The (ir)resistible rise of agency rents

Bruno Biais, Toulouse School of Economics (CNRS-CRM & PWRI-IDEI) and

Augustin Landier, Toulouse School of Economics (TSE)[1]

September 2012

Abstract

The rents agents can extract from principals increase with the magnitude of incentive problems, which the literature usually takes as given. We endogenize it, by allowing agents to choose more or less opaque and complex technologies. In our overlapping generations model, agents compete with their predecessors. We study if virtuous old-timers can keep young managers' rent-seeking in check. With dynamic contracts, long horizons help principals incentivize agents. Hence, old agents are imperfect substitutes for young ones. This mutes down competition between generations, especially if compensation deferral is strong. As a result, young managers can opt for more opaque and complex technologies, and therefore larger rents, than their predecessors. Thus, in equilibrium, complexity and rents rise over time.

[1] This research was conducted within the Paul Woolley Research Initiative on Capital Market Dysfunctionalities at IDEI, Toulouse. Many thanks for insightful discussions to Guillaume Plantin, Thomas Mariotti, David Sraer and Jean Tirole and participants at the fourth Game Theory Society world meeting, especially Roger Myerson.

1 Introduction

In the presence of moral hazard, principals must leave rents to agents, to provide them with incentives. The more complex and opaque the task delegated to the agent, the more difficult it is to monitor him, the larger his rents. Furthermore, while operating the project, the agent acquires knowledge and network connections, which are key to the success of the project, but are also portable. In this context the agent could leave the firm and use these skills and connections for his own profit. To induce the agent to remain in the firm, the principal must leave him rents, especially if wealth diversion is facilitated by the opacity and complexity of the task.[2]

Analyses of rents under moral hazard have been developed, notably, by Holmstrom (1979), Grossman and Hart (1983), and Holmstrom and Tirole (1997). Models in which the agent can abscond, taking with him a fraction of the value of the firm, have been developed, notably, by Townsend (1979), Diamond (1984), Bolton and Scharfstein (1990). In all these analyses however, the severity of the moral hazard or cash diversion problem is taken as given. Yet, in practice, agents can take actions that will affect the magnitude of these problems. For example, they can opt for activities, sectors or products that are extremely complex or opaque. They can even devote resources to increasing such complexity and opacity. Agents can also opt for activities relying primarily on network connections or specific knowledge, which they could carry with them if absconding.[3]

The first innovation of the present paper is to consider a setting in which, when agents are born, they choose the characteristics of the task they propose to conduct for principals. Correspondingly, they choose the magnitude of the incentive problem to which principals will be confronted. Thus, the agents set the potential rents they could obtain if hired.

The agents' desire to capture rents, however, could be kept in check by market forces and competition among managers. If each principal could run an auction with several, otherwise identical, managers, he could select the agent with the smallest incentive problem, and hence the smallest rent.

[2]A particularly perverse instance of this problem arose in the case of AIG. As was mentioned in the New York Times issue of March 16, 2009: "A.I.G. employees concocted complex... If they leave... they might simply turn around and trade against A.I.G.'s book. Why not? They know how bad it is. They built it."

[3]For example, Hochberg, Ljungqvist and Lu (2007) show empirically the importance of newtworks in venture capital, while Oyer (2008) notes his results are consistent with finance sector managers acquiring skills and developing networks of connections shortly after taking jobs on Wall Street.

Anticipating this, managers would have to opt for activities with very limited agency problems, in order to be hired. We show, however, the existence of natural forces limiting competition between agents, thus creating the scope for rent capture. And in this context we endogenize the dynamics of agency problems.

We consider an OLG model. At each period a new principal and a new manager are born.[4] Each lives for two periods. At the beginning of his life, the young manager chooses (at a cost) the parameter b specifying the magnitude of the agency problem. The young principal meets the young agent, observes his b, and decides whether to hire him or not. When making this choice, the principal bears in mind that she could instead hire the agent born at the previous generation, which would be particularly attractive if that agent had a low b. Thus, if the young and old agents were otherwise perfectly substitutable, competition between generations would deter young agents from choosing higher bs than their predecessors. Consequently, opacity, complexity and rents would remain low. Incentive problems, however, endogenously induce imperfect substitutability between successive generations. The intuition is the following: To reduce rents, principals defer compensation (as in Becker and Stigler, 1974, and Rogerson, 1985). This makes it relatively unattractive for a young principal to hire an old agent. First, to poach the old agent, the young principal would have to pay him the rent he has been promised by his current employer. Second, old agents have a shorter horizon than young ones. This reduces the ability of the young principal to defer these agents' compensation to reduce their rents. Thus, other things equal, it is cheaper to incentivize young agents than old ones. Because old agents are imperfect substitutes for young ones, the latter can afford to choose technologies with greater agency problems than their predecessors', and still be hired. This gives rise to an upward trend in complexity, opacity, incentive problems and rents.

While the economic mechanims we analyze can be at play in various settings, our assumptions are particularly relevant for the financial sector – which offers plenty of opportunies to create complicated and opaque products. Indeed, the implications from our theoretical model, and in particular the gradual rise of rents are is in line with stylized facts from the financial sector. Philippon and Resheff (2011) document a strong increase in excess wages, or rents, in that sector, since the 1980s. They also

[4] In Section 6, we show that our analysis extends to the case where, each period, N managers and N investors are born.

find that job complexity and rents are correlated, and that the increase in these variables over the last three decades has been gradual. Our theoretical results are in line with these findings as well as with more anecdotal evidence. For example, Greg Smith, an executive director at Goldman Sachs, decided to resign after 12 years in that institution. In the article he wrote on that topic in *The New-York Times* on March 14, 2012, he undescored the decline in standards: "The firm has veered so far from the place I joined right out of college that I can no longer in good conscience say that I identify with what it stands for" as well as the rise of complexity, i.e., the increasing tendency to "pitch lucrative and complicated products to clients even if they are not the simplest investments or the ones most directly aligned with the client's goals."[5]

Our analysis also makes a theoretical point which, to be the best of our knowledge, had not been made before: While, at an individual level, long–term contracting and backloading of compensation is beneficial, at an aggregate level, it can generate inefficiencies in equilibrium. This is because, when different managers have different horizons, they can have different long–term contracts and therefore be imperfect substitutes, which mutes down competition in the labor market. In that sense, backloading of compensation in individually optimal dynamic contracts, exerts negative externalities on other principals in the labor market.

Because our analysis uncovers that equilibrium can be inefficient, it calls for public policy intervention. We consider the case of a regulator, who can set transparency and simplicity standards, monitor agents, and punish those who don't comply. We focus on proportional monitoring costs. That is, we assume costs are linear in the expected number of agents that are monitored –and thus don't include a fixed component. We show that, even in the absence of fixed monitoring costs, it can be optimal to abstain from monitoring during several periods and then engage in intense monitoring for one period. The intuition is the following: To induce agents to comply with transparency and simplicity standards, it is necessary to choose a large monitoring probability. Small monitoring probabilities have no effect on agents' choices between compliance and non compliance. Therefore monitoring, when it occurs, must be on a large scale. On the other hand, intense monitoring during one period compels all young agents in that period to opt for low bs. This resets agency problems at a low level. While lack of monitoring after that period leads to a decline in standards, this decline is only gradual. It is

[5] See http://www.nytimes.com/2012/03/14/opinion/why-i-am-leaving-goldman-sachs.html?_r=1&src=me&ref=general

therefore optimal to wait until standards have declined enough before incurring again the large cost of monitoring.

Our analysis of suboptimal occupational choices motivated by the desire to earn rents is in the same spirit as Murphy, Shleifer and Vishny (1991). But while Murphy, Shleifer and Vishny (1991) emphasize macro–determinants of rents, such as the size of the country or market as well as political factors, we focus more on micro–determinants, such as agency conflicts between investors and managers and imperfect substitutability between different vintages of managers.

Our work is also related to Myerson (2012). In both papers overlapping generations of managers are hired by investors, and agency problems (moral–hazard or cash–diversion) lead to compensation deferral. While, both papers show how the dynamic provision of incentives can generate cycles, they focus on different objects. Myerson (2012) shows how long–term contracting designed to cope with moral hazard can generate gradual growth followed by steep recessions. In contrast we show how long–term contracting creates imperfect substitutability between generations, giving rise to an increase in rents.

The next section presents our model. Section III analyzes optimal compensation contracts. Section IV characterizes the dynamics of equilibrium rents. Section V discusses policy intervention. Section VI presents extensions and discusses robustness. Section VII concludes.

2 Model

Investors and managers: Consider an overlapping generations model, where, each period, one investor and one manager are born. Both are risk neutral, have limited liability, live for two periods and have discount factor $\beta \in (0,1)$. Because we consider an overlapping generations model, successive generations of managers coexist in the market at a given point in time. This creates the scope for interactions, and in particular competition, between generations.

Each investor is initially endowed with one unit of investment good. She can invest it in a default technology, which she can operate herself and which returns 1 unit of consumption good per period during two periods. Alternatively, she can delegate the management of her capital to the agent. When entrusted with one unit of capital, the agent can generate return equal to $R > 1$ units of consumption good per period during two periods. For simplicity the choice between self–investment and delegated investment is irreversible.

Managers have zero initial endowment. At the beginning of his life, each young manager must choose among a range of investment techniques indexed by $b \in [b_{\min}, 1)$, with $b_{\min} > 0$.[6]

Each technique corresponds to a specific type of skills, knowhow and human capital. The cost of acquiring skills b is equal to cb, with $c \geq 0$. b's are ordered according to the complexity and opacity of the task. Low values of b correspond to simple and transparent investment techniques. Higher values of b correspond to complex and opaque investment techniques. c is the cost of designing complex products and strategies. Importantly, the choice of b is irreversible. The idea is tha managers acquire skills, human capital, relations and knowledge of investment techniques at an early stage in their career. Then, they use this informational capital.[7] Finally, for simplicity, we assume R does not depend on b. In Section 6 we show that our qualitative results are robust to letting R increase in b, as long as this increase is not too strong.

Sequence of play: Within each period $t \geq 1$, the timing of actions is the following:

- Stage 1: The young manager chooses $b_t \in [b_{\min}, 1)$.
- Stage 2: The young investor meets the young manager, observes his type (b_t), and decides whether to make him a take–it–or–leave–it offer or reject him.
- Stage 3: If the investor decides to reject the young manager or the latter rejects the offer, then the investor decides whether to self–invest or to approach the old manager (born at $t-1$). In the latter case, the investor meets the old manager, observes his type (b_{t-1}) and makes him a take–it–or–leave–it offer. Also, at this stage, if the previous investor is not currently employing an agent, she can approach a manager and make him a take–it–or–leave–it offer. Similarly, if the previous manager did not get hired, he can approach the young investor and apply for a job.
- Stage 4: Investment takes place.

[6] Assuming $b_{\min} > 0$ makes Lemma 3 immediate, but this assumption is not necessary for our analysis.

[7] See Oyer (2008) for empirical evidence on long term effects of initial career paths in the financial sector.

- Stage 5: Each manager must decide whether to remain in the firm and obtain the wage promised by the manager,[8] or abscond. If the manager absconds, he does not get paid, but obtains δbR, with $\delta \leq 1$. This corresponds to the profits he can earn by working on his own, using the valuable contacts and knowhow obtained while on the job. In that case, the return obtained by the investor is lowered to $R(1-b)$.
- Stage 6: Consumption takes place.

For simplicity (and to limit the number of cases in the analysis below), we assume

$$R(1 - \delta b_{\min}) > 1,$$

i.e., $b_{\min}$ is sufficiently low that, in a one–period context, self-investment by the principal is dominated by the delegation of investment to an agent with the lowest possible b.

Discussion of the assumptions: The assumption that investors have all the bargaining power is for simplicity. In Section 6, we present an extension of the model where the manager have some (but not all of) the bargaining power. It yields the same qualitative insights as the simpler present case.

The assumption that agents' types are observed by investors only when they meet them is also made for simplicity. As will be explained in Section 4, it simplifies the structure of beliefs and thus facilitates the analysis of the game.

The assumption that the manager can abscond amounts to a simple cash–diversion model as in Biais, Mariotti, Plantin and Rochet (2007), in line with Townsend (1979), Diamond (1984), Bolton and Scharfstein (1990) and DeMarzo and Fishman (2007). The larger b, the more opaque and complex the investment technology, the greater the fraction of the return the agent can extract when absconding, the lower the return left to the investor in this case. For simplicity we assume that, after managers abscond, they can't be hired by another investor. While we assume $\delta \leq 1$, all our major qualitative results are upheld if $\delta = 1$. Thus, our analysis holds both if cash–diversion is inefficient and if it is ex–post efficient.

[8] For simplicity also we assume the investor can commit to the contract offered.

While, in general, the return on the project when the manager remains in the firm could be a random variable taking several values, here, for simplicity, we assume it can take only one value, R. Extending the analysis to the case of more than one value would be straightforward and would not alter our qualitative results. Note also that the present cash diversion model is isomorphic to a simple moral hazard model, where the agent is instructed to exert maximal effort, as in Holmstrom and Tirole (1997). Thus, δb in our model can also be interpreted in similar terms as the private benefits from shirking (B) in Holmstrom and Tirole (1997).

First best: To conclude this section, we describe the first best benchmark, i.e., the situation where the benevolent social planner can observe and decide what investors and managers do. Since utilities are linear, there is a unique Pareto optimum regarding real decisions, and the points on the Pareto frontier differ only in terms of purely redistributive transfers between investors and managers. Since $R > 1$, the optimum is reached when investors delegate the management of their capital. Since $c \geq 0$ and R is indedendant of b, the socially efficient level of b is 0. The corresponding utilitarian welfare for each generation is

$$(1 + \beta)R. \tag{1}$$

3 Moral hazard and managerial compensation

In this section we analyze the compensation contract offered at time t by an investor hiring a young manager with given type b_t. We first analyze the contract offered by the investor hiring the manager for two periods. Next, we show that this dominates hiring the agent for one period only, and then another agent for the second period.

The compensation contract offered at time t by the investor is a pair of wages, w_t^t and w_{t+1}^t, paid to the manager if he remains in the firm at the end of periods t and $t+1$. In the cash–diversion context we consider, the optimal contract is such that the manager takes the efficient action and does not abscond. The corresponding incentive compatibility conditions are

$$w_{t+1}^t \geq \delta b_t R \tag{2}$$

and

$$w_t^t + \beta w_{t+1}^t \geq \delta b_t R. \tag{3}$$

The optimal contract maximizes the investor's net return subject to the incentive compatibility and limited liability constraints. It is spelled out in our first lemma.

Lemma 1: *At time t, for a given choice of b_t, and when the manager is hired for two periods, (3) binds and there is a continuum of optimal contracts, indexed by $\lambda_t \in [1, \frac{1}{\beta})$,*

$$\{w_t^t, w_{t+1}^t\} = \{(1 - \beta\lambda_t)\delta b_t R, \lambda_t \delta b_t R\}.$$

Remark 1: Complexity and Rents. Lemma 1 implies that the present value of the fund manager's earnings is

$$w_t^t + \beta w_{t+1}^t = \delta b_t R, \tag{4}$$

while the present value of the investor's returns is

$$R(1 + \beta) - \delta b_t R. \tag{5}$$

Thus, the investor's net return is decreasing in b_t, while the manager's earning is increasing. As the complexity and opacity of the investment technology increase, the investor must give the manager a larger and larger fraction of the total revenue of the investment.

Remark 2: Backloading and inefficiency. The larger λ_t, the greater the fraction of the compensation that is deferred to the second period. For example, when $\lambda_t = \frac{1}{\beta}$, managerial compensation is entirely deferred to $t + 1$. Both the principal and the agent are indifferent between all values of $\lambda_t \in [1, \frac{1}{\beta})$. In the next section, however, we show that the contractual choices of one generation have external effects on the next ones. Thus, different choices of λ_t lead to different inefficiencies.

Remark 3: Compensation and seniority. Lemma 1 implies that $w_t^t < w_{t+1}^t$, since

$$\frac{1}{1 + \beta} < 1 \leq \lambda_t.$$

Thus, for a given generation, compensation rises with seniority, in line with stylized facts. In the next section, however, we show that the structure of the equilibrium we analyze can imply that, when comparing generations, senior managers from the previous generations earn less than junior managers from the current generation.

Lemma 1 characterizes the optimal contracts when the manager is hired for two periods. What if, instead, the investor hires the manager for one period only, and then hires another manager at the second period. In this case, the incentive compatibility condition for the first period is

$$w_t^t \geq \delta b_t R.$$

At the second period, the investor could hire the young manager, with type b_{t+1}. This would require matching the wage offered by the young investor at time $t+1$ and also providing the young manager with the incentives not to abscond. Thus the investor would have to pay that agent at least

$$\delta b_{t+1} R.$$

Overall, the present value of the returns obtained by the investor would be no larger than

$$R(1+\beta) - \delta(b_t + \beta b_{t+1})R. \tag{6}$$

(5) is larger than (6). This reflects that, with repeated moral hazard, long term contracts help the principal reduce agency rents. In line with previous analyses of dynamic contracting (Becker and Stigler, 1974, and Rogerson, 1985), the future promised rents (here w_{t+1}) is also useful to incentivize current effort (and thus helps limiting w_t).[9] This cannot be achieved when the principal hires a sequence of agents compensated with short–term contracts. Hence, it is cheaper for the investor to hire the manager for two periods. We state this result in the following lemma:

Lemma 2: *Hiring the manager for two periods dominates hiring him for one period and then hiring another manager.*

4 The dynamics of equilibrium rents

We now turn to the dynamics of the b's chosen by managers before they are matched with investors.

[9] See also Biais, Mariotti, Plantin and Rochet (2007) or DeMarzo and Fishman (2007)).

Given an initial b_0, an equilibrium is a sequence $\mathcal{E} = \{b_t^*, w_t^{t^*}, w_{t+1}^{t^*}\}_{t\geq 1}$, satisfying the following conditions:

- **Optimization:** At each time t, the young manager chooses b_t^*, to maximize his gains and the investor makes an optimal hiring decision.
- **Rational expectations:** Investors and managers at time t have rational expectations about equilibrium dynamics at all times $\tau > t$, and when each market participant expects the others to follow the equilibrium, he/she himself/herself finds it optimal to also play according to $\mathcal{E}$.
- **Market clearing:** At each generation, the young manager is hired by the young investor.

From Lemma 1, the young manager's gains, if he opts for b_t and is employed, are

$$(\delta R - c)b_t. \tag{7}$$

The young manager chooses b_t to maximize (7), subject to the constraint that the investor is willing to hire him. Equilibrium requires that, when the agent anticipates all the others played as specified by $\mathcal{E}$, he will choose $b_t = b_t^*$.

If $\delta R < c$, then the agent opts for $b_t = 0$. To raise the possibility that the agent would choose $b_t > 0$, we assume hereafter the cost c is not too large, i.e.,

$$\delta R > c.$$

To characterize the optimal decision of the investor meeting a young manager with type b_t, we need to compare her payoff if she opts for the equilibrium action (hiring the young manager at Stage 2) to what she gets if she takes an out–of–equilibrium action.

The first out–of–equilibrium option for the investor is self–investment at Stage 3 of period t. She does not choose that option if her gains from hiring the young manager

$$R(1+\beta) - \delta b_t R. \tag{8}$$

are at least as large as the gains from self–investment $(1+\beta)$. This is is the case if

$$b_t \leq b_{\max} \equiv \min[\frac{R-1}{R}\frac{1+\beta}{\delta}, 1]. \tag{9}$$

The second out–of–equilibrium option for the investor is to "poach" an old employed agent at Stage 3 of period t and then hire the generation t manager, unemployed at t and available at $t+1$. She would have to give the former at least as much as his current wage, which she rationally expects to be $w_t^{t-1*} = \delta\lambda_{t-1}^* b_{t-1}^* R$, and would have to incentivize the latter. To do so, she would have to pay him at least $\delta b_t R$ at $t+1$. On the other hand, the old manager at $t+1$ cannot ask for more than that, since he could not obtain more from the time $t+1$ investor. Hence, overall, if she were to opt for that deviation, the time t investor would expect to get

$$R(1+\beta) - \delta(\lambda_{t-1}^* b_{t-1}^* + \beta b_t)R. \tag{10}$$

The third out–of–equilibrium option for the investor is to poach an old employed manager at t and then a young one at $t+1$. When evaluating the gains from this out–of–equilibrium move, the investor rationally expects both the previous and the subsequent generation to follow $\mathcal{E}$. Indeed, the extensive form of the game is such that the next generation's managers and investors make their decisions before observing if their predecessor deviated. Therefore they make their decisions based on the belief that their predecessor sticked to $\mathcal{E}$, and find it optimal to do the same themselves. Thus, when deviating, the generation t investor expects to pay $\delta\lambda_{t-1}^* b_{t-1}^* R$ to the old manager she hires at time t, and $\delta b_{t+1}^* R$ to the young manager she hires at time $t+1$. Consequently, the deviating investor expects to earn

$$R(1+\beta) - \delta(\lambda_{t-1}^* b_{t-1}^* + \beta b_{t+1}^*)R. \tag{11}$$

Equating (8), evaluated at b_t^*, to the maximum of (10) and (11), as long as the no–self–investment constraint (9) does not bind, the equilibrium choice of the generation t young manager is

$$b_t^* = \lambda_{t-1} b_{t-1}^* + \beta \min[b_t^*, b_{t+1}^*]. \tag{12}$$

(12) and $\lambda_{t-1} \geq 1$ directly imply our next lemma.

Lemma 3: *At time t, if (9) does not bind, then*

$$b_t^* > b_{t-1}^*.$$

Lemma 3 and (12) imply that, as long as the no–self–investment constraint (9) does not bind,

$$b_t^* = \frac{\lambda_{t-1}}{1-\beta} b_{t-1}^*. \tag{13}$$

Thus, we obtain our first proposition.

Proposition 1: *Set the initial manager's type to $b_0 \geq b_{\min}$. There exists a continuum of equilibria, indexed by $\lambda \in [1, 1/\beta]$, such that, for all $t > 0$, b_t^* is strictly increasing in t, until it reaches $b_{\max}$ at time $T_{\max}$ and*

$$b_t^* = \min[(\frac{\lambda}{1-\beta})^t b_0, b_{\max}]. \tag{14}$$

The rise of rents: For simplicity, the proposition only presents the equilibria in which λ_t is constant through time. There also exists equilibria in which λ_t varies as time goes by, but they yield the same qualitative insights as those described in Proposition 1. Interestingly, even if λ_t is constant through time, b_t is non–stationary. Indeed, (14) implies that, until $T_{\max} - 1$, b_t grows at rate

$$\rho_t = \frac{b_t^* - b_{t-1}^*}{b_{t-1}^*} = \frac{\lambda + \beta - 1}{1-\beta} > 0. \tag{15}$$

Thus opacity, complexity and rents rise in equilibrium. This is because investors can offer long–term contracts to young managers, not to old ones. This makes the former attractive relative to the latter, other things equal. Hence, at time t, the young manager can afford to choose $b_t > b_{t-1}$ and yet be employed.

Backloading exerts contractual externalities: Note that ρ_t is increasing in λ. While, at each time t, the investor is indifferent between all $\lambda_t \in [1, \frac{1}{\beta})$, his choice affects competition between successive generations of managers. The greater λ_t, the more compensation is backloaded, the lower the competition between generations, the stronger the growth of rents, ρ_t. The lowest possible equilibrium growth rate of rents, arises for $\lambda = 1$, where

$$\rho_t = \frac{\beta}{1-\beta}. \tag{16}$$

Note that, even for $\lambda_t = 1$, $\rho_t > 0$.

Patience and rents: The more patient the investor, the more he suffers from the cost of incentivizing a new agent next period, the less competition between generations there is, the larger is b_t^*.

Welfare: The utilitarian welfare for generation t,

$$R(1+\beta) - cb_t.$$

is lower than welfare in the first best, (1). This is because managers incur wasteful costs (cb_t) to come up with complex and opaque technologies, that increase rents but don't create value for society.

Note however that, when b_T reaches $b_{\max}$, utilitarian welfare becomes

$$R(1+\beta) - c\frac{R-1}{R}\frac{1+\beta}{\delta} = (1+\beta)(R - \frac{c}{\delta R}(R-1)),$$

which is greater than welfare under self–investment under our assumption that $R > 1$ and $\delta R > c$. Thus, while managers choose to incur wasteful costs, these don't wipe out all the efficiency gains from investment delegation.

5 Policy

The analysis above shows that the equilibrium choice of b is inefficient. This begs the question whether policy intervention can improve efficiency. To analyze this issue we assume there is a benevolent regulator, maximizing utilitarian welfare. To model regulatory intervention we introduce an additional stage in the game. Each period t, at stage 0 the regulator announces a cap $\underline{b}_t$, and an inspection probability α_t, to which we assume he can commit. Then, at stage 1, the young agent chooses b_t, and, with probability α_t, is inspected by the regulator. In case of inspection, if $b_t > \underline{b}_t$ the agent is prevented from working. Otherwise, he enters the market and meets the investor, and the game unfolds. In particular, compensation is determined as in Lemma 1. For simplicity, we hereafter focus on the equilibrium where $\lambda_t = 1, \forall t$. Finally, we assume the monitoring technology is linear, i.e., there exists a constant $\gamma > 0$ such that the cost of monitoring with probability α_t during one period is $\alpha_t \gamma$. In this section we compare the performance of three policies:

- Laissez faire, in which the regulator never intervenes.

- Permanent monitoring, in which the regulator sets a constant cap $\underline{b}$, and monitors with constant probability α to ensure that agents always comply with the regulatory cap.

- Periodic monitoring, which operates as follows: during $T-1$ periods (with T finite and strictly larger than 1), laissez faire prevails, i.e., $\alpha = 0$. Then, at the T^{th} period, the regulator intervenes, sets the maximum $\underline{b}$ for this period, and monitors the agent with probability α_T, sufficiently large to ensure compliance. Then a new cycle starts.

5.1 Permanent monitoring

We analyze for which values of the permanent monitoring policy $(\underline{b}, \alpha)$ it is an equilibrium for agents to always comply. The expected gain of the agent if he complies is

$$\delta \underline{b} R.$$

When the agent chooses $\hat{b} > \underline{b}$ and is not inspected he is matched with an investor. The latter is willing to hire him at t, rather than poaching an old agent (with type $\underline{b}$) at t and then hiring the generation $t+1$ agent (also with type $\underline{b}$) if

$$R(1+\beta) - \delta \hat{b} R \geq R(1+\beta) - (1+\beta)\delta \underline{b} R.$$

Binding the constraint yields

$$\hat{b} = (1+\beta)\underline{b}.$$

The agent prefers to comply if his gain under compliance exceeds is expected gain under non compliance. Binding this constraint we have

$$\underline{b} = (1-\alpha)\hat{b}. \tag{17}$$

Lemma 4: *An optimal permanent monitoring policy* $(\underline{b}, \alpha)$*, is such that*

$$\alpha = \frac{\beta}{1+\beta} \text{ and } \underline{b} = b_{\min}.$$

Consistent with intuition, the lemma states that the agent complies if the probability of inspection (α) is sufficiently large. That $\underline{b} = b_{\min}$ stems from the fact that the cost of monitoring is independent of the cap $\underline{b}$. Thus, if the regulator opts for a permanent monitoring policy, it is optimal to minimize the cap.

The steady state utilitarian welfare of each generation under a stationary inspection policy incentivizing permanent compliance is

$$W(1) \equiv R(1+\beta) - \frac{\beta}{1+\beta}\gamma - cb_{\min}. \tag{18}$$

On the other hand, the steady state utilitarian welfare under laissez faire is

$$W(\infty) \equiv R(1+\beta) - cb_{\max}. \tag{19}$$

Comparing $W(1)$ and $W(\infty)$ we obtain the following proposition.

Proposition 2: *Welfare is larger under a stationary policy incentivizing permanent compliance than under laissez faire if*

$$\gamma < \frac{1+\beta}{\beta}c(b_{\max} - b_{\min}). \tag{20}$$

The proposition states that welfare is greater under permanent monitoring than under laissez faire if the inspection cost, γ, is relatively low compared to the cost of opting for complex investment techniques, c, and if $b_{\max}$ is large while $b_{\min}$ is low.

5.2 Periodic monitoring

Now turn to the analysis of periodic monitoring. To evaluate the efficiency of a periodic monitoring policy of period T we consider the average welfare of the T generations, which we denote by $W(T)$. First present the equilibrium dynamics of b between two regulatory interventions. Second, we study the condition under which the agent complies at the time of policy intervantion. Third, we show that an optimal periodic monitoring policy does not let b reach $b_{\max}$. Finally we compare $W(1)$ and $W(\infty)$ to $W(T)$.

The dynamics of b with periodic monitoring: **Lemma 5:** *If the agent complies when the regulator intervenes, for any integer k, $b_{kT} = \underline{b}$, and during laissez faire periods equilibrium dynamics are as follows*

$$b_{kT+n} = \min[\frac{\underline{b}}{(1-\beta)^n}, b_{\max}], \forall n \in \{1, ...T-2\}, \textit{if } T > 2,$$

while

$$b_{(k+1)T-1} = \min[(\frac{1}{(1-\beta)^{T-2}} + \beta)\underline{b}, b_{\max}].$$

The monitoring probability ensuring compliance: At the time of intervention, $t = kT$, the regulator sets b below the previous level, i.e., $\underline{b} < b_{t-1}$. If the generation t agent complies, he gets

$$\delta \underline{b} R.$$

If he deviates, he chooses the maximum level of complexity at which he is still employed, i.e., $\hat{b}$ such that

$$R(1+\beta) - \delta\hat{b}R = R(1+\beta) - \delta b_{t-1}R - \beta\delta\min[\hat{b}, b^*_{t+1}]R. \tag{21}$$

The left–hand–side is the profit of the investor if she hires at time t the manager who deviated to $\hat{b}$. The right–hand–side is her expected profit if she poaches an old manager at time t, and then, at time $t+1$, hires either the (unemployed) generation t manager (and pays him $\delta\hat{b}R$) or the young manager (whom she expects to follow the candidate equilibrium strategy and choose b^*_{t+1}). (21) simplifies to

$$\hat{b} = b_{t-1} + \beta\min[\hat{b}, b^*_{t+1}]. \tag{22}$$

Given rational expectations about $\hat{b}$, the regulator will choose the lowest possible level of monitoring ensuring compliance, so that (17) holds. Following similar steps as in the previous subsection, our next proposition obtains.

Proposition 3: *The optimal periodic monitoring policy with period T is such that*

$$\underline{b} = b_{\min} \textit{ and } \alpha(T) = 1 - \frac{1}{\frac{1}{(1-\beta)^{T-2}} + \beta\frac{2-\beta}{1-\beta}} \textit{ for } T > 2 \textit{ while } \alpha(2) = 1 - \frac{1}{(1+\beta)^2}.$$

The monitoring probability $\alpha(T)$ is increasing in T. The longer the regulator waits, the larger the b of the last generation before the regulatory intervention. The more tempting it is for agents not to comply, hoping to earn large rents. Again, that $\underline{b} = b_{\min}$ stems from the fact that the cost of monitoring is independent of the cap $\underline{b}$.[10]

An optimal periodic monitoring policy does not let b reach $b_{\max}$: Proposition 4: *The periodic monitoring policy of period T such that $b_{T-1} = b_{\max}$ is dominated by laissez faire or the periodic monitoring policy with period $T-1$.*

Comparison of $W(1)$ and $W(\infty)$ to $W(T)$: Lemma 5 and Proposition 3, yield the following corollary:

Corollary 1: *For any finite T strictly larger than 2, average welfare under periodic monitoring is*

$$W(T) = R(1+\beta) - \frac{1}{T}\{cb_{\min}[(1-\beta)\frac{(\frac{1}{1-\beta})^{T-1}-1}{\beta} + (\frac{1}{1-\beta})^{T-2} + \beta] + (1 - \frac{1}{(\frac{1}{1-\beta})^{T-2} + \beta\frac{2-\beta}{1-\beta}})\gamma\}. \quad (23)$$

For $T = 2$ it is equal to

$$R(1+\beta) - \frac{1+(1+\beta)}{2}cb_{\min} - \frac{1}{2}(1 - \frac{1}{(1+\beta)^2})\gamma. \quad (24)$$

We now compare welfare under periodic monitoring to its counterpart under permanent monitoring and under laissez faire. The next proposition states a preliminary result.

Proposition 5: *Welfare under monitoring with $T = 2$ is larger than welfare under laissez faire and than welfare under permanent monitoring iff*

$$\frac{(1+\beta)^2}{\beta}cb_{\min} < \gamma < c\frac{b_{\max} - \frac{1+(1+\beta)}{2}b_{\min}}{\frac{1}{2}(1 - \frac{1}{(1+\beta)^2})}.$$

[10] What determines the agent's propensity to comply is the ratio of b just before the intervention to that requested by the regulator. $\underline{b}$ appears both on the numerator an the denominator of that ratio, and therefore simplifies out.

The next proposition offers a comparison for general values of T.

Proposition 6:

Fix the parameters $R, \beta, b_{\max}, b_{\min}$. There exist thresholds $0 < \gamma_1 < \gamma_2 < \gamma_3 < \gamma_4$ and B such that:

- *If $\gamma < \gamma_1$, the optimal cyclical policy consists of constant monitoring imposing $\underline{b} = b_{\min}$.*
- *If $\gamma > \gamma_4$, the optimal cyclical policy consists in laissez-faire.*
- *If $b_{\min} < B$ and $\gamma_1 < \gamma < \gamma_2$, the optimal cyclical policy consists of periodic interventions, at intervals of two or more periods, resetting b to $\underline{b} = b_{\min}$.*

TBA Illustration with numerical values.

6 Extensions

Coming back to the model without regulatory intervention, we now show that our qualitative results hold after realxing several of our assumptions.

6.1 When R increases with b

In this subsection, we show that our qualitative insights are robust to letting R increase with b. To do so we assume $R(b) = \mu + \kappa b$. When $\kappa = 0$, this is the model analyzed above. To examine the local robustness of our resuts we focus on the case where κ is small. The efficient choice of b maximizes $R(b) - cb$. In keeping with our focus on the case where κ is small, we suppose $\kappa < c$. Thus it is efficient to set $b = 0$.

Bearing in mind that at stage 2, when the young investor meets the young agent, b_t and $R(b_t)$ are set, the optimal contract is

$$(w_t^t, w_{t+1}^t) = ((1-\beta)\delta b_t R(b_t), \delta b_t R(b_t)),$$

so that fund managers' earnings are

$$w_t^t + \beta w_{t+1}^t = \delta b_t R(b_t), \tag{25}$$

and the investor's returns are

$$R(b_t)(1+\beta) - \delta b_t R(b_t). \tag{26}$$

At period t, the young manager chooses b_t to maximize

$$\delta b_t R(b_t) - c b_t = \delta b_t(\mu + \kappa b_t) - c b_t,$$

subject to the constraint that the period t investor is willing to hire him. Without the constraint, the first order condition would be

$$\delta\mu + 2\delta\kappa b_t - c = 0.$$

Assuming $\delta\mu > c$, the unconstrained optimum b for the agent is

$$b_t = \frac{\delta\mu - c}{2\delta\kappa}.$$

While it would be socially optimal to have $b_t = 0$, the agent would prefer a positive b_t, to extract rents. Consider the case when $\frac{\delta\mu - c}{2\delta\kappa} \geq b_{\max}$ then b_t is set by the condition under which the investor is willing to hire the young agent. The investor prefers to do so rather than starting with employing an old one, and then hiring the unemployed generation t manager, if

$$R(b_t)(1+\beta) - \delta b_t R(b_t) \geq R(b_{t-1}) + \beta R(b_t) - \delta(b_{t-1}R(b_{t-1}) + \beta b_t R(b_t)).$$

That is

$$R(b_t) - R(b_{t-1}) \geq \delta\{b_t R(b_t) - [b_{t-1}R(b_{t-1}) + \beta b_t R(b_t)]\}.$$

When this holds as an equality we have

$$b_t = \frac{b_{t-1}}{1-\beta}\frac{R(b_{t-1})}{R(b_t)} + \frac{R(b_t) - R(b_{t-1})}{\delta(1-\beta)R(b_t)}.$$

When $\kappa = 0$, we are back to the previous case. When κ is small, $R(b_{t-1})$ is close to $R(b_t)$, and we still have $b_t \geq b_{t-1}$.

6.2 When the agent has some bargaining power

In the above analysis we assumed the principal made take–it–or–leave–it offers to the agent. We now relax this assumption and show that our analysis is robust to assuming that the agent has some bargaining power.

Consider the principal and the agent born at time t. As above, when meeting the agent at time t the principal makes him a take–it–or–leave–it offer (w_t^t, w_{t+1}^t). In contrast with the above analysis, after the end of period t, after having decided to remain in the firm and received w_t^t, the agent can, with probability ϕ make a take–it–or–leave–it counter–offer $\hat{w}$ to the principal. If the principal accepts, then if the agent remains in the firm at the end of period $t+1$, the principal must pay him $\hat{w}$ instead of w_{t+1}^t. On the other end, if the principal refuses the counter–offer of the agent, then the latter leaves the firm, and the principal only obtains $R(1-b_t)$ at the end of period $t+1$. To extract as much surplus as possible from the principal, the agent asks for $\hat{w} = Rb_t$, which leaves the principal indifferent between accepting and rejecting the counter–offer. Thus, the principal solves the following maximization program:

$$\max_{w_t^t, w_{t+1}^t} R(1+\beta) - [w_t^t + \beta(\phi\hat{w} + (1-\phi)w_{t+1}^t)]$$

subject to the incentive compatibility condition at time t

$$w_t^t + \beta(\phi\hat{w} + (1-\phi)w_{t+1}^t) \geq \delta b_t R,$$

the incentive compatibility condition at time $t+1$

$$w_{t+1}^t \geq \delta b_t R,$$

and the limited liability constraint. Suppose the latter does not bind, proceeding as in Lemma 1, the incentive compatibility condition at time t is binding. Hence the expected gain of the agent when meeting the principal is

$$w_t^t + \beta(\phi\hat{w} + (1-\phi)w_{t+1}^t) = \delta b_t R,$$

and the expected gain of the principal $R(1+\beta) - \delta b_t R$, as in Lemma 1. To check under what condition the limited liability condition holds, substitute the value of $\hat{w}$ into the binding time $t+1$ incentive compatibility condition. This yields

$$w_t^t + \beta(\phi R b_t + (1-\phi)w_{t+1}^t) = \delta b_t R.$$

That is

$$w_t^t + \beta(1-\phi)w_{t+1}^t = b_t R(\delta - \beta\phi).$$

For this to hold we must have $\delta > \beta\phi$. As in Lemma 1, there is a continuum of optimal compensation contracts. For simplicity focus on the contract for which $w_t^t = 0$, so that

$$w_{t+1}^t = \frac{\delta - \beta\phi}{\beta(1-\phi)} b_t R.$$

Thus we can state the following result.

Proposition 7: *If $\delta > \beta\phi$, at time t, for a given choice of b_t, there exists an optimal contract*

$$\{w_t^t, w_{t+1}^t\} = \{0, \frac{\delta - \beta\phi}{\beta(1-\phi)} b_t R\}.$$

This contract is similar to that characterized in Lemma 1. Thus, even if the agent has some bargaining power, as long as $\delta > \beta\phi$, equilibrium is qualitatively similar to that characterized in Proposition 1 for the case where the investor has all the bargaining power.

6.3 Intra–generational competition

In the model analyzed above, for simplicity, there is only one agent per generation. Our qualitative results are upheld, however, when there are several agents, as long as intra-generational competition is not perfect. To illustrate this point, consider the following extension of our basic model.

The sequence of play is almost the same as in the basic model, but, at each generation N managers and N agents are born. Then, at Stage 1 of period t, each young manager chooses his b. We focus on symmetric equilibria where all generation t young managers choose the same b_t^*. At Stage 2 of period t, each young investor is matched with one young manager. She observes his type and can make him a take it or leave it offer. If they don't strike a deal, then at Stage 3, as in the basic model, the young investor can approach an old employed agent. Alternatively, in contrast with the basic model, the investor can approach a young manager who has just been employed. In that case, she observes the type b of this manager and can make him a take it or leave it offer. From that point on, the game unfolds as in the basic model.

In equilibrium, all agents choose the same b, and investors rationally anticipate that. Hence, there is no point for the young investor to contact another young manager at Stage 3. She would get exactly the same gain as with the agent she had been initially matched with. Thus, as in the basic model, all generation t managers choose their b to leave the investor indifferent between hiring them or their predecessors, while intra–generational competition does not impose any additional constraint on managers. Hence the equilibrium is exactly the same as in Proposition 1.

7 Conclusion

TBA

References

Biais, Bruno, Thomas Mariotti, Guillaume Plantin and Jean Charles Rochet, 2007, *Review of Economic Studies*

Becker, Gary and George Stigler, 1974, "Law enforcement, malfeasance, and compensation of enforcers," *Journal of Legal Studies,* 3:1-18.

Bolton, Patrick, and David Scharfstein, 1990, "A theory of predation based on agency problems in financial contracting," *American Economic Review*, 80, 94–106.

DeMarzo, P.M., and M.J. Fishman, 2007, "Optimal Long-Term Financial Contracting," *Review of Financial Studies*, 20, 2079–2128.

Diamond, Douglas, 1984, "Financial intermediation and delegated monitoring," *Review of Economic Studies*, 51, 393–414.

Grossman, S., and O. Hart, 1983, "An analysis of the principal–agent problem," *Econometrica*, 51, 7–45.

Hochberg, Y., A. Ljungqvist, and Y. Lu, 2010, "Networking as a barrier to entry and the competitive supply of venture capital", *Journal of Finance*, 65, 829-859.

Holmstrom, B., 1979, "Moral Hazard and Observability", *The Bell Journal of Economics*, 10, 74–91.

Holmstrom, Bengt, and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics*, 112, 663–691.

Murphy Shleifer Vishny,1991, "The Allocation of Talent: Implications for Growth," *Quarterly Journal of Economics*, 106, 503–530.

Myerson, Roger, 2012, "A model of moral–hazard credit cycles," Working paper, University of Chicago.

New York Times, 2009, http://www.nytimes.com/2009/03/17/business/17sorkin.html?_r=1)

Oyer, P., 2008, "The Making of an Investment Banker: Stock Market Shocks, Career Choice, and Lifetime Income," *Journal of Finance*, 63, 2601–2628.

Rogerson, W., 1985, "Repeated Moral Hazard," *Econometrica*, 58, 69-76.

Philippon, T. and A. Resheff, 2012, "Wages and Human Capital in the U.S. Financial Industry: 1909-2006," forthcoming *Quarterly Journal of Economics*.

Townsend, Robert, 1979, "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, 22, 265–293.

Proofs

Proof of Lemma 1:

The program of the investor is

$$\max_{w_t^t, w_{t+1}^t} R(1+\beta) - w_t^t - \beta w_{t+1}^t, \text{s.t.}, \ w_{t+1}^t \geq \delta b_t R \text{ and } w_t^t + \beta w_{t+1}^t \geq \delta b_t R.$$

The Lagrangian is

$$\mathcal{L} = R(1+\beta) - w_t^t - \beta w_{t+1}^t + \mu_t(w_t^t + \beta w_{t+1}^t - \delta b_t R) + \mu_{t+1}(w_{t+1}^t - \delta b_t R),$$

where μ_t and μ_{t+1} are the multipliers of the time t and $t+1$ incentive constraints, respectively. The first order condition with respect to w_t is:

$$-1 + \mu_t = 0.$$

Hence the incentive compatibility constraint at time $t+1$ binds, i.e.,

$$w_t^t + \beta w_{t+1}^t = \delta b_t R,$$

and the set of optimal values for w_t^t is the interval $[0, (1-\beta)\delta b_t R]$.

QED

Proof of Lemma 3:

Since $\lambda_{t-1} \geq 1$ and $b_t^* \geq b_{\min} > 0$, (12) implies $b_t > b_{t-1}$.

QED

Proof of Proposition 1:

To complete the analysis of the equilibrium, we must rule out a last possible deviation by managers. If the generation t manager chooses $b_t = b_t^*$, he gets hired and works for two periods. It could be tempting for the time t manager to deviate to $b_t = \hat{b} > b_t^*$, aware that he would not be hired at time t, but hoping he would earn large rents at $t+1$.

To evaluate his gains under this deviation, the agent must form beliefs about the reaction of the next generation. Our assumption that managers and investors make choices at stages 1 and 2 facilitates this analysis. Since they make their decision before having the opportunity to observe previous actions, they stick to the equilibrium even after deviations.

When deviating to $\hat{b}$ at period t, the manager must ensure he will be hired at period $t+1$. At that time, when making the equilibrium choice and hiring the young manager, the young investor obtains

$$R(1+\beta) - \delta b^*_{t+1} R.$$

If, instead, the period $t+1$ investor hires the previous deviator, and, at the following period, the (now old) time $t+1$ manager, he obtains

$$R(1+\beta) - \delta \hat{b} R - \beta \delta b^*_{t+1} R.$$

To be employed at time $t+1$, the deviator must choose $\hat{b}$ such that

$$\hat{b} \leq (1-\beta) b^*_{t+1}. \tag{27}$$

The equilibrium choice of b^*_t dominates the deviation to $\hat{b}$ if

$$\delta b^*_t R \geq \beta \delta \hat{b} R. \tag{28}$$

Binding (27) and substituting it into (28), the condition under which the agent is better off taking the equilibrium action rather than deviating to $\hat{b}$ is

$$b^*_t \geq \beta(1-\beta) b^*_{t+1}.$$

Substituting b^*_{t+1} from (14), the condition is

$$b^*_t \geq \beta(1-\beta)\lambda \frac{b^*_t}{1-\beta}.$$

That is

$$\frac{1}{\beta} \geq \lambda,$$

which holds since $\lambda \in [0, 1/\beta]$.

QED

Proof of Lemma 5:

First consider the case $T > 2$. At time $t = kT + n$, where $n \in \{1, ...T - 2\}$, young managers choose the highest possible b_t at which they are still employed. Following the same reasoning as in the previous analyses, binding the employability constraint yields

$$R(1+\beta) - \delta b_t R = R(1+\beta) - \delta b_{t-1} R - \beta\delta \min[b_t, b_{t+1}] R,$$

which simplifies to

$$R(1+\beta) - \delta b_t R = R(1+\beta) - \delta b_{t-1} R - \beta\delta b_t R.$$

Thus

$$b_t = \frac{b_{t-1}}{1-\beta}.$$

This implies

$$b_{kT+n} = \frac{b_{kT+n-1}}{1-\beta}, \forall n \in \{1, ...T-2\}.$$

For the last generation before regulatory intervention, $t = (k+1)T - 1$, things are slightly different since the young manager anticipates that $b_{t+1} = \underline{b}$. Hence, the manager in that generation chooses b_t such that

$$R(1+\beta) - \delta b_t R = R(1+\beta) - \delta b_{t-1} R - \beta\delta \underline{b} R.$$

That is

$$b_t = b_{t-1} + \beta \underline{b}.$$

Since $t - 1 = kT + T - 2$, $b_{t-1} = \frac{\underline{b}}{(1-\beta)^{T-2}}$. Thus we have

$$b_t = (\frac{1}{(1-\beta)^{T-2}} + \beta)\underline{b}. \tag{29}$$

Second, consider the case $T = 2$. At $t = 1$, we have $b_1 = \underline{b}$, at $t = 2$, we have $b_2 > \underline{b}$, ... b_2 binds the employability constraint:

$$R(1+\beta) - \delta b_2 R = R(1+\beta) - \delta \underline{b} R - \beta\delta \underline{b} R.$$

Hence

$$b_2 = (1+\beta)\underline{b}, \tag{30}$$

which is consistent with (29) evaluated at $T = 2$.

QED

Proof of Proposition 3:

First consider the case $T > 2$. In this case (22) writes as

$$\hat{b} = b_{t-1} + \beta \min[\hat{b}, \frac{\underline{b}}{1-\beta}]. \tag{31}$$

If $\min[\hat{b}, \frac{\underline{b}}{1-\beta}] = \hat{b}$, that is $\hat{b} < \frac{\underline{b}}{1-\beta}$, then (31) yields

$$\hat{b} = \frac{b_{t-1}}{1-\beta}$$

which is in contradiction with $\hat{b} < \frac{\underline{b}}{1-\beta}$ since $\underline{b} < b_{t-1}$. Hence $\min[\hat{b}, \frac{\underline{b}}{1-\beta}] = \frac{\underline{b}}{1-\beta}$, and $\hat{b}$ is set by

$$\hat{b} = b_{t-1} + \beta\frac{\underline{b}}{1-\beta}.$$

Thus (17) writes as

$$\underline{b} = (1-\alpha)[b_{t-1} + \beta\frac{\underline{b}}{1-\beta}].$$

That is

$$\alpha = 1 - \frac{1}{\frac{b_{t-1}}{\underline{b}} + \frac{\beta}{1-\beta}} = 1 - \frac{1}{\frac{b_{kT-1}}{\underline{b}} + \frac{\beta}{1-\beta}}.$$

Note that the right–hand–side is between 0 and 1. Substituting b_{kT-1} from Lemma 5, this simplifies to

$$\alpha = 1 - \frac{1}{\frac{1}{(1-\beta)^{T-2}} + \beta\frac{2-\beta}{1-\beta}}.$$

Second, consider the case $T = 2$. Subsituting (30) in (22), we have

$$\hat{b} = (1+\beta)\underline{b} + \beta\min[\hat{b}, (1+\beta)\underline{b}].$$

If $\hat{b} \leq (1+\beta)\underline{b}$, then this equation is equivalent to

$$\hat{b} = \frac{1+\beta}{1-\beta}\underline{b},$$

which is a contradiction. Hence, $\hat{b} > (1+\beta)\underline{b}$ and the employability constraint yields

$$\hat{b} = (1+\beta)^2\underline{b}.$$

Thus (17) yields

$$\alpha = 1 - \frac{1}{(1+\beta)^2}.$$

QED

Proof of Proposition 4:

Either $W(T) \leq W(\infty)$, and the proposition holds, or $W(T) > W(\infty)$. Consider the latter case. Define

$$b_{average}(T) = \frac{1}{T}[\sum_{t=1}^{T} b_t],$$

with the convention that policy intervention occurs at time 1, so that $b_1 = \underline{b}$. We have

$$W(T) = R(1+\beta) - cb_{average}(T) - \frac{\gamma}{T}\alpha(T).$$

We now prove that

$$W(T) < \frac{T-1}{T}W(T-1) + \frac{1}{T}W(\infty). \tag{32}$$

(32) is equivalent to

$$T[R(1+\beta) - cb_{average}(T) - \frac{\gamma}{T}\alpha(T)] < (T-1)[R(1+\beta) - cb_{average}(T-1) - \frac{\gamma}{T-1}\alpha(T-1)] + W(\infty). \tag{33}$$

Now, since the last b in the periodic monitoring polcy of period T is equal to $b_{\max}$, we have that

$$Tb_{average}(T) - (T-1)b_{average}(T-1) = b_{\max}.$$

Hence, (33) holds if

$$T\{R(1+\beta) - \frac{\gamma}{T}\alpha(T)\} - cb_{\max} < (T-1)[R(1+\beta) - \frac{\gamma}{T-1}\alpha(T-1)] + W(\infty). \tag{34}$$

Substituting $W(\infty)$ from (19) this inequaliy is equivalent to

$$-\gamma[\alpha(T) - \alpha(T-1)] < 0, \tag{35}$$

which holds since $\alpha(T) > \alpha(T-1)$.

Now, since $W(\infty) < W(T)$, (32) implies that

$$W(T-1) > W(T),$$

which completes the proof.

QED

Proof of Proposition 5:

(24) is greater than (19), iff

$$\frac{1+(1+\beta)}{2} cb_{\min} + \frac{1}{2}(1 - \frac{1}{(1+\beta)^2})\gamma < cb_{\max}. \tag{36}$$

That is

$$\gamma < c \frac{b_{\max} - \frac{1+(1+\beta)}{2} b_{\min}}{\frac{1}{2}(1 - \frac{1}{(1+\beta)^2})}.$$

Note that the right–hand–side of is positive, since $b_{\max} \geq (1+\beta)b_{\min}$, because of (**??**).

(24) is greater than (18), iff

$$\frac{\beta}{2} cb_{\min} < [\frac{\beta}{1+\beta} - \frac{1}{2}(1 - \frac{1}{(1+\beta)^2})]\gamma.$$

$$\frac{\beta}{2} cb_{\min} < \frac{1}{(1+\beta)^2}[\beta(1+\beta) - \frac{1}{2}((1+\beta)^2 - 1)]\gamma.$$

$$\frac{\beta}{2} cb_{\min} < \frac{1}{(1+\beta)^2}[\beta(1+\beta) - \frac{1}{2}(2\beta + \beta^2)]\gamma.$$

$$\frac{\beta}{2} cb_{\min} < \frac{\beta}{(1+\beta)^2}[(1+\beta) - (1 + \frac{\beta}{2})]\gamma.$$

$$\frac{(1+\beta)^2}{\beta} cb_{\min} < \gamma.$$

QED

Proof of Proposition 6:

Call N the smallest integer such that $n > N \Rightarrow \frac{b_{\min}}{(1-\beta)^n} > b_{\max}$. Thus, we know that for any value of γ, a cycle of length longer than N induces lower welfare than a cycle of length N. Thus, looking for the optimal policy is equivalent to solving the following program:

$$\max_{T \in \{1,2,\ldots,N,+\infty\}} W_T$$

where we note W_1 the welfare of permanent monitoring and $W_{+\infty}$ the welfare of laissez-faire.

$$W_T(\gamma)=\begin{cases} R(1+\beta)-\frac{\beta}{1+\beta}\gamma-cb_{\min} & \text{for} \quad T=1 \\ R(1+\beta)-\frac{1+(1+\beta)}{2}cb_{\min}-\frac{1}{2}(1-\frac{1}{(1+\beta)^2})\gamma & \text{for} \quad T=2 \\ R(1+\beta)-\frac{1}{T}\{cb_{\min}[(1-\beta)\frac{(\frac{1}{1-\beta})^{T-1}-1}{\beta}+(\frac{1}{1-\beta})^{T-2}+\beta] & \\ +(1-\frac{1}{(\frac{1}{1-\beta})^{T-2}+\beta\frac{2-\beta}{1-\beta}})\gamma\} & \text{for} \quad T>2 \\ R(1+\beta)-cb_{\max} & \text{for} \quad T=+\infty \end{cases}$$

$$W_T(0)=\begin{cases} R(1+\beta)-cb_{\min} & \text{for} \quad T=1 \\ R(1+\beta)-\frac{1+(1+\beta)}{2}cb_{\min} & \text{for} \quad T=2 \\ R(1+\beta)-\frac{1}{T}\{cb_{\min}[(1-\beta)\frac{(\frac{1}{1-\beta})^{T-1}-1}{\beta}+(\frac{1}{1-\beta})^{T-2}+\beta]\} & \text{for} \quad T>2 \\ R(1+\beta)-cb_{\max} & \text{for} \quad T=+\infty \end{cases}$$

It follows that $\forall T\in\{2,...,N,+\infty\}:W_1(0)>W_T(0)$. Thus, by continuity, this inequality holds in a neighborhood of $\gamma=0$.I.e., $\exists\gamma_1>0;\gamma<\gamma_1\Rightarrow W_1=\max_{T\in\{1,2,...,N,+\infty\}}W_T$.

Similarly, we see that

$$\lim_{\gamma\to+\infty}W_T(\gamma)=\begin{cases} -\infty & \text{for} \quad T\in\{1,2,...,N\} \\ R(1+\beta)-cb_{\max} & \text{for} \quad T=+\infty \end{cases}$$

It follows that $\forall T\in\{1,...,N\}:\lim_{\gamma\to+\infty}W_{+\infty}(\gamma)>\lim_{\gamma\to+\infty}W_T(\gamma)$. Thus, by continuity, this inequality holds in a neighborhood of $\gamma=+\infty$.I.e., $\exists\gamma_4>0;\gamma>\gamma_4\Rightarrow W_{+\infty}=\max_{T\in\{1,2,...,N,+\infty\}}W_T$.

Now, let's prove the existence of (γ_1,γ_2). One just needs to notice that:

$$\gamma_2=\frac{(1+\beta)^2}{\beta}cb_{\min}<\gamma_3=c\frac{b_{\max}-\frac{1+(1+\beta)}{2}b_{\min}}{\frac{1}{2}(1-\frac{1}{(1+\beta)^2})}$$

is equivalent to:

$$b_{\min}<\frac{b_{\max}}{\frac{(1+\beta)^2}{2\beta}+1-\frac{1}{2\beta})+\frac{\beta}{2}}=B.$$

From Proposition 5, we can conclude that for $\gamma_1<\gamma<\gamma_2$:

$$Arg\max_{T\in\{1,2,...,N,+\infty\}}W_T(\gamma)\notin\{1,+\infty\}$$

QED