Information Linkages and Correlated Trading*

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Abstract

In a market with informationally connected traders, the dynamics of volume, price informativeness, price-volatility, and price-impacts are severely affected by the number of information linkages every trader experiences with his neighbors. We show that in the presence of information linkages among traders, volume and price informativeness increase and price-impacts lower. Moreover, we find that information linkages boost or damage the traders' profitability according to whether such linkages convey negatively or positively correlated signals. Finally, our model predicts patterns of trades correlation consistent with patterns of trades correlation identified in the empirical literature: 'neighbor' trades are positively correlated and 'distant' trades are negatively correlated.

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Introduction

One pervasive feature in financial markets is the existence of information linkages among market participants. Traders and investors are socially connected and have access to comparable sources of information. Many writers describe the financial community as one of overlapping groups of people who share similar opinions, either because they are endowed with comparable signals about fundamentals and/or communicate regularly with one another [e.g., Shiller (1984, 2005), Hertz (1998)], or simply because they are exposed to similar cultural biases [e.g., Guiso, Sapienza and Zingales (2006)]. Many information-based explanations of asset price movements hinge upon the assumption that individuals do not experience information linkages at all. In this paper we relax this assumption and explore the resulting implications along traditional dimensions - asset price volatility, liquidity, trading volume, market efficiency, and gains from informed trading.

Our notion of information linkages is tightly related to the recent empirical literature on the value of local information and social interactions in financial markets. For example, Coval and Moskowitz (1999) provide strong evidence that proximity influences investors' portfolio choices. Hong, Kubik and Stein (2005) document that US fund managers quartered in the same city exhibit similar portfolio choices. They argue that these correlated portfolio choices arise (i) through peer-to-peer communication; or (ii) simply because fund managers in a given area commit themselves to investment decisions based upon common sources of information - such as a local newspaper or TV station. Similarly, Feng and Seasholes (2004) find that in the Chinese stock market, trades are positively correlated for geographically close investors, but *negatively* correlated for geographically distant investors.

A rational explanation of these findings must necessarily rely on some pronounced heterogeneity in the investors' signals and beliefs. Thus, at the heart of our analysis is the idea that in asset markets, there are groups of traders whose signals and beliefs are more correlated with some and less correlated with other groups of traders. A general measure of (informational) distance between two traders is the amount of information these two traders share. To elevate this notion of local information sharing to a market-wide level, we consider a model with strategic traders who are differentially informed about the long-term value of an asset, but also experience information sharing through local connections. These local connections give rise to overlapping reference groups which may include only one's closest neighbors or even the entire market. Indeed, there are no obvious arguments suggesting whether information linkages should be best thought of as being local or global. Our framework is kept as general as possible to allow us to think about a wide spectrum of possibilities.

Why do we need a model to explain that traders with comparable information trade similarly? First, we wish to examine the implications of our model on traditional market variables such as volume and liquidity. For example, do information linkages among traders boost stock market volatility? What is the economic link between this volatility, volume and the existence of information linkages in the market?

Second, we are interested in the *equilibrium* patterns of trade correlations among traders. In equilibrium, every trader makes use of the information available at the information links he has with his peers; but he also knows that by trading aggressively, he would reveal part of this information to distant peers. What is the ultimate effect on the correlation between 'close' and 'distant' trades? We will explain that our model generates predictions about these patterns that are quite distinct from the predictions of theories relying on traders who 'herd' on each other.

Third, we wish to understand the profitability implications of these information linkages among traders: do asset markets with many information linkages lead to an increase in the traders' profitability? This question has important practical scopes. For example, suppose that the gains from informed trading deteriorates as the number of information linkages increases. (Below, we will explain that this is indeed the prediction of our model.) In this case, we would expect to see a small number of traders in markets with many information linkages, and vice-versa - a testable prediction.

More in detail, the predictions of our model can be streamlined quite clearly:

(i) The equilibrium price process, trading activity, return volatility, and traders' profitability are severely affected by the number of information linkages among traders.

(ii) Compared to an economy without information linkages, an economy with information linkages is characterized by higher liquidity, lower return volatility, and higher volume.

(iii) The correlation among trades is heterogeneous, both temporally and spatially. Precisely, the correlation among trades is very high at the beginning of the trading period. The same correlation decreases with the unfolding of the trading period but it does exhibit different patterns, according to the informational distance among traders. Such an additional heterogeneity features the following patterns:

  (iii.1) For traders who are sufficiently close (close neighbors, say), the correlation among opinions and trades is persistently high over the entire trading period.

  (iii.2) Traders opinions and trades diverge with their relative informational distance. Eventually, the correlation between trades is negative for relatively distant traders. A significant and persistent divergence in trade occurs even when the number of information linkages is so high to make any two traders' opinions very close at the beginning of the trading period.

(iv) Finally, information linkages can boost the gains from informed trading if they convey negatively correlated signals. If the signals available at the information linkages are positively correlated, however, the mere existence of these linkages considerably and consistently damages the traders' profitability.

The first property makes our model economically meaningful and easy to interpret. Consider, for example, its second prediction about the high volume. Its economic intuition is simple. Heterogeneity in private information is a source of monopolistic power for traders. And information linkages destroy part of this monopolistic power. As a result, every trader trades to preempt the actions of his peers, and market-wide volume increases. (Our predictions about liquidity and volatility can be understood in a similar vein.) As we explain in section 3.1, an economy *without* information linkages might lead to a similar conclusion only when the traders initial beliefs are extremely and implausibly highly.

Next, consider the third property about the pattern of correlation among trades. This pattern is the one we would like to see in light of the empirical evidence. The economic interpretation for the positive correlation for close trades is intuitive, as in economies without information linkages, the correlation among trades may only become negative eventually [see Foster and Viswanathan (1996)]. Perhaps more intriguingly, our model also matches the empirical evidence on the *negative* correlation between distant trades [see our previous discussion of Feng and Seasholes (2004) findings]. In our model, this property emerges because of the market clearing mechanism. The mechanism is as follows. Over time, the equilibrium asset price can only embody more and more information. Hence, the price reaction to unanticipated trades increases over time, which makes traders stand on opposite sides of the market, on average. This makes the correlation of trades decrease over time. In other words, the correlation of the traders' opinions is simply that part of their private signals' correlation that is not already in the aggregate order flow. This correlation tends to become negative, especially for distant traders. However, the boost in the correlation among information endowments 'kicks in' for close traders. In fact, we find that for these close traders, the information sharing effect dominates the previous negative correlation effect.

Note that a negative correlation for 'distant traders' does not necessarily emerge in the presence of traders who herd, irrespective of whether these traders herd rationally or not. Indeed, an herding behavior relies on sequential moves, and can only lead to an overall positive correlation among trades. Our model also predicts a positive correlation among close trades - a property that matches the empirical evidence and also shared by herding-based explanations (see, for example, Bikhchandani and Sharma (2001)). Naturally, the point of these remarks is not to rule out that herding behavior occurs in financial markets. Rather, our point here is that herding should not be taken as the only explanation of positively correlated trades among 'close' traders.

The fourth prediction about the gains from informed trading can be explained as follows. In our model, traders face a crucial trade-off. On the one hand, the existence of information linkages entails a loss in the traders' monopolistic power. On the other hand, these information linkages improve the quality of the traders' inference about the fundamental value of the asset they trade. If the signals available at the information linkages are positively correlated, then, the losses generated by the first effect are *always* smaller than the gains associated with the second effect. This property arises under a wide range of conditions on initial beliefs heterogeneity and the market structure - as summarized by the number of traders and batch auctions, and the initial correlation of the signals made available at the traders' initial locations. As it turns out, these results are reversed if the signals available at the information linkages are negatively correlated.[1]

The framework this paper relies on builds upon the seminal papers of Foster and Viswanathan (1996) and Back, Cao and Willard (2000), who developed a multi-traders generalization of the Kyle's (1985) model. In these two papers, every trader is endowed with one signal about the long-term value of an asset, but the correlation between any two signals is the same for all traders. Our model is built upon the same economic construct underlying these two papers. But the existence of information linkages destroys the homogeneous correlation of the traders' information endowments, and induces patterns of signals correlation varying with the traders' informational proximity. As a result, some traders may agree more with some and less with other peers. For example, our model predicts that in some cases, two traders may not be directly connected to the same information linkages, but still exhibit highly correlated information endowments. This phenomenon occurs when two traders share some information with a third trader who is sharing information with each of the initial two traders.

Finally, note that there do exist rational explanations of patterns of heterogenous trade.

[1]In the credit markets literature, Pagano and Jappelli (1993) identified conditions under which banks find it profitable to exchange information about their customers' quality. Under uncertainty about the borrower's quality, credit bureaus allow lenders to improve their knowledge about new customers, at the cost of giving up to competitors one's informational rent about existing customers. While similar trade-offs enter into our profits calculations, in our model information sharing is not necessarily the result of any information exchange activity.

Notably, Brennan and Cao (1997) considered a *rational expectations* model (not a model with *strategic* agents) in which agents' geographical distance makes them able to estimate the value of a given asset through different precisions. For example, a good piece of public news makes privately more precisely informed traders (local) react less than less precisely privately informed traders (foreign), which might lead to negative correlation between distant traders. Feng and Seasholes (2004) pointed out that the Brennan and Cao mechanism can explain their empirical findings of a negative correlation among distant trades. Our paper offers an alternative mechanism, in which patterns of heterogenous trade correlation arises even if the precision of the private signals is the same for all the traders.

Our paper is also related to the most recent theoretical literature. Stein (2006) developed a model in which competitors can find it fruitful to engage in truthful conversations when these conversations boost the quality of their initial estimates about an asset payoff. His model thus provides elegant foundations to a particular mechanism leading to the information linkages we consider in our paper. Also, in independent work, Ozsoylev (2006) developed a model in which every investor is able to observe the expectations of his neighbors. Thus, his model is very similar in spirit to our local information linkages mechanism. However, there are two important differences between these two models. First, Ozsoylev considers a rational expectations model in which investors do not have market power. The assumption of no-market power allows the author to investigate asymmetric information networks. In our model, traders do enjoy some monopolistic information power. Therefore, to avoid dimensionality issues related to forecasting the forecasts of others, we consider a symmetric network in which every trader relates to the remaining traders in the same manner - and still has patterns of signal correlation varying with the proximity of his peers. The second difference between Ozsoylev's model and ours is that our model is dynamic. For all these reasons, Ozsoylev's model and ours should be considered as being complementary.

The paper is organized as follows. In the next section, we develop the information structure of the model. In Section 2 and 3, we derive the dynamic equilibrium and in Section 3, we analyze its properties. Section 4 concludes. The appendix contains all technical details omitted in the main text.

1 Information structure

1.1 The asset market and traders' location

We consider a market for one risky asset organized in $N \geq 1$ batch auctions. This asset pays a random payoff $f \sim N(0, \sigma_{f,0}^2)$ at the end of the trading period. The crucial feature of our model is that a number of imperfectly competitive traders experience information linkages. Precisely, we assume that these traders are physically located around a circle. By convention, they are ordered clockwise, as to say that trader i has trader $i+1$ to his left and trader $i-1$ to his right (see Figure 1). For reasons developed below, we assume that there are M such traders, where M is an odd number.

A number of signals about the fundamental value f are observed at each trader's location. Let $\mathbf{s}_0 = (s_{1,0}, \cdots, s_{M,0})^\top$ be the $M \times 1$ vector of the signals available in the market. We assume that every single signal $s_{i,0}$ is available at the i-th trader's location. Moreover, the signal $s_{i,0}$ available at the i-th trader's location is also observed by his neighbors. We consider "double-sided" information linkages. That is, the signal available at any trader location is observed by G traders to the right and by G traders to the left of any given trader. For example, the i-th trader may share the signal at his location with traders $i-1$ and $i+1$ (see Figure 1). In this case, traders $i-1$, i and $i+1$ observe $s_{i,0}$; trader i observes $s_{i-1,0}$, $s_{i,0}$ and $s_{i+1,0}$, and so forth.

This network of information linkages tilts the initial geography of signals in such a way that the i-th trader information set is $\mathbf{s}_{i,0} = (s_{i-G,0}, \cdots, s_{i,0}, \cdots, s_{i+G,0})^\top$, $G \in [0, (M-1)/2]$.[2] To easy notation, we let $\hat{G} = 2G+1$ be the number of signals every trader has access to. In the absence of information linkages, we have that $\hat{G} = 1$ and, hence, $\mathbf{s}_{i,0} = s_{i,0}$ for all i. In principle, the maximum level of information linkages could be $\hat{G} = M$, in which case $\mathbf{s}_{i,0} = \mathbf{s}_0$ for all i. However, such a complete information sharing economy may fail to have a linear equilibrium as the number of auctions N gets large and the uncertainty related to liquidity trades (to be introduced later) gets small, as initially conjectured by Holden and Subrahmanyam (1992) and formally shown by Back, Cao and Willard (2000). Therefore, we shall limit ourselves to cases in which $\hat{G} < M$.

Our information structure can be interpreted in a variety of ways. For example, every signal $s_{i,0}$ can be thought of as being broadcast to the trader's i location through a local newspaper or TV station. Then, informationally linked traders are those traders who have access to the same

[2]In this paper, we shall make an abuse in notation and write $G \in A$ for $G \in A \bigcap \mathbb{N}$, where A is some set and $\mathbb{N}$ denotes the set of integers. A similar abuse in notation will occur for other objects related to G - such as the number of traders located on some specific arcs of the circle.

source of information. In the extreme case in which $\hat{G} = 1$, every trader gathers information from a unique local source of news, and there are no information linkages among traders. As $\hat{G}$ increases, these sources of news overlap across traders. In this example, the number $2G$ of information linkages every trader experiences with his peers is interpreted as the media coverage of information providers.

Another interpretation of our information structure relates to a sort of social distance among traders. Such a social distance can stand for geographical, cultural, demographic or linguistic distance. In general, it is simply the expression of differences in beliefs among traders. Accordingly, a varying degree of closeness of opinions and views among traders can be the result of repeated interactions among them. Importantly, then, our model can be understood as one that takes the architecture of this opinion formation as given, focusing instead on the resulting asset pricing implications. In a recent paper, Stein (2006) laid down the foundations for a honest exchange of ideas to arise among strategic players. Such an exchange arises due to a new mechanism, related to a complementarity in the production of ideas: for example, to produce a useful idea about an asset payoff, it is necessary to have access to the peer's previous ideas on the same topic. In our model, information sharing can be understood as the result of such fruitful conversations among close traders.

Next, we describe the distribution of the initial signals available at the traders' location. To keep the presentation as general and tractable as possible, we follow Foster and Viswanathan (1996) and assume that all the signals are jointly normally distributed with mean zero and variance-covariance matrix equal to $\Psi_0 \equiv E\left[(s_{1,0}, \cdots, s_{M,0})^\top (s_{1,0}, \cdots, s_{M,0})\right]$. The unconditional distribution of the signals is symmetric in that: (i) each signal has variance Λ_0, (ii) the covariance between any two signals is Ω_0, and (iii) the covariance between each signal and the fundamental value is c_0. The joint distribution of the vector $(f, s_{1,0}, \cdots, s_{M,0})^\top$ is given by:

$$\begin{bmatrix} f \\ \mathbf{s}_0 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \sigma_f^2 & c_0\mathbf{1}^\top \\ c_0\mathbf{1} & \Psi_0 \end{bmatrix}\right), \quad \Psi_0 \equiv \begin{bmatrix} \Lambda_0 & \Omega_0 & \cdots & \Omega_0 \\ & \Lambda_0 & & \Omega_0 \\ & & \ddots & \vdots \\ & & & \Lambda_0 \end{bmatrix}. \tag{1}$$

Let $(x_{i,n})_{n=1}^N$ be the sequence of orders submitted by the i-th trader over the trading period. These orders are submitted so as to

$$\max_{(x_{i,n})} E\left[\sum_{n=1}^N (f - p_n)\, x_{i,n} \middle| F_{i,n}\right] \equiv W_{i,n}, \tag{2}$$

where $F_{i,n}$ is the information set available to trader i at the batch auction number n. (We will provide a precise description of this information set in Section 2.1.) On top of these orders, there is a sequence of exogenous liquidity trades $(u_n)_{i=1}^{N}$, where $u_n \sim NID\left(0, \sigma_u^2\right)$ for all n. The aggregate order flow is given by:

$$y_n = \sum_{i=1}^{M} x_{i,n} + u_n, \quad n = 1, \cdots, N. \tag{3}$$

Finally, the $(M+1)$-th market participant is a market maker who commits himself to offset the order flow according to the Semi-Strong efficiency rule:

$$p_n = E\left(f | \, y_1, \cdots, y_n\right), \quad n = 1, \cdots, N. \tag{4}$$

As we shall explain in the next section, our information structure simplifies each trader's dynamic inference about other traders' signals, and allows us to focus on linear equilibria, thereby avoding dimensionality issues related to forecasting the forecasts of others. This simplification occurs for two reasons: (i) the distribution of $[f \ \mathbf{s}_0]$ in eq. (1) is still symmetric, as in Foster and Viswanathan (1996); (ii) the circular location of the traders makes the patterns of signals correlation the same for each of these traders. At the same time, a circular information structure allows one to study a market of differentially informed traders who experience heterogeneous patterns of signals' correlation.

1.2 Average signals

Let $\bar{s}_{i,0}$ denote trader i's average signal:

$$\bar{s}_{i,0} = \hat{G}^{-1} \sum_{k=-G}^{G} s_{i+k,0}. \tag{5}$$

We call $E\left(f|\mathbf{s}_0\right)$, the expectation of the fundamental asset value conditional on the information disseminated among traders, the *full information liquidation value*. Define the average of the individual average signals $\bar{s} = M^{-1}\sum_{i=1}^{M} \bar{s}_{i,0}$, and the two constants $\kappa = c_0\left(\Lambda_0 + (M-1)\,\Omega_0\right)^{-1}$ and $\theta = \kappa M$. By eq. (1),

$$E\left(f | \, \mathbf{s}_0\right) = \theta \bar{s}. \tag{6}$$

Therefore $\bar{s}$ is a sufficient statistic for the full information liquidation value. Note that θ is well defined, provided the matrix Ψ_0 is invertible. This invertibility condition requires the following restriction on the model parameters:

$$\Lambda_0 > -\left(M-1\right)\Omega_0. \tag{7}$$

We denote the unconditional variance-covariance matrix of the average signals $(\bar{s}_{i,0})_{i=1}^{M}$ with $\bar{\Psi}_0 \equiv E\left[(\bar{s}_{1,0}, \cdots, \bar{s}_{M,0})^{\top}(\bar{s}_{1,0}, \cdots, \bar{s}_{M,0})\right]$. The elements of this matrix depend on the number of information linkages in the market. Accordingly, we set $\bar{\Psi}_0 \equiv \bar{\Psi}_0(G)$, where

$$\bar{\Psi}_0(G) \equiv \begin{bmatrix} \bar{\Lambda}_0(G) & \bar{\Omega}_0(1,G) & \cdots & \bar{\Omega}_0\left(\frac{M-1}{2},G\right) & \cdots & \bar{\Omega}_0(-1,G) \\ & \bar{\Lambda}_0(G) & & & & \bar{\Omega}_0(-2,G) \\ & & & \ddots & & \vdots \\ & & & & & \bar{\Lambda}_0(G) \end{bmatrix}, \tag{8}$$

and the elements

$$\begin{aligned} \bar{\Lambda}_0(G) &= var(\bar{s}_{i,0}) \\ \bar{\Omega}_0(k,G) &= cov(\bar{s}_{i+k,0}, \bar{s}_{i,0}), \quad k = \mp 1, \mp 2, \cdots, \mp \tfrac{M-1}{2} \end{aligned}$$

denote the unconditional variance of the average signals ($\bar{\Lambda}_0(G)$), and the unconditional covariance between the average signals of any two traders who are located k positions apart ($\bar{\Omega}_0(k,G)$) ($k \neq 0$). Naturally, $\bar{\Lambda}_0(G)$ is the same for each trader, due to the symmetric unconditional distribution of the signals in eq. (1). Furthermore, we have that $\bar{\Omega}_0(k,G) = \bar{\Omega}_0(-k,G)$, which follows by the circular information structure described in Section 1.1.

Next, we define the unconditional covariance between the sum of any trader's $\bar{s}_{i,0}$ with the remaining traders' average signals as:

$$\bar{\Gamma}_0(G) = cov\left(\sum_{j \neq i} \bar{s}_{j,0}, \bar{s}_{i,0}\right). \tag{9}$$

Due to the geographical location of the traders, $\bar{\Gamma}_0(G)$ is the same for each trader. Finally, the covariance between the average signal $\bar{s}_{i,0}$ and the fundamental value f is simply

$$\bar{c}_0 = cov(f, \bar{s}_{i,0}) = c_0,$$

and does not depend on the number of information linkages.

1.3 Correlations

In this section we provide the expressions for the elements of the variance-covariance matrix $\bar{\Psi}_0(G)$ in eq. (8). First, the unconditional variance of the average signals $\bar{s}_{i,0}$ is simply,

$$\bar{\Lambda}_0(G) = \frac{\Lambda_0 + 2G\Omega_0}{\hat{G}}. \tag{10}$$

Consider, next, any two traders i and $j = i + k$ with $k \neq 0$ and the unconditional covariance between the average signals these two traders have access to, i.e. the off-diagonal elements $\bar{\Omega}_0(k, G)$ in eq. (8). If $\hat{G} < M$, the covariance between the average signals depends (i) on the overall number of information linkages in the market and, hence, on G and (ii) k, the distance between the i-th trader and $(i + k)$-th trader. The dependence on k occurs because the number of signals every trader shares with the remaining traders depends on their relative position on the circle. For example, assume that $2G < (M - 1)/2$. In this case, trader i shares $2G$ signals with trader $i + 1$, $2G - 1$ signals with trader $i + 2$ and in general $2G + 1 - k$ signals with trader $i + k$. Eventually, trader i shares no signals with trader $i + 2G + 1$ and beyond. As Figure 2 illustrates, the covariance between the *average* signals does in general depend on k.

To derive, explicitly, the correlation among the traders' information endowments, we need to distinguish between two cases according to whether $2G$ is less or greater than $(M - 1)/2$. If $2G \leq (M - 1)/2$, one has that any two traders i and $j = i + k$ do not share any signal when they are located sufficiently apart, as in Figure 2. Formally, we have that $\mathbf{s}_{i+k,0} \cap \mathbf{s}_{i,0} = \{\emptyset\}$ if $|k| > 2G$, which implies $\bar{\Omega}_0(k, G) = \Omega_0$. However, if these two traders are sufficiently close, they share information. Formally, we have that $s_{i+k,0} \in \mathbf{s}_{i,0}$ for all $|k| \leq 2G$. In Appendix A, we show that,

$$\text{For } 2G \leq \tfrac{M-1}{2}, \quad \bar{\Omega}_0(k, G) = \begin{cases} \bar{\Lambda}_0(G) - \hat{G}^{-2}(\Lambda_0 - \Omega_0)\, k, & \text{for } k \in [1, 2G+1] \\ \Omega_0, & \text{for } k \in \left[2G+1, \tfrac{M-1}{2}\right] \end{cases} \tag{11a}$$

If the number of information linkages among traders is large, compared to the number of traders itself, traders may experience an additional boost in the correlation of their information endowments. This is because a high number of information linkages may induce any two traders to share signals even when they do not share the signals available at their location, a property of the model we label "double overlap". Such a "double overlap" indeed always occurs if $2G \geq (M - 1)/2$. Consider, for example, Figure 3. In this example, trader i shares the signal available at his own location with trader $i - \ell$, but he does not share this signal with trader $i + k_2$. However, traders $i + k_2$ and $i - \ell$ share at least the signals that are available at their location. Hence, traders i and $i - \ell$ have correlated information endowments, even if they are located so far apart that they do not experience direct information linkages. In the appendix, we show that the "double overlap" modifies the correlation structure in (11a) as follows:

$$\text{For } 2G \geq \tfrac{M-1}{2}, \quad \bar{\Omega}_0(k,G) = \begin{cases} \bar{\Lambda}_0(G) - \hat{G}^{-2}(\Lambda_0 - \Omega_0)\,k, & \text{for } k \in \left[1, 2\left(\tfrac{M-1}{2} - G\right)\right] \\ 2\bar{\Lambda}_0(G) - \hat{G}^{-2}M(\Lambda_0 - \Omega_0) - \Omega_0, & \text{for } k \in \left[2\left(\tfrac{M-1}{2} - G\right), \tfrac{M-1}{2}\right] \end{cases} \tag{11b}$$

To summarize, eqs. (10), (11a) and (11b) imply that the elements of the variance-covariance matrix $\bar{\Psi}_0(G)$ in eq. (8) depend on the number of information linkages. The elements on the main diagonal, $\bar{\Lambda}_0(G)$, are the same. The off-diagonal elements, instead, are decreasing in the the traders' relative distance k, according to the pattern in eqs. (11a)-(11b). Naturally, the unconditional covariance $\bar{\Gamma}_0(G)$ in eq. (9) is such that $\bar{\Gamma}_0(G) = \sum_{k\neq i}\bar{\Omega}_0(k,G)$ and is the same for every trader i. In particular, in the appendix we show that eqs. (11a)-(11b) imply that

$$\bar{\Gamma}_0(G) = (M-1)\,\Omega_0 + \frac{2G}{\hat{G}}(\Lambda_0 - \Omega_0), \qquad \text{for all } G \in \left[0, \tfrac{M-1}{2}\right]. \tag{12}$$

The fact that the unconditional covariance $\bar{\Gamma}_0(G)$ is the same for every trader simplifies the traders' inference about the forecasts of others, and allow us to focus on linear equilibria, as we shall explain.

1.4 Forecasts

We now describe how, conditionally on the information set $\mathbf{s}_{i,0}$, any trader i forecasts (i) the final liquidation value, and (ii) the sum of the remaining traders' average signals. By eq. (1),

$$E(f|\,\mathbf{s}_{i,0}) = \hat{G}\eta_1\bar{s}_{i,0}, \tag{13}$$

where $\eta_1 \equiv c_0(\Lambda_0 + 2G\Omega_0)^{-1}$. As we explained in Section 1.3, the correlation between the average signals of any two traders varies with their relative location [see eqs. (11a)-(11b)]. Therefore, every trader's expectation of the remaining traders' average signals depends on the relative distance k between the trader and his peers. However, the expectation of the sum of all remaining traders' average signals is independent of k. In the appendix, we show that it equals

$$E\left(\textstyle\sum_{j\neq i}\bar{s}_{j,0}|\mathbf{s}_{i,0}\right) = E\left(\textstyle\sum_{j\neq i}\bar{s}_{j,0}|\bar{s}_{i,0}\right) = (M-1)\,\phi_1\bar{s}_{i,0}, \tag{14}$$

where the regression coefficient is

$$\phi_1 \equiv \frac{\bar{\Gamma}_0(G)}{(M-1)\,\bar{\Lambda}_0(G)},$$

and $\bar{\Lambda}_0(G)$ and $\bar{\Gamma}_0(G)$ are given by eqs. (10) and (12).

2 Equilibrium characterization

2.1 Market maker's inference

Let $z_{i,t} = y_t - x_{i,t}$ be the residual order flow as of trader i. We let $F_{i,n} = \{\mathbf{s}_{i,0}, (z_{i,t})_{t=1}^{n-1}, (x_{i,t})_{t=1}^{n-1}\}$ and $F_{M+1,n} = \{(y_t)_{t=1}^{n}\}$ denote the information sets available to the i-th trader and the market maker at the n-th batch auction. The market maker sets prices according to the Semi-Strong efficiency condition in eq. (4). We denote the market maker's update of the *individual* signals estimate as

$$t_{i,n} = E\left(s_{i,0} \middle| F_{M+1,n} \right).$$

Given the symmetric location of the initial traders and signals, the previous expectation is independent of the i-th trader's specific location. Therefore, we set $t_{i,n} = t_n$. A simple but important point is that t_n is also the updated estimate of each *individual average* signal available to any trader i,

$$E\left(\bar{s}_{i,0} | F_{M+1,n}\right) = \hat{G}^{-1} E\left[\sum_{k=-G}^{G} s_{i+k,0} \middle| F_{M+1,n} \right] = \hat{G}^{-1} \sum_{k=-G}^{G} t_{i+k,n} = t_n. \tag{15}$$

The relation between p_n (the market maker's updated estimate of the asset value) and t_n (the market maker's updated estimate of the individual average signal) is given by

$$p_n = \theta t_n. \tag{16}$$

Let $s_{i,n}$ denote the i-th trader residual informational advantage (relative to the market maker) on the signal made available to his location after n rounds of trading,

$$s_{i,n} \equiv s_{i,0} - E\left(s_{0,n} \middle| F_{M+1,n} \right) = s_{i,0} - t_n.$$

Trader i informational advantage on his average signal, $\bar{s}_{i,n}$, has a similar interpretation. By eq. (15), it is:

$$\bar{s}_{i,n} = \bar{s}_{i,0} - E\left(\bar{s}_{0,n} \middle| F_{M+1,n} \right) = \bar{s}_{i,0} - t_n. \tag{17}$$

Finally, let us define the market maker's updates of the residual variances,

$$\begin{aligned} \sigma_{f,n}^2 &= var\left[E\left(f \middle| \mathbf{s}_0 \right) \middle| F_{M+1,n} \right] = var\left(\theta \bar{s} - p_n \middle| F_{M+1,n} \right) \\ \Lambda_n &= var\left(s_{i,0} \middle| F_{M+1,n} \right) = var\left(s_{i,n} \middle| F_{M+1,n} \right) \\ \Omega_n &= cov\left(s_{i,0}, s_{j,0} \middle| F_{M+1,n} \right) = cov\left(s_{i,n}, s_{j,n} \middle| F_{M+1,n} \right) \end{aligned} \tag{18}$$

where $\sigma_{f,n}^2$ is the residual variance of the full information fundamental value after n rounds of trading; Λ_n and Ω_n are the residual variance and covariance of the individual signals available at the traders' location.

2.2 Dimensionality issues

We focus on equilibria in which each trader's forecasts of (i) the fundamental value f and (ii) the forecasts of others are linear in the trader's average signal. In these equilibria, all higher order forecasts of other traders' forecasts are also linear in the same average signals. Consequently, average signals constitute sufficient statistics for both the fundamental value and the forecasts of others. Furthermore, we focus on equilibria independent from the forecasts' history. As we now show, our information structure makes the strategic gaming in our model comparable to that introduced by Foster and Viswanathan (1996). Specifically, we assume that in equilibrium, traders' demand and the market maker's learning about the fundamental value take the following form:

$$x_{i,n} = \hat{G}\beta_n \bar{s}_{i,n-1} \tag{19}$$

$$p_n = p_{n-1} + \lambda_n y_n \tag{20}$$

for some deterministic sequences $(\beta_n)_{n=1}^N$ and $(\lambda_n)_{n=1}^N$ to be determined in equilibrium.

Moreover, the market maker's learning about individual (and average) signals evolves according to

$$t_n = t_{n-1} + \zeta_n y_n, \tag{21}$$

for some deterministic sequence $(\zeta_n)_{n=1}^N$ to be determined in equilibrium. The relation between the updating parameters ζ_n and λ_n is given by:

$$\lambda_n = \theta\zeta_n. \tag{22}$$

At the n-th trading round, any trader i forecasts the fundamental value that is not predicted by the market maker after $n-1$ rounds, using his information $F_{i,n}$. By the assumption that the trading strategies are linear [see eq. (19)], and the market maker's update in eqs. (17) and (21),

$$x_{i,n} = \hat{G}\beta_n \bar{s}_{i,n-1} = \hat{G}\beta_n \left(\bar{s}_{i,0} - t_{n-1}\right) = \hat{G}\beta_n \left(\bar{s}_{i,0} - \textstyle\sum_{r=1}^{n-1} \zeta_r y_r\right).$$

Therefore, the residual order flow $(z_{i,t})_{t=1}^{n-1}$ is redundant, and we set $F_{i,n} = \{\mathbf{s}_{i,0}, (y_t)_{t=1}^{n-1}\}$. In other words, and as in Foster and Viswanathan (1996), any trader i can manipulate other the remaining traders' beliefs about the fundamental value only through the aggregate order flow. As a result, every trader forecasts the asset value as follows:

$$E\left(f - p_{n-1} \middle| F_{i,n}\right) = \hat{G}\eta_n \bar{s}_{i,n-1}, \tag{23}$$

for some deterministic sequence $(\eta_n)_{n=1}^N$ to be determined in equilibrium. That is, $\bar{s}_{i,n-1}$ is sufficient for any trader i to forecast the fundamental value, before submitting his order at time n. Note that eq. (23) is the dynamic analog to eq. (13). Similarly, trader i forecasts (the sum of) other traders' forecasts of the fundamental value according to

$$E\left(\sum_{j\neq i}\bar{s}_{j,n-1}\middle| F_{i,n}\right) = (M-1)\,\phi_n\bar{s}_{i,n-1}, \tag{24}$$

for some deterministic sequences $(\phi_n)_{n=1}^N$. As is clear, focusing on the linear strategies in eq. (19) plays a key role in resolving the dimensionality issue, since it implies that the forecasts of the forecasts of others are linear in each trader's average signal.

2.3 Deviations

Linearity also rules out increasing state history over time. The argument hinges on the fact that linear strategies in eq. (19) are played *in equilibrium*. When we turn to consider deviations from the optimal play by trader i, we need to take into account that $\bar{s}_{i,n-1}$ is no longer a sufficient statistic to predic the fundamental value and the other traders' forecast, as in eqs. (23)-(24). In fact, $\bar{s}_{i,n-1}$ is sufficient only if trader i played the strategy (19) in the first $n-1$ trading rounds.

Let us denote deviation from the equilibrium path with a prime ($'$). Trader i deviation from the equilibrium play (19) to $(x'_{i,k})_{k=1}^{n-1}$ during the first $n-1$ auctions would generate the aggregate order flow $\{y'_k = y_k - (x_{i,k} - x'_{i,k})\}_{k=1}^{n-1}$. Since the market maker's update of the estimate of the fundamental value and the average signals are linear in the order flow due to equations (20)-(21), trader i deviation modifies the market maker's learning process as well, resulting in $(p'_k)_{k=1}^{n-1}$ and $(t'_k)_{k=1}^{n-1}$. Given past suboptimal play, it turns out that the residual average signal along the equilibrium path $\bar{s}_{i,n-1}$ and the price deviation $\left(p_{n-1} - p'_{n-1}\right)$ are *jointly* sufficient to forecast the fundamental value as well as the forecasts of other traders. This result allows to conjecture that trader i's value function in eq. (2) after n auctions takes the form:

$$W_{i,n} = \alpha_n\bar{s}_{i,n}^2 + \psi_n\bar{s}_{i,n}\left(p_n - p'_n\right) + \mu_n\left(p_n - p'_n\right)^2 + \delta_n, \tag{25}$$

for some deterministic sequences $(\alpha_n)_{n=1}^N$, $(\psi_n)_{n=1}^N$, $(\mu_n)_{n=1}^N$ and $(\delta_n)_{n=1}^N$ to be determined in equilibrium. Past suboptimal play is captured by the second and third term in the value function in eq. (25). Moreover, trader i deviation coincides with the equilibrium strategy in eq. (19) plus an additional term reflecting the price deviation induced by suboptimal play in the previous $n-1$ rounds:

$$x'_{i,n} = \hat{G}\beta_n\bar{s}_{i,n-1} + \gamma_n\left(p_{n-1} - p'_{n-1}\right). \tag{26}$$

The necessary and sufficient conditions for an equilibrium hinge upon the mutual consistency between the conjectured value function in eq. (25) and the traders' deviation in eq. (26).

We have:

Proposition 1. *There exists a symmetric linear recursive Bayesian equilibrium in which trading strategies and prices are as in eqs.* (19)-(20); λ_n *is the unique real, positive solution to*:

$$\begin{aligned} 0 \;=\;& \frac{\theta(M-\hat{G})\left(\Lambda_n-\Omega_n\right)\sigma_u^4}{\hat{G}M^2\sigma_{f,n}^4}\lambda_n^4+\frac{\sigma_u^2\psi_n\bar{\Lambda}_n}{\sigma_{f,n}^2}\lambda_n^3-\frac{\theta\sigma_u^2[2\Lambda_n+(M-1)\,\Omega_n-\frac{2G}{\hat{G}}\left(\Lambda_n-\Omega_n\right)]}{M\sigma_{f,n}^2}\lambda_n^2 \\ & -\psi_n\bar{\Lambda}_n\lambda_n+\frac{\sigma_{f,n}^2}{\theta}; \end{aligned} \tag{27}$$

and the trading strategy coefficients β_n *and* γ_n *are given by*:

$$\beta_n = \frac{\theta\lambda_n\sigma_u^2}{\hat{G}M\sigma_{f,n}^2}; \tag{28}$$

$$\gamma_n = \frac{\left(1-2\lambda_n\mu_n\right)\left[1-\theta^{-1}\hat{G}\left(M-1\right)\beta_n\lambda_n\right]}{2\lambda_n\left(1-\lambda_n\mu_n\right)}. \tag{29}$$

The value function coefficients satisfy the recursions:

$$\begin{aligned} \alpha_{n-1} &= \alpha_n\left[1-\theta^{-1}\hat{G}\left(1+(M-1)\phi_n\right)\beta_n\lambda_n\right]^2+\hat{G}^2\beta_n\left[\eta_n-\beta_n\lambda_n\left(1+(M-1)\phi_n\right)\right]; \\ \psi_{n-1} &= \psi_n\left[1-\lambda_n\gamma_n-\theta^{-1}\hat{G}\left(M-1\right)\beta_n\lambda_n\right]\left[1-\theta^{-1}\hat{G}\left(1+(M-1)\phi_n\right)\beta_n\lambda_n\right] \\ &\quad+\hat{G}\left\{\gamma_n\left[\eta_n-\beta_n\lambda_n\left(1+(M-1)\phi_n\right)\right]-\beta_n\gamma_n\lambda_n+\beta_n\left[1-\theta^{-1}\hat{G}\left(M-1\right)\beta_n\lambda_n\right]\right\}; \\ \mu_{n-1} &= \mu_n\left[1-\lambda_n\gamma_n-\theta^{-1}\hat{G}\left(M-1\right)\beta_n\lambda_n\right]^2+\gamma_n\left[1-\lambda_n\gamma_n-\theta^{-1}\hat{G}\left(M-1\right)\beta_n\lambda_n\right]; \\ \delta_{n-1} &= \delta_n+\theta^{-2}\alpha_n\lambda_n^2\sigma_u^2+\theta^{-2}\hat{G}^2\alpha_n\lambda_n^2\beta_n^2 var\left(\left.\textstyle\sum_{j\neq i}\bar{s}_{j,n-1}\right|F_{i,n}\right); \end{aligned} \tag{30}$$

where $\alpha_N=\psi_N=\mu_N=\delta_N=0$ *and*

$$\phi_n = \frac{\bar{\Gamma}_{n-1}}{\left(M-1\right)\bar{\Lambda}_{n-1}}; \tag{31}$$

$$\eta_n = \frac{\theta\left(\bar{\Gamma}_{n-1}+\bar{\Lambda}_{n-1}\right)}{\hat{G}M\bar{\Lambda}_{n-1}}; \tag{32}$$

$$var\left(\left.\textstyle\sum_{j\neq i}\bar{s}_{j,n-1}\right|F_{i,n}\right) = M\left[\Lambda_{n-1}+\left(M-1\right)\Omega_{n-1}\right]-\left[1+\phi_n^2\left(M-1\right)^2\right]\bar{\Lambda}_{n-1}-2\bar{\Gamma}_{n-1},$$

where $\bar{\Lambda}_n(G) = var(\bar{s}_{i,n}|F_{M+1,n})$, $\bar{\Gamma}_n(G) = \sum_{k\neq i}\bar{\Omega}_n(k,G)$ *and, finally,* $\bar{\Omega}_n(k,G) = cov(\bar{s}_{i,n},\bar{s}_{i+k,n}|F_{M+1,n})$*. Furthermore the following inequality must hold:*

$$\lambda_n(1-\lambda_n\mu_n) > 0,$$

and the following recursion on the full information residual variance must hold:

$$\sigma^2_{f,n} = \left(1-\theta^{-1}\hat{G}M\beta_n\lambda_n\right)\sigma^2_{f,n-1}.$$

In our model, not only are traders concerned with learning from the information that other traders possess. This learning process is also complicated by every trader's geographical location and the number of information linkages every trader has with his neighbors. As Proposition 1 reveals, trading strategies and value functions are heavily affected by the heterogeneous correlation structure arising from all these information linkages - a fact that we will examine in great detail in the next section.

3 Market dynamics implications

This section analyzes the properties of the equilibrium predicted by the model. We make the standard assumption that the fundamental value of the asset equals the sum of all the signals disseminated in the economy,

$$f = \sum_{i=1}^{M} s_{i,0}. \tag{33}$$

We normalize the variance of the fundamental value to one, i.e. $\sigma^2_f = 1$ and take $\sigma^2_u = N^{-1}$. That is, we analyze a situation in which the uncertainty about the fundamentals equals uncertainty about non-fundamentals across all batch auctions.

The additive structure in eq. (33) implies that (i) the covariance between any signal and the fundamentals equals $c_0 = \Lambda_0 + (M-1)\Omega_0$ and, hence, that the parameter θ in the full information liquidation value [see eq. (6)] is $\theta = M$; and (ii) the correlation between any two individual signals equals

$$\rho = \frac{1}{M(M-1)}\left(\frac{1}{\Lambda_0} - M\right). \tag{34}$$

Therefore, the correlation of the individual signals available at the information linkages, ρ, the number M of traders, the number $2G$ of information linkages each of these traders has with his peers, and the length of the trading period N, are the only free parameters in the model. Indeed, the unconditional variance of the fundamental value f is such that $M\Lambda_0 + M(M-1)\Omega_0 = \sigma_f^2 \equiv 1$ and, obviously, $\Omega_0 = \rho\Lambda_0$. Hence, the unconditional variance of the individual signals, Λ_0, can be computed once we have M and ρ.

We set $M = 7$ and consider $N = 10$ batch auctions. (We have also experimented with different numbers of traders and auctions, but obtained qualitatively very similar results.) The resulting equilibrium is now parametrized by (i) the initial correlation between the signals at any two locations (ρ), and (ii) the number of information linkages for each trader ($2G$). We use backward induction to solve for the equilibrium (see Appendix C for details). In the next section, we describe the properties of the equilibrium arising for a negative correlation of the signals available at the traders' location, $\rho = -15\%$, for $\rho = 10\%$ (low correlation) and $\rho = 90\%$ (high correlation). Moreover, we analyze the cases of $G = 0$ (no information linkages), $G = 1$ (information linkages) and $G = 2$ (many information linkages and double overlap).

3.1 Volume, liquidity, and volatility

We study how the existence of information linkages affects the trading volume, liquidity and asset return volatility. As in Admati and Pfleiderer (1988), we decompose the (expected) volume at the n-th auction in terms of the contribution of the market maker, the M traders, and the liquidity traders. Precisely, we identify each component with its conditional standard deviation, and set $Vol_n = Vol_{M,n} + Vol_{I,n} + Vol_{U,n}$, where

$$\begin{aligned} Vol_{M,n} &= \sqrt{\hat{G}^2\beta_n^2 M\left[\Lambda_{n-1} + (M-1)\,\Omega_{n-1}\right] + \sigma_u^2}; \\ Vol_{I,n} &= \hat{G}\beta_n\sqrt{M\left[\Lambda_{n-1} + (M-1)\,\Omega_{n-1}\right]}; \end{aligned}$$

and $Vol_{U,n} = \sigma_u$, for all n. Furthermore, we compute the asset return volatility by also conditioning on the market maker's information set. By eqs. (19)-(20),

$$var\left(p_n - p_{n-1} | F_{M+1,n-1}\right) = \lambda_n^2 \cdot Vol_{M,n}^2.$$

The central prediction of the model is its ability to predict a high volume in the presence of information linkages (see Figures 5 and 6). This result holds even with a moderate value of the initial correlation ρ. We developed the economic intuition for this result in the introduction and so in the interest of space we do not repeat it here.

Can high volume be consistent with models without information linkages? Figure 5 (for $\rho = 90\%$ and $G = 0$) illustrates that a model without information linkages can be consistent with high volume. At the same time, these models can only do so if the signal correlation among traders is extremely high in the first place. Arguably, there is no evidence that this has occurred. Our model proposes an alternative simple mechanism based on information linkages between traders.

The trading activity due to informed traders,[3] $Vol_{M,n}$, is depicted in Figure 5. As is clear, information linkages increase the incentives to preempt other traders thus resulting in higher volume. Not surprisingly, Figure 4 shows that these incentives are larger -and thus the effect of information linkages on volume is more pronounced- the lower is the initial correlation among individual signals, or equivalently, the larger is each trader's monopolistic power.

Figure 6 plots the evolution of the fundamental residual variance, $\sigma^2_{f,n}$, over time.[4] Information linkages make the variance of the liquidation value decrease more rapidly. Moreover, the improvement in market efficiency -as measured by the ratio $1/\sigma^2_{f,n}$- is more pronounced with low initial correlation. These findings are directly related to the volume dynamics previously documented. The mechanism is that information linkages make competition among informed traders more intense. As a consequence, traders impound more information into their orders thus reducing the residual variance.

The price responsiveness to the order flow, λ_n, is displayed in Figure 7. Overall, the effect of information linkages is to increase market liquidity, i.e. the ratio $1/\lambda_n$. Again, the higher trade aggressiveness in the presence of information linkages (see Figure 5) decrease the adverse selection faced by the market maker since the order flow is more informative (see Figure 6). Thus the market maker reduces the price impact of the order flow, and the market is more liquid.

The dynamics of asset return volatility are shown in Figure 8. For a given number of infomation linkages, both the evolution of the price impact and the expected (informed) order flow affect the volatility dynamics. Since both components are shown to decrease over time, so does volatility. Moreover, the presence of information linkages produces an increased volume and liquidity. Figure 8 reveals that the overall effect is to increase volatility during the first trading rounds, and to decrease it afterwards.

3.2 Correlated trading and volume

Figure 9 shows the time variation in the average signal correlation every trader has with his neighbors, or $\bar{\rho}_n(k,G) \equiv \bar{\Omega}_n(k,G)/\bar{\Lambda}_n(G)$ $(k=1,2,3)$. This correlation is inversely related

[3] For reasons of space we do not plot the other components of total volume, most notably the volume traded by the market maker which exhibit a similar pattern.

[4] Figure 6 reveals that $\sigma^2_{f,0} < \sigma^2_f$ (in our case, $\sigma^2_f \equiv 1$). When signals are as in eq. (33), this inequality *has* to be expected for $\sigma^2_{f,0} = \frac{\theta^2}{M}\left[\Lambda_0 + (M-1)\,\Omega_0\right] = \frac{M}{\rho^{-1}+M-1}$.

to the monopolistic power deriving from the information shared by every trader with neighbors. Moreover, we have that:

$$corr\left(x_{i,n}, x_{i+k,n} \middle| F_{M+1,n-1}\right) = \bar{\rho}_{n-1}\left(k, G\right).$$

Therefore, $\bar{\rho}_{\cdot}\left(\cdot\right)$ also measures how information linkages make neighbors's trades "resemble" one another. In general, traders are worse off as $\bar{\rho}_{\cdot}\left(\cdot\right)$ increases. For a given G, $\bar{\rho}_n\left(k, G\right)$ is increasing in ρ for all n and k as one might have expected. Moreover, for given k and G, $\bar{\rho}_n\left(\cdot\right)$ decreases over time. As noted in Foster and Viswanathan (1996) this is related to the market maker's learning about the (average of the) traders' signals over time. Furthermore, for all k, $\bar{\rho}_n\left(k\right)$ increases with G for the first trading rounds. In other terms, information linkages generally entail a loss in the traders' monopolistic power. Intuitively, this is because the presence of information linkages induces an increase in the initial correlation $\bar{\rho}_0\left(\cdot\right)$ between all traders' average signals. Noteworthy, in the presence of information linkages, (i) it takes fewer batch auctions for the correlation between distant traders to become negative with respect to the case $G = 0$, and (ii) correlation between close neighbors remains positive throughout the trading game. As we mention in the Introduction, this behavior is consistent with findings in Feng and Seasholes (2004).

How is volume correlated among traders? Figure 10 depicts the time variation in the correlation between individual traders' volume.[5] The above dynamics of the correlation between individual trades are useful to interpret the evolution of the correlation between traders' volume. In the absence of information linkages, individual trades are nearly uncorrelated for low initial signal correlation (see Figure 9). This explains why volume is uncorrelated as well for $\rho = 0.1$. On the other hand for high values of ρ individual trades are positively correlated, and this correlation is relatively strong over the first auctions while it vanishes as N increases. Not surprisingly, volume correlation follows the same dynamics. In the presence of information linkages, the correlation between individual trades is positive for close neighbors while it decays to negative values for distant traders. Moreover, such decay is more pronounced for $G = 1$ than for $G = 2$. For close neighbors the volume correlation exhibits the same dynamics as the trades correlation. However, when the distance between traders increases, volume correlation becomes U-shaped and few information linkages ($G = 1$) lead to stronger correlation with respect to more information linkages ($G = 2$). This is because for $G = 1$, the increased correlation of individual trades due to the presence of information linkages does not offset the decrease stemming from the market maker's learning. Individual trades are strongly negatively correlated for most of the

[5] We do not have a closed-form solution for the correlation between the individual traders' volume. We computed this correlation through simulations (see Appendix C for details).

trading rounds and volume correlation peaks up. On the other hand, when $G = 2$ the increase in individual trades correlation due to information linkages dominates the decrease due to the market maker learning process. As a result, individual trades –and therefore volume– are weakly correlated for most of the batch auctions.

3.3 Traders' profitability

Information linkages do damage traders' profitability according to whether the correlation ρ is positive or negative. Such a result holds in an unambigously huge number of parametrizations of the model - in terms of the number of batch auctions N, the number of traders M, the number of information linkages $2G$ available for each trader, and the initial correlation ρ between individual signals (when this is positive). Figure 11 illustrates this point in an exemplary manner. Figure 11 also reveals another striking result. Namely, for any fixed G, traders' profitability is nonmonotonic in the initial correlation ρ. More generally, our numerical results suggest that for any fixed tuple (M, N, G), there exists a value of the initial correlation ρ which maximizes the traders' profitability.

What are the origins of these results? Consider the relation in eq. (34). It reveals that the initial correlation ρ is related to two distinct effects. First, as ρ increases, every trader loses more and more bits of information endowments available only to him. At the same time, an increase in ρ is obviously associated with a drop in σ_ϵ^2, and thus helps imperfectly informed traders to improve their estimates of the fundamental asset value. When ρ is low, the losses in monopolistic power associated with the first effect dominate over the precision gains deriving from the second effect. When ρ is high, the losses in monopolistic power are dominated by the precision gains. (Figure 11 shows this tradeoff, for each G.)

In our model, an increase in the number of information linkages $2G$ measures similar effects. On the one hand, a trader with many information linkages has a better estimate of the fundamental value of the asset. On the other hand, these information linkages also make our trader lose part of his monopolistic informational power. But in contrast with the simple situation in which there exists a level of correlation maximizing traders' profitability, here there does not exist such an "optimal" number of information linkages maximizing the traders' power.

To develop some intuition about this result, consider again the average signal $\bar{s}_{i,0}$ trader i has about the fundamental [see eq. (5)]. Assume temporarily that $s_{i,0} = f + \epsilon_i$. By replacing this assumption into the average signal in eq. (5), we find that

$$\bar{s}_{i,0} = f + \frac{1}{2G+1} \sum_{k=-G}^{G} \epsilon_{i+k}. \tag{35}$$

By the Law of Large Numbers, the probability limit $p \lim_{G \to \infty} \bar{s}_{i,0} = f$, for every trader $i = 1, \cdots, M$. Yet this result implies that the monopolistic power of the traders eventually shrinks to zero, since G goes to infinity with M. Alternatively, it may be the case that the number of traders required for such a reduction does not exist in the first place.

Finally, the convergence of the average signal $\bar{s}_{i,0}$ to the truth f takes place at the usual $\sqrt{G}$-rate. In contrast, the boost in precision obtained with an increase in ρ takes a place at a much faster rate. For example, suppose that $\rho = 0.1$. Then we have, $var\left(s_{i,0}\right) = 1 + \frac{1-\rho}{\rho} = 10$, and $var\left(\bar{s}_{i,0}\right) = 1 + \frac{1}{2G+1}\frac{1-\rho}{\rho}$. So an increase in the initial correlation from $\rho = 0.1$ to $\rho = 0.5$ makes the variance $var\left(s_{i,0}\right)$ decrease from 10 to 2. This is indeed a noticeable boost in precision associated with some market power deteration. In contrast, a five-fold reduction in the variance $var\left(\bar{s}_{i,0}\right)$ through activation of new information linkages necessitates $2G = 8$ such linkages! This is a much more noticeable reduction in market power.

4 Conclusion

This paper develops a dynamic model of trading with differentially informed traders experiencing information linkages related to the long-term value of an asset. We show that in the presence of information linkages, volume and price informativeness increase, asymmetric information costs decrease, and traders' profitability is damaged or not according to whether the signals available at the traders' location are positively correlated or not. Finally, the model predicts a relation between patterns of trades correlation and traders' geographical location: 'neighbor' trades are positively correlated and 'distant' trades are negatively correlated. These predicitions are consistent with the robust phenomena documented in the empirical literature, and cannot be simultaneously accommodated within traditional models dealing with differentially informed traders who experience symmetric signal distributions.

Appendix

A. Derivation of selected formulae

Derivation of eqs. (10)-(12). To derive eq. (10), we use the definition of the average signal in eq. (5). A simple computation leaves:

$$var\left(\bar{s}_{i,0}\right)=\frac{\hat{G}\Lambda_0+2G\hat{G}\Omega_0}{\hat{G}^2},$$

or equivalently (10). Next, we derive eqs. (11a) and (11b). These equations correspond to two cases: *a)* $2G \leq (M-1)/\,2$ and *b)* $2G \geq (M-1)/\,2$, which we now study separately.

Case a) ($2G \leq (M-1)/\,2$). Consider traders i and $j=i+k$, $k\neq 0$. We have: $s_{i+k,0}\notin \mathbf{s}_{i,0}$ for all $|k|>G$. Therefore

$$\bar{\Omega}_0(k,G)\equiv cov\left(\bar{s}_{i,0},\bar{s}_{i+k,0}\right)=\hat{G}^{-2}\sum_{l=-G}^{G}\sum_{m=-G}^{G}cov\left(s_{i+l,0},s_{i+k+m,0}\right)=\Omega_0,$$

for all $|k|>2G$, which is the second line in (11a). If instead $|k|\leq 2G$, $s_{i+k,0}\in \mathbf{s}_{i,0}$ for all $|k|\leq G$ and $\mathbf{s}_{i+k,0}\cap\mathbf{s}_{i,0}\neq\{\emptyset\}$. In particular, trader i shares $(2G+1-k)$ signals with trader $i+k$. Each of these signals contributes for $\left(\Lambda_0+2G\Omega_0\right)/\left(2G+1\right)^2$ to $\bar{\Omega}_0(k,G)$. Shared signals thus contribute for

$$\frac{\Lambda_0+2G\Omega_0}{(2G+1)^2}\cdot(2G+1-k)$$

to $\bar{\Omega}_0(k,G)$. The remaining (not shared) k signals contribute for

$$\frac{(2G+1)\,\Omega_0}{\left(2G+1\right)^2}\cdot k$$

to $\bar{\Omega}_0(k,G)$. Therefore,

$$\bar{\Omega}_0(k,G)=\frac{\left(\Lambda_0+2G\Omega_0\right)\left(\hat{G}-k\right)+\hat{G}\Omega_0 k}{\hat{G}^2}.$$

Grouping terms in the previous expression yields the first line in (11a).

Case b) ($2G \geq (M-1)/\,2$). This case differs from the previous one due to the double overlap discussed in the main text (see Figure 3). In this case, the number of signals shared by traders i and $i+k$ is:

$$L(k,G)\equiv 2G+1-k+n(k,G),\qquad k=1,\cdot\cdot\cdot,\tfrac{M-1}{2}. \tag{A1}$$

The term $n(k,G)$ arises because traders on trader i's r.h.s. semicircle might be sharing signals with traders located between $i+1$ and $i+(M-1)/2$ on the l.h.s. semicircle (see Figure 3); and obviously the i-th trader shares signals with traders located between $i-1$ and $i-G$ as well. The double overlap occurs if and only if trader $i+k$ on the l.h.s. semicircle and trader $i-\ell$ with $\ell \in [1, \frac{M-1}{2}]$ on the r.h.s. semicircle are such that ℓ and k satisfy:

$$\begin{cases} \frac{M-1}{2} - (\ell - 1) + \frac{M-1}{2} - k \leq G \\ G \geq \ell \geq 1 \\ \frac{M-1}{2} \geq k \geq 1 \end{cases}$$

The first inequality in the previous restrictions requires trader $i-\ell$ to share his signal with trader $i+k$. The second and third constraints restrict trader $i-\ell$ to be on the r.h.s. semicircle and trader $i+k$ to be on the l.h.s. semicircle relative to trader i. Thus, for fixed k $(1 \leq k \leq (M-1)/2)$, the double overlap occurs if and only if

$$G \geq \ell \geq M - G - k, \qquad k = 1, \cdots, \tfrac{M-1}{2},$$

and $\ell \geq 1$. Clearly, $\min_k (M-G-k) = \frac{M-1}{2} - G + 1 \geq 1$. Hence, the constraint that $\ell \geq 1$ is redundant. By the previous inequalities, it immediately follows that:

$$n(k,G) = \max\left[G - (M-G-k) + 1, 0\right].$$

By replacing this result into eq. (A1) leaves:

$$L(k,G) = \begin{cases} 4G + 1 - (M-1), & \frac{M-1}{2} \geq k \geq 2\left(\frac{M-1}{2} - G\right) \\ 2G + 1 - k, & 1 \leq k \leq 2\left(\frac{M-1}{2} - G\right) \end{cases}$$

For all $k \in \left[1, 2\left(\frac{M-1}{2} - G\right)\right]$, $\bar{\Omega}_0(k,G)$ is thus exactly as in case a) for $k \in [1, 2G]$, and the first line of eqs. (11b) follows. For all $k \in \left[2\left(\frac{M-1}{2} - G\right), \frac{M-1}{2}\right]$, tedious but straightforward computations lead to the second line of eqs. (11b).

Finally, we demonstrate that eq. (12) holds true. As usual, we consider the two cases in which $2G \gtreqless (M-1)/2$. If $0 \leq 2G \leq (M-1)/2$, there are $[M - (4G+1)]$ traders $i+k$ such that $\mathbf{s}_{i+k,0} \cap \mathbf{s}_{i,0} = \{\emptyset\}$. In correspondence of these indexes, $cov\left(\bar{s}_{i+k,0}, \bar{s}_{i,0}\right) = \Omega_0$. Therefore,

$$\bar{\Gamma}_0(G) = 2 \sum_{k=1}^{2G} \bar{\Omega}_0(k,G) + [M - (4G+1)]\, \Omega_0.$$

The $2G$ covariances in the summation can be computed through the first line in (11a). Eq. (12) follows by the expression of $\bar{\Lambda}_0(G)$ in eq. (10). Next, consider the case $(M-1)/2 \leq 2G \leq M-1$.

We have:

$$\bar{\Gamma}_0(G) = 2\left\{\sum_{k=1}^{M-1-2G} \bar{\Omega}_0(k,G) + \left(2G - \frac{M-1}{2}\right)\left[2\bar{\Lambda}_0(G) - \frac{M(\Lambda_0 - \Omega_0)}{\hat{G}^2} - \Omega_0\right]\right\}.$$

By plugging eqs. (11b) and (10) into the previous equation, we find that the expression of $\bar{\Gamma}_0(G)$ is the same as the one obtained in the case $0 \leq 2G \leq (M-1)/2$, and eq. (12) follows. ■

Derivation of eq. (14). Consider the projection of $s_{i+k,0}$ onto $\mathbf{s}_{i,0}$ for $|k| \leq G$. We have $s_{i+k,0} \in \mathbf{s}_{i,0}$ for $k = 0, \mp 1, \cdots, \mp G$. Hence,

$$E(s_{i+k,0}|\,\mathbf{s}_{i,0}) = s_{i+k,0}, \quad k = 0, \mp 1, \cdots, \mp G.$$

Next, consider a signal $s_{i+k,0}$ a given trader i does not have access to, i.e. $s_{i+k,0}$, for $k = \mp(G+1), \cdots, \mp\frac{M-1}{2}$. Let $\Psi_{0,G} = E(\mathbf{s}_{i,0}\mathbf{s}_{i,0}^\top)$ be the $\hat{G} \times \hat{G}$ variance-covariance matrix of the vector $\mathbf{s}_{i,0}$. $\Psi_{0,G}$ is a $\hat{G} \times \hat{G}$ submatrix extracted from Ψ_0, and its inverse can be obtained with the same strategy of proof as in Foster and Viswanathan (1996) (p. 1479). Let $K \equiv [(\Lambda_0 - \Omega_0)(\Lambda_0 + 2G\Omega_0)]^{-1}$, yielding,

$$\Psi_{0,G}^{-1} = K \cdot \begin{bmatrix} \Lambda_0 + (2G-1)\Omega_0 & -\Omega_0 & \cdots & -\Omega_0 \\ & \Lambda_0 + (2G-1)\Omega_0 & & -\Omega_0 \\ & & \ddots & \vdots \\ & & & \Lambda_0 + (2G-1)\Omega_0 \end{bmatrix}.$$

For $|k| > G$, we have that $cov(s_{i+k,0}, \mathbf{s}_{i,0}) = \Omega_0 \mathbf{1}_{\hat{G}}$, and by the Projection Theorem,

$$\begin{aligned} E(s_{i+k,0}|\mathbf{s}_{i,0}) &= \Omega_0 \mathbf{1}^\top (\Psi_{0,G})^{-1} \mathbf{s}_{i,0} \\ &= \Omega_0 \frac{\Lambda_0 + (2G-1)\Omega_0 - 2G\Omega_0}{(\Lambda_0 - \Omega_0)(\Lambda_0 + 2G\Omega_0)} \mathbf{1}^\top \mathbf{s}_{i,0} \\ &= \frac{\hat{G}\Omega_0}{\Lambda_0 + 2G\Omega_0} \bar{s}_{i,0}, \quad k = 0, \mp 1, \cdots, \mp G. \end{aligned}$$

Therefore,

$$E(s_{i+k,0}|\mathbf{s}_{i,0}) = \begin{cases} s_{i+k,0} & \text{for } k = 0, \mp 1, \cdots, \mp G \\ \dfrac{\hat{G}\Omega_0}{\Lambda_0 + 2G\Omega_0} \bar{s}_{i,0} & \text{for } k = \mp(G+1), \cdots, \mp\frac{M-1}{2} \end{cases}$$

We now compute, for a given trader i, his forecast of the sum of the remaining traders' average signals onto $\mathbf{s}_{i,0}$. We obviously have $\bar{s}_{i,0} \in \mathbf{s}_{i,0}$ and $\sum_{j=1}^{M} \bar{s}_{j,0} = \sum_{j=1}^{M} s_{j,0}$. Therefore,

$$\begin{aligned}
E\left(\sum_{j\neq i} \bar{s}_{j,0} \middle| \mathbf{s}_{i,0}\right) &= \sum_{j=1}^{M} E\left(s_{j,0} \middle| \mathbf{s}_{i,0}\right) - \bar{s}_{i,0} \\
&= \mathbf{1}_G^{\top}\mathbf{s}_{i,0} + \sum_{|k|>G} E\left(s_{i+k,0} \middle| \mathbf{s}_{i,0}\right) - \bar{s}_{i,0} \\
&= (2G+1)\,\bar{s}_{i,0} + \frac{(M-(2G+1))\,\Omega_0}{\Lambda_0 + 2G\Omega_0}(2G+1)\,\bar{s}_{i,0} - \bar{s}_{i,0} \\
&= \frac{(M-1)(2G+1)\,\Omega_0 + 2G\,(\Lambda_0 - \Omega_0)}{\Lambda_0 + 2G\Omega_0}\bar{s}_{i,0}.
\end{aligned}$$

Eq. (14) follows by rearranging terms in the last equality, and by the definition of ϕ_1 in the main text. ■

Derivation of eq. (16). By Semi-Strong market efficiency, $p_n = E\left(f \middle| F_{M+1,n}\right)$. By eq. (6), $\bar{s}$ is a sufficient statistic for $E\left(f \middle| \mathbf{s}_0\right)$. Therefore,

$$\begin{aligned}
p_n &= E\left(f \middle| F_{M+1,n}\right) \\
&= E\left\{ E\left[E\left(f \middle| \mathbf{s}_0\right) \middle| \bar{s}, F_{M+1,n}\right] \middle| F_{M+1,n}\right\} \\
&= E\left[E\left(\theta\bar{s} \middle| \bar{s}, F_{M+1,n}\right) \middle| F_{M+1,n}\right] \\
&= \frac{\theta}{M} E\left(\sum_{i=1}^{M} \bar{s}_{i,0} \middle| F_{M+1,n}\right) \\
&= \theta t_n,
\end{aligned}$$

where the last line follows by eq. (15). ■

We will repeatedly use the results recorded in the following lemma.

Lemma 1. *We have,*

$$\sigma_{f,n}^2 = \frac{\theta^2}{M}\left[\Lambda_n + (M-1)\,\Omega_n\right]. \tag{A2}$$

Furthermore, the following recursions hold,

$$\Omega_{n-1} - \Omega_n = \Lambda_{n-1} - \Lambda_n; \tag{A3a}$$
$$\sigma^2_{f,n-1} - \sigma^2_{f,n} = \theta^2 \left(\Lambda_{n-1} - \Lambda_n\right); \tag{A3b}$$
$$\bar{\Lambda}_{n-1}(G) - \bar{\Lambda}_n(G) = \Lambda_{n-1} - \Lambda_n, \qquad \textit{all } G; \tag{A3c}$$
$$\bar{\Omega}_{n-1}(k,G) - \bar{\Omega}_n(k,G) = \Lambda_{n-1} - \Lambda_n, \qquad \textit{all } k, G; \tag{A3d}$$
$$\bar{\Gamma}_{n-1}(G) - \bar{\Gamma}_n(G) = (M-1)\left(\Lambda_{n-1} - \Lambda_n\right), \qquad \textit{all } G. \tag{A3e}$$

Proof. First, we derive eq. (A2). By the Law of Iterated Expectations,

$$E\left(\theta\bar{s}\middle| F_{M+1,n}\right) = E\left[E\left(f\middle|\mathbf{s}_0\right)\middle| F_{M+1,n}\right] = E\left(f\middle| F_{M+1,n}\right) = p_n.$$

Hence,

$$\begin{aligned}
\sigma^2_{f,n} &= E\left[\left(\frac{\theta}{M}\sum_{i=1}^{M}\bar{s}_{i,0} - \frac{\theta}{M}\sum_{i=1}^{M} t_{i,n}\right)^2\middle| F_{M+1,n}\right] \\
&= \frac{\theta^2}{M^2}E\left[\left(\sum_{i=1}^{M}\bar{s}_{i,n}\right)^2\middle| F_{M+1,n}\right] \\
&= \frac{\theta^2}{M^2}E\left[\left(\sum_{i=1}^{M}s_{i,n}\right)^2\middle| F_{M+1,n}\right] \\
&= \frac{\theta^2}{M}\left[\Lambda_n + (M-1)\,\Omega_n\right].
\end{aligned}$$

Next, we derive eqs. (A3a)-(A3e). Let $\tau_n \equiv \mathit{cov}\left(s_{i,n-1}, y_n\middle| F_{M+1,n-1}\right)$. Clearly, τ_n is independent of i, and $\Psi_n \equiv E\left[\left(s_{1,0}-t_n,\cdots,s_{M,0}-t_n\right)^\top\left(s_{1,0}-t_n,\cdots,s_{M,0}-t_n\right)\middle| F_{M+1,n}\right]$. By the Projection Theorem,

$$\Psi_n = \Psi_{n-1} - \frac{\tau_n^2}{\mathit{var}\left(y_n\middle| F_{M+1,n-1}\right)}\mathbf{1}\mathbf{1}^\top,$$

which gives the recursions:

$$\begin{aligned}
\Lambda_n &= \Lambda_{n-1} - \frac{\tau_n^2}{\mathit{var}\left(y_n\middle| F_{M+1,n-1}\right)}; \qquad \text{(A4)}\\
\Omega_n &= \Omega_{n-1} - \frac{\tau_n^2}{\mathit{var}\left(y_n\middle| F_{M+1,n-1}\right)};
\end{aligned}$$

or equivalently (A3a). Taking one lag in equation (A2) yields:

$$\sigma^2_{f,n-1} = \frac{\theta^2}{M}\left[\Lambda_{n-1} + (M-1)\,\Omega_{n-1}\right],$$

giving the recursion:

$$\sigma_{f,n-1}^2 - \sigma_{f,n}^2 = \frac{\theta^2}{M} \left[\Lambda_{n-1} - \Lambda_n + (M-1) \left(\Omega_{n-1} - \Omega_n \right) \right] = \theta^2 \left(\Lambda_{n-1} - \Lambda_n \right),$$

where the last equality follows by eq. (A3a). Now consider the variance of average signals $\bar{\Lambda}_n(G)$. By eq. (10), $\bar{\Lambda}_n(G)$ can be expressed in terms of the elements in the individual signals variance-covariance matrix Ψ_n as:

$$\bar{\Lambda}_n(G) = \frac{(\Lambda_n + 2G\Omega_n)}{\hat{G}},$$

and eq. (A3c) follows by eq. (A3a), and by simple computations. We now provide the update for $\bar{\Omega}_n(k,G)$, thus completing the specification of the variance-covariance matrix $\bar{\Psi}_n(G) \equiv E\left[\left(\bar{s}_{1,0} - t_n, \cdots, \bar{s}_{M,0} - t_n \right)^\top \left(\bar{s}_{1,0} - t_n, \cdots, \bar{s}_{M,0} - t_n \right) \Big| F_{M+1,n} \right]$. By eq. (A3a) and the expression of the off-diagonal elements in $\bar{\Psi}_n(G)$ [see eqs. (11a) and (11b) evaluated at n],

$$\bar{\Omega}_{n-1}(k,G) - \bar{\Omega}_n(k,G) = \Lambda_{n-1} - \Lambda_n, \quad \text{all } k, G.$$

Finally, the following recursion is readily obtained by eq. (12),

$$\bar{\Gamma}_{n-1}(G) - \bar{\Gamma}_n(G) = (M-1)\left(\Lambda_{n-1} - \Lambda_n \right). \blacksquare$$

So the off-diagonal elements of $\bar{\Psi}_n(G) \equiv E\left[\left(\bar{s}_{1,0}, \cdots, \bar{s}_{M,0} \right)^\top \left(\bar{s}_{1,0}, \cdots, \bar{s}_{M,0} \right) \Big| F_{M+1,n} \right]$ depend on G, while the difference $\bar{\Psi}_{n-1}(G) - \bar{\Psi}_n(G)$ does not,

$$\bar{\Psi}_{n-1}(G) - \bar{\Psi}_n(G) = \left(\Lambda_{n-1} - \Lambda_n \right) \mathbf{1}\mathbf{1}^\top.$$

To easy notation, we now supress the dependence of the various coefficients on G.

Derivation of eq. (21). By the definition of t_n and s_n,

$$t_n - t_{n-1} = E\left(s_{i,0} - t_{n-1} \middle| F_{M+1,n} \right) = E\left(s_{i,n-1} \middle| F_{M+1,n} \right) = \zeta_n y_n,$$

where ζ_n is the regression coefficient of $s_{i,n-1}$ (or equivalently of $\bar{s}_{i,n-1}$) on y_n, viz

$$\zeta_n = \frac{cov\left(s_{i,n-1}, y_n \middle| F_{M+1,n-1} \right)}{var\left(y_n \middle| F_{M+1,n-1} \right)} = \frac{\tau_n}{var\left(y_n \middle| F_{M+1,n-1} \right)}. \blacksquare \qquad \text{(A5)}$$

Derivation of eq. (22). We have:

$$E\left(\frac{1}{M} \sum_{i=1}^{M} \bar{s}_{i,n-1} \middle| F_{M+1,n} \right) = \frac{1}{M} E\left(\sum_{i=1}^{M} s_{i,n-1} \middle| F_{M+1,n} \right) = \zeta_n y_n.$$

By eq. (16), $p_{n-1} = \theta t_{n-1}$. Hence,

$$\bar{s} - \frac{1}{\theta} p_{n-1} = \bar{s} - t_{n-1} = \frac{1}{M} \sum_{i=1}^{M} (\bar{s}_{i,0} - t_{n-1}) = \frac{1}{M} \sum_{i=1}^{M} \bar{s}_{i,n-1}.$$

Therefore,

$$E\left(\theta \bar{s} - p_{n-1} \middle| F_{M+1,n}\right) = \theta E\left(\frac{1}{M} \sum_{i=1}^{M} \bar{s}_{i,n-1} \middle| F_{M+1,n} \right) = \theta \zeta_n y_n.$$

But by eq. (6), $E\left[E\left(f|\,\mathbf{s}_0\right)|\,F_{M+1,n}\right] = E\left(\theta \bar{s}|\,F_{M+1,n}\right)$. Hence, by the Law of Iterated Expectations,

$$E\left(f - p_{n-1}|F_{M+1,n}\right) = E\left[E\left(f|\,\mathbf{s}_0\right) - p_{n-1}|F_{M+1,n}\right] = E\left(\theta \bar{s} - p_{n-1}|\,F_{M+1,n}\right) = \theta \zeta_n y_n,$$

and eq. (22) follows by eq. (20) and $E\left(\theta \bar{s}|\,F_{M+1,n}\right) = p_n$. ■

The following lemma is needed to derive eqs. (23)-(24).

Lemma 2. *For all $i, j = 1, \cdots, M$ and $n = 2, \cdots, N$,*

$$E\left(s_{j,n-1}|\,F_{i,n}\right) = E\left(s_{j,n-1}|\,s_{i,n-1}\right).$$

Proof. By eqs. (19), (21) and (22), the aggregate order flow can be recursively written as

$$y_n = \left(\frac{\beta_n a_{n-1}}{\beta_{n-1}} \right) y_{n-1} + \varepsilon_n,$$

where $a_{n-1} \equiv 1 - \theta^{-1} \hat{G} M \beta_{n-1} \lambda_{n-1}$ and $\varepsilon_n \equiv u_n - \beta_n \beta_{n-1}^{-1} u_{n-1}$. Solving backward yields,

$$y_n = b_{n-1} y_1 + \sum_{j=1}^{n-2} b_j \varepsilon_{n-j} + \varepsilon_n,$$

where $b_j \equiv \beta_n \beta_{n-j}^{-1} \prod_{h=1}^{j} a_{n-h}$. Hence, $\{y_n\}_{n \geq 1}$ is a Gaussian process. Therefore, it is sufficient to show that $cov[s_{j,n-1}, (y_1, \cdots, y_{n-1})^\top] = \mathbf{0}_{(n-1)\times 1}$. For all $k \leq n-1$,

$$\begin{aligned} cov\left(y_k, s_{j,n-1}\right) &= E\left(y_k \cdot s_{j,n-1}\right) \\ &= E\left[E\left(y_k \cdot s_{j,n-1}|\,y_1, \cdots, y_{n-1}\right)\right] \\ &= E\left[y_k \cdot E\left(s_{j,n-1}|\,y_1, \cdots, y_{n-1}\right)\right] \\ &= 0, \end{aligned}$$

where the first line follows because $E\left(y_1, \cdots, y_{n-1}\right)^\top = \mathbf{0}_{(n-1)\times 1}$, the second line holds by the Law of Iterated Expectations, and the last line follows by the definition of $s_{j,n-1}$. ■

Derivation of eqs. (23)-(24). We have:

$$\begin{aligned}
E\left(f-p_{n-1}\right|F_{i,n}) &= E\left[E\left(f-p_{n-1}\right|\mathbf{s}_0)\right|\mathbf{s}_{i,0},F_{M+1,n-1}] \\
&= E\left(\theta\bar{s}-p_{n-1}\right|\mathbf{s}_{i,0},F_{M+1,n-1}) \\
&= \theta E\left(\frac{1}{M}\sum_{i=1}^{M}\bar{s}_{i,n-1}\middle|\mathbf{s}_{i,n-1},F_{M+1,n-1}\right) \\
&= \theta E\left(\frac{1}{M}\sum_{i=1}^{M}s_{i,n-1}\middle|\mathbf{s}_{i,n-1},F_{M+1,n-1}\right) \\
&= \frac{\theta}{M}E\left(s_{i,n-1}+\sum_{j\neq i}s_{j,n-1}\middle|\bar{s}_{i,n-1}\right) \\
&= \frac{\theta}{M}\left(1+\frac{\bar{\Gamma}_{n-1}}{\bar{\Lambda}_{n-1}}\right)\bar{s}_{i,n-1},
\end{aligned}$$

by the Law of Iterated Expectations, the fact that $\theta\bar{s}-p_{n-1}=\dfrac{\theta}{M}\sum_{i=1}^{M}\bar{s}_{i,n-1}$ [see the derivation of eq. (22)], and Lemma 1. This is eq. (23) with $\eta_n=\theta(\hat{G}M)^{-1}(1+\bar{\Lambda}_{n-1}^{-1}\bar{\Gamma}_{n-1})$. By the same arguments,

$$E\left(\sum_{j\neq i}\bar{s}_{j,n-1}\middle|F_{i,n}\right)=\frac{\bar{\Gamma}_{n-1}}{\bar{\Lambda}_{n-1}}\bar{s}_{i,n-1},$$

which is eq. (24) with $\phi_n=(M-1)^{-1}\bar{\Lambda}_{n-1}^{-1}\bar{\Gamma}_{n-1}$. ■

B. Proof of Proposition 1

We proceed in three steps. In the first step, we derive a recursive expression for the price deviation induced by traders' suboptimal play. In the second step, we derive the traders' optimality conditions. In the third step, we compute market maker updates.

Step 1: Price deviation

We show that the price deviation induced by suboptimal play of a given trader i has the following recursive structure:

$$p_n-p_n'=\left(p_{n-1}-p_{n-1}'\right)\left[1-\theta^{-1}\hat{G}\left(M-1\right)\beta_n\lambda_n\right]+\hat{G}\lambda_n\beta_n\bar{s}_{i,n-1}-\lambda_n x_{i,n}'. \tag{B1}$$

Indeed, let $y_n'=\sum_{j\neq i}x_{j,n}+x_{i,n}'+u_n$ and t_n' be the aggregate order flow and the market maker's update when trader i deviates to $x_{i,n}'$. By eq. (21),

$$t_n=\sum_{k=1}^{n}\zeta_k y_k.$$

Similarly $t'_n = \sum_{k=1}^{n} \zeta_k y'_n$. Therefore, by eq. (17),

$$\begin{aligned} \bar{s}_{j,n-1} - \bar{s}'_{j,n-1} &= \left(\bar{s}_{j,0} - t_{n-1}\right) - \left(\bar{s}_{j,0} - t'_{n-1}\right) \\ &= \sum_{k=1}^{n-1} \zeta_k y'_k - \sum_{k=1}^{n-1} \zeta_k y_k \\ &= \frac{1}{\theta}\left(\sum_{k=1}^{n-1} \lambda_k y'_k - \sum_{k=1}^{n-1} \lambda_k y_k\right) \\ &= \frac{1}{\theta}\left(p'_{n-1} - p_{n-1}\right), \end{aligned} \tag{B2}$$

where the third line follows by eq. (22), and the fourth line follows because eq. (20) implies that $p_n = \sum_{k=1}^{n} \lambda_k y_k$. Thus, by eq. (17) and eq. (21),

$$\bar{s}_{i,n} = \bar{s}_{i,0} - t_n = \bar{s}_{i,n-1} - (t_n - t_{n-1}) = \bar{s}_{i,n-1} - \zeta_n y_n.$$

Substituting for the equilibrium order flow, using the property that $\zeta_n = \lambda_n/\theta$ in eq. (22), and taking expectations yields:

$$\begin{aligned} E\left(\bar{s}_{i,n}\middle| F_{i,n}\right) &= \bar{s}_{i,n-1} - \frac{\hat{G}\beta_n\lambda_n}{\theta}\left[\bar{s}_{i,n-1} + E\left(\textstyle\sum_{i\neq j} \bar{s}_{i,n-1}\middle| F_{i,n}\right)\right] \\ &= \left[1 - \frac{\hat{G}\beta_n\lambda_n}{\theta}\left(1 + (M-1)\,\phi_n\right)\right]\bar{s}_{i,n-1}, \end{aligned} \tag{B3}$$

where the last line follows by eq. (24). Using the equilibrium strategy in eq. (19) and the price recursion in eq. (20), we find that the price deviation has the following expression:

$$\begin{aligned} p_n - p'_n &= p_{n-1} - p'_{n-1} + \lambda_n\left(y_n - y'_n\right) \\ &= p_{n-1} - p'_{n-1} + \lambda_n\left[\textstyle\sum_{j\neq i} \hat{G}\beta_n\left(\bar{s}_{j,n-1} - \bar{s}'_{j,n-1}\right) + \hat{G}\beta_n\bar{s}_{i,n-1} - x'_{i,n}\right]. \end{aligned}$$

Substituting for $(\bar{s}_{j,n-1} - \bar{s}'_{j,n-1})$ from eq. (B2) in the previous equation gives eq. (B1).

Step 2: Traders' strategies

First, we show that the value function in eq. (25) and the strategy in eq. (26) are mutually consistent. Any trader i faces the following recursive problem:

$$\begin{aligned} W_{i,n-1} &= \max_{x'_{i,n}} E\left[(f - p_n)\,x'_{i,n} + W_{i,n}\middle| F_{i,n}\right] \\ &= \max_{x'_{i,n}} E\left[\left(f - p_{n-1} - \lambda_n x'_{i,n} - \lambda_n \textstyle\sum_{j\neq i} x_{j,n}\right)x'_{i,n} + W_{i,n}\middle| F_{i,n}\right]. \end{aligned} \tag{B4}$$

Given the trading strategy conjectured in eq. (25), the optimality conditions of the previous problem lead to:

$$\begin{aligned} 0 \;=\;& E\left(f-p'_{n-1}\middle| F_{i,n}\right)-\hat{G}\beta_n\lambda_n E\left(\textstyle\sum_{j\neq i}\bar{s}'_{j,n-1}\middle| F_{i,n}\right)\\ &-2\lambda_n x'_{i,n}-\lambda_n\psi_n E\left(\bar{s}_{i,n}\middle| F_{i,n}\right)-2\lambda_n\mu_n E\left(p_n-p'_n\middle| F_{i,n}\right)\ \text{(first order conditions);}\end{aligned}$$

and

$$-\lambda_n+\lambda_n^2\mu_n<0\ \text{(second order conditions).}$$

Because $\left(\bar{s}_{j,n-1},\left(p_{n-1}-p'_{n-1}\right)\right)\in F_{i,n}$, the first order conditions can be reorganized as follows:

$$\begin{aligned} 0=&\,E\left(f-p_{n-1}\middle| F_{i,n}\right)+\left(p_{n-1}-p'_{n-1}\right)-\hat{G}\beta_n\lambda_n\textstyle\sum_{j\neq i}\left(\bar{s}'_{j,n-1}-\bar{s}_{j,n-1}\right)\\ &-\hat{G}\beta_n\lambda_n E\left(\textstyle\sum_{j\neq i}\bar{s}_{j,n-1}\middle| F_{i,n}\right)-2\lambda_n x'_{i,n}-\lambda_n\psi_n E\left(\bar{s}_{i,n}\middle| F_{i,n}\right)-2\lambda_n\mu_n E\left(p_n-p'_n\middle| F_{i,n}\right).\end{aligned}$$

By replacing eqs. (B1), (B2) and (B3) in the previous equation, and by rearranging terms, we obtain eq. (26), where γ_n is as in eq. (29) and

$$\beta_n=\frac{\eta_n-\hat{G}^{-1}\lambda_n\psi_n}{\lambda_n\left[1+\left(1-\theta^{-1}\lambda_n\psi_n\right)\left(1+(M-1)\phi_n\right)\right]}. \tag{B5}$$

Next, we use eq. (26), and find that the expected profit in any single auction is:

$$\begin{aligned} &E\left[\left(f-p_n\right)x'_{i,n}\middle| F_{i,n}\right]\\ &=\hat{G}^2\beta_n\left[\eta_n-\beta_n\lambda_n\left(1+(M-1)\phi_n\right)\right]\bar{s}^2_{i,n-1}\\ &\quad+\gamma_n\left[1-\lambda_n\left(\gamma_n+\theta^{-1}\hat{G}(M-1)\beta_n\right)\right]\left(p_{n-1}-p'_{n-1}\right)^2\\ &\quad+\left\{\gamma_n\left(\eta_n-2\beta_n\lambda_n\right)+\beta_n\left[1-(M-1)\lambda_n\left(\gamma_n\phi_n+\theta^{-1}\hat{G}\beta_n\right)\right]\right\}\hat{G}\bar{s}_{i,n-1}\left(p_{n-1}-p'_{n-1}\right). \end{aligned}\tag{B6}$$

By taking the conditional expectation of the value function in eq. (25) leaves:

$$E\left(W_{i,n}\middle| F_{i,n}\right)=\alpha_n E\left(\bar{s}^2_{i,n}\middle| F_{i,n}\right)+\psi_n\left(p_n-p'_n\right)E\left(\bar{s}_{i,n}\middle| F_{i,n}\right)+\mu_n\left(p_n-p'_n\right)^2+\delta_n. \tag{B7}$$

By eqs. (B1) and (B3), both $E\left(\bar{s}_{i,n}|F_{i,n}\right)$ and $\left(p_n-p'_n\right)$ are linear in $\left(p_{n-1}-p'_{n-1}\right)$ and $\bar{s}_{i,n-1}$. To identify all coefficients of the value function, we are therefore left with finding the conditional expectation $E(\bar{s}^2_{i,n}|F_{i,n})$. By eqs. (17) and (21),

$$\begin{aligned} &E\left(\bar{s}^2_{i,n}\middle| F_{i,n}\right)\\ &=\bar{s}^2_{i,n-1}+\zeta_n^2 E\left(y_n^2\middle| F_{i,n}\right)-2\zeta_n\bar{s}_{i,n-1}E\left(y_n\middle| F_{i,n}\right)\\ &=\left[1-\theta^{-1}\hat{G}\beta_n\lambda_n\left(1+(M-1)\phi_n\right)\right]^2\bar{s}^2_{i,n-1}+\theta^{-1}\lambda_n^2\sigma_u^2+\theta^{-1}\hat{G}^2\beta_n^2\lambda_n^2 var\left(\textstyle\sum_{j\neq i}\bar{s}_{j,n-1}\middle| F_{i,n}\right), \end{aligned}\tag{B8}$$

where

$$\begin{aligned} var\left(\left.\sum_{j\neq i}\bar{s}_{j,n-1}\right| F_{i,n}\right) &= var\left(\left.\sum_{j\neq i}\bar{s}_{j,n-1}\right| \bar{s}_{i,n-1}\right) \\ &= E\left[\sum_{j\neq i}\bar{s}_{j,n-1} - (M-1)\,\phi_n \bar{s}_{i,n-1}\right]^2 \\ &= var\left(\left.\sum_{j\neq i}\bar{s}_{j,0}\right| F_{M+1,n-1}\right) - (M-1)^2\,\phi_n^2 var\left(\left.\bar{s}_{i,0}\right| F_{M+1,n-1}\right) \\ &= M\left[\Lambda_{n-1} + (M-1)\,\Omega_{n-1}\right] - \left[1 + (M-1)^2\,\phi_n^2\right]\bar{\Lambda}_{n-1} - 2\bar{\Gamma}_{n-1}, \quad \text{(B9)} \end{aligned}$$

and the first equality follows by the arguments utilized to show Lemma 2 in appendix B. Next, plug eq. (B9) into eq. (B8), then plug the resulting expression into eq. (B7). Also, replace the expression for $p_n - p'_{n-1}$ from eq. (B1) into eq. (B7). Finally, plug the resulting expression for eq. (B7) and eq. (B6) into eq. (B4) and identify terms to obtain the recursions for the coefficients α_n, μ_n, ψ_n and δ_n in eq. (30).

Step 3: Market maker updates

Finally, we consider the market maker's problem. By plugging the equilibrium trades in eq. (19) into the order flow in eq. (3) gives

$$y_n = \sum_{i=1}^{M}\hat{G}\beta_n\bar{s}_{i,n-1} + u_n = \hat{G}M\beta_n\left[\frac{1}{M}\sum_{i=1}^{M}\left(\bar{s}_{i,0} - t_{n-1}\right)\right] + u_n. \quad \text{(B10)}$$

where the second line follows by the market maker's update in eq. (17). By the definition of $\bar{s}$, and the equality $t_{n-1} = \theta^{-1}p_{n-1}$ in eq. (16),

$$y_n = \hat{G}M\beta_n\left(\bar{s} - \frac{p_{n-1}}{\theta}\right) + u_n = \frac{\hat{G}M\beta_n}{\theta}\left(\theta\bar{s} - p_{n-1}\right) + u_n. \quad \text{(B11)}$$

By eqs. (B11) and (6), and the Law of Iterated Expectations,

$$cov\left(f, y_n\right| F_{M+1,n-1}) = cov\left(\theta\bar{s}, y_n\right| F_{M+1,n-1}).$$

Therefore,

$$\begin{aligned} cov\left(f, y_n\right| F_{M+1,n-1}) &= cov\left(\theta\bar{s} - p_{n-1}, y_n\right| F_{M+1,n-1}) \\ &= \frac{\hat{G}M\beta_n}{\theta} var\left(\theta\bar{s} - p_{n-1}\right| F_{M+1,n-1}) \\ &= \frac{\hat{G}M\beta_n}{\theta}\sigma_{f,n-1}^2, \quad \text{(B12)} \end{aligned}$$

where the first line follows because $p_{n-1} \in F_{M+1,n-1}$, the second line is obtained through the order flow in eq. (B11), and the third line is due to the expression of the residual variance in eq.

(18). We now re-write the recursion of Λ_n in terms of equilibrium parameters. Using the order flow in eq. (B10),

$$\begin{aligned}\tau_n &\equiv cov\left(s_{i,n-1}, y_n\right| F_{M+1,n-1}) \\ &= \hat{G}\beta_n cov\left(s_{i,n-1}, \sum_{i=1}^{M} s_{i,n-1}\right| F_{M+1,n-1}\Big) \\ &= \hat{G}\beta_n\left(\Lambda_{n-1} + (M-1)\,\Omega_{n-1}\right) \\ &= \frac{M\hat{G}\beta_n}{\theta^2}\sigma^2_{f,n-1},\end{aligned}$$

where the third line follows by the expression of the residual variances in eq. (18), and the last line holds by the expression of $\sigma^2_{f,n}$ in eq. (A2). Therefore, by eqs. (A4), (A5), the expression for τ_n found above, and eq. (22),

$$\Lambda_n = \Lambda_{n-1} - \zeta_n\tau_n = \Lambda_{n-1} - \frac{M\hat{G}\beta_n\lambda_n}{\theta^3}\sigma^2_{f,n-1}. \tag{B13}$$

Again by the above expression for τ_n, eqs. (22) and (A5),

$$\lambda_n = \theta\zeta_n = \frac{\theta\tau_n}{var\left(y_n\right| F_{M+1,n-1})} = \frac{M\hat{G}\beta_n\sigma^2_{f,n-1}}{\theta\cdot var\left(y_n\right| F_{M+1,n-1})}.$$

But

$$\begin{aligned}var\left(y_n\right| F_{M+1,n-1}) &= \left(\theta^{-1}\hat{G}M\beta_n\right)^2 var\left(\theta\bar{s} - p_{n-1}\right| F_{M+1,n-1}) + \sigma^2_u \\ &= \left(\theta^{-1}\hat{G}M\beta_n\right)^2 \sigma^2_{f,n-1} + \sigma^2_u.\end{aligned}$$

Therefore, the price sensitivity in eq. (20) can be represented as:

$$\lambda_n = \frac{\theta M\hat{G}\beta_n\sigma^2_{f,n-1}}{\left(\hat{G}M\beta_n\right)^2\sigma^2_{f,n-1} + \theta^2\sigma^2_u}. \tag{B14}$$

After n trading rounds, the full information fundamental value has residual variance given by:

$$\sigma^2_{f,n} = \sigma^2_{f,n-1} - \lambda_n cov\left(f, y_n|F_{M+1,n-1}\right) = \left(1 - \theta^{-1}\beta_n\lambda_n\hat{G}M\right)\sigma^2_{f,n-1}, \tag{B15}$$

where the last equality follows by the eq. (B12) and the first equality holds as $E\left(y_n|F_{M+1,n-1}\right) = 0$, and by the Law of Iterated Expectations,

$$\begin{aligned}cov\left(f, y_n|F_{M+1,n-1}\right) &= E\left(f\cdot y_n|F_{M+1,n-1}\right) \\ &= E\left[E\left(f\cdot y_n|F_{M+1,n}\right)|F_{M+1,n-1}\right] \\ &= E\left(p_n\cdot y_n|F_{M+1,n-1}\right) \\ &= E\left(p_{n-1}\cdot y_n|F_{M+1,n-1}\right) + E\left(\lambda_n\cdot y_n^2|F_{M+1,n-1}\right) \\ &= \lambda_n\cdot var\left(y_n|F_{M+1,n-1}\right).\end{aligned}$$

Next, we plug eq. (B14) into eq. (B15) and obtain:

$$\sigma_{f,n}^2 = \frac{\theta^2\sigma_u^2\sigma_{f,n-1}^2}{\left(\hat{G}M\beta_n\right)^2\sigma_{f,n-1}^2 + \theta^2\sigma_u^2}. \tag{B16}$$

By combining eqs. (B14) and (B16) we find an alternative expression for λ_n,

$$\lambda_n = \frac{\hat{G}M\beta_n}{\theta}\frac{\sigma_{f,n}^2}{\sigma_u^2}, \tag{B17}$$

or equivalently eq. (28). By solving eq. (B16) for $\sigma_{f,n-1}^2$ gives

$$\sigma_{f,n-1}^2 = -\frac{\theta^2\sigma_u^2\sigma_{f,n}^2}{\left(\hat{G}M\beta_n\right)^2\sigma_{f,n}^2 - \theta^2\sigma_u^2}. \tag{B18}$$

By combining eqs. (B13), (B17) and (B18), we find that Λ_n solves,

$$\Lambda_{n-1} - \Lambda_n = -\frac{\sigma_u^4}{\left(\hat{G}M\beta_n\right)^2\sigma_{f,n}^2 - \theta^2\sigma_u^2}\lambda_n^2 = -\frac{\lambda_n^2\sigma_u^2\sigma_{f,n}^2}{\theta^2\left(\lambda_n^2\sigma_u^2 - \sigma_{f,n}^2\right)}, \tag{B19}$$

where the last equality follows because $\left(\hat{G}M\beta_n\right)^2\sigma_{f,n}^2 = \theta^2\lambda_n^2\sigma_u^4\sigma_{f,n}^{-2}$ [due to eq. (28)]. Also, eqs. (A3c) and (A3e) imply that

$$\frac{\bar{\Gamma}_{n-1}}{\bar{\Lambda}_{n-1}} = \frac{\bar{\Gamma}_n + (M-1)(\Lambda_{n-1} - \Lambda_n)}{\bar{\Lambda}_n + (\Lambda_{n-1} - \Lambda_n)}. \tag{B20}$$

Furthermore, by eqs. (31)-(32):

$$\frac{\bar{\Gamma}_{n-1}}{\bar{\Lambda}_{n-1}} = (M-1)\,\phi_n = \frac{\hat{G}M}{\theta}\eta_n - 1. \tag{B21}$$

By combining eqs. (28) and (B5), we find that

$$\theta\sigma_u^2\lambda_n^2\left[1 + \left(1 - \theta^{-1}\lambda_n\psi_n\right)\left(1 + (M-1)\,\phi_n\right)\right] = \left(\hat{G}\eta_n - \lambda_n\psi_n\right)M\sigma_{f,n}^2.$$

Substituting eqs. (31)-(32) in the previous equation leaves

$$\theta\sigma_u^2\lambda_n^2\left[1 + \left(1 - \theta^{-1}\lambda_n\psi_n\right)\left(1 + \frac{\bar{\Gamma}_{n-1}}{\bar{\Lambda}_{n-1}}\right)\right] = \left[\frac{\theta}{M}\left(1 + \frac{\bar{\Gamma}_{n-1}}{\bar{\Lambda}_{n-1}}\right) - \lambda_n\psi_n\right]M\sigma_{f,n}^2.$$

The quartic equation $F(\lambda_n) = 0$ in eq. (27) is obtained by substituting eqs. (B19), (B20), (A2), (10) and (12) evaluated at n into the previous equation, and by tedious computations. To show that eq. (27) admits a unique positive solution, note that the constant and the coefficient of λ_n^4 are both positive, and that the coefficient of λ_n^2 is negative.[6] On the other hand, the sign of ψ_n determines the sign of the terms in λ_n^3 and λ_n. However, regardless of whether ψ_n is positive or negative, there are only two sign changes. By Descartes' rule, eq. (27) has at most two real positive roots. By eq. (B15), $\sigma_{f,n}^2 < \sigma_{f,n-1}^2 \Leftrightarrow \theta^{-1}\beta_n\lambda_n\hat{G}M < 1$. By eq. (28) this restriction becomes $\lambda_n^2 < \sigma_{f,n}^2\sigma_u^{-2}$ or equivalently $\lambda_n < \sigma_{f,n}\sigma_u^{-1}$. By eq. (27), $F(\lambda = 0) = \frac{\theta}{M}[\Lambda_n + (M-1)\Omega_n] > 0$, $F\left(\lambda_n = \sigma_{f,n}\sigma_u^{-1}\right) = -\frac{\theta}{M^2}(\Lambda_n + (M-1)\Omega_n) < 0$ and $F(\lambda = +\infty) = +\infty$; hence, there is one and only one positive root between 0 and $\sigma_{f,n}\sigma_u^{-1}$. ■

C. Computation of the equilibrium

We solve for the equilibrium using backward induction. By eq. (A3a), $\Lambda_n - \Omega_n = \Lambda_0 - \Omega_0$. We fix a terminal value for Λ_N and compute $\Omega_N = \Lambda_N + \Omega_0 - \Lambda_0$. $\sigma_{f,N}^2$ then follows by eq. (A2). Since $\alpha_N = \psi_N = \mu_N = \delta_N = 0$, we solve for λ_N in eq. (27), which yields β_N and γ_N through eqs. (28)-(29). To compute the value function coefficients as of at time $N-1$, one needs to express Λ_{N-1} and Ω_{N-1} in terms of variables known at time N. Below, we show that:

$$\Lambda_{n-1} = \frac{\theta\Lambda_n - \hat{G}(M-1)\lambda_n\beta_n(\Lambda_n - \Omega_n)}{\theta - \hat{G}M\lambda_n\beta_n}. \tag{C1}$$

Then, Λ_{N-1} is obtained by evaluating eq. (C1) at $n = N$, and Ω_{N-1} is obtained by the equality $\Omega_{N-1} = \Lambda_{N-1} + \Omega_0 - \Lambda_0$. Finally, we retrieve regression coefficients ϕ_N and η_N through eqs. (31)-(32) and the equality $\bar{\Gamma}_{N-1} = \bar{\Gamma}_N + (M-1)(\Lambda_{N-1} - \Lambda_N)$ [see eq. (A3e)]. The value function coefficients at time $N-1$ are therefore uniquely determined by eq. (30).

The above procedure is then applied at each trading round $n \in [1, N]$ yielding the inital value of Λ_0 implied by the choice of the terminal value of Λ_N. The resulting initial value of Λ_0 is then compared to the one we posited as initial parameter, and the procedure is repeated for different choices of Λ_N until convergence is achieved.

Derivation of eq. (C1). Taking one lag in eq. (A2) and substituting the result into eq. (B13) yields:

$$\Lambda_{n-1} = \Lambda_n + \frac{\hat{G}\beta_n\lambda_n}{\theta}[\Lambda_{n-1} + (M-1)\Omega_{n-1}].$$

[6] Let $\rho_n = \Omega_n/\Lambda_n$ be the correlation coefficient between individual signals. Since $|\rho_n| \leq 1$, then $\Lambda_n - \Omega_n \geq 0$ and the coefficient for λ_n^4 is non-negative. Moreover $\Lambda_n + (M-1)\Omega_n$ is positive due to (7), and a fortiori $2\Lambda_n + (M-1)\Omega_n > 0$ since $\Lambda_n > 0$.

Since $\Omega_{n-1} - \Omega_n = \Lambda_{n-1} - \Lambda_n$ [see eq. (A3a)],

$$\Omega_{n-1} = \Omega_n + \frac{\hat{G}\beta_n\lambda_n}{\theta}\left(\Lambda_{n-1} - \Omega_{n-1} + M\Omega_{n-1}\right) = \frac{\theta\Omega_n + \hat{G}\beta_n\lambda_n\left(\Lambda_{n-1} - \Omega_{n-1}\right)}{\theta - M\hat{G}\beta_n\lambda_n}.$$

By solving for Ω_{n-1} we find that

$$\Omega_{n-1} = \frac{\theta\Omega_n + \hat{G}\beta_n\lambda_n\left(\Lambda_{n-1} - \Omega_{n-1}\right)}{\theta - \hat{G}\beta_n\lambda_n M}. \tag{C2}$$

Finally,

$$\Lambda_{n-1} = \Lambda_n + \Omega_{n-1} - \Omega_n = \frac{\theta\Lambda_n - \hat{G}\beta_n\lambda_n\left[M\left(\Lambda_n - \Omega_n\right) - \left(\Lambda_{n-1} - \Omega_{n-1}\right)\right]}{\theta - \hat{G}\beta_n\lambda_n M},$$

where we have used eq. (C2). Eq. (C1) follows by replacing $\Lambda_n - \Omega_n = \Lambda_{n-1} - \Omega_{n-1}$ in the previous equation. ■

Computation of the correlation among volumes. By definition, the correlation among the volume generated by any two traders i and j is,

$$\rho_{\mathrm{V},n}\left(i,j\right) \equiv \frac{cov\left(\left|x_{i,n}\right|, \left|x_{j,n}\right|\right)}{var\left(\left|x_{i,n}\right|\right)}, \quad \text{for } i \neq j. \tag{C3}$$

To produce the results in Figure 10, we set $\rho = 0.1$ or $\rho = 0.9$. We simulated $S = 5000$ values of the initial signals $(s_{i,0})_{i=1}^{M}$; and we computed the dynamics of the individual trades over the ten batch auctions. Thus, we obtained a sequence $(x_{i,n}^{s})_{s=1}^{S}$ of trades for each trader i and each batch auction n. We estimated $\rho_{\mathrm{V},n}\left(i,j\right)$ in eq. (C3) through the estimator $\hat{\rho}_{S,\mathrm{V},n}\left(i,j\right)$ given below,

$$\hat{\rho}_{S,\mathrm{V},n}\left(i,j\right) \equiv \frac{\sum_{s=1}^{S}\left(\left|x_{i,n}^{s}\right| - \frac{1}{S}\sum_{s=1}^{S}\left|x_{i,n}^{s}\right|\right)\left(\left|x_{j,n}^{s}\right| - \frac{1}{S}\sum_{s=1}^{S}\left|x_{j,n}^{s}\right|\right)}{\sqrt{\sum_{s=1}^{S}\left(\left|x_{i,n}^{s}\right| - \frac{1}{S}\sum_{s=1}^{S}\left|x_{i,n}^{s}\right|\right)^2 \cdot \sum_{s=1}^{S}\left(\left|x_{j,n}^{s}\right| - \frac{1}{S}\sum_{s=1}^{S}\left|x_{j,n}^{s}\right|\right)^2}}.$$

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Figures

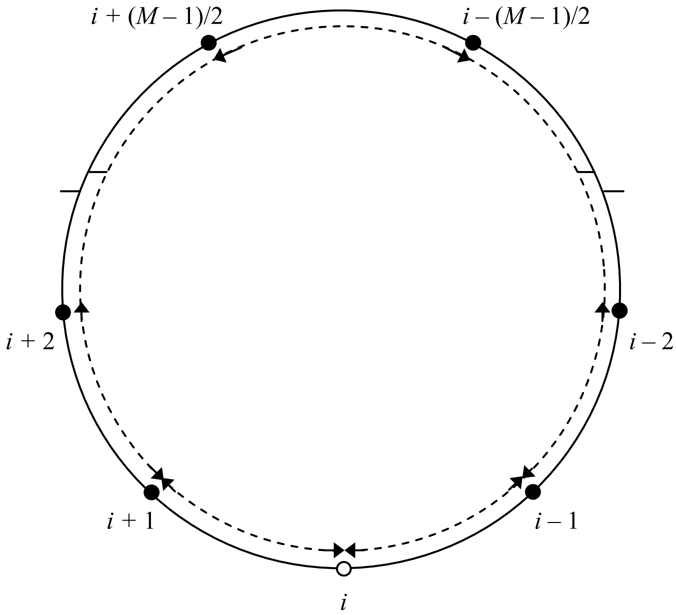
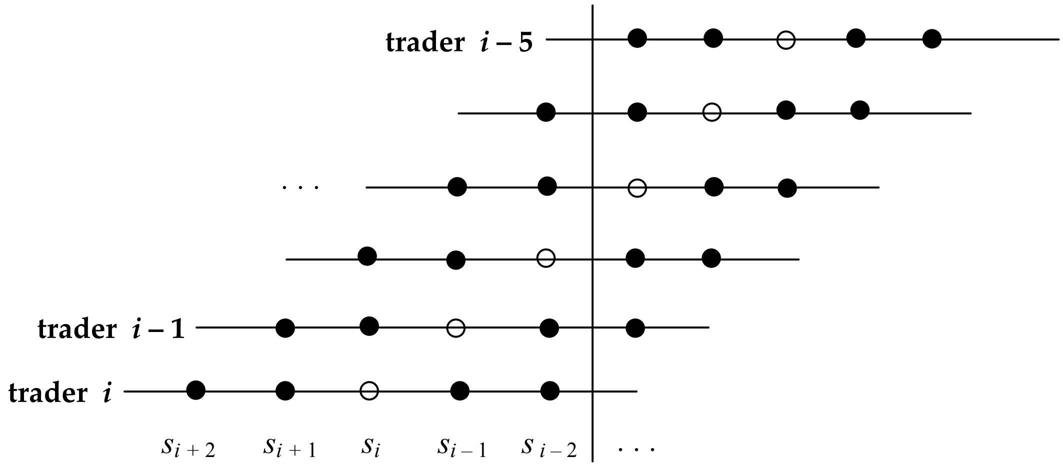


Figure 1: Geographical location of traders

This figure depicts an example of information linkages among M traders who are physically located on a circle. Every filled circle represents the signal available at the location of each trader. Every trader has (M–1)/2 traders to his left and (M–1)/2 traders to his right. In this example, every trader has 2G=2 information linkages. The signal available at the location of the i–th trader (the empty circle) is also observed by one trader on his left and one trader on his right.



Signals available at the location of the i–th trader

Figure 2: Overlapping information sets
This figure illustrates how signals overlap across the traders, when there are $M \geq 11$ such traders and each of them experience $2G=4$ information linkages with his peers. The empty circles denote the signals that are available at the location of the traders. The filled circles denote the signals every trader observes on top of the signal available at his location. The signals that are at the left (resp. right) of the empty circles are those signals that are available at the location of left (resp. right) neghbors.

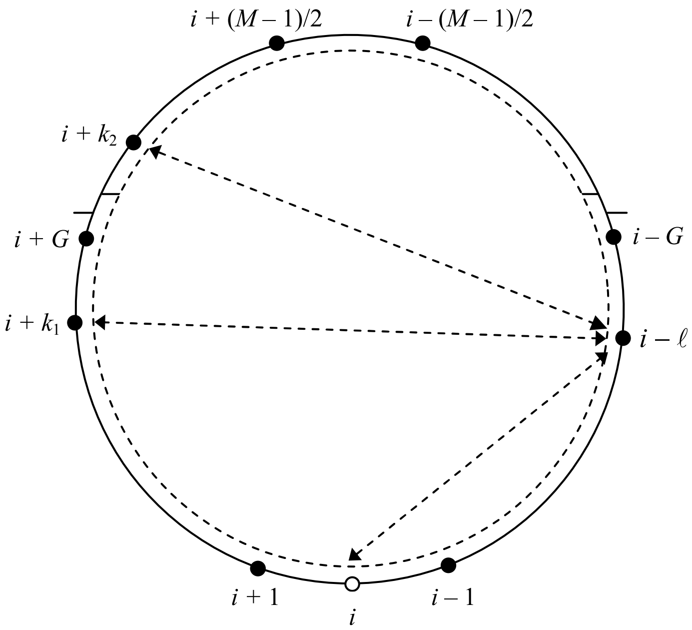


Figure 3: "Double overlap"

This figure illustrates the "double overlap" feature of the model, which emerges whenever the number of information linkages among traders is large, i.e. $2G \geq (M\text{-}1)/2$. In this particular example, trader i does not share the signal available at his location with trader $i + k_2$, but traders $i + k_2$ and $i - \ell$ do share at least the signals available at their location. As a result, traders i and trader $i + k_2$ do indeed share information.

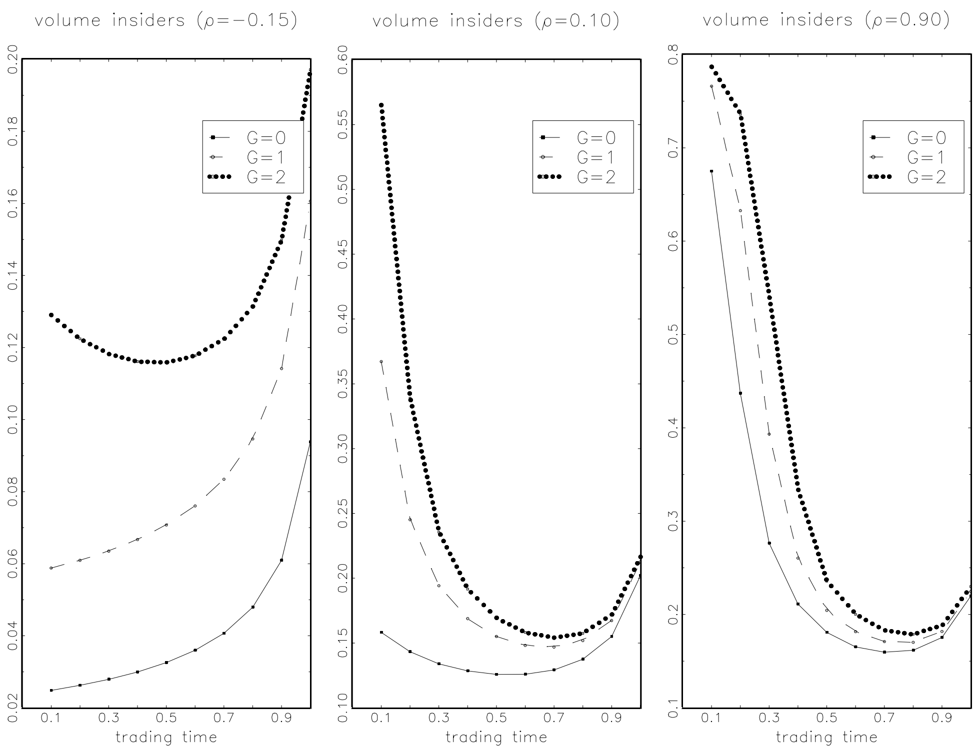


Figure 4: Volume

The dynamics of (traders) volume in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity traders variance across all periods and ten trading rounds. The left-hand side panel depicts the dynamics of volume when the initial correlation among the signals available at the traders' information linkages is negative (ρ=-0.15). The remaining panels depict the dynamics of volume when this correlation is positive but low (ρ=0.10) (middle panel) and high (ρ=0.90) (right-hand side panel). In each panel, we display the dynamics of volume arising when each trader has a number of information linkages equal to 2*G*, with *G*=0,1 and 2.

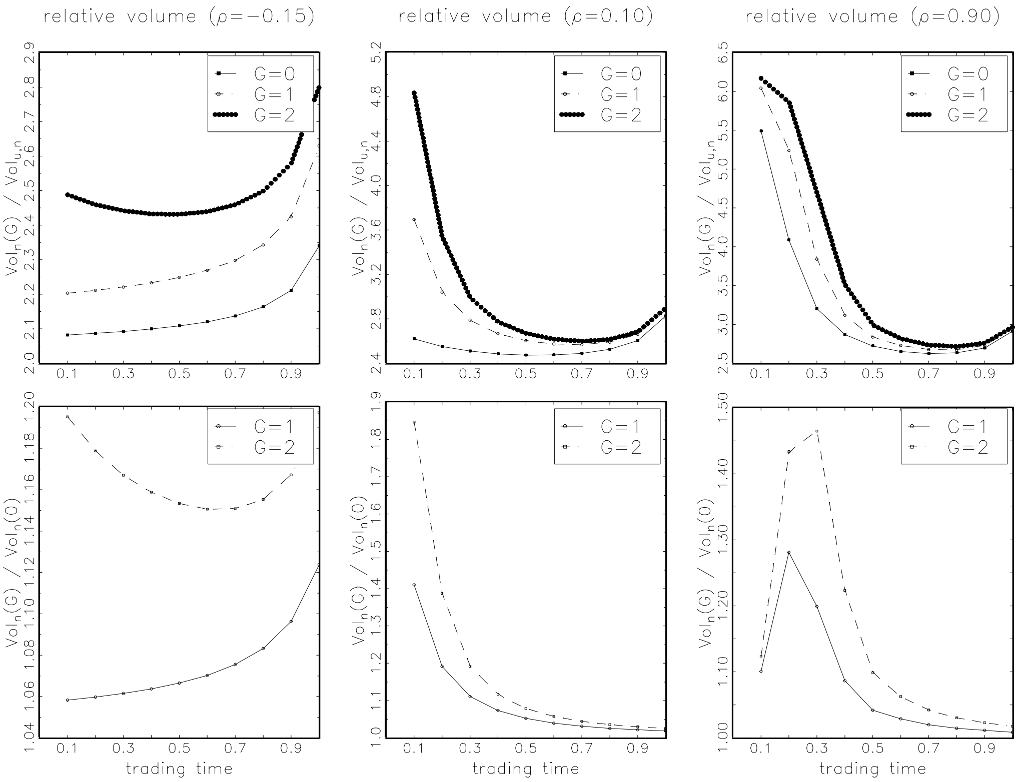


Figure 5: Relative volume

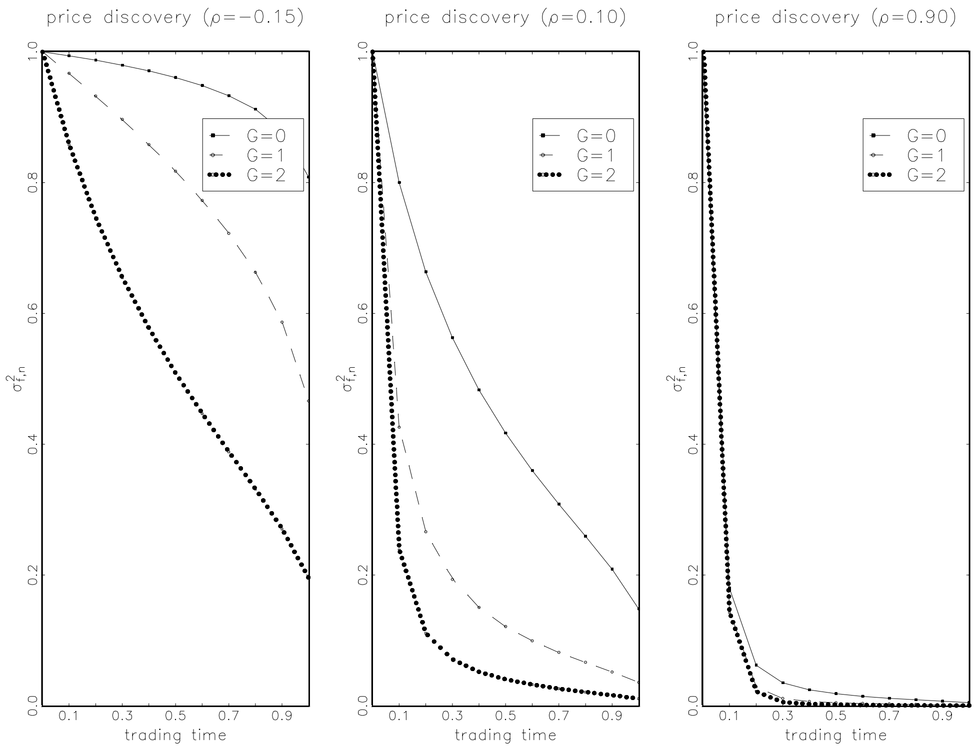


Figure 6: Efficiency

The price-discovery process in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panel depicts the price-discovery process arising when the initial correlation among the signals available at the traders' information linkages is negative (ρ=-0.15). The remaining panels depict price-discovery process arising when this correlation is positive but low (ρ=0.10) (middle panel) and high (ρ=0.90) (right-hand side panel). In each panel, we display the price-discovery process arising when each trader has a number of information linkages equal to 2G, with G=0,1 and 2.

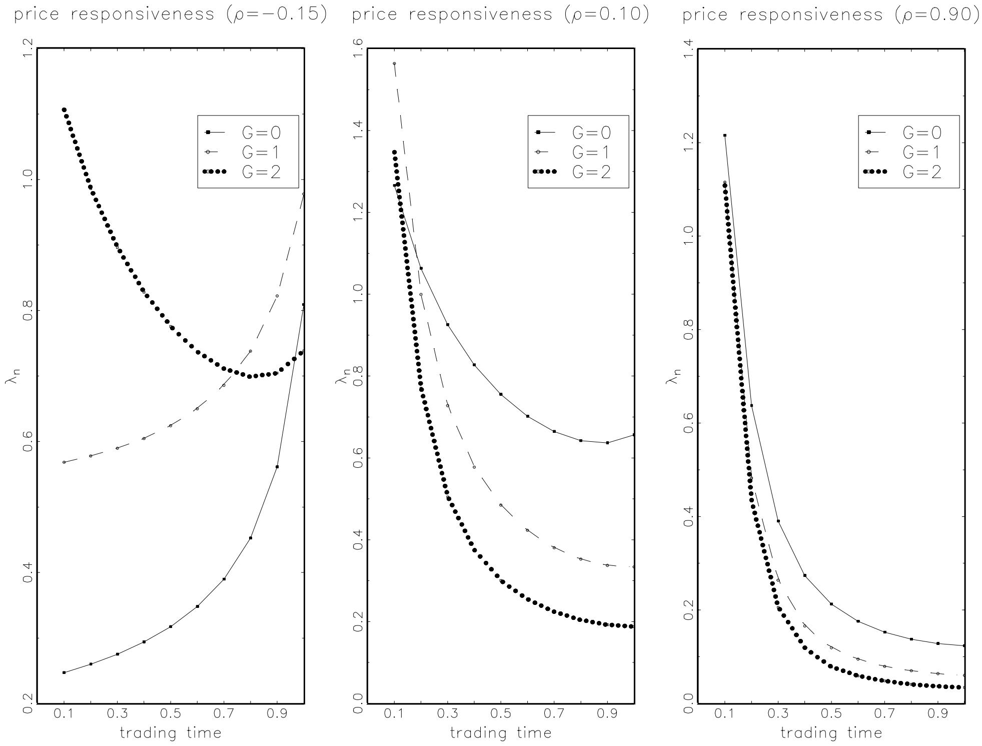


Figure 7: Price impacts

Price-impacts in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panel depicts the dynamics of price-impacts arising when the initial correlation among the signals available at the traders' information linkages is negative (ρ=-0.15). The remaining panels depict the dynamics of price-impacts when this correlation is positive but low (ρ=0.10) (middle panel) and high (ρ=0.90) (right-hand side panel). In each panel, we display the dynamics of price-impacts arising when each trader has a number of information linkages equal to 2G, with G=0,1 and 2.

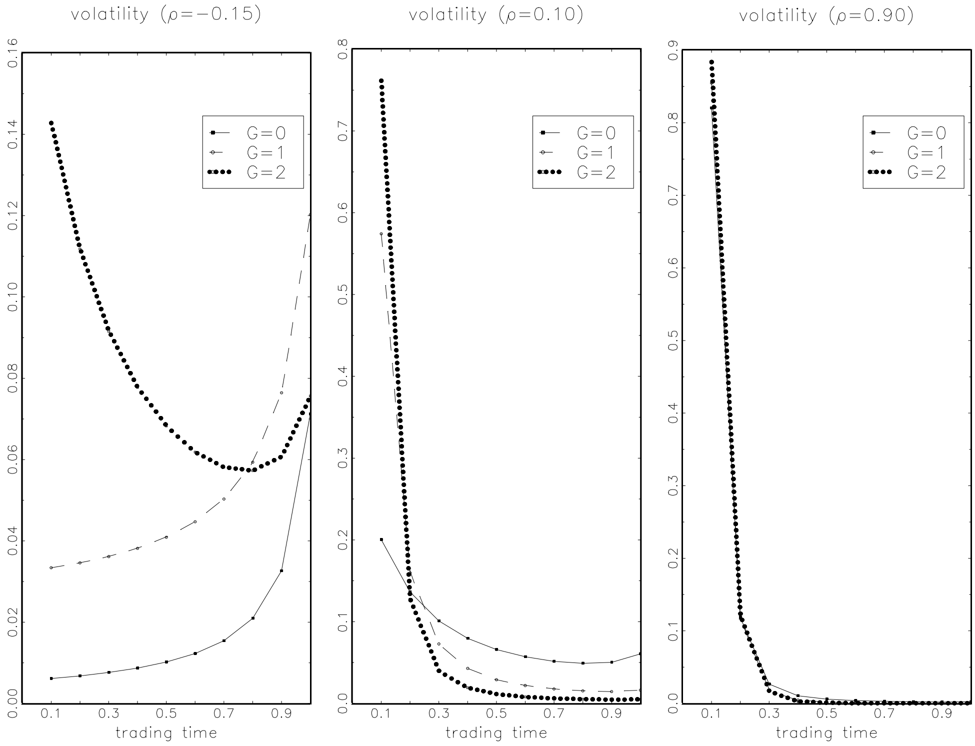


Figure 8: Volatility

Asset return volatility in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panel depicts the dynamics of return volatility arising when the initial correlation among the signals available at the traders' information linkages is negative (ρ=-0.15). The remaining panels depict the dynamics of return volatility when this correlation is positive but low (ρ=0.10) (middle panel) and high (ρ=0.90) (right-hand side panel). In each panel, we display the dynamics of return volatility arising when each trader has a number of information linkages equal to 2G, with G=0,1 and 2.

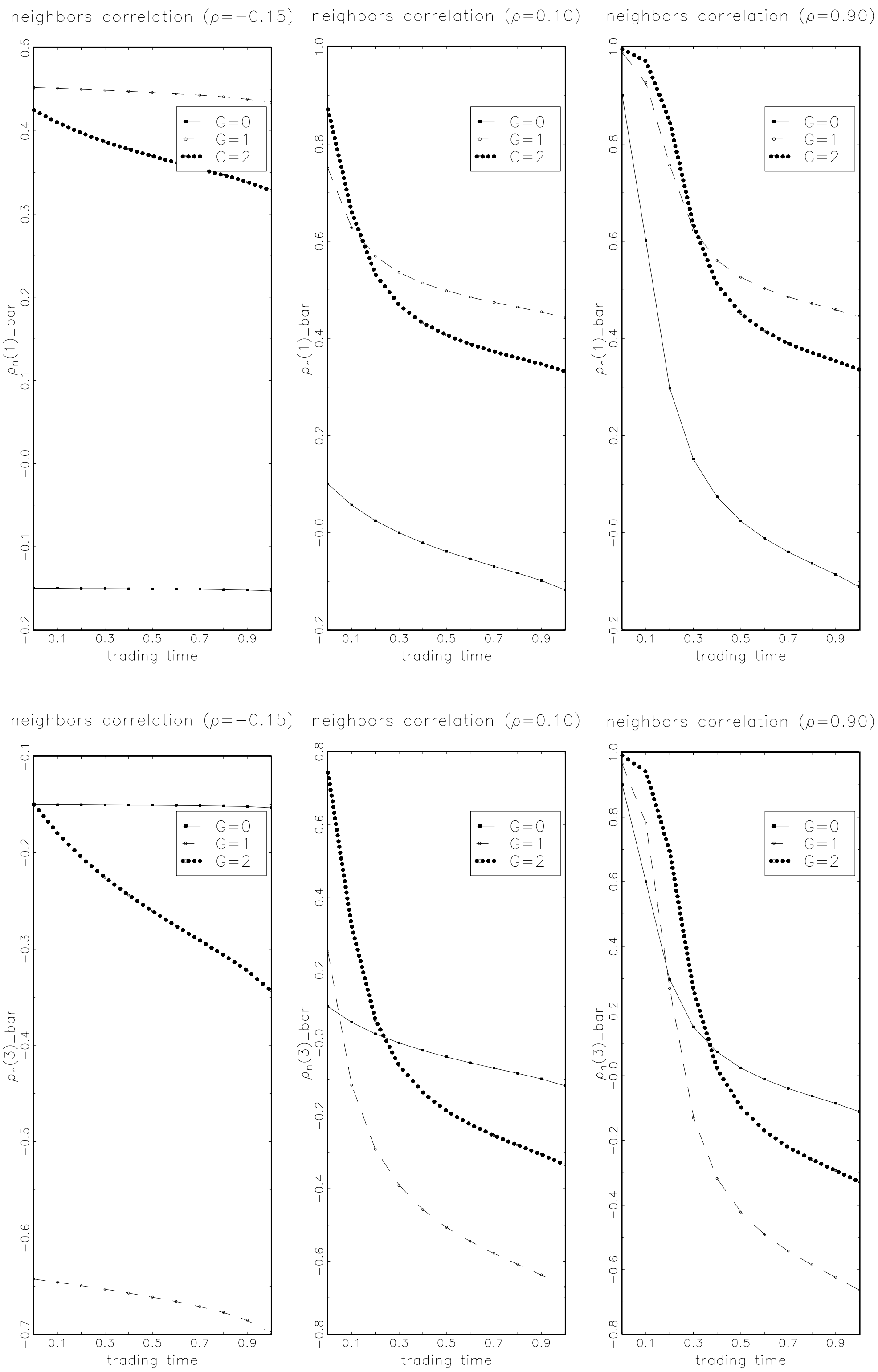


Figure 9: Heterogeneity in correlation among trades

Correlation among trades in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panels depict the dynamics of correlation among trades arising when the initial correlation among the signals available at the traders' information linkages is negative (ρ=-0.15). The remaining panels depict the dynamics of correlation among trades when this correlation is positive but low (ρ=0.10) (middle panels) and high (ρ=0.90) (right-hand side panels). The top panels depict the correlation dynamics between two close neighbors, i.e. the dynamics of trade correlation between traders *i* and *i*-1. The bottom panels depict the correlation dynamics between two distant traders, i.e. the dynamics of correlation between traders i and *i*-3. In each case, we display trades the dynamics of correlation arising when each trader has a number of information linkages equal to 2*G*, with *G*=0,1 and 2.

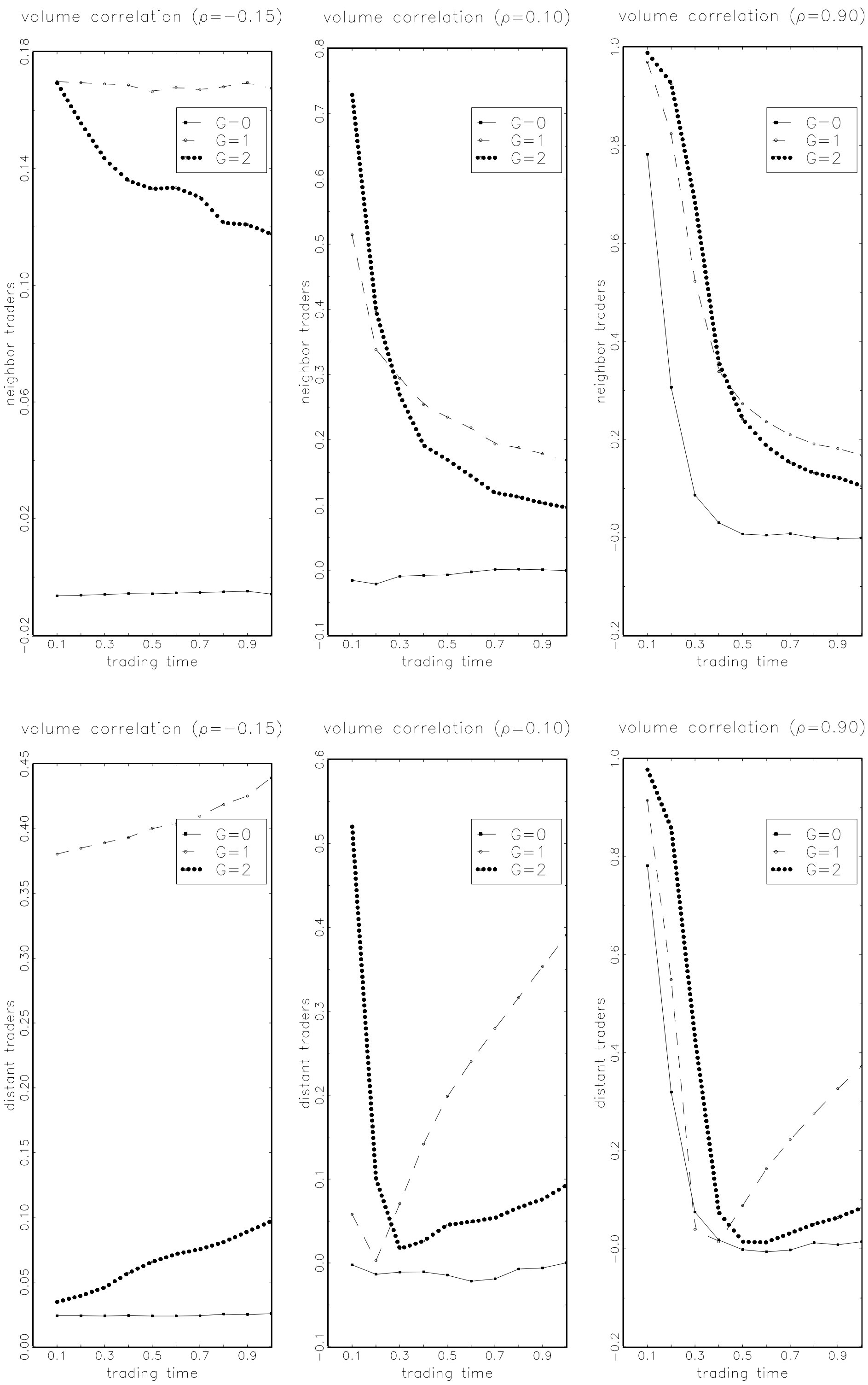


Figure 10: Heterogeneity in correlation among volume

Correlation among volume in a market with information linkages, seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. The left-hand side panels depict the dynamics of correlation among volume arising when the initial correlation among the signals available at the traders' information linkages is negative (ρ=-0.15). The remaining panels depict the dynamics of correlation among volume when this correlation is positive but low (ρ=0.10) (middle panels) and high (ρ=0.90) (right-hand side panels). The top panels depict the correlation dynamics between two close neighbors, i.e. the dynamics of volume correlation between traders i and i-1. The bottom panels depict the dynamics of volume correlation between two distant traders, i.e. the dynamics of correlation between traders i and i-3. In each case, we display trades the dynamics of correlation arising when each trader has a number of information linkages equal to 2G, with G=0,1 and 2.

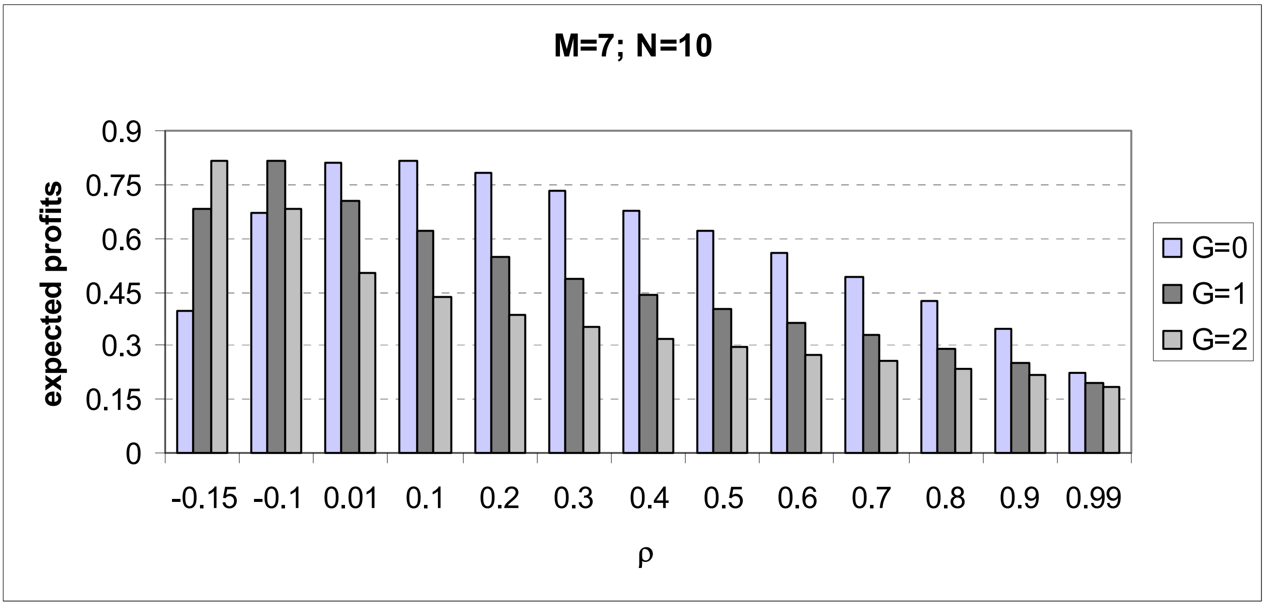


Figure 11: Traders' expected profits

The traders' expected profits in a market with seven informed traders, one unit of initial variance of information, one unit of liquidity trader variance across all periods and ten trading rounds. Displayed are the expected profits arising when each trader has a number of information linkages equal to $2G$, with G=0,1 and 2; and the initial correlations among individual signals equals ρ=-0.15, -0.10, 0.01, 0.1, 0.2,⋯,0.9, and 0.99.