthly	-4.3%	-3.8%	-2.8%	-2.1%	-1.7%
Quarterly	-2.0%	-1.6%	-1.0%	-0.7%	-0.4%
Annually	-0.2%	-0.1%	0.0%	0.0%	0.0%

Table 2: Annualized utility costs of ignoring momentum for a constrained investor. Panel A portrays the utility costs of ignoring momentum for the value-weighted index and Panel B for the equally-weighted index. The investor is subject to borrowing and short-sales constraints and trades in discrete time. The trading frequency is either monthly, quarterly, or annually. The utility costs are presented in annualized terms. The results are portrayed for risk aversion levels $\gamma = 2, 4, 6, 8$, and 10. The main text provides further details.

the utility gains are remarkable at the monthly trading frequency for the equally-weighted index, and still noticeable for the value-weighted index. An investor with a risk aversion coefficient equal to 2, and trading at this frequency on the equally-weighted (value-weighted) index, is willing to give up 4.5% (30bp) per year of her initial wealth to exploit the information contained in the performance variable. As expected, the gains drop rapidly as the trading frequency decreases. The gains at the quarterly frequency are 2.1% (equally-weighted) and 10bp (value-weighted), and they vanish at the yearly frequency. Thus, while confirming the findings of Campbell, Chan, and Viceira (2003) at quarterly and annual levels, we ascribe them to the trading frequency used. On a monthly basis, we find that investors can benefit from exploiting the information contained in the performance variable.

6 Conclusions

We develop a new model that is able to capture the stylized pattern of autocorrelations in stock index returns, and derive the optimal strategic asset allocation, focusing on the practically relevant case of an investment horizons of up to 5 years.

We calibrate the model to two global US indices (the CRSP value- and equally-weighted indices) and draw three main conclusions. First, once we account for momentum and mean-reversion as opposed to only mean-reversion, the optimal allocation to equities is no longer monotonic in the investment horizon: it first decreases, as momentum tends to make stocks riskier, and subsequently increases, for mean-reversion leads to less risky stocks in the long-run. It takes up to five years for the equally-weighted index, and up to one year for the value-weighted index, before the strategic allocation exceeds the myopic allocation. These results challenge the known advice that the allocation to equities must increase monotonically with the investor's horizon. Second, we estimate the utility costs of ignoring momentum. We find that the annualized costs are substantial and relatively insensitive to the investors horizon. Third, we estimate the costs of behaving myopically as opposed to strategically and find, when the model is calibrated to the value-weighted index, that the costs of acting myopically are gradually increasing in the investor's horizon, although they are relatively small for horizons of up to 5 years. This is in marked contrast to the case in which the model is calibrated to the equally-weighted index, which exhibits strong momentum effects. In this case, the value of hedging is substantial at short investment horizons, but remains constant up to 5 to 6 years. At this point, mean-reversion becomes dominating, and the value of hedging starts gradually to increase. We show that the utility gains from conditioning the investment strategy on the performance variable persist if the investor is subject to borrowing and short-sales constraints and trades at a monthly frequency. At an annual frequency, our estimates indicate that investors cannot benefit anymore from the information contained in the performance variable.

This paper can be extended in various directions. One natural extension of our model is to assess the impact of transaction costs, discrete time trading, and parameter uncertainty. Second, we treat momentum as a time-series phenomenon, although there is evidence that cross-serial dependencies may render another source of momentum profits (see Lewellen (2002)). Koijen, Rodriguez, and Sbuelz (2006) extend the model developed in this paper to include multiple assets and allow for the possible cross-correlations. This leads to an analysis of optimal momentum strategies in the presence of mean-reversion that are likely to improve the naive strategies usually employed. Third, there is abundant empirical evidence indicating that volatilities and correlations are time-varying. Simple rolling estimates suggest that autocorrelations also shift considerably over time, which we consider to be yet another interesting avenue to explore.

References

Y. Aït-Sahalia and M.W. Brandt, 2001, Variable Selection for Portfolio Choice, *The Journal of Finance* 56, 1297-1351.

K. Amin, J. Coval, and H. Seyhun, 2004, Index Option Prices and Stock Market Momentum, *The Journal of Business* 77, 835-874.

A. Ang and G. Bekaert, 2006, Stock Return Predictability: Is it There?, forthcoming *Review of Financial Studies*.

M. Baker, 1998, Fund Managers Attitudes to Risk and Time Horizons: The Effect of Performance Benchmarking, *European Journal of Finance* 4, 257-278.

P. Balduzzi and A.W. Lynch, 1999, Transaction costs and predictability: some utility cost calculations, *Journal of Financial Economics* 52, 47-78.

R.J. Balvers and D. Mitchell, 1997, Autocorrelated returns and optimal intertemporal portfolio choice, *Management Science* 43, 1537-1551.

R.J. Balvers and Y. Wu, 2006, Momentum and Mean Reversion across National Equity Markets, *Journal of Empirical Finance* 13, 24-48.

N. Barberis, 2000, Investing for the Long Run when Returns are Predictable, *The Journal of Finance* 55, 225-264.

J. van Binsbergen, M.W. Brandt, R.S.J. Koijen, 2006, Optimal Decentralized Investment Management, *NBER Working Paper No.* 12144.

J. Boudoukh, R. Michaely, M. Richardson, and M.R. Roberts, On the Importance of Measuring Payout Yield: Implications for Empirical Asset Pricing, forthcoming *The Journal of Finance*.

M.W. Brandt, 1999, Estimating Portfolio and Consumption Choice: A Conditional Euler Equations Approach, *The Journal of Finance* 14, 1609-1645.

J.Y. Campbell, Y.L. Chan, and L.M. Viceira, 2003, A Multivariate Model of Strategic Asset Allocation, *Journal of Financial Economics* 67, 41-80.

J.Y. Campbell and S.B. Thompson, Predicting the Equity Premium Out of Sample: Can Anything Beat the Historical Average?, *NBER Working Paper No.* 11468.

J.Y. Campbell and L.M. Viceira, 1999, Consumption and Portfolio Decisions When Expected Returns are Time-Varying, *The Quarterly Journal of Economics* 114, 433-496.

J.Y. Campbell and L.M. Viceira, 2005, The Term Structure of the Risk-Return Tradeoff, *Financial Analysts Journal* 61, 34-44.

J.Y. Campbell and M. Yogo, 2005, Efficient Tests of Stock Return Predictability, forthcoming *Journal of Financial Economics*.

J. Conrad and G. Kaul, An Anatomy of Trading Strategies, 1998, *Review of Financial Studies* 11, 489-520.

E. F. Fama, and K.R. French, 1988, Permanent and Temporary Components of Stock Prices, *Journal of Political Economy* 96, 246-273.

A. Goyal and I. Welch, 2003, Predicting the Equity Premium with Dividend Ratios, *Management Science* 5, 639-654.

A. Goyal and I. Welch, 2006, A Comprehensive Look at The Empirical Performance of Equity Premium Prediction, forthcoming *Review of Financial Studies*.

N. Jegadeesh, and S. Titman, 1993, Returns to Buying Winners and Selling Losers: Implications for Stock MarketEfficiency, *The Journal of Finance* 48, 65-91.

J. Khil and B.-S. Lee, 2002, A Time-Series Model of Stock Returns with a Positive Short-Term Correlation and a Negative Long-Term Correlation, *Review of Quantitative Finance and Accounting* 18, 381-404.

R.S.J. Koijen, J.-C. Rodriguez, and A. Sbuelz, 2006, Optimal Long-Term Momentum and Mean-Reversion Strategies, *Working paper*.

M. Lettau and S. van Nieuwerburgh, 2006, Reconciling the Return Predictability Evidence, forthcoming *Review of Financial Studies*.

J. Lewellen, 2002, Momentum and autocorrelation in stock returns, *Review of Financial Studies* 15, 533-563.

J. Liu, 2006, Portfolio Choice In Stochastic Environments, forthcoming *Review of Financial Studies*.

A. W. Lo, and A. C. MacKinlay, 1988, Stock market prices do not follow random walks: Evidence from a simple specification test, *Review of Financial Studies* 1, 41-66.

F.A. Longstaff, and E.S. Schwartz, 2001, Valuing American Options by Simulation: A Simple Least-Squares Approach, *Review of Financial Studies*, 14, 113-148.

T.J. Moskowitz and M. Grinblatt, 1999, Do Industries Explain Momentum?, *The Journal of Finance* 4, 1249-1290.

J. Poterba, and L. Summers, 1988, Mean Reversion in Stock Returns: Evidence and Implications, *Journal of Financial Economics* 22, 27-60.

A. Sangvinatsos and J.A. Wachter, 2005, Does the Failure of the Expectation Hypothesis Matter for Long-Term Investors?, *The Journal of Finance* 60, 179-230.

A. Shleifer and R. Vishny, 1997, The Limits of Arbitrage, *The Journal of Finance* 52, 35-55.

J.A. Wachter, 2002, Portfolio and Consumption Decisions under Mean-Reverting Returns: An Exact Solution for Complete Markets, *Journal of Financial and Quantitative Analysis* 37, 63-92.

A Optimal and affine portfolio strategies

In this appendix, we derive the optimal strategic allocation to stocks and the cash account, and the corresponding value function. We first solve for the induced utility function for investment strategies which are affine in the state variables. Next, we show that the optimal policy constitutes a special case of this class of strategies. Using the more general result on strategies which are affine in the state variables, we can determine the utility costs of sub-optimal strategies in Section 5.2. Similar problems have been solved in Sangvinatsos and Wachter (2005) and Liu (2006).

We introduce the following reduced form notation for the dynamics of the financial market model presented in Section 2:

$$\begin{aligned} \frac{\mathrm{d}S_t}{S_t} &= (\zeta' Y_t)\,\mathrm{d}t + \sigma_S' \mathrm{d}Z_t, && \text{(A.1)} \\ \mathrm{d}Y_t &= (\kappa_0 + K Y_t)\,\mathrm{d}t + \Sigma \mathrm{d}Z_t, && \text{(A.2)} \end{aligned}$$

with:

$$\zeta = \begin{pmatrix} \phi \\ 1-\phi \end{pmatrix}, \kappa_0 = \begin{pmatrix} 0 \\ \alpha\mu_0 \end{pmatrix}, K = \begin{pmatrix} -(1-\phi) & (1-\phi) \\ 0 & -\alpha \end{pmatrix}, \tag{A.3}$$

and $\Sigma = (\sigma_S, \sigma_\mu)'$ and $Y_t = (M_t, \mu_t)$.

We next determine the induced utility function of an investment strategy which is affine in the state variables:

$$\pi(\tau, Y) = \theta_0(\tau) + \theta_1(\tau)' Y, \tag{A.4}$$

with $\tau = T - t$ the remaining investment horizon. Both the optimal investment strategy (as we show later formally) and the sub-optimal policies considered in Section 5.2 are of the form of (A.4). The corresponding wealth dynamics is given by:

$$\frac{\mathrm{d}W_t}{W_t} = (\pi(\tau, Y_t)\left(\mu_t + \phi\left(M_t - \mu_t\right) - r\right) + r)\,\mathrm{d}t + \pi(\tau, Y_t)\sigma_S' \mathrm{d}Z_t, \tag{A.5}$$

and the utility derived from such a policy is defined by:

$$J\left(W_t, Y_t, \tau\right) = \mathbb{E}_t\left(\frac{W_T^{1-\gamma}}{1-\gamma}\right) \tag{A.6}$$

The induced utility function is a conditional expectation and consequently a martingale. This implies that the drift of the value function satisfies the partial differential equation (PDE):

$$\mathcal{L}J + J_t = 0, \tag{A.7}$$

with $\mathcal{L}$ indicating the infinitesimal generator and subscripts denoting partial derivatives, in this case with respect to time.

It is well-known that the utility function induced by affine policies in an affine financial market model is exponentially quadratic in the state vector Y_t, i.e.:

$$\begin{aligned} J\left(W_t, Y_t, \tau\right) &= \frac{W_t^{1-\gamma}}{1-\gamma} H\left(Y_t, \tau\right), & \text{(A.8)} \\ H\left(Y_t, \tau\right) &\equiv \exp\left(\frac{1}{2} Y_t'(\tau) Y_t + B(\tau)' Y_t + C(\tau)\right), & \text{(A.9)} \\ H\left(Y_t, 0\right) &= 1, & \text{(A.10)} \end{aligned}$$

with $A \in \mathbb{R}^{2\times 2}$, $B \in \mathbb{R}^{2\times 1}$, $C \in \mathbb{R}$. Substitution of the induced utility function in (A.8) in the PDE in (A.7) and exploiting the affine structure of the investment strategy in (A.4), we obtain a system of ordinary differential equations (ODEs) for the value function coefficients:[12]

$$\begin{aligned} \dot{A}(\tau) &= 2(1-\gamma)\theta_1\zeta' - \gamma(1-\gamma)\sigma_S'\sigma_S\theta_1\theta_1' + K'\left(A(\tau) + A(\tau)'\right) \\ &\quad + (1-\gamma)\theta_1\sigma_S'\Sigma'\left(A(\tau) + A(\tau)'\right) \\ &\quad + \frac{1}{4}\left(A(\tau) + A(\tau)'\right)\Sigma\Sigma'\left(A(\tau) + A(\tau)'\right), \end{aligned} \tag{A.11}$$

$$\begin{aligned} \dot{B}(\tau)' &= (1-\gamma)\left(\theta_0\zeta' - r\theta_1'\right) - \gamma(1-\gamma)\sigma_S'\sigma_S\theta_0\theta_1' \\ &\quad + \frac{1}{2}\kappa_0'\left(A(\tau) + A(\tau)'\right) + B(\tau)'K \\ &\quad + \frac{1}{2}(1-\gamma)\theta_0\sigma_S'\Sigma'\left(A(\tau) + A(\tau)'\right) \\ &\quad + (1-\gamma)B(\tau)'\Sigma\sigma_S\theta_1' + \frac{1}{2}B(\tau)'\Sigma\Sigma'\left(A(\tau) + A(\tau)'\right), \end{aligned} \tag{A.12}$$

$$\begin{aligned} \dot{C}(\tau) &= (1-\gamma)(1-\theta_0)r - \frac{1}{2}\gamma(1-\gamma)\theta_0^2\sigma_S'\sigma_S + \kappa_0'B(\tau) + \frac{1}{2}B(\tau)'\Sigma\Sigma'B(\tau) \\ &\quad + \frac{1}{4}\operatorname{trace}\left(\left(A(\tau) + A(\tau)'\right)\Sigma\Sigma'\right) + (1-\gamma)\theta_0\sigma_S'\Sigma'B(\tau), \end{aligned} \tag{A.13}$$

[12] We drop the indices of the policy coefficients θ_0 and θ_1 for notational convenience.

together with the boundary conditions:

$$A(0) = 0_{2\times 2},\ B(0) = 0_{2\times 1},\ C(0) = 0. \tag{A.14}$$

Solving for A, B, and C restitutes the utility function J induced by the affine policy (A.4).

Next, we derive the optimal strategic allocation and show that it is affine in the state variables. The Bellman equation corresponding to the optimization problem specified in (9) subject to (10) is given by:

$$\max_{\pi}(\mathcal{L}J + J_t) = 0, \tag{A.15}$$

which leads to the following first order condition:

$$J_W W_t(\zeta' Y_t - r) + J_{WW} W_t^2 \pi^\star(\tau, Y_t)\sigma_S'\sigma_S + W_t\sigma_S'\Sigma' J_{WY} = 0. \tag{A.16}$$

We conjecture the value function to be of the exponentially quadratic form as in (A.8). This results immediately in the optimal strategic allocation given in Proposition 1 with $\theta_0^\star$ and $\theta_1^\star$ given by:

$$\theta_0^\star(\tau) = \frac{-r}{\gamma\sigma_S'\sigma_S} + \frac{\sigma_S'\Sigma' B(\tau)}{\gamma\sigma_S'\sigma_S}, \tag{A.17}$$

$$\theta_1^\star(\tau) = \frac{1}{\gamma\sigma_S'\sigma_S}\begin{pmatrix}\phi \\ 1-\phi\end{pmatrix} + \frac{1}{2\gamma\sigma_S'\sigma_S}\left(A(\tau) + A(\tau)'\right)\Sigma\sigma_S. \tag{A.18}$$

These coefficients of the optimal strategic allocation can be used in turn to determine the value function coefficients.

B Optimal portfolio choice in discrete time

We use the simulation-based approach to portfolio choice as it has been introduced by Brandt et al. (2005). We consider an investor which optimizes its portfolio subject to short-sale and borrowing constraints. The discrete time problem is then given by

$$\max_{(x_s)_{s=t}^{T-1}\in\mathcal{K}} \mathbb{E}_t\left(\frac{1}{1-\gamma}W_T^{1-\gamma}\right), \tag{B.1}$$

with W_t indicating wealth, subject to the dynamic budget constraint

$$W_{t+1} = W_t \left(x_t \frac{S_{t+1}}{S_t} + (1 - x_t) \exp(r) \right), \tag{B.2}$$

with

$$\mathcal{K} = \{x \mid x \geq 0, x^\top \iota \leq 1\}. \tag{B.3}$$

The principle of dynamic programming is used to determine the optimal portfolio strategy. Starting at time $T-1$, we first solve

$$\max_{x \in \mathcal{K}} \mathbb{E}_{T-1} \left(\frac{1}{1-\gamma} \left(x_{T-1} \frac{S_T}{S_{T-1}} + (1 - x_{T-1}) \exp(r) \right)^{1-\gamma} \right), \tag{B.4}$$

where the homogeneity of the power utility index is exploited. The main complication is that this conditional expectation cannot be calculated analytically. In line with Brandt et al. (2005) and Longstaff and Schwartz (2001), we approximate the conditional expectation via a projection on a set of basis functions in the state variables, i.e.

$$\mathbb{E}_{T-1} \left(\frac{1}{1-\gamma} \left(x_{T-1} \frac{S_T}{S_{T-1}} + (1 - x_{T-1}) \exp(r) \right)^{1-\gamma} \right) \simeq \zeta^\top f\left(Y_{T-1}\right), \tag{B.5}$$

where ζ denote the projection coefficients and $Y = (M, D)^\intercal$. In order to estimate the projection coefficients, ζ, we simulate M paths of both state variables and stock returns on the basis of the discrete time model. We indicate the paths with $(\omega_1, ..., \omega_M)$. Next, the projection coefficients are estimated via a cross-sectional regression across all paths, which results in the following estimator

$$\begin{aligned} \hat{\zeta} &= \left(\sum_{i=1}^{M} f\left(Y_{T-1}(\omega_i)\right) f\left(Y_{T-1}(\omega_i)\right)^\top \right)^{-1} \\ &\times \left(\frac{1}{1-\gamma} \sum_{i=1}^{M} f\left(Y_{T-1}(\omega_i)\right) \left(x_{T-1} \frac{S_T}{S_{T-1}}(\omega_i) + (1 - x_{T-1}) \exp(r) \right)^{1-\gamma} \right), \end{aligned} \tag{B.6}$$

where $f\left(X_{T-1}(\omega_i)\right)$ is a column vector containing the values of the basis functions, evaluated at the state variables in branch ω_i. Next, we consider a grid of possible portfolios at every branch at every point in time and solve for the optimal portfolio, with a grid space of one percent. This avoids numerical problems possibly arising when we apply the iterative approach used in Brandt et al. (2005). Along these lines, we can proceed backwards and solve for the optimal investment strategy at each point in time. This procedure results in the optimal initial strategic allocation and an estimate of the value function. This method is in particular well suited to deal with restricted information sets, since we can easily reduce the state vector. This allows us to estimate the utility gain from conditioning the investment strategy on both the fundamental and the performance variable as opposed to only the fundamental variable.