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Portfolio choice, behavioral preferences and equity home bias[☆]

Alessandro Magi^{*}

Department of Statistical Sciences, University of Bologna, Via Belle Arti 41, 40126 Bologna, Italy

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Abstract

We provide a plausible explanation of aggregate portfolio behavior, in a framework where economic agents have behavioral (*narrow framing*) preferences. The representative agent derives utility not only from consumption (standard models) but also from risky financial wealth fluctuations. Moreover, the investor frames the stock market risk narrowly and has loss averse preferences. We numerically solve, for the foreign equity share, a simple model of international portfolio choice, providing a possible explanation for the equity home bias puzzle. Only economic agents able to process correctly information deriving from stock markets exploit the diversification opportunities provided by international financial markets.

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1. Introduction

Recently, many contributions have emphasized that standard CRRA preferences have different problems in explaining some stock market puzzles¹ and at the same time, behavioral theories have increasingly gained credit as an alternative explanation to these puzzles. In particular, some

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^{*} Tel.: +39 0541 434319; fax: +39 0541 434305.

E-mail address: a.magi@unibo.it.

¹ See Mehra and Prescott (1985) (equity premium puzzle), French and Poterba (1991) (equity home bias puzzle) and Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995) (stock market participation puzzle). For complete surveys see Kocherlakota (1996) and Campbell (2000, 2003).

recent papers² propose that people are loss averse over changes in the value of their stock market holdings. The basic idea is that, even if stock market risk is just one of many risks that determine their overall wealth risk, people still get utility directly from stock market fluctuations (*narrow framing*) and are more sensitive to losses than to gains (*loss aversion*). Christelis, Jappelli and Padula (2006) argue that individuals' cognitive abilities may strongly affect investors' financial choices, pointing out that cognitive ability is closely related to the ability to process information. In fact, evidence from psychology shows that poor cognitive skills are associated with low ability of processing information (Spaniol & Bayen, 2005): cognitive skills act as an additional constraint that optimizing individuals face when making their financial decisions.

In order to illustrate one of the possible applications of behavioral preferences, we focus on the so-called *equity home bias puzzle*. Standard portfolio theory (mean/variance and consumption-based asset pricing models) states that it would be optimal for investors to hold a large fraction of their equity portfolio invested in foreign stocks³; but available empirical evidence is at odds with this theoretical prediction, showing that the most important components of household equity portfolios are domestic stocks: most countries hold a small share of foreign stocks in their equity portfolios. In particular, French and Poterba (1991) and Tesar and Werner (1995) estimated the percentage of aggregate stock market wealth invested in domestic equities in the beginning of the 1990s to have been well above 90% for U.S. and Japan,⁴ and around 80% for U.K. and Germany.⁵ During the 1990s the foreign equity participation by US investors has increased: Tesar and Werner (1998) show that in 1996 only around 10% of total U.S. equity holdings was invested abroad. But this level, if compared with what theoretical models predict, is too low.

In the asset pricing/macroeconomics literature, many and different explanations have been provided about this puzzle. Lewis (1999) offers an extensive survey of potential explanations, ranging from the possibility for domestic stocks to better hedge home risks than foreign stocks, the presence of non-tradable consumption goods, diversification costs exceeding the gains, the effects of uncertainty about the economic environment and the role of measurement errors in the data. But Lewis concludes that “overall, equity home bias in portfolio levels remains a puzzle”. In other words, economists agree about the fact that, at the moment, no explanation is conclusive and fully satisfactory.

In this paper, exploiting a behavioral finance based-approach,⁶ we argue that people with poor capabilities of processing information do not diversify their financial investments. And it is reasonable to suppose that, among individuals with poor capabilities of processing information, we find in particular people with a low level of education, while those with a higher level of education (an undergraduate degree or more) have higher capabilities. With this rationale in mind, we numerically solve a simple dynamic model of international portfolio choice, providing a possible explanation for the equity home bias puzzle. Barberis et al. (2006) solve a similar model finding numerically the parameter values for which an agent with a recursive utility function that

² Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), Barberis and Huang (2001, 2004), Barberis and Huang (2007), Barberis, Huang and Thaler (2006).

³ See the seminal contributions by Levy and Sarnat (1970), Solnik (1974) and French and Poterba (1991); for recent exhaustive review articles see Lewis (1999) and Obstfeld and Rogoff (2000).

⁴ 94% for U.S. and 98% for Japan (French & Poterba, 1991).

⁵ This phenomenon in financial asset holdings, as documented by Golub (1990) and Tesar and Werner (1995), is also present in the bond market.

⁶ For a complete survey of behavioral finance contributions see Barberis and Thaler (2003); for a review specifically focused on aggregate stock market behavior see Stracca (2002a).

allows for narrow framing would not participate in a stock market offering a high mean return and low correlation with other risks. But they do not solve explicitly the model neither for the equity asset share (in a context of stock market participation puzzle) nor for the foreign equity share (in a context of equity home bias puzzle).

The paper is organized as follows. In Section 2 we discuss the basics of some behavioral finance models, finding their origins in the applied psychology literature. We also propose an *original* justification for the use of narrow framing preferences; moreover, we briefly see the Barberis–Huang (BH) model, which introduces the basic analytical formulations used in the paper. In Section 3, adopting the preferences introduced in the BH approach, we build and solve a simple international portfolio choice model in order to investigate and explain the equity home bias puzzle. The most important outcomes and implications of the model are presented and discussed in Section 4. Finally, Section 5 concludes the paper.

2. Narrow framing of risks, utility functions and financial wealth fluctuations

2.1. Theoretical basics

In traditional models⁷ the economic agent typically adopts the following behavior: he merges the new choices he faces with those already faced, then he controls if the new “gamble” improve or not the future distribution of wealth and/or consumption. But recently, experimental evidence on financial decision making under uncertainty (the seminal paper is Tversky & Kahneman, 1981) has shown that individuals often do not behave as in traditional models. In many cases, when people evaluate risk, they often engage in *narrow framing*: that is, they often evaluate risks in isolation, separately from other risks they are already facing. As remarked by Barberis and Huang (2004), “. . . *narrow framing means that the agent derives utility directly from the outcome of a specific gamble he is offered, and not just indirectly via its contribution to his total wealth. Equivalently, he derives utility from the gamble’s outcome over and above what would be justified by a concern for his overall wealth risk*”.

The classic demonstration of such behavior is due to the seminal paper of Tversky and Kahneman (1981). We have a clear contradiction: economic agents are faced with two concurrent decisions and they make a sub-optimal choice, opting for a dominated strategy. What happens is that instead of focusing on the *combined* outcome of the two decisions (i.e. on the outcome that determines their final wealth), individuals focus on the outcome of each decision *separately*. There are different situations where we can find a similar behavior. For example, we can think about the so-called stock market non-participation and the equity home bias: in both cases profitable diversification opportunities are rejected.

Kahneman (2003) argues that when an agent evaluates a new gamble, the distribution of the gamble, separately considered, is much more “accessible” than the distribution of his overall wealth once the new gamble has been merged with his other risks. The expression “accessible” refers to the fact that many decisions are based on easily interpretable information; and this consideration is based on the idea that many choices are made intuitively rather than through effortful reasoning.

Consistently with the explanation of Kahneman (2003) and in support of it, we can recall the seminal contribution provided by Simon (1982). Simon remarks that individuals’ cognitive

⁷ We refer to models based on traditional Von Neumann–Morgenstern utility functions, defined on consumption and/or wealth.

resources are limited: this element forces individuals to simplify the space of the choice problem, which appears unmanageable for his excessive complexity.

On these premises, we can naturally think of financial markets as a field where we can apply the theoretical approach we are discussing about. In fact, few sectors of human activity are characterized by a so huge quantity of information as it occurs in stock markets (Slovic, 1972). Such information is highly accessible by everyone because it can be daily reached by means of newspapers, tv-news, internet, etc. But the crucial point here is the correct and optimal processing of information.

The coming of new technologies, providing quickly information about world stock market movements, has highly contributed to increase individuals' difficulties in exploiting at the maximum the huge amount of information available to them. In fact, although a large amount of information means more accuracy in evaluating alternative choices, as argued by Simon (1982), an amount of information in excess, given the individual's bounded cognitive resources, makes the decision space unmanageable; and in order to simplify this space, economic agents make narrow framing (and not "overall") evaluations. As a consequence, this behavior implies that individuals make the choice that is *apparently* the best one. The overall evaluation of the problem would lead to a better choice than that effectively made, but the lack of the "optimal" skills in processing information leads to the sub-optimal choice. The overall framing is involuntary declined in favor of the narrow one because of the lack of such optimal skills in processing information. We observe that this framework can rationalize a key assumption in Sims (2003): individuals only devote small fractions of their capacity in observing and processing information.⁸

2.2. *The Barberis–Huang (BH) approach*

Barberis et al. (2001) [BHS henceforth] propose a new approach in order to solve some financial market puzzles. They introduce a new source of utility for the representative agent, besides the usual one (consumption). The basic idea is the following: economic agents derive direct utility not only from consumption but also from fluctuations in the value of their financial wealth, and such fluctuations heavily affect investors' risk aversion, regardless of their correlation with consumption growth. This idea has its origins in the seminal work by Kahneman and Tversky (1979), which introduced the so-called *prospect theory*, based on prospect-type utility: economic agent derives utility not from consumption/wealth levels but from their changes, evaluated with respect to a *reference* level. Therefore, the utility function is defined on gains and losses and captures the so-called *loss aversion*, i.e. the fact that the agent is more sensitive to reductions in his wealth rather than to increases of the same magnitude.

Obviously, this approach is in contrast with standard asset pricing models,⁹ which assume that economic agents only care of their future utility deriving from consumption levels. But in the economic literature there are many contributions that, by means of theoretical arguments and experimental works, show that standard explanations of individual attitudes towards risk are widely questionable and often wrong (see Rabin, 1998, 2002). As stressed by Rabin (2002), "...*Our attitudes towards risk are driven instead primarily by attitudes towards change in wealth levels.*" In the BHS approach, the motivating idea is that after a big loss in the stock market, the

⁸ We point out that information-based explanations such as ours are in line with recent papers on the so-called "rational inattention" (Sims, 2003; Reis, 2005).

⁹ For a survey on C-CAPM models see Campbell (2003) and Kocherlakota (1996).

investor may experience a sense of regret over his decision to invest in stocks. As a consequence, “he may interpret this loss as a sign that he is a second-rate investor, thus dealing his ego a painful blow; and he may feel humiliation in front of friends and family when word leaks out” (Barberis et al., 2001).

The BHS model has been modified by the same authors in a subsequent contribution (we refer to it as “BH model”). The analytical novelty introduced by Barberis and Huang (2004) is the use of recursive utility (Epstein–Zin–Weil utility).¹⁰ The standard formulation is the following,

$$U_t = W(C_t, \mu(U_{t+1}|I_t)) \quad (1)$$

where $\mu(U_{t+1}|I_t)$ is the certainty equivalence of future utility U_{t+1} , given the time t information. The function $W(\cdot)$ is an aggregator which combines future utility U_{t+1} with current consumption C_t in order to generate current utility U_t . Usually, in this kind of literature, the aggregator function assumes the CES (constant elasticity of substitution) form,

$$W(C, x) = [(1 - \beta)C^\rho + \beta x^\rho]^{1/\rho} \quad (2)$$

with $0 < \beta < 1$, $0 \neq \rho < 1$, while for the certainty equivalence we assume a functional form with homogeneity of degree one,

$$\mu(kx) = k\mu(x), \quad k > 0.$$

By adopting this preference formulation, the maximization problem of the representative investor modifies as follows:

$$\text{Max } U_t = W(C_t, \mu(U_{t+1}|I_t) + b_0 E_t [v(G_{S,t+1})]) \quad (3)$$

$$\text{s.t. } W_{t+1} = (W_t - C_t)R_{W,t+1} \quad (4)$$

where

$$W(C, x) = [(1 - \beta)C^\rho + \beta x^\rho]^{1/\rho} \quad (5)$$

$$\mu(x) = [E(x^\delta)]^{1/\delta}, \quad 0 \neq \delta < 1 \quad (6)$$

$$G_{S,t+1} = \theta_{S,t}(W_t - C_t)(R_{S,t+1} - R_f) \quad (7)$$

$$v(G) = \begin{cases} G & \text{for } G \geq 0 \\ \lambda G & \text{for } G < 0 \end{cases}, \quad \lambda > 1 \quad (8)$$

We are in a pure exchange economy, with no labor income. We have two financial assets: a risky asset (stock) with gross rate of return $R_{S,t+1} = 1 + r_{S,t+1}$ between t and $t + 1$ and a risk-free asset with safe return R_f ; $\theta_{S,t}$ is the fraction of post-consumption wealth invested in the risky asset.

What is the novelty in the utility function? The first term is the usual one we find in standard asset pricing models. The novelty is the second term, $v(G_{t+1})$, which represents utility deriving from individual stock wealth changes: in other words, utility deriving from fluctuations in individual's risky financial wealth.¹¹ In particular, G_{t+1} is the gain or loss obtained by the agent on his equity

¹⁰ See Epstein and Zin (1989, 1991) and Weil (1989).

¹¹ We only consider risky asset fluctuations: the time $t + 1$ risk-free return is known with certainty at time t , and therefore there is no risk in the changes of the safe asset.

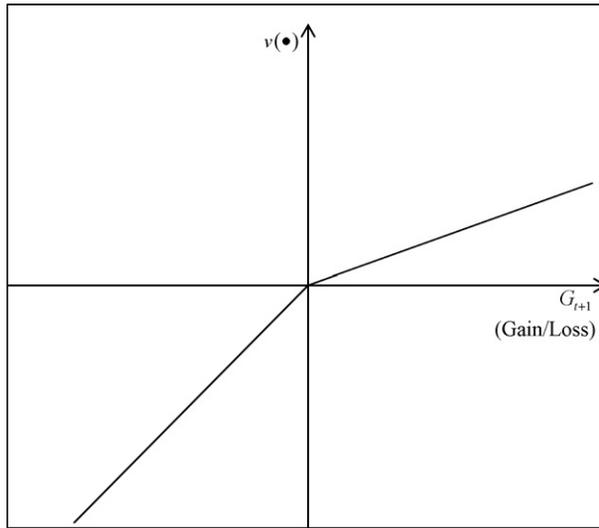


Fig. 1. Utility function with loss aversion.

investments between t and $t+1$. The utility (disutility) deriving to the investor from this gain (loss) is measured by the function $v(\cdot)$. We note that we have both the narrow framing of risks, introduced by the presence of $v(\cdot)$, and the loss aversion, introduced by the particular (piecewise linear) form of $v(\cdot)$. As we can see by (7), the *reference level* for measuring the gain/loss is given by the initial value of financial asset parameterized with the risk-free asset. The idea is that investor will be satisfied if $R_{S,t+1} > R_f$ and unsatisfied vice versa (Barberis et al., 2001).

The certainty equivalence has been specified in a very simple form (Eq. (6)), widely used in the literature. $R_{W,t+1}$ measures the return of the overall individual wealth, i.e. of the individual's "market portfolio", between t and $t+1$. Obviously, the composition of this portfolio depends on the number of assets we take into account. In general, we could consider two assets only, but also n assets.

Now, let's analyze the particular form of the function $v(G_{t+1})$. In Fig. 1 we have a simple graphical representation of it. We note that its form makes clear the fundamental feature of representative agent's preferences, i.e. the higher sensitivity to stock wealth setbacks rather than to increases of the same magnitude: it is the so-called loss aversion.

Another very important question is the frequency by which the investor evaluates his financial situation, checking his stock market performances. Following the results obtained by Benartzi and Thaler (1995), the BH approach considers the year as standard evaluation period. The time horizon of equity investments is usually longer, 3–5 years, but it is reasonable assuming that economic agent seriously checks his financial market performances at least once a year. This assumption is confirmed by some elements: we file taxes once a year, receive our most comprehensive mutual fund reports once a year, and institutional investors scrutinize their money managers' performances most carefully on an annual basis (Barberis et al., 2001).

What about b_0 ? In the BH model this parameter plays a very important role. It is a non-negative parameter which permits us to control for the importance of utility deriving from financial wealth changes relatively to the utility deriving from consumption. At the same time, b_0 can also be interpreted as the *narrow framing* degree of a risky investment. If we set $b_0 = 0$ we have the standard consumption-based asset pricing model.

3. A model of international portfolio choice with narrow framing of risks

3.1. The model

We can adopt “BH preferences” by using them in a simple international portfolio choice framework, in order to investigate the equity home bias puzzle. In the behavioral finance literature with narrow framing preferences (a very recent literature), among partial equilibrium analyses, works in such a sense do not exist. Barberis et al. (2006) investigate the stock market non-participation puzzle (with U.S. data) but they do not solve the model explicitly for the equity asset share.¹²

We consider a simple two-country economy, for instance Country A and the Rest of the World (RoW). In a partial equilibrium framework, we only analyse the choices of the Country A representative agent (and we suppose that Country A is the domestic one). We have no production and no labor income: the only income source is of financial nature. The time horizon of the economy is infinite.

In our economy we have three financial assets: economic agent divides his financial wealth between them. The first asset is a risk-free domestic bond (or “cash on hand” with a safe return); the second one is the domestic stock market (a risky asset of Country A); the third one is a foreign equity. Every asset is supplied in a fixed quantity: hence, we do not investigate the supply side of the stock market. Both risky assets have an exogenous log-rate of return of the following type:

$$\log(R_{D,t+1}) = g_D + \sigma_D \varepsilon_{D,t+1} \quad (9)$$

$$\log(R_{F,t+1}) = g_F + \sigma_F \varepsilon_{F,t+1} \quad (10)$$

with

$$\begin{pmatrix} \varepsilon_{D,t} \\ \varepsilon_{F,t} \end{pmatrix} \approx N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix} \right] \text{ i.i.d.}$$

where mean log-returns and standard deviations, g_i and σ_i , are given and stochastic shocks ε_i are exogenous. The risk-free asset has a rate of return known with certainty; the dividend stream is exogenous and is embedded in the log-returns. Moreover, we suppose the exchange rate between the two countries to be equal to 1 and not subject to fluctuations (fixed exchange rates). All variables are expressed in real terms.

Moreover, in solving the model we make the following further assumptions:

- (1) The risk-free asset share is given, for an amount equal to $\bar{\theta}_f$: hence, the investor chooses how to divide his risky wealth between domestic and foreign equity.
- (2) The representative agent makes narrow framing on foreign asset only.

The last assumption might be seen as a simplification, because the same thing might also hold for the other risky asset, the domestic one. As an explanation for this assumption, we can

¹² Stracca (2002b) discusses the possibility of resolving the equity home bias puzzle by means of *prospect theory*, but he does not build and solve any portfolio model explicitly. An interesting paper related to our work is Gomes (2005): he numerically solves a multi-period portfolio problem with loss-averse investors, but without narrow framing.

suppose domestic asset to be a more familiar risk than other financial risks, with a probability distribution that is “easy” to combine with the distribution of other risks (and in particular this holds for less educated people, because more educated people are able to make “optimal” evaluations). The same thing does not hold for foreign equity. This interpretation agrees with several behavioral finance contributions. For example, [Huberman \(2001\)](#) argues that there is a high propensity to invest in “familiar” financial assets: such assets give the investor the illusion to control his own investments better than in other cases (see also [Goetzmann & Kumar, 2002](#); [Kelly, 1995](#)).

By using the recursive preferences of the BH approach, the investor faces the following (recursive) utility function:

$$U_t = W(C_t, \mu(U_{t+1}) + b_0[E_t(G_{F,t+1})]) \quad (11)$$

With reference to Eqs. (5) and (6) we fix $\rho = \delta = 1 - \gamma$, where γ is the relative risk aversion coefficient. In the asset pricing literature, under GEU preferences, setting the exponent of the aggregator function equal to the exponent of the certainty equivalence function is usual. Since we have that

$$W(C, x) = [(1 - \beta)C^{1-\gamma} + \beta x^{1-\gamma}]^{1/1-\gamma} \quad \text{and} \quad \mu(x) = [E(x^{1-\gamma})]^{1/1-\gamma}$$

the utility function maximized by the representative agent can be rewritten as

$$U_t = \{(1 - \beta)C_t^{1-\gamma} + \beta([E_t(U_{t+1}^{1-\gamma})]^{1/1-\gamma} + b_0 E_t[v(G_{F,t+1})])^{1-\gamma}\}^{1/1-\gamma} \quad (12)$$

Now, it is necessary to adapt the standard wealth accumulation constraint to our simple portfolio choice model. Taking into account the three financial assets and their returns, the agent maximizes Eq. (12) subject to

$$W_{t+1} = (W_t - C_t)R_{W,t+1}$$

with

$$R_{W,t+1} = (\bar{\theta}_f R_f + \theta_{D,t} R_{D,t+1} + \theta_{F,t} R_{F,t+1}),$$

where $\theta_{D,t}$ and $\theta_{F,t}$ are, respectively, the shares of individual financial wealth invested in domestic and foreign stock market, while $\bar{\theta}_f$ is the constant share invested in the risk-free (domestic) asset. Hence, it is evident that $\bar{\theta}_f + \theta_{D,t} + \theta_{F,t} = 1$. Moreover,

$$G_{F,t+1} = \theta_{F,t}(W_t - C_t)(R_{F,t+1} - R_f) \quad v(G) = \begin{cases} G & \text{for } G \geq 0 \\ \lambda G & \text{for } G < 0 \end{cases}, \quad \lambda > 1. \quad (13)$$

3.2. Optimality conditions for the consumption/portfolio problem

Given the recursive nature of the intertemporal maximization problem, it can be solved by using dynamic programming techniques. We have the following value function:

$$U_t = V(W_t) = \underset{C_t, \theta_{i,t}}{\text{Max}} W(C_t, \mu[V(W_{t+1})] + b_0 E_t v(G_{F,t+1})) \quad (14)$$

Taking into account the aggregator function $W(C,x)$, Eq. (14) changes as follows:

$$V(W_t) = \text{Max}_{C_t, \{\theta_{i,t}\}_{i=1}^n} \{(1 - \beta)C_t^{1-\gamma} + \beta[\mu[V(W_{t+1})] + b_0 E_t v(G_{F,t+1})]^{1-\gamma}\}^{1/1-\gamma} \quad (15)$$

We can obtain a particular formulation for the value function in order to show that consumption and portfolio decisions are separable. In fact, we observe that given the form of $G_{F,t+1}$,

$$G_{F,t+1} = \theta_{F,t}(W_t - C_t)(R_{F,t+1} - R_f), \quad (16)$$

we can guess for the value function the following form,

$$V(W_t) = A_t W_t, \quad (17)$$

where A_t is interpretable as the solution of a maximization problem; the validity of this conjecture will be verified ex-post.

By substituting Eqs. (16) and (17) evaluated at $t + 1$, into (15), and exploiting the intertemporal budget constraint, we have

$$\begin{aligned} V(W_t) = \text{Max}_{C_t, \theta_{i,t}} \{ & (1 - \beta)C_t^{1-\gamma} + \beta[\mu[A_{t+1}(W_t - C_t)R_{W,t+1}] \\ & + b_0 E_t v[\theta_{F,t}(W_t - C_t)(R_{F,t+1} - R_f)]]^{1-\gamma}\}^{1/1-\gamma} \end{aligned}$$

By exploiting the homogeneity of degree one of $\mu(\cdot)$ and $v(\cdot)$ we get

$$\begin{aligned} V(W_t) = \text{Max}_{C_t, \theta_{i,t}} \{ & (1 - \beta)C_t^{1-\gamma} + \beta[(W_t - C_t)\mu[A_{t+1}R_{W,t+1}] \\ & + (W_t - C_t)b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]]^{1-\gamma}\}^{1/1-\gamma} \end{aligned}$$

Collecting $(W_t - C_t)$ and raising to $1 - \gamma$ what is in square brackets, we have

$$\begin{aligned} V(W_t) = \text{Max}_{C_t, \theta_{i,t}} \{ & (1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}[\mu(A_{t+1}R_{W,t+1}) \\ & + b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]]^{1-\gamma}\}^{1/1-\gamma} \end{aligned} \quad (18)$$

From Eq. (18) we can see that optimal consumption and portfolio choices are separable. In particular, since portfolio problem is

$$P_t^* = \text{Max}_{\theta_{i,t}} [\mu(A_{t+1}R_{W,t+1}) + b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]], \quad (19)$$

we can rewrite the value function as follows:

$$V(W_t) = \text{Max}_{C_t} \{(1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}(P_t^*)^{1-\gamma}\}^{1/1-\gamma}. \quad (20)$$

Obviously, Eq. (20) is the optimal consumption choice problem (we are assuming to know the optimal portfolio solution P_t^*): but this problem may be re-formulated differently. If $\alpha_t \equiv C_t/W_t$,

we can write the problem (20) in the following way:

$$A_t = \text{Max}_{\alpha_t} [(1 - \beta)\alpha_t^{1-\gamma} + \beta(1 - \alpha_t)^{1-\gamma}(P_t^*)^{1-\gamma}]^{1/1-\gamma}. \quad (21)$$

For obtaining Eq. (21) we have to divide for W_t both sides of the Eq. (20). We have:

$$\frac{V(W_t)}{W_t} = \text{Max}_{C_t/W_t} \frac{1}{W_t} [(1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}(P_t^*)^{1-\gamma}]^{1/1-\gamma};$$

by assuming $V(W_t)/W_t \equiv A_t$ and raising both sides to $1 - \gamma$ we can rewrite the last equation as

$$\begin{aligned} A_t^{1-\gamma} &= \text{Max}_{C_t/W_t} \left(\frac{1}{W_t} \right)^{1-\gamma} [(1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}(P_t^*)^{1-\gamma}] \\ &= \text{Max}_{C_t/W_t} \left[(1 - \beta) \left(\frac{C_t}{W_t} \right)^{1-\gamma} + \beta \left(\frac{W_t - C_t}{W_t} \right)^{1-\gamma} (P_t^*)^{1-\gamma} \right]. \end{aligned}$$

Now, exploiting the fact that $\alpha_t \equiv C_t/W_t$, we get

$$A_t^{1-\gamma} = \text{Max}_{\alpha_t} [(1 - \beta)\alpha_t^{1-\gamma} + \beta(1 - \alpha_t)^{1-\gamma}(P_t^*)^{1-\gamma}],$$

and raising both sides to $1/(1 - \gamma)$ we obtain Eq. (21). By substituting $\alpha_t \equiv C_t/W_t$ in (21), it is possible to verify the conjecture (17) further: $A_t W_t = V(W_t) = \text{Eq. (18)}$.¹³

Now, let's consider the problem (21). For obtaining the optimal first order condition we have to derive with respect to α_t . We have:

$$\begin{aligned} &\frac{1}{1 - \gamma} [(1 - \beta)\alpha_t^{1-\gamma} + \beta(1 - \alpha_t)^{1-\gamma}(P_t^*)^{1-\gamma}]^{1/1-\gamma-1} \\ &\quad \times [(1 - \gamma)(1 - \beta)\alpha_t^{-\gamma} - (1 - \gamma)\beta(1 - \alpha_t)^{-\gamma}(P_t^*)^{1-\gamma}] \\ &= 0 \Rightarrow [(1 - \gamma)(1 - \beta)\alpha_t^{-\gamma} - (1 - \gamma)\beta(1 - \alpha_t)^{-\gamma}(P_t^*)^{1-\gamma}] \\ &= 0 \Rightarrow (1 - \beta)\alpha_t^{-\gamma} = \beta(1 - \alpha_t)^{-\gamma}(P_t^*)^{1-\gamma} \end{aligned} \quad (22)$$

where P_t^* , as previously seen, is the portfolio problem solution.¹⁴ We solve Eq. (22) for P_t^* and then, substituting the expression obtained in (21), we get

$$A_t = (1 - \beta)^{1/1-\gamma} \alpha_t^{-\gamma/1-\gamma}, \quad (23)$$

and at time $t + 1$ we have

$$A_{t+1} = (1 - \beta)^{1/1-\gamma} \alpha_{t+1}^{-\gamma/1-\gamma}. \quad (24)$$

Now we can substitute (24) in the portfolio problem (19), obtaining

$$P_t^* = \text{Max}_{\theta_{i,t}} [\mu[(1 - \beta)^{1/1-\gamma} \alpha_{t+1}^{-\gamma/1-\gamma} R_{W,t+1}] + b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]]$$

¹³ See the Mathematical Appendix A.1.

¹⁴ This is only the "assumed" solution: we will see that for solving problem (19) it is necessary an iterative procedure which must take into account the first order condition for consumption, i.e. Eq. (22).

$$P_t^* = \text{Max}_{\theta_{i,t}} [(1 - \beta)^{1/1-\gamma} \mu(\alpha_{t+1}^{-\gamma/1-\gamma} R_{W,t+1}) + b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]] \quad (25)$$

where we have again used the homogeneity of degree one of function $\mu(\cdot)$ in order to bring out of it $(1 - \beta)^{1/(1-\gamma)}$. Writing this function in the CES form,

$$\mu(x) = [E(x^{1-\gamma})]^{1/1-\gamma},$$

we can rewrite the portfolio problem as follows:

$$P_t^* = \text{Max}_{\theta_{i,t}} [(1 - \beta)^{1/1-\gamma} [E_t(\alpha_{t+1}^{-\gamma/1-\gamma} R_{W,t+1}^{1-\gamma})]^{1/1-\gamma} + b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]] \quad (26)$$

where

$$R_{W,t+1} = (\bar{\theta}_f R_f + \theta_{D,t} R_{D,t+1} + \theta_{F,t} R_{F,t+1}).$$

3.3. Numerical solution of the model

How do we solve the portfolio problem stated by Eq. (26)? Obviously, we have to maximize with respect to portfolio shares θ_i , but the difficulty is that such shares are expressed as functions of α , i.e. as functions of the optimal solution (the *policy function*) for consumption, and we do not have it. A feasible solution strategy is the following one.

We guess a possible solution for problem (21): in other words, we guess a value for α . This also means to guess a value for A (see (23)). We solve (26) for such a value of α and substitute the resulting P_t^* in the first order condition for consumption (22), in order to generate a new candidate α . Then, with the last α obtained, we solve the problem (26) again and we go on with this iterative procedure until convergence occurs.¹⁵

The economic agent have to choose between foreign and domestic asset: it is sufficient to maximize with respect to one of the two assets, since being $\bar{\theta}_f + \theta_{D,t} + \theta_{F,t} = 1$, once that, say $\theta_{F,t}$, is determined, the unknown share ($\theta_{D,t}$) is automatically determined. We maximize with respect to foreign equity, $\theta_{F,t}$. In doing so, we have to guess some (constant) values for α_t and $\theta_{F,t}$, because we will not find the standard policy functions, but time-invariant solutions. Assuming the *guess*

$$(\alpha_t, \theta_{F,t}) = (\alpha, \theta_F),$$

the portfolio problem (26) becomes as follows:

$$P_t^* = \text{Max}_{\theta_F} [(1 - \beta)^{1/1-\gamma} \alpha^{-\gamma/1-\gamma} [E_t(R_{W,t+1}^{1-\gamma})]^{1/1-\gamma} + b_0 E_t v[\theta_F(R_{F,t+1} - R_f)]] \quad (27)$$

where

$$R_{W,t+1} = (\bar{\theta}_f R_f + \theta_D R_{D,t+1} + \theta_F R_{F,t+1})$$

with

$$\theta_D = 1 - \bar{\theta}_f - \theta_F.$$

¹⁵ This procedure has been implemented using the software of numerical computation MatLab.

Table 1

Equity portfolio shares (%)—our elaboration: Country A–Rest of the World

| β | γ | λ | b_0 | θ_D (%) | θ_F (%) |
|---------|----------|-----------|-------|----------------|----------------|
| 0.98 | 3 | 2.25 | 0 | 32 | 28 |
| 0.98 | 3 | 2.25 | 0.10 | 37 | 23 |
| 0.98 | 3 | 2.25 | 0.20 | 45 | 15 |
| 0.98 | 3 | 2.25 | 0.30 | 54.4 | 5.6 |

We observe that in solving the portfolio problem (27) we have some analytical problems arisen by the solution of the expected value

$$E_t(R_{W,t+1}^{1-\gamma})$$

i.e., substituting the expression for $R_{W,t+1}$ (also consider Eqs. (9) and (10)),

$$E_t\{[\bar{\theta}_f R_f + (1 - \bar{\theta}_f - \theta_F) e^{g_D + \sigma_D \varepsilon_{D,t+1}} + \theta_F e^{g_F + \sigma_F \varepsilon_{F,t+1}}]^{1-\gamma}\}.$$

We use the numerical quadrature (Tauchen & Hussey, 1991), a widely used procedure in the asset pricing literature.¹⁶

4. The model in action: some applications

We solve the model numerically, by considering some real economies. In doing so, we use parameter values from Michaelides (2003) and Campbell (2003), for the time periods 1973–2001 and 1919–1998, respectively. These values, which refer to rates of return, standard deviations and correlations, are based on the MSCI index. We take into account data relative to United States, Italy, United Kingdom and an area of 12 countries called “Euro”: Austria, Belgium, Switzerland, Germany, Denmark, Spain, France, United Kingdom, Italy, Netherlands, Norway and Sweden. We remark that the following applications have the aim to show that the model holds for different economies and different stockholders.

We start with a simple example. We assume that in our two-country economy, “Country A” and “Rest of the World” (RoW), we have the following parameters: $R_f = 1.02$, $g_D = 0.06$, $\sigma_D = 0.20$, $g_F = 0.06$, $\sigma_F = 0.20$, $\bar{\theta}_f = 0.40$, $\beta = 0.98$. For simplicity, we consider a zero correlation between the two returns. The two stocks have the same return and the same volatility: hence, in a frictionless economy, we should expect a “fifty to fifty” share of the residual wealth: $\theta_D = 0.30$ and $\theta_F = 0.30$. In fact, this is what happens with $b_0 = 0$. Then, as Table 1 shows, with narrow framing and loss aversion, what emerges is that the foreign asset is held decreasingly as the degree of narrow framing b_0 increases. The mechanism in action is the usual one: the investor assigns more importance to his aversion towards financial losses (*loss aversion*) than to his willingness to exploit diversification opportunities deriving from national and international equity markets. Moreover, the prospective buy of stocks is evaluated in isolation, and not merged with the risks already faced. These facts contribute to make foreign financial investment very risky and induce the investor to hold domestic equity mostly. The foreign equity is perceived to be less attractive than it would be if the investor

¹⁶ In particular, we use the so-called Gauss–Hermite quadrature, typically adopted when we deal with distributions which go from minus infinity to infinity (see Judd, 1998). The MatLab codes used for solving the model are available, upon request, from the author. About the other expected value in Eq. (27), see the Appendix A.2.

Table 2

Parameter values: ITA–USA (1973–2001)

| | ITA | USA |
|------------------|-------|-------|
| R_f | 1.03 | |
| g^D | 0.074 | |
| σ_D | 0.39 | |
| g^F | | 0.077 |
| σ_F | | 0.185 |
| $\omega_{D,F}$ | 0.50 | 0.50 |
| $\bar{\theta}_f$ | 0.50 | |

had the optimal skills for processing information and hence would be able to make a combined evaluation of the two assets.

4.1. Italy–United States

Now, we see how the model works when we make some applications with data drawn from real economies. Suppose that domestic economy is the Italian one while foreign economy is the U.S. one.¹⁷ Obviously, the idea that Italian investor can diversify his investments only in U.S. assets is a simplification, but this permits us to point out the crucial features of the model.

In order to see the magnitude of the home bias in Italy, we can have a look at the Survey on Household Income and Wealth (SHIW) (1998, 2000, 2002). The three surveys show that for the period 1998–2002, on average, Italian households' financial portfolio has been composed for 99% by domestic assets (of every type) and for 1% by foreign assets.¹⁸

Table 2 shows the parameters of the model for this case. The value relative to the risk-free asset share, $\bar{\theta}_f$, comes from SHIW data.¹⁹ However, in general, the choice of preference parameters reflects what we find in the literature (see Magi, 2004 for a short survey). We use parameters consistent with the prevailing literature and with econometric estimates and experimental evidence available on this topic (and this holds for the next two cases as well). For example, about λ , we have estimates around to 2.5 (Tversky & Kahneman, 1992; Barberis & Thaler, 2003): we use a value equal to 3. Otherwise, for b_0 we use a range of values, in order to test the behavior of the model for different b_0 's.

As Table 3 shows, when the narrow framing degree is zero the model behaves as predicted by standard theories: the largest fraction of own risky wealth is invested in foreign asset (38% against 12%). If we introduce “BH preferences”, as the narrow framing degree increases the foreign asset share goes down. We notice that the model produces this outcome despite a similar mean return, a volatility in favor of U.S. equity and a correlation not so high.

¹⁷ In the cases we analyse, the first country is always the domestic economy, the second country the foreign one.

¹⁸ We refer to total financial assets and not to equity assets only, because of homogeneity problems among different typologies of investments (home and foreign). For instance, in the case of foreign investments, mutual funds are a single item and are not divided in equity and non-equity funds. Such a division there is for domestic investments, but only in the 2002 wave (see Bank of Italy, 1998, 2000, 2002).

¹⁹ If we consider the three surveys, on average what emerges is that in Italian households' portfolio the share of total risk-free assets (T-Bills plus liquid assets) is about 64% (our elaboration based on Bank of Italy data). But some long maturity T-Bills, BTP for example, have to be considered as risky assets, henceforth $\bar{\theta}_f = 0.50$. See also, in particular for a comparison Italy–U.S., Faiella and Neri (2004).

Table 3

Equity portfolio shares (%)—our elaboration: ITA–USA (1973–2001)

| β | γ | λ | b_0 | θ_D (%) | θ_F (%) |
|---------|----------|-----------|-------|----------------|----------------|
| 0.98 | 3 | 3 | 0 | 12 | 38 |
| 0.98 | 3 | 3 | 0.10 | 37 | 13 |
| 0.98 | 3 | 3 | 0.15 | 43 | 7 |
| 0.98 | 3 | 3 | 0.20 | 43.1 | 6.9 |
| 0.98 | 3 | 3 | 0.30 | 49.94 | 0.06 |
| 0.98 | 4 | 3 | 0 | 11 | 39 |
| 0.98 | 4 | 3 | 0.10 | 42.7 | 7.3 |
| 0.98 | 4 | 3 | 0.20 | 49.95 | 0.05 |
| 0.98 | 5 | 3 | 0 | 9 | 41 |

Table 4

Parameter values: UK–USA (1919–1998)

| | UK | USA |
|------------------|--------|-------|
| R_f | 1.0125 | |
| g_D | 0.077 | |
| σ_D | 0.22 | |
| g_F | | 0.071 |
| σ_F | | 0.185 |
| $\omega_{D,F}$ | 0.50 | 0.50 |
| $\bar{\theta}_f$ | 0.20 | |

Table 2 shows that half of Italian representative agent's financial investments is composed by the risk-free asset: the residual half is divided between the two risky assets, with the largest share for the domestic stock market. Because of narrow framing, economic agents evaluate the foreign equity as less attractive than it would be if the investor would make a correct evaluation of it, combined with the other risky asset; at the same time, the investor assigns more weight to the domestic asset, declining a convenient diversification opportunity. Hence, the model is able to match available empirical evidence and is at odds with standard portfolio theory.

Table 5

Equity portfolio shares (%)—our elaboration: UK–USA (1919–1998)

| β | γ | λ | b_0 | θ_D (%) | θ_F (%) |
|---------|----------|-----------|-------|----------------|----------------|
| 0.98 | 3 | 3 | 0 | 43 | 37 |
| 0.98 | 3 | 3 | 0.10 | 52 | 28 |
| 0.98 | 3 | 3 | 0.20 | 66 | 14 |
| 0.98 | 3 | 3 | 0.25 | 72.5 | 7.5 |
| 0.98 | 3 | 3 | 0.30 | 78.3 | 1.7 |
| 0.98 | 4 | 3 | 0.10 | 61 | 19 |
| 0.98 | 4 | 3 | 0.20 | 62 | 18 |
| 0.98 | 4 | 3 | 0.30 | 73 | 7 |

Table 6

Parameter values: USA–EURO (1973–2001)

| | USA | EURO |
|------------------|-------|-------|
| R_f | 1.02 | |
| g_D | 0.077 | |
| σ_D | 0.185 | |
| g_F | | 0.086 |
| σ_F | | 0.27 |
| $\omega_{D,F}$ | 0.60 | 0.60 |
| $\bar{\theta}_f$ | 0.20 | |

4.2. United Kingdom–United States

For this case we use parameters which refer to data relative to a long time period, 1919–1998 (see Campbell, 2003). Table 4 shows such parameters, while in Table 5 we have the most important results obtained from the numerical solution: the behavior of the model is in line with his theoretical premises. When $b_0 = 0$ foreign equity share is quite high (37%), very close to the domestic one (43%). Hence, the investor diversifies his financial investments, as predicted by standard models. The introduction and the progressive increase of the narrow framing degree reduces the U.K. investor's propensity to invest abroad. We observe that in order to generate such a behavior, with respect to the previous case, a greater value of the narrow framing degree is necessary.

4.3. United States–Euro

Now, we suppose that domestic economy is represented by the U.S., while the foreign one is represented by an area called “EURO”, which embeds Austria, Belgium, Switzerland, Germany, Denmark, Spain, France, United Kingdom, Italy, Netherlands, Norway and Sweden. In order to get risk and return parameters for this area, we have simply calculated an arithmetic average; the same thing holds for the correlation parameter.²⁰ The idea here is to construct a representative European area, where the U.S. investor can diversify his financial investments. However, within this area, we stress that by taking into account the six biggest country only, we get outcomes similar to the “twelve-group” (Table 6).

Under the assumptions of the model, U.S. investor's behavior confirms the tendency already seen in the other cases (see Table 7). In particular, in order to match the empirical evidence, we need a level of narrow framing lower than the level of the two previous cases. However, in the three cases discussed here, we obtain the same qualitative results by using the same preference parameter values. In this way we can infer, for the different economies, the appropriate values for the behavioral parameters.

At this point, what is important to emphasize is that the two parameters which characterize the model as a model of behavioral finance (with narrow framing preferences) are always consistent with the values used in other studies and simulations.²¹ In particular, in order to match empirical evidence, the parameter of narrow framing (b_0) does not need to be greater than 0.30 (in some

²⁰ We continue to refer to the parameters used in Michaelides (2003).

²¹ See Barberis et al. (2001), Barberis and Huang (2001, 2004), Barberis and Huang (2007), Barberis et al. (2006) and Magi (2004).

Table 7

Equity portfolio shares (%)—our elaboration: USA–EURO (1973–2001)

| β | γ | λ | b_0 | θ_D (%) | θ_F (%) |
|---------|----------|-----------|-------|----------------|----------------|
| 0.98 | 3 | 3 | 0 | 56.4 | 23.6 |
| 0.98 | 3 | 3 | 0.05 | 64.4 | 15.6 |
| 0.98 | 3 | 3 | 0.10 | 78.3 | 1.7 |
| 0.98 | 3 | 3 | 0.15 | 78.5 | 1.5 |
| 0.98 | 3 | 3 | 0.20 | 79.95 | 0.05 |
| 0.98 | 4 | 3 | 0.05 | 68.4 | 11.6 |
| 0.98 | 4 | 3 | 0.10 | 77 | 3 |
| 0.98 | 5 | 3 | 0.05 | 76.2 | 3.8 |

previous studies it is often greater than 0.30 but is usually lower than 1, see Barberis et al., 2001; Barberis & Huang, 2001). Hence, the present paper can be seen as a further contribution for investigating the “black-box” represented by behavioral preference parameters (b_0 and λ), which attempt “to capture” the individual preferences more strictly related to individual psychology and irrationality.

5. Concluding remarks

In this paper we have provided a contribution based on behavioral preferences, for explaining what we observe in the data about household equity portfolios. Standard portfolio theory states that for economic agents holding a foreign equity share higher than that actually held would be optimal. What happens is that convenient diversification opportunities are declined. Why?

We find a satisfactory answer for this question by numerically solving a simple model of international portfolio choice, adopting the so-called “narrow framing” preferences. Basically, we stress that the mechanism in action is based on the individual’s limited capabilities of processing information: foreign asset is perceived as less attractive than it would be if the investor had the optimal information skills and hence would be able to evaluate the two risky assets jointly. What follows is a low foreign equity share. The model matches available empirical evidence and is at odds with standard portfolio theory.²² We can also see the distinctive feature of this approach: in a descriptive context, it explains the actual (sub-optimal) choices of investors and not how they should behave in order to reach the optimum.

Appendix A. Mathematical

A.1. Proof of the relation $V(W_t) = A_t W_t$

We start by Eq. (21) and substitute into $\alpha_t \equiv C_t/W_t$, in order to verify that $A_t W_t = V(W_t)$, i.e. that $A_t W_t$ is equal to Eq. (18). We have:

²² A quite similar (but more detailed) idea is proposed in two recent complementary papers by Guiso, Sapienza and Zingales (2005a,b): they emphasize the role of trust and culture for holding equities and participating to the stock market.

$$\begin{aligned}
A_t &= \text{Max}_{\alpha_t} [(1 - \beta)\alpha_t^{1-\gamma} + \beta(1 - \alpha_t)^{1-\gamma}(P_t^*)^{1-\gamma}]^{1/1-\gamma} \\
&= \text{Max}_{C_t/W_t} \left[(1 - \beta) \left(\frac{C_t}{W_t} \right)^{1-\gamma} + \beta \left(1 - \frac{C_t}{W_t} \right)^{1-\gamma} (P_t^*)^{1-\gamma} \right]^{1/1-\gamma} \\
&= \text{Max}_{C_t/W_t} \left[(1 - \beta) \frac{C_t^{1-\gamma}}{W_t^{1-\gamma}} + \beta \frac{(W_t - C_t)^{1-\gamma}}{W_t^{1-\gamma}} (P_t^*)^{1-\gamma} \right]^{1/1-\gamma}
\end{aligned}$$

Collecting $1/W_t^{1-\gamma} = (1/W_t)^{1-\gamma}$, we get

$$= \text{Max}_{C_t/W_t} \left\{ \left(\frac{1}{W_t} \right)^{1-\gamma} [(1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}(P_t^*)^{1-\gamma}] \right\}^{1/1-\gamma} =$$

Raising both members in braces to $1/(1 - \gamma)$, we have

$$= \text{Max}_{C_t/W_t} \frac{1}{W_t} [(1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}(P_t^*)^{1-\gamma}]^{1/1-\gamma}.$$

Therefore

$$A_t = \text{Max}_{C_t/W_t} \frac{1}{W_t} [(1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}(P_t^*)^{1-\gamma}]^{1/1-\gamma}.$$

Now, we multiply both sides for W_t (the same operation is also necessary under the Max, i.e. for the variable with respect to which we are maximizing):

$$\begin{aligned}
A_t W_t &= \text{Max}_{C_t} [(1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma}(P_t^*)^{1-\gamma}]^{1/1-\gamma} \\
&= \text{Max}_{C_t} \{ (1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma} (\text{Max}_{\theta_{i,t}} [\mu(A_{t+1} R_{W,t+1}) \\
&\quad + b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]]^{1-\gamma}) \}^{1/1-\gamma} \\
&= \text{Max}_{C_t, \theta_{i,t}} \{ (1 - \beta)C_t^{1-\gamma} + \beta(W_t - C_t)^{1-\gamma} [\mu(A_{t+1} R_{W,t+1}) \\
&\quad + b_0 E_t v[\theta_{F,t}(R_{F,t+1} - R_f)]]^{1-\gamma} \}^{1/1-\gamma} = V(W_t)
\end{aligned}$$

We note that the last expression is simply Eq. (18): hence, it follows that $A_t W_t = V(W_t)$.

A.2. Calculation of $E_t v[\theta_F(R_{F,t+1} - R_f)]$ in Eq. (27)

In the portfolio problem (27), before solving for the portfolio weight θ_F , we must calculate the expected value on the right-hand side, $E_t v[\theta_F(R_{F,t+1} - R_f)]$, and luckily we can do it explicitly, obtaining a closed-form solution. We have:

$$\begin{aligned}
E_t v[\theta_F(R_{F,t+1} - R_f)] &= E_t v[\theta_F(e^{g_F + \sigma_F \varepsilon_{F,t+1}} - R_f)] \\
&= E_t [I_{R_{F,t+1} \geq R_f} \theta_F (e^{g_F + \sigma_F \varepsilon_{F,t+1}} - R_f) \\
&\quad + I_{R_{F,t+1} < R_f} \theta_F \lambda (e^{g_F + \sigma_F \varepsilon_{F,t+1}} - R_f)] =
\end{aligned}$$

where I is an indicator function. We can isolate θ_F :

$$\theta_F E_t [I_{R_{F,t+1} \geq R_f} (e^{g_F + \sigma_F \varepsilon_{F,t+1}} - R_f) + I_{R_{F,t+1} < R_f} \lambda (e^{g_F + \sigma_F \varepsilon_{F,t+1}} - R_f)] =$$

Now, we only develop the expression under the expected value, i.e. what we have after θ_F .

$$\begin{aligned} E_t [I_{g_F + \sigma_F \varepsilon_{F,t+1} \geq \log(R_f)} (e^{g_F + \sigma_F \varepsilon_{F,t+1}} - R_f)] + E_t [I_{g_F + \sigma_F \varepsilon_{F,t+1} < \log(R_f)} \lambda (e^{g_F + \sigma_F \varepsilon_{F,t+1}} - R_f)] \\ = e^{g_F} E_t (I_{\varepsilon_{F,t+1} \geq \hat{\varepsilon}_F} e^{\sigma_F \varepsilon_{F,t+1}}) - R_f E_t (I_{\varepsilon_{F,t+1} \geq \hat{\varepsilon}_F}) + \lambda e^{g_F} E_t (I_{\varepsilon_{F,t+1} < \hat{\varepsilon}_F} e^{\sigma_F \varepsilon_{F,t+1}}) \\ - \lambda R_f E_t (I_{\varepsilon_{F,t+1} < \hat{\varepsilon}_F}) \end{aligned}$$

where we are using the indicator function and

$$\hat{\varepsilon}_F = \frac{\log(R_f) - g_F}{\sigma_F}.$$

Now, we have to solve the four members of the last expression: we have to calculate the expected values with the indicator functions. In doing so, we take into account that, in general,

$$E(I_{\varepsilon < \hat{\varepsilon}}) = N(\hat{\varepsilon})$$

$$E(I_{\varepsilon < \hat{\varepsilon}} e^{a\varepsilon}) = e^{a^2/2} N(\hat{\varepsilon} - a)$$

where $N(\cdot)$ is the cumulative distribution function of the standard normal distribution. Moreover, we exploit the fact that if a random variable x is distributed as a standard normal, then

$$E(e^{a+bx}) = e^{a+b^2/2}$$

In our case we have,

$$E(e^{a+bx}) = e^{g_S + \sigma_S^2/2} N(\sigma_S - \hat{\varepsilon}_S) - R_f N(-\hat{\varepsilon}_S) + \lambda e^{g_S + \sigma_S^2/2} N(\hat{\varepsilon}_S - \sigma_S) - \lambda R_f N(\hat{\varepsilon}_S)$$

By using $N(x) = 1 - N(-x)$ e $N(-x) = 1 - N(x)$, we can write

$$\begin{aligned} E(e^{a+bx}) &= e^{g_S + \sigma_S^2/2} - R_f + e^{g_S + \sigma_S^2/2} [\lambda N(\hat{\varepsilon}_S - \sigma_S) - N(\hat{\varepsilon}_S - \sigma_S)] - R_f [\lambda N(\hat{\varepsilon}_S) - N(\hat{\varepsilon}_S)] \\ &= e^{g_S + \sigma_S^2/2} - R_f + (\lambda - 1) [e^{g_S + \sigma_S^2/2} N(\hat{\varepsilon}_S - \sigma_S) - R_f N(\hat{\varepsilon}_S)]. \end{aligned}$$

Now, recalling θ_F , we get the final result:

$$E_t v[\theta_F (R_{F,t+1} - R_f)] = \theta_F \{e^{g_F + \sigma_F^2/2} - R_f + (\lambda - 1) [e^{g_F + \sigma_F^2/2} N(\hat{\varepsilon}_F - \sigma_F) - R_f N(\hat{\varepsilon}_F)]\}.$$

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