Endowment–Dependent Reference Consumption in Aggregate Household Portfolios*

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Preliminary

Abstract

Reference–dependent preference models assume that agents derive utility from deviations of consumption from benchmark levels, rather than from consumption levels. These references can be constant (HARA utility), habit– or wealth–determined, as advocated by Regret Theory. We study the latter hypothesis by relating references to wealth levels. Deriving closed–form expressions, we estimate a fully structural multi-variate Brownian system in optimal consumption, portfolio and wealth using aggregate household financial and tangible wealth data. Our results reveal a significant positive effect of wealth on reference consumption and more realistic estimates of the benchmark and risk aversion compared to HARA, and to habit–determined reference models.

JEL classification: G11, G12.

Keywords: Portfolio choice, Reference–dependent utility, Estimation of diffusion processes.

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1 Introduction

Reference Dependent (RD) utility differs from conventional Neoclassical frameworks in that agents are concerned about deviations of consumption from a reference level, rather than about consumption levels themselves. There is by now a large and growing body of evidence in favor of RD utility as representation of agents' preferences over both risk-less and stochastic outcomes.[1]

RD preferences have powerful implications for financial decision making. First, consumption levels that are above and close to references imply both a larger marginal utility cost of small fluctuations in consumption and higher risk aversion. The agents correspondingly act as "portfolio insurers" in selecting assets; they ensure that realized wealth levels will be high enough to minimize fluctuations in marginal utility. Furthermore, Regret Theory admits kinks at references, and convex utility below them. Consequently, an agent with consumption below references will act as a risk–taker, generating strong discontinuities in optimal portfolio rules (Berkelaar et al., 2004; Gomes, 2005). Moreover, stochastic references can imply additional contributors to marginal utility risk other than consumption risk. To the extent that these risks may be priced by the market, potential explanations of high, and time-varying risk premia could be obtained. These implications have led research to use RD preferences in order to address pricing anomalies (e.g. Constantinides, 1990; Benartzi and Thaler, 1995; Campbell and Cochrane, 1999; Barberis et al., 2001, among others).

Analysis of RD preferences has focused on issues of definitions of deviations from references (e.g. ratio vs differences), on functional forms (e.g. power vs negative exponential transform

[1]For example, one application of RD preferences, Prospect Theory, solves some well–known puzzles of Neoclassical theory identified through experimental data by appending two additional hypotheses to RD. First, Diminishing Sensitivity (DS) assumes that large (positive or negative) deviations from references have stronger impacts than smaller ones. Second, Loss Aversion (LA) supposes that negative deviations about references have stronger impacts compared to positive ones. These specifications can solve observed anomalies of Neoclassical theory, including the Endowment Effect (the loss from giving up a good is larger than gain from its acquisition), the Status Quo Bias (excessive retention of current status), Preference for Improvements over Tradeoffs, and Disadvantage Bias (excessive weight given to same difference between two options when viewed as difference between two disadvantages rather than between two advantages). See Kahneman and Tversky (1979); Tversky and Kahneman (1991, 1992); Bateman et al. (1997) for discussions.

of deviations) and on the presence or not of Loss Aversion. Yet, perhaps more fundamentally, researchers do not agree as to what exactly should constitute an adequate reference. Unfortunately, RD frameworks provide little guidance as to how that reference should be defined.

A simple approach is parametric; whereas standard Neoclassical approaches assumes that the reference is zero, as in CARA and CRRA utility functions, HARA utility instead allows for a constant, positive reference. However, the parametric formulation has counterintuitive implications. References are fixed at the outset, and decline in relevance as consumption grows in dynamic settings.

An alternative allows references to reflect current conditions, i.e. state, at the time of the decision, and to grow at a similar rate to consumption. One example is to let references be determined by past decisions, as is proposed by the Habit literature (e.g. Sundaresan, 1989; Constantinides, 1990; Detemple and Zapatero, 1991; Heaton, 1995; Campbell and Cochrane, 1999; Li, 2001, 2005; Tallarini and Zhang, 2005, among others). In this setting, cumulated lagged consumption defines a habit stock which constitutes the current references. Applications in asset pricing have found that this approach holds promises in replicating known dynamics of the asset market. However, success with respect to mean excess return on stocks is limited; since the single determining factor remains consumption risk, and because that risk is minimal, high observed premia can only be reproduced by high prices of consumption risk (i.e. risk aversion).

An alternative interpretation of references is provided by Prospect Theory. It focuses on the role of the agent's assets in shaping benchmark consumption:

> "Strictly speaking, value should be treated as a function of two arguments: the asset position that serves as a reference point, and the magnitude of the change (positive or negative) from that reference point." (Kahneman and Tversky, 1979, p. 277)

This suggests a natural interpretation of references as depending on the agent's contemporaneous wealth, i.e. the market value of all the agent's assets. Moreover, under perfect market assumptions, wealth is equal to the expected discounted stream of future consumption. This would be consistent with an expectations–based interpretation of references:

> "Gains and Losses, of course, are defined relative to some neutral reference point. The reference point usually corresponds to the current asset position ... However, the location of the reference point, and the consequent coding of outcomes as gains or losses can be affected by the formulation of the offered prospects, and by the expectations of the decision maker." (Kahneman and Tversky, 1979, p. 274)

Interpreting references as wealth–dependent makes intuitive sense. Clearly, optimal consumption growth should be related to wealth growth, thereby ensuring the practical relevance of references. Moreover, wealth is much volatile and correlated with risky returns than consumption. As such, this could provide additional potential for marginal utility volatility and correlation with returns.

The purpose of this paper is to analyze empirically the conjecture of Regret Theory that wealth should be a determinant of references. Contrary to standard preference–based literature on asset pricing dynamics, we focus on empirical portfolio, rather than on pricing implications. In Section 2, we consider the Regret Theory conjecture by assuming that references are an affine transform of wealth. This specification has the advantage of nesting both CRRA, and HARA utility as special cases. In parallels with the Habit literature, and unlike generalized Loss Aversion, we constrain consumption to remain above references, and maintain risk aversion throughout the consumption domain. This can be interpreted as corresponding to a highly loss–averse agent, and is consistent with empirical findings on the degree of loss aversion.

Next, we solve for the optimal rules which are then substituted in the budget contraint to derive the closed–form expressions for instantaneous changes in consumption, asset holding values, and wealth. This multi–variate Brownian motion constitutes the fully structural econometric model which we estimate. In Section 3, we resort to a change–in–variables approach to address potential discretization bias of estimating continuous–time models with discretely–sampled data. We also incorporate the returns process with the optimal quantities process in a single–step Maximum Likelihood estimation to correct inference.

The estimation results presented in Section 4 highlight several interesting points. First, both the null of CRRA and HARA preferences are strongly rejected when tested against wealth–determined references. The estimated wealth–dependence parameter is significantly positive, suggesting that references are increasing in wealth. Importantly, references are a stationary and realistic share of consumption, oscillating between 5.5 and 8.5% of nondurables and services. Risk aversion estimates however remain similar to those obtained under CRRA and HARA i.e. excessively high.

In Section 5, we also estimate an internal Habit model for comparison purposes. The estimates for the habit parameters are all significant and of the correct sign. The corresponding references are similar to those obtained for other Habit models, whether internal or external, and point to a stationary reference corresponding to 90% of consumption on average. Since the curvature parameter is roughly the same as for the other models, a low surplus consumption ratio implies huge risk aversion estimates, nearly 8 times larger than those obtained initially. Nonetheless, the risk aversion level of the indirect utility function hovers close to the upper limit of the acceptable range.

Overall, in Section 6 our analysis highlights the relevance of a consumption reference that remains significant and meaningful throughout the sample period. The conjecture of Regret Theory that references are wealth–determined is verified in our aggregate portfolio data set.

Finally, wealth–determined references yield lower estimates of reference level, and consequently more realistic estimates of risk aversion.

2 Model

This section outlines the model. We subsequently characterize optimal consumption and asset holdings. Finally, we obtain the closed-form expressions for the differential equations governing consumption, asset values and wealth by substituting optimal consumption and portfolio in the budget constraint.

2.1 Economic environment and preferences

In order to emphasize the role of alternative preference specifications, we consider a complete-markets and representative-agent framework similar that studied by Merton (1971) or by Lucas (1978). The stochastic environment is characterized by continuous information with filtration on $\boldsymbol{Z}_t \in \mathbb{R}^n$, a standard Brownian motion. The investment set consists of n risky securities and one risk-less asset. Denote by $\boldsymbol{\mu}_p \in \mathbb{R}^n$ and by $\boldsymbol{\sigma}_p \in \mathbb{R}^{n\times n}$ the constant drift and diffusion parameters for the risky returns, and by $r \in \mathbb{R}$ the short rate.

The representative agent's objective is to select consumption $C_t \in \mathbb{R}^+$ and portfolio weights $\boldsymbol{v}_t \in \mathbb{R}^n$ so as to solve:

$$J_0 = \max_{\{C_t, \boldsymbol{v}_t\}_t} \mathrm{E}_0 \int_0^\infty \exp(-\rho t) U(C_t, X_t) dt, \quad \rho > 0 \tag{1}$$

subject to

$$dW_t = \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + r]W_t - C_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \tag{2}$$

where J_0 is a value function, E_0 is a conditional expectations operator, ρ is a subjective discount rate, X_t is a reference level, and W_t is the agent's (total) wealth. The agent's instantaneous utility $U_t = U(C_t, X_t)$ is characterized by:

$$U_t = \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}, \quad \gamma \geq 0, \quad C_t \geq X_t \tag{3}$$

where γ is a curvature parameter that, under the special case of reference independence, i.e. $X_t = 0, \forall t$, refers to relative risk aversion. Otherwise, consumption risk aversion $RR_{c,t}$ is time–varying and given by:

$$RR_{c,t} = \frac{\gamma}{S_t}, \quad S_t = \frac{C_t - X_t}{C_t} \tag{4}$$

where S_t is the surplus consumption ratio. As usual, the case of $\gamma = 1$ can be handled through the logarithmic utility function.

Following the RD literature, X_t can be interpreted as a reference (benchmark) consumption level. Preferences (1), (3) state that an agent evaluates the desirability of a consumption stream from the deviations in differences over and above the benchmark levels at each periods of time. From standard practices in the Habit literature, since γ is non–negative but otherwise unrestricted, the restriction $C_t \geq X_t$ is necessary for utility to be well defined. Regret Theory with Loss Aversion may allow for kinks at references $C_t = X_t$ and convexities below references $C_t < X_t$. In our setup, the restriction $C_t \geq X_t$ can be interpreted as indicating that the agent is very averse to loss.[2] Alternative specifications consider other functional forms for

[2]For example, a typical Loss Aversion specification is given by:

$$U_t = \begin{cases} \alpha C_t + (1-\alpha)\frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}, & \text{if } C_t \geq X_t; \\ \alpha C_t - \lambda(1-\alpha)\frac{|C_t - X_t|^{1-\gamma}}{1-\gamma}, & \text{if } C_t < X_t, \end{cases}$$

where $\lambda \geq 1$ captures the degree of loss aversion. The Habit convention $\alpha = 0, C_t \geq X_t$ can be thought of as a highly loss averse agent, i.e. $\lambda \gg 1$ (Yogo, 2005a, p. 9). Values of λ above 2 are typically estimated and reported in the Prospect Theory literature (Tversky and Kahneman, 1992; Benartzi and Thaler, 1995; Berkelaar et al., 2004; Yogo, 2005a).

$U_t = U(C_t, X_t)$, such as CES, or ratios of levels to references (Abel, 1990; Bakshi and Chen, 1996; Garcia et al., 2005). We resort to the simpler differences–*cum*–power specification to emphasize parallels with the Habit literature (Sundaresan, 1989; Constantinides, 1990; Heaton, 1995; Campbell and Cochrane, 1999).

References X_t can be either constant or time-varying. The restriction $X_t = 0, \forall t$ corresponds to the familiar CRRA case of preferences over levels, rather than over differences. Imposing $X_t = X_0, \forall t$ yields HARA utility. In both cases, references are interpreted as a deep parameter, given at outset and unaffected by the state of the world at time of decision. As mentioned earlier, this interpretation is contradicted by the experimental evidence highlighted by Regret Theory that references are more likely time-varying and reflect the state variables at the time of the decision (see footnote 1).

One such time–varying and state–dependent specification for references is suggested by Regret Theory and relates references to the current value of assets (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991), implying that $X_t = X(W_t)$. We follow this route by specifying an affine references function:

$$X_t = \eta_0 + \eta_w W_t \geq 0. \tag{5}$$

In what follows, we will refer to X_t in (5) as Wealth–determined References (WDR). A similar linear effect of wealth on references is used in Regret Theory settings by Berkelaar et al. (2004) and by Gomes (2005).[3] A natural restriction would be for C_t, X_t to be con–integrated. In our case, this would be tantamount to imposing $\eta_w > 0$, which ensures that the reference level of consumption increases in wealth, a plausible assumption. Moreover, (5) allows for negative

[3]These authors append a slow–moving effect to (5) by considering $X_t = (1 - \eta_w) R_f X_{t-1} + \eta_w W_t$, where R_f is the gross risk-free rate of return (e.g. Gomes, 2005, eq. (6)). Under wealth–independent references ($\eta_w = 0$), references grow exogenously at the risk–free rate. We do not pursue this strategy here, but instead consider an affine transform of wealth only to facilitate comparisons with CRRA, and HARA utility.

η_0, as long as the reference is positive on the relevant range. Negative references have the unfortunate consequence of allowing for finite marginal utility at zero consumption, and thus potential negative optimal consumption. Imposing $X_t \geq 0$ rules out this possibility. Note that $\eta_w = 0$ yields the HARA class, whereas $\eta_0 = \eta_w = 0$ yields CRRA utility.[4]

Preferences (3), (5) belong to the state–dependent utility class, where the state is defined to be wealth. Other Wealth–Dependent utility functions have been proposed in the literature. Whereas some researchers have restricted preferences to be over specific elements of wealth such as financial (Barberis et al., 2001), or tangible wealth (Grossman and Larocque, 1990; Detemple and Giannikos, 1996; Chetty and Szeidl, 2004; Aït-Sahalia et al., 2004; Yogo, 2005b), we follow Bakshi and Chen (1996); Smith (2001); Gong and Zou (2002); Kandel and Kuznitz (2004) in making utility a function of *total* wealth instead.

Preferences (5) differ from standard Habit models in that references are forward–, instead of backward–looking. To see this, note that under perfect markets, wealth is simply the expected discounted value of future consumptions streams, such that references (5) can be written as:

$$X_t = \eta_0 + \eta_w \frac{1}{\pi_t} E_t \int_t^\infty \pi_s C_s ds, \tag{6}$$

where π_t is a state-price density. In comparison, the Habit literature assumes that references are a function of past consumption profiles:

$$X_t = e^{-at} X_0 + b \int_0^t e^{a(s-t)} C_s ds, \quad a, b \geq 0. \tag{7}$$

[4]The restriction that reference *is* wealth, i.e. $\eta_0 = 0, \eta_w = 1$ is also nested in (5). However, to the extent that both optimal and empirical consumption are strictly less than wealth, this would imply that $C_t < X_t, \forall t$, rendering utility (3) ill–defined. Under generalized Regret Theory, consumption below references would correspond to the convex segment of utility $\forall t$. The sufficiency conditions of global concavity in the state would consequently be violated (e.g. Seierstad and Sydsaeter, 1977).

The parameter a represents the rate of discounting applied to lagged consumption, whereas b measures the importance of habit. Internal habit assume that C_s is a lagged control of the agent; external habit does not. Subsequent analysis will refer to X_t in (7) as Habit–determined References (HDR).

First, as for the case of internal (but contrary to external) habit, the reference stock X_t is a current state, but future X_{t+s} are controlled by the agent through his choice of C_t. As such, the preferences (5) entail time non–separability, in that current choices affect future wealth, and future references, and thus future marginal utilities. Note however that, unlike habit, this control over references is partial to the extent that future references depend on future realizations of the portfolio return; future references are stochastic, regardless of consumption choices.

Second, as for the case of Non–Expected Utility (NEU, Epstein and Zin, 1989; Weil, 1989), (6) entail state– as well as time–non–separabilities in that preferences are non–linear in probabilities through the power transform (3). These preferences are useful in addressing empirical anomalies in that they allow for an additional beta related to total wealth (i.e. market) risk to supplement consumption beta in the pricing equations (Duffie and Epstein, 1992). Moreover, Alvarez and Jermann (2005) show that such state non–separabilities are useful in magnifying the permanent components of pricing kernels so as to jointly reproduce stocks as well as short– and long–term bonds dynamics, even if the consumption process is i.i.d.. In contrast, the Habit kernel defined by (7) yields a single–factor pricing equation, regardless of whether habit is internal or external, in which all the risk is captured by consumption covariances with returns, and the persistence of pricing kernels by consumption dynamics.

Finally, WDR (6) naturally incorporate the notion that expectations about future consumption streams should determine references (Köszegi and Rabin, 2005).[5] The presence of expectations of future consumption streams in the within–period utility function is shown to have a very strong impact on portfolio decisions by Kandel and Kuznitz (2004). In particular when the impact on marginal utility of consumption is positive, lower positions in risky assets, and larger savings can be generated. This is exactly the case in our setting. In Figure 1, an increase in wealth from W_0 to W_1 implies *ceteris paribus* a clockwise rotation of the marginal utility schedule and higher marginal utility and curvature at any consumption level C_0.

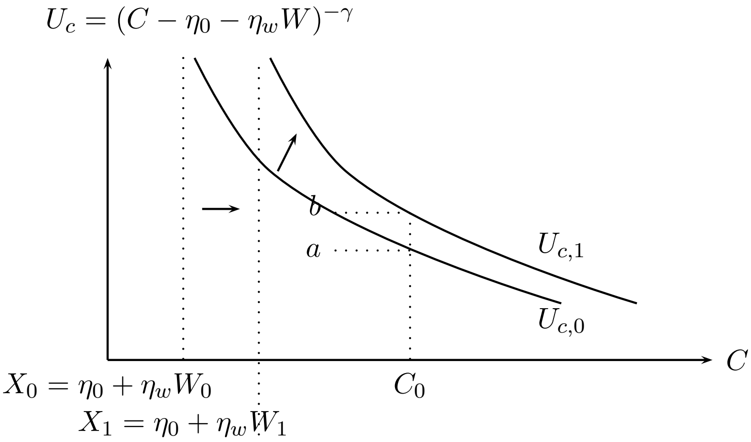

Figure 1: Effect of increase in wealth on marginal utility: Wealth–determined references

Next, we analyze the closed–form solutions for the agent's problem (1), under RD preferences (3). For the present, since analytical expressions under Habit–determined references (7) are well known, we focus instead on those for the WDR (5). We will return to the Habit restrictions later in Section 4 when we compare our results.

[5]Unlike Köszegi and Rabin (2005), these expectations in our case are not lagged, but contemporaneous to the choice of $C_t, \boldsymbol{v}_t$. Note that these non-separabilities are fully recognized by the agent, i.e. his decisions are dynamically consistent. This can be associated with the conditions of Personal Equilibrium required by Köszegi and Rabin (2005) that the agent's expectations determining his references are consistent with his choice of controls.

2.2 Optimal Consumption and Portfolio Rules

The agent's problem (1)–(3), (5) could be solved using standard dynamic programming methods. It turns out however that a simpler alternative is available. Comparisons of (5) and (7) highlight the linearity in consumption in both the Wealth– and Habit–determined RD models. Schroder and Skiadas (2002) show that closed-form expressions for linear Habit models (the primal problem) are conveniently obtained by simple modifications to the standard solutions in models without habit (the dual problem). Their analysis is cast in terms of Habit–determined references, but it can be readily extended to our WDR setup. First, by appropriately redefining the agent's problem, expressions for the dual consumption, dual short rate and dual risk premia can be obtained. Second, these expressions are then substituted back into the known solutions to the dual problem. Third, the solutions to the primal problem are obtained by adding in the wealth-in-the-utility term to the second-step solutions.

In what follows let Y_t refer to a variable in the primal problem and let $\hat{Y}_t$ refer to its dual problem counterpart. We start by defining the dual variables as follows:

$$\hat{C}_t \equiv C_t - \eta_w W_t, \tag{8}$$

$$\hat{U}_t \equiv \frac{(\hat{C}_t - \eta_0)^{1-\gamma}}{1-\gamma} = U_t. \tag{9}$$

Next, replace for C_t in budget constraint (2) by using (8) to obtain:

$$\begin{aligned} dW_t &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + r]W_t - \hat{C}_t - \eta_w W_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \\ &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + (r - \eta_w)]W_t - \hat{C}_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \\ &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + \hat{r}]W_t - \hat{C}_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \end{aligned} \tag{10}$$

Observe that wealth, portfolio, and the risk premia $(\boldsymbol{\mu}_p - r)$ remain unchanged, whereas the short rate is replaced by $\hat{r} \equiv r - \eta_w$, and consumption is replaced by $\hat{C}_t$ as given in (8).[6] Moreover, dual utility (9) suppresses any explicit dependence on wealth, and is simply a HARA over dual consumption $\hat{C}_t$. Under the iso-morphism result of Schroder and Skiadas (2002), we can:

1. use the known solutions of Merton for the dual problem $(\hat{C}_t, \hat{\boldsymbol{v}}_t)$ as functions of $W_t, \hat{r}, \boldsymbol{\mu}_p - r$,
2. correct the short rate in these solutions using $\hat{r} = r - \eta_w$,
3. get back the expression for C_t by inverting (8); the expression for $\boldsymbol{v}_t$ is the same as that for $\hat{\boldsymbol{v}}_t$.

Following this iso-morphism approach reveals that the indirect utility $J_t = J(W_t)$, the optimal consumption C_t and the value of risky assets $\boldsymbol{V}_t \equiv \boldsymbol{v}_t W_t$ are respectively given by:

$$J_t = \frac{(G + FW_t)^{1-\gamma}}{1-\gamma}, \tag{11}$$

$$\begin{aligned} C_t = & -\eta_0 \left\{ \left(\frac{\gamma - 1}{\gamma} \right) \left(\frac{\rho/(\gamma-1) + 0.5M/\gamma}{r - \eta_w} \right) - \frac{1}{\gamma} \right\} \\ & + \left\{ \left(\frac{\gamma-1}{\gamma} \right) (r + \rho/(\gamma - 1) + 0.5M/\gamma) + \frac{\eta_w}{\gamma} \right\} W_t, \end{aligned} \tag{12}$$

$$\boldsymbol{V_t} = \left(\frac{-\eta_0}{r - \eta_w} \right) \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} + \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} W_t, \tag{13}$$

[6] This suggests that if π_t is the primal state-price density, then $\hat{\pi}_t \equiv e^{\eta_w t}\pi_t$ is its dual counterpart, since a standard no-arbitrage argument establishes that:

$$\begin{aligned} \hat{r} &= -\mu_{\hat{\pi}}/\hat{\pi}_t = r - \eta_w \\ \hat{\boldsymbol{\mu}}_{\boldsymbol{p}} - \hat{r} &= -\left(1/\hat{\pi}_t\right) \boldsymbol{\sigma_p} \boldsymbol{\sigma}'_{\hat{\boldsymbol{\pi}}} = \boldsymbol{\mu_p} - r, \end{aligned}$$

where,

$$
\begin{aligned}
F &\equiv \left\{ \left(\frac{\gamma-1}{\gamma} \right) \left(r - \eta_w + \frac{\rho}{\gamma-1} + 0.5M/\gamma \right) \right\}^{\gamma/(\gamma-1)}, \\
G &\equiv -\eta_0 \left\{ \left(\frac{\gamma}{\gamma-1} \right) F^{-1/\gamma} - \frac{\rho}{F(\gamma-1)} - \frac{0.5M}{\gamma F} \right\}^{-1}, \\
M &\equiv (\boldsymbol{\mu}_p - r)' \boldsymbol{\Sigma}_{pp}^{-1} (\boldsymbol{\mu}_p - r) \geq 0, \\
\boldsymbol{\Sigma}_{ij} &\equiv \mathrm{E}[\boldsymbol{\sigma}_i d\boldsymbol{Z}_t d\boldsymbol{Z}_t' \boldsymbol{\sigma}_j']
\end{aligned}
$$

It can be shown that these solutions correspond exactly to those obtained using the more traditional dynamic programming approach.

Equation (11) highlights interesting characteristics of the value function. First, the particular form of wealth dependence that we are considering supposes that the Bernoulli transform is applied to an affine function of consumption and wealth. Since optimal consumption is also affine in wealth, this functional has the property that the value function belongs to the HARA class. Note in particular that $\eta_0 = 0$ implies $G = 0$, such that the value function becomes iso-elastic despite the wealth dependence.

Second, we can analyze risk aversion using the marginal utility of wealth schedule, $J_{w,t}$, and the distance of wealth level W_t from the wealth reference level X_w at the optimum. In particular, straightforward manipulations reveal that:

$$X_w \equiv \frac{-G}{F} = \frac{\eta_0}{r - \eta_w} \geq 0, \tag{14}$$

$$\frac{-W_t J_{ww,t}}{J_{w,t}} = \frac{\gamma W_t}{W_t - X_w} \geq 0. \tag{15}$$

The constant reference level of wealth (14) can take on negative or positive values depending on the parameters η_0, η_w and on the interest rate r, with $X_w > 0$ implying *ceteris paribus* higher, and counter-cyclical risk aversion at the optimum. Again, non–negative X_w rules out finite marginal utility of wealth at zero wealth levels, and potentially negative optimum wealth. Note that the impact of $\eta_w > 0$ on the optimal reference level is to increase (decrease) X_w if $\eta_0 > 0$ ($\eta_0 < 0$). Furthermore, regardless of the sign of the reference wealth, relative risk aversion is asymptotically constant and equal to the curvature index γ. Otherwise, levels, and dynamic properties of risk aversion depend crucially on the distance of wealth to reference $W_t - X_w$.

Next, the optimal rules (12) and (13) are affine in wealth. As for the standard HARA utility, imposing $\eta_0 = 0$ results in the iso-elastic case of both rules being proportional to net worth. Otherwise, wealth dependence affects both the intercept $(C_t, \boldsymbol{V}_t)$ and the slope (C_t) of the closed-form solutions. To isolate these effects, it is useful to resort to our previous analysis of the value function. Note that we can indeed rewrite the optimal rules (12), (13) as:

$$
\begin{aligned}
C_t &= -\left\{ \left(\frac{\gamma - 1}{\gamma} \right) X_w \left[\rho/(\gamma - 1) + 0.5M/\gamma \right] - \frac{\eta_0}{\gamma} \right\} \\
&\quad + \left\{ \left(\frac{\gamma - 1}{\gamma} \right) \left[r + \rho/(\gamma - 1) + 0.5M/\gamma \right] + \frac{\eta_w}{\gamma} \right\} W_t, && (16) \\
(\boldsymbol{\mu}_p - r)' \boldsymbol{V}_t &= \frac{M}{\gamma} \{ -X_w + W_t \}, && (17)
\end{aligned}
$$

where X_w is given by (14). For the rest of this section's analysis, assume that the investor is at least moderately risk averse, i.e $\gamma > 1$.

First, turning to consumption, a positive X_w decreases the intercept in (16). From (15), $X_w > 0$ implies a steeper marginal utility of wealth at the optimum, and consequently, greater J_w risk. The risk-averse investor reacts to this by increasing wealth away from reference X_w, by decreasing consumption and increasing savings. For $\eta_0 > 0$, a WDR investor has a higher X_w and this effect on consumption is even larger.

Secondly, regardless of X_w, the WDR investor always has a higher marginal propensity to consume out of wealth. The reason is that higher wealth implies a higher reference consumption

X_t, and consequently, a higher U_c risk. The risk-averse WDR investor therefore reacts by increasing consumption away from reference more than a HARA or CRRA investor.

Third, (17) expresses the expected excess return (in \$ terms) on the optimal total wealth portfolio. As usual, higher curvature γ results in more conservative positions. Again, reference levels of wealth influence the intercept terms. A positive X_w implies more J_w risk at any wealth levels. The risk-averse agent hedges away these risks by selecting more conservative portfolios. Negative reference values however reduce risk aversion and increase the asset values held in risky assets.

Clearly linearity for the optimal rules (12), and (13) implies that the change in consumption and portfolio are $dC_t = c_w dW_t$, and $d\boldsymbol{V}_t = \boldsymbol{v}_w dW_t$, where $c_w, \boldsymbol{v}_w$ are constants defined by (12) and (13). Furthermore, we can substitute the solutions in the budget constraint (2) to obtain the closed-form expression for instantaneous changes in wealth. Consequently the instantaneous changes in consumption, the value invested in assets, and wealth are:

$$
\begin{aligned}
dC_t &= \left\{ \left(\frac{\gamma - 1}{\gamma} \right) (r + \rho/(\gamma - 1) + 0.5M/\gamma) + \frac{\eta_w}{\gamma} \right\} dW_t, && (18) \\
d\boldsymbol{V}_t &= \left\{ \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} \right\} dW_t, && (19) \\
dW_t &= \left[\frac{-\eta_0 + (r - \eta_w)W_t}{\gamma(r - \eta_w)} \right] \left\{ \left[\left(\frac{\gamma + 1}{\gamma} \right) 0.5M + r - \eta_w - \rho \right] dt \right. \\
&\quad \left. + (\boldsymbol{\mu}_p - r)' \boldsymbol{\Sigma}_{pp}^{-1} \boldsymbol{\sigma}_p d\boldsymbol{Z}_t \right\}. && (20)
\end{aligned}
$$

Having derived the closed-form expressions for the laws of motion of the endogenous variables, we now turn to the estimation of the structural parameters based on these processes.

3 Estimation

3.1 Econometric Model

Estimation focuses on the multivariate Brownian motion given by (18)–(20), which can be written as:

$$
\begin{aligned}
dC_t &= c_w dW_t, && (21)\\
d\boldsymbol{V}_t &= \boldsymbol{v}_w dW_t, && (22)\\
dW_t &= [\mu_0 + \mu_w W_t]dt + [\sigma_0 + \sigma_w W_t]dZ_t, && (23)
\end{aligned}
$$

where $c_w, \boldsymbol{v}_w, \mu_0, \mu_w, \sigma_0, \sigma_w$ are constant loadings that depend only on the deep parameters. In principle, estimation the model could be undertaken over returns or over quantities data. We select the second approach for a number of reasons.

First, estimating optimal allocations imposes considerably more theoretical restrictions that are related to the deep parameters on the joint first and second moments. With respect to deep parameters, standard analyses of returns treat the equilibrium quantities in the pricing kernels as exogenous; the theoretical restrictions are imposed on the prices of risk exclusively, with conditional second moments left unrestricted. In comparison, the allocations analysis produces theoretical restrictions on both first and second moments of changes in consumption, asset holdings and wealth (through the restrictions on $c_w, \boldsymbol{v}_w, \mu_0, \mu_w, \sigma_0, \sigma_w$), while returns are treated as exogenous. In the absence of prior information on η_w in particular, these additional restrictions will be useful in identifying the preference parameters of interest. Second, empirical studies of aggregate optimal consumption and asset holdings are much less frequent than asset pricing studies. We believe that focusing on quantities rather than on returns thus provides another perspective that complements existing results (Lo and Wang, 2001, also argue in favor of using the informational content of quantities more thoroughly).

Transformation One major problem in estimating (21)–(23) is that there exists no closed-form transition density for multi-variate Brownian motions with affine drifts and diffusions. Indeed, analytical expressions for the likelihood function exist only for a limited class of Itô processes (Melino, 1996). Unfortunately, our multi-variate process does not belong to this class. Alternative solutions include discrete (Euler) approximations, and simulating the continuous-time paths between the discretely-sampled data, either through classical (Durham and Gallant, 2002) or through Bayesian (Eraker, 2001) approaches.

Our solution to this problem is different and considerably simpler to implement. It is based on a homoscedasticity-inducing transformation for general Brownian motions. It will be shown that this approach also stationarizes the drift term. Consequently, a standard discretized approximation is appropriate, efficient, and unbiased. In particular, a straightforward application of Itô's lemma reveals the following.

Lemma 1 *Let $Y_t \in \{C_t, \boldsymbol{V}_t\}$ be defined as follows:*

$$Y_t = y_0 + y_w W_t, \tag{24}$$

$$dW_t = (\mu_0 + \mu_w W_t)dt + (\sigma_0 + \sigma_w W_t)dZ_t, \tag{25}$$

where $\boldsymbol{y}, \boldsymbol{\mu}, \boldsymbol{\sigma}$ are constants defined in (12) *and* (13)*, and in* (20)*, and consider the following transformation:*

$$\tilde{Y}_t = \frac{\log[y_w \sigma_0 + \sigma_w (Y_t - y_0)]}{\sigma_w}, \tag{26}$$

Then, $\tilde{Y}_t$ has constant drift and diffusion given by:

$$d\tilde{Y}_t = \left[\frac{\mu_w}{\sigma_w} - 0.5\sigma_w\right] dt + dZ_t. \tag{27}$$

Proof. First, (24) and (25) reveal that:

$$\begin{aligned} dY_t &= [y_w\mu_0 + \mu_w(Y_t - y_0)]dt + [y_w\sigma_0 + \sigma_w(Y_t - y_0)]dZ_t && (28) \\ &= \mu(Y_t)dt + \sigma(Y_t)dZ_t. && (29) \end{aligned}$$

Next, by Itô's lemma, we have for $\tilde{Y}_t = \tilde{Y}(Y_t)$:

$$d\tilde{Y}_{j,t} = \left[\mu(Y_t)\tilde{Y}'(Y_t) + 0.5\sigma(Y_t)^2\tilde{Y}''(Y_t)\right] dt + \sigma(Y_t)\tilde{Y}'(Y_t)dZ_t \tag{30}$$

Observe that $\mu_0/\mu_w = \sigma_0/\sigma_w$ to substitute in (30) to obtain (27). ∎

The transformation (26) requires that its first derivative with respect to the Itô process Y_t is the inverse of the diffusion. It is usually introduced in order to stationarize the diffusion (Shoji and Ozaki, 1998; Aït-Sahalia, 2002; Durham and Gallant, 2002). In our case, both drift and diffusion are affine and have intercept and slope coefficients that are closely inter-related. Consequently the theoretical restrictions implied by the model are such that the transformation *also* stationarizes the drift term. This is fortunate since the resulting transformed model is easily estimated by maximum likelihood. In particular, the discretization of (27):

$$\Delta\tilde{Y}_t = \left[\frac{\mu_w}{\sigma_w} - 0.5\sigma_w\right] + \epsilon_t \tag{31}$$

where ϵ_t is a standard Gaussian term, can be consistently and efficiently estimated by MLE (e.g. Gourieroux and Jasiak, 2001, pp. 287–288).

Likelihood function The optimal rules in (12) and (13) take the moments of the returns' distribution $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$, as well as the risk-free rate r as given. These moments could be estimated in an external round, using a two-step method, and substituted back into the optimal rules to obtain the predicted rules. Instead, we perform a single-step procedure and incorporate the mean and covariance matrix of the risky returns into the calculation of the likelihood function.[7] This approach has the advantage of factoring in the parametric uncertainty concerning $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$ into the calculation of the standard errors of the deep parameters. Specifically, denote by $\tilde{\boldsymbol{Y}}_t \equiv [\tilde{C}_t, \tilde{\boldsymbol{V}}_t, \tilde{W}_t]'$ the $n+2$ vector of transformed variables, the model to be estimated is the following:

$$\begin{pmatrix} \Delta\tilde{\boldsymbol{Y}}_t \\ \Delta\boldsymbol{P}_t/\boldsymbol{P}_{t-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_p \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_y \\ \boldsymbol{\epsilon}_p \end{pmatrix}, \quad \begin{pmatrix} \boldsymbol{\epsilon}_y \\ \boldsymbol{\epsilon}_p \end{pmatrix}, \sim \text{N.I.D.} \left(\begin{pmatrix} \boldsymbol{0}_y \\ \boldsymbol{0}_p \end{pmatrix}, \begin{pmatrix} \boldsymbol{I}_y & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_{pp} \end{pmatrix} \right), \tag{32}$$

where $\boldsymbol{\mu}_y$ is given by (31), and $\boldsymbol{I}_y$ is an $n+2$ identity matrix.

First, in accordance with the maintained assumption of the model, all the innovations are Gaussian. Second, as mentioned earlier, the transformation in Lemma 1 implies that the quantities innovations are standardized white noise. Third, consistent with the model, the covariance matrix is block diagonal, i.e. we impose the absence of cross-correlations between innovations in quantities and returns. Any potential covariance between the two is fully taken into account in the closed-form solutions; allowing for additional correlations is not theoretically justified.

With these elements in mind, the contributions to the likelihood function (with constant term omitted) are given by:

$$f_t = -0.5 \log[\det(\boldsymbol{\Sigma})] + \log[\det(\boldsymbol{K}_t)] - 0.5\boldsymbol{\epsilon}_t'\boldsymbol{\Sigma}^{-1}\boldsymbol{\epsilon}_t \tag{33}$$

[7] Following standard practices, the risk-free rate r is calibrated to its mean value.

where $\mathbf{\Sigma}$ is defined implicitly in (32), while $\boldsymbol{K}_t \equiv \text{Diag}([K_{c,t}, \boldsymbol{K}_{v,t}, K_{w,t}, 1, \ldots 1])$ and $K_{y,t} = 1/[y_w \sigma_0 + \sigma_w(Y_t - y_0)]$ is a Jacobian correction term associated with the transformation (31). The parameter vector is then $\boldsymbol{\theta} \equiv \{\gamma, \rho, \eta_0, \eta_c, \eta_w, \boldsymbol{\mu}_p, \boldsymbol{Q}_{pp}\}$, where $\boldsymbol{Q}_{pp} \equiv \text{Chol}(\mathbf{\Sigma}_{pp})$ is the n-dimensional triangular Cholesky root of the returns covariance matrix.

Hypothesis tests We consider two benchmarks in assessing the performance of the WDR model. As mentioned earlier, CRRA utility is obtained by imposing that $\eta_0, \eta_w = 0$, whereas HARA utility imposes $\eta_w = 0$. To the extent that it has been studied extensively in asset pricing models, CRRA utility constitutes a natural benchmark. HARA utility, although less popular, has the advantage of optimal rules which are not proportional to wealth (see the previous discussion). Both the theoretical restrictions and the model selection tests will be performed and discussed below.

We also consider an alternative internal habit–based specification of references. However, since the WDR and habit models are not nested, we do not perform a formal statistical inference. Rather, we discuss each model's relative performance in terms of the realism of the derives reference and risk aversion series.

3.2 Data

Our data set consists of post-war U.S. quarterly observations on aggregate consumption, asset holdings and corresponding returns indices. The time period covered ranges from 1963:Q2 to 2005:Q3, for a total of 170 observations. All quantities are expressed in real, per-capita terms, where the aggregate price index is taken to be the implicit GDP deflator (base year: 2000). Similarly, all returns are converted in real terms by subtracting the inflation index.

Consumption The consumption series is the aggregate expenditure on Non-Durables and Services. The source of the data is the Bureau of Economic Analysis NIPA series. This series has been used in most asset pricing studies.

Assets The aggregate portfolio holdings are defined as follows:

$$\begin{aligned} \boldsymbol{V}_t &= [V_{0,t}, V_{1,t}, V_{2,t}, V_{3,t}, V_{4,t}] \\ &= [\text{Deposits, Bonds, Stocks, Home, Mortgages}]. \end{aligned}$$

Each asset holdings are obtained from the Flow of Funds Accounts made available by the Board of Governors of the Federal Reserve (Table L.100). They correspond to the level values of asset holdings by households and non-profit organizations (see also Lettau and Ludvigson, 2003). More precisely the individual assets (mnemonic) and financial wealth are:

- Deposits ($V_{0,t}$, FL15400005): Includes foreign, checkable, time, savings deposits and money market fund shares.
- Bonds ($V_{1,t}$, FL153061005): U.S. government securities (Treasury and Agency).
- Stocks ($V_{2,t}$, FL153064105): Corporate equities (at market value) directly held by households, does not include indirectly held mutual funds.
- Home ($V_{3,t}$, FL155035015): Real estate (at market value) corresponding to all owner–occupied housing, whether primary or second homes, plus vacant land.
- Mortgage ($V_{4,t}$, FL153165105) Value of mortgage liabilities faced by households, including home equity lines of credit.
- Wealth: Deposits + Bonds + Stocks + Home + Mortgages ($W_t = V_{0,t} + V_{1,t} + V_{2,t} + V_{3,t} + V_{4,t}$).

Deposits will thus be taken to represent the risk-free asset, whereas both long-term government bonds, corporate equity, home equity, and mortgages are proxies for the risky assets.

The choice of specific portfolio holdings was dictated by a number of practical elements. First, these assets correspond to some of the largest asset holdings for U.S. households, and their returns have been studied extensively in the asset pricing literature, thus providing useful benchmarks for our analysis. Second and related, these assets have corresponding returns series. Those returns are required to evaluate the distributional parameters $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$ that are used to compute the theoretical asset holdings. Other portfolio holdings such as pension and life insurance reserves are also important in relative size. However, no clear returns indices are available for these assets.[8]

Our definition of financial wealth has been used in other theoretical models of portfolio choice (e.g. Campbell et al., 2003). However, contrary to Campbell et al. (2003), we incorporate the important real estate assets so as to encompass tangible assets in total wealth. We verify later in Section 4.4 the robustness of our results to using the narrower purely financial definition of total wealth. A main advantage of defining wealth in this way is that it is observable and the definition provides more structure on the econometric model since one of the theoretical asset holding is defined residually.[9]

Table 1 reports the sample moments for the consumption and asset holdings in percentage of wealth (those series are plotted in Figure 2). A first observation is that the shares of wealth allocated to consumption, deposits and stocks are roughly of the same order of magnitude. Net

[8]For example, pension reserves are typically invested differently by fund managers whether they are defined benefit or defined contribution. Finding a unique pricing index for this series in the absence of detailed information on the funds' composition is impractical.

[9]In particular, (13) reveals that, for the risk-free asset:

$$V_{0,t} = v_{00} + v_{w,0} W_t, \quad v_{00} = -v_{10} - v_{20}, \quad v_{w0} = 1 - v_{1w} - v_{2w}. \tag{34}$$

Our definition abstracts from other elements, such as durables, mutual funds, and human wealth. Unfortunately, real returns indices on both durable goods and mutual funds are difficult to evaluate, and these assets were omitted from our selected holdings series $\boldsymbol{V}_t$. Moreover, human wealth is not observable, whether in levels or in rates of returns and thus also eliminated.

home equity is larger, whereas bonds on the other hand represent a much lower share of wealth and are smoother.

Table 1: Summary statistics: Consumption, portfolio and wealth

		Levels (2000$)		Shares of wealth			
Variable	Series	min	max	min	max	mean	std
C_t	consumption	8'002	23'350	26.6%	38.9%	32.9%	3.0%
$V_{0,t}$	cash	7'187	17'815	19.7%	38.3%	29.9%	4.8%
$V_{1,t}$	bonds	1'003	3'630	1.7%	7.5%	4.3%	1.2%
$V_{2,t}$	stock	5'169	34'990	14.9%	46.9%	29.1%	9.2%
$V_{3,t}$	home	12'640	57'261	39.9%	81.6%	58.0%	10.0%
$V_{4,t}$	mortgage	-24'557	-4'270	-35.0%	-14.1%	-21.3%	5.1%
W_t	wealth	27'904	74'768				

Wealth is $W_t = \sum_{i=0}^{4} V_{i,t}$. Sample period is 1963:Q1–2005:Q3 (170 quarterly observations).

Returns We follow Campbell et al. (2003) in constructing the returns series that correspond to our assets definition. The return on cash is taken to be the real return on 3-months Treasury Bills. The return on bonds is proxied by the real return on 5-years T-Bills. The stock returns are evaluated as the value-weighted returns on the S&P–500 index. In order to compute the return on homes, we used the capital gains based on median sales prices for new single–family houses. Finally, the mortgage rate was proxied by the effective interest rate on conventional, closed home mortgage loans. Again, the inflation series is computed from the GDP deflator.

Table 2 presents sample moments of the real returns. These series have been widely discussed in the asset pricing literature, so we only briefly outline their main features. First, we observe that all risky assets warrant a positive premia. Equity returns however are clearly larger, and much more volatile. The return on home equity is moderate, and quite volatile, especially given the large and stable rate of interest on mortgages.[10] In comparison, the return on medium–term government bonds are slightly lower, though much less volatile.

[10]Consistent with our perfect market hypothesis, we do not incorporate fiscal distortions, such as the deductability of interest payments on mortgages in our analysis.

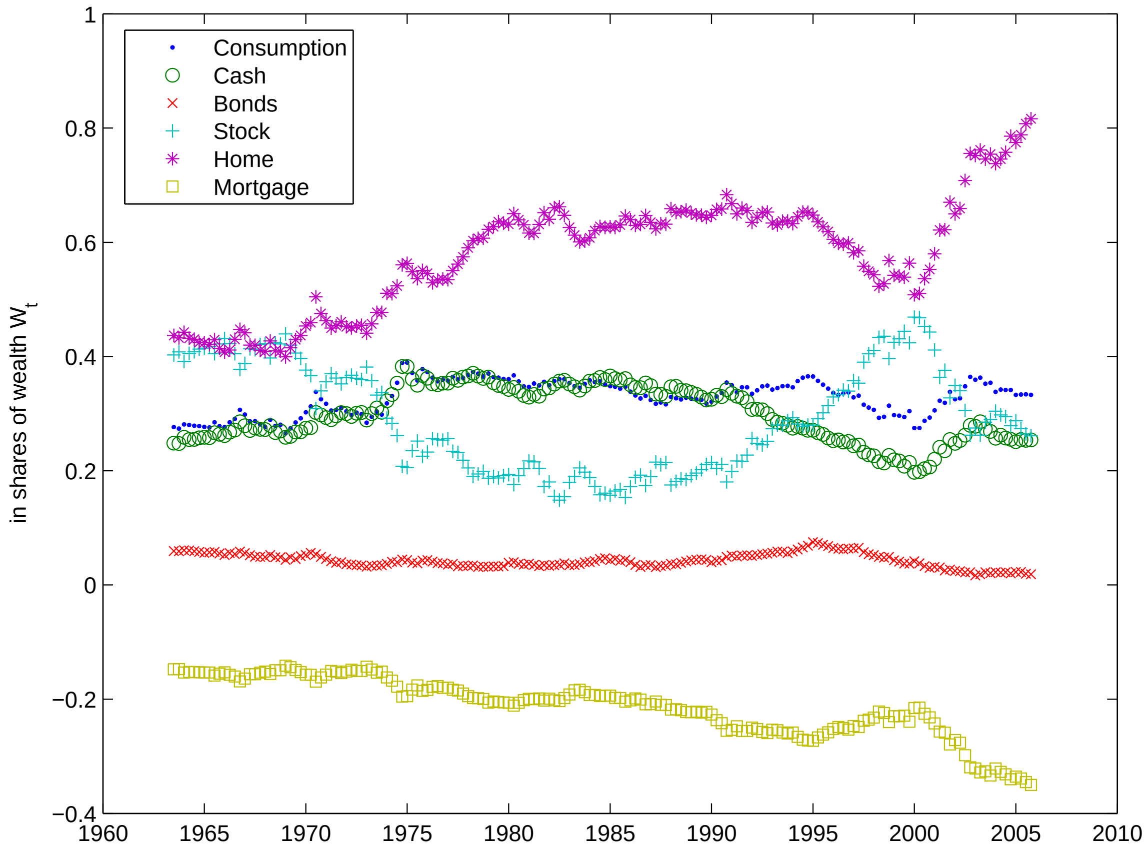

Figure 2: Consumption and Asset Shares of Wealth

The next section presents the estimation results for the WDR model (18)–(20). We start by discussing the estimation details, followed by the estimated parameters and the derives series of interest.

4 Results: Wealth–determined references

4.1 Estimation details

The model to be estimated is highly non-linear in both parameters, and in variables (because of the transformation). Consequently, we experimented with numerous identification strategies to ensure convergence, and global optimization of the likelihood function. Following standard practices, the short rate r was calibrated to the mean real quarterly return on T-Bills. Also, the

Table 2: Summary statistics: Returns

		In real, annual rates			
Variable	Series	min	max	mean	std
$r_{0,t}$	30-d T-Bills	-4.50%	7.39%	1.80%	2.27%
$r_{1,t}$	5-y T-Bills	-4.23%	9.38%	3.02%	2.45%
$r_{2,t}$	S&P-500	-70.20%	137.02%	10.89%	30.47%
$r_{3,t}$	$\%\Delta(P_{home,t})$	-35.81%	55.29%	3.49%	16.94%
$r_{4,t}$	Mortg. rate	-2.72%	10.60%	4.61%	2.46%

Sample period is 1963:Q1–2005:Q3 (170 quarterly observations).

subjective discount rate ρ was calibrated to a realistic annual rate of 3.5%. As for estimation, the strategy we used was based on a sequential estimation procedure.

In a first step, we estimated the most restricted CRRA model ($\eta_0 = \eta_w = 0$). We used the unconditional means of returns to generate the starting values for the returns process. Then, we performed a grid search over the starting value for remaining preference parameter γ to control for potential multiplicity of optimum. To facilitate convergence, we used a sequential combination of Simplex and Newton-based algorithms in which outputs were used as starting values until final convergence was obtained.

Then, in a second step, we used these estimated results as starting values for the HARA model ($\eta_0 \neq 0, \eta_w = 0$) and performed a grid search for the starting value of η_0, using the one which yielded the highest likelihood at the optimum. Finally, we used the HARA results as starting values in the WDR model ($\eta_0 \neq 0, \eta_w \neq 0$), again performing a grid search over potential starting values for η_w, and resorting to successive use of different algorithms to ensure global optimization.[11]

[11] The CRRA results were fairly robust to the choice of starting value for γ. The HARA and WDR models however were quite sensitive. We verified for global optimum by spanning the starting values for γ, η_0, η_w.

4.2 Parameter estimates

Table 3 presents the estimated parameters for the CRRA (columns 1 and 2), HARA (columns 3 and 4) and WDR (columns 5 and 6) models. Panel A reports the preference parameters, while the drift and diffusion parameters for the risky returns process are presented in Panels B and C. For each model, the first column gives the point estimate while the second reports the associated T–statistic.

Table 3: Estimation results

Model parameter	CRRA estimate	CRRA T-stat	HARA estimate	HARA T-stat	WDR estimate	WDR T-stat
			A. Preference parameters			
γ	27.423	5.0921	27.672	7.9935	25.72	32.409
η_0			27.351	44.991	-131.28	-4.805
η_w					0.0257	5.3718
			B. Drifts of risky returns process			
μ_1	0.0069	8.8238	0.0069	9.379	0.0069	14.951
μ_2	0.0222	8.9944	0.0226	7.6794	0.0222	6.3392
μ_3	0.0118	4.9843	0.0119	4.8872	0.0116	4.8029
μ_4	0.009	9.6367	0.0092	10.443	0.0091	16.596
			C. Diffusions of risky returns process			
$Q_{1,1}$	0.0061	14.5570	0.0061	14.3220	0.0061	14.4270
$Q_{1,2}$	0.0047	0.6916	0.0045	0.6456	0.0052	0.7596
$Q_{1,3}$	-0.013	-3.4391	-0.0133	-3.4422	-0.0121	-3.2271
$Q_{1,4}$	0.0067	12.755	0.0067	12.6260	0.0066	12.7750
$Q_{2,2}$	0.0720	18.403	0.0720	18.3870	0.0719	18.4080
$Q_{2,3}$	0.0043	1.1979	0.0043	1.1817	0.0046	1.2807
$Q_{2,4}$	0.0002	0.5706	0.0002	0.5393	0.0002	0.5893
$Q_{3,3}$	0.0408	18.5630	0.0409	18.2120	0.0407	17.8580
$Q_{3,4}$	-0.0005	-2.1037	-0.0005	-2.6366	-0.0005	-13.2810
$Q_{4,4}$	0.0031	44.8950	0.0030	54.2550	0.0031	19.1930
LLF	-7208.4		-7103.6		-7100.0	
nb. obs.	1870		1870		1870	

Estimation results for model (18)–(20) where assets are financial and real estate assets: $V_t = [CASH, BONDS, STOCK, HOME, MRTG]$. Fixed parameter $\rho = (1+0.035)^{1/4} - 1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\boldsymbol{Q}_{pp} \equiv \text{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process. Number of observation is number for full system. Estimation period is 1963:Q1–2005:Q3 (170 quarterly observations).

A first observation is that the curvature parameter γ is positive, and significant. The parameter is almost identical for the CRRA and HARA models and slightly lower for the WDR model. Second, the fixed reference η_0 is positive and very significant for HARA. Consequently, the null of CRRA is strongly rejected when tested against the HARA utility. This estimated parameter however becomes negative, while remaining significant under WDR preferences.

Third, the wealth dependence parameter η_w is positive, consistent with the theoretical restriction, and also significant, thereby rejecting the null of HARA against WDR. A LR test of the CRRA restrictions against the WDR alternative finds that the former is strongly rejected with a test statistic [p–value] of 219 [0]. Overall, these results indicate that the nulls of CRRA and HARA are consistently rejected when tested against the WDR specification in explaining the dynamics of aggregate portfolios. The other drift and diffusion parameters in Panels B, C are mainly instrumental to our analysis and are not discussed in details. It should be noted that, as could be expected since returns are exogenous, they are reasonably robust to the choice of preference model.

4.3 Derived series

Reference levels Figure 3 plots the induced reference levels X_t (Panel A) and the surplus consumption ratio $S_t = (C_t - X_t)/C_t$ for the three models (Panel B) .

First, the reference consumption level for CRRA is zero. Second, the reference under HARA utility is constant $X_t = \eta_0 = 27.35$. This level appears excessively small relative to actual consumption which in our sample increases from 8'000\$ to 23'000\$ (see Table 1), such that the share of reference declines from 0.0034 to 0.0012, and the surplus consumption share is virtually one. Heuristically, it would seem counter-intuitive to argue that the reference non-durables and services consumption level (in real terms) is less than 30\$ per quarter for the average US household, and that this level has not changed between 1963 and 2005.

Conversely, under WDR utility, the reference consumption $X_t = \eta_0 + \eta_w W_t$ increases in wealth, since η_w was estimated to be significantly positive. Our results suggest that refer-

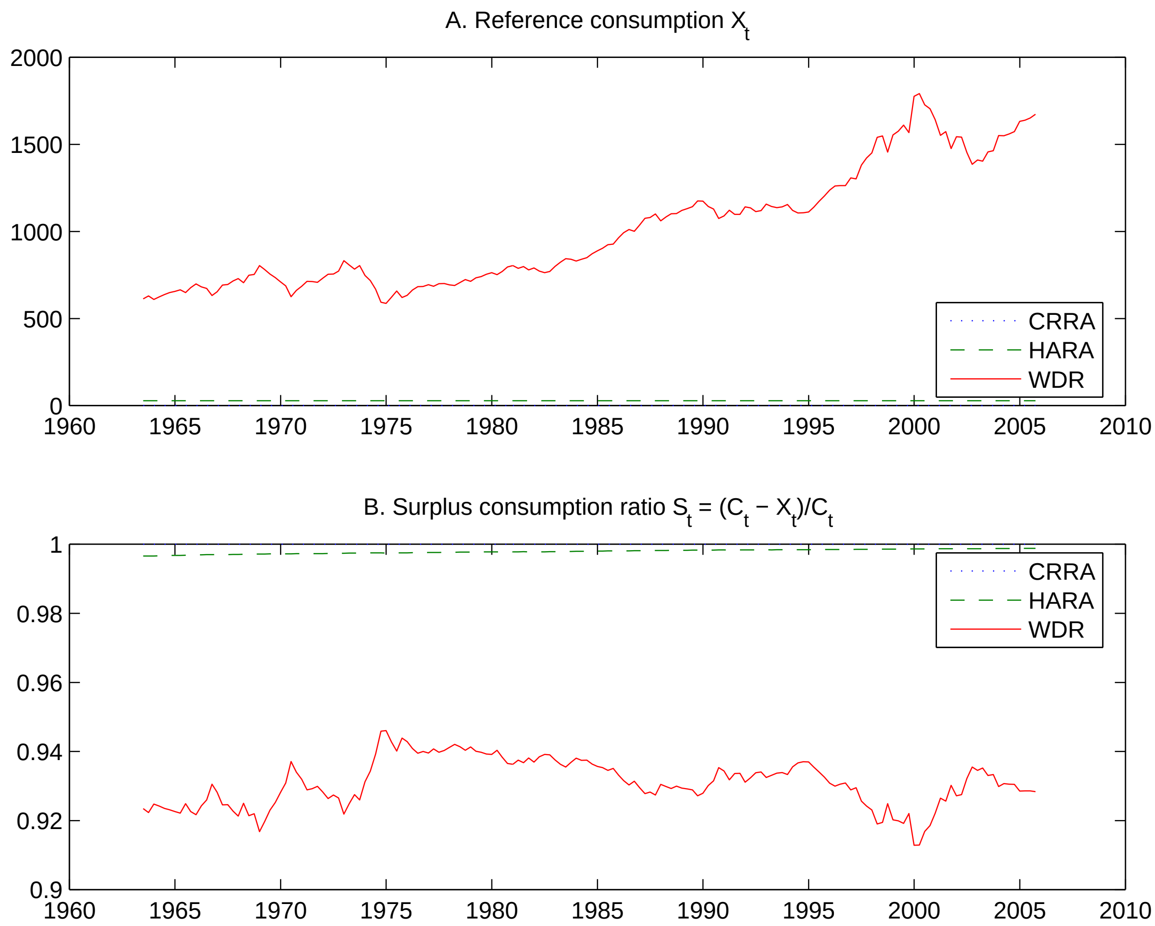

Figure 3: Reference and surplus consumption, wealth–determined references

ence consumption increased from 612$ in 1963:Q2 to 1'672$ in 2005:Q3, levels which seem heuristically plausible while satisfying the theoretical restriction (5) for non-negative optimal consumption. As a result the reference consumption level is a stationary share of consumption, varying between 5.5% and 8.5%. Hence, to the extent that a reference level of consumption should be meaningful, it should correspond to a non–negligible share of actual consumption. In this light, the ones obtained under WDR certainly appears more realistic. Note also that the surplus consumption is counter-cyclical. Consumption being less pro-cyclical than wealth implies that reference consumption falls more than actual consumption during downturns.

Second, the reference level for wealth obtained from the indirect utility function X_w is again zero for CRRA, and is constant and approximately equal under HARA (6'181$) and WDR

(6'168$). Again, the level appears realistic compared to actual wealth levels (see Table 1) while satisfying the theoretical condition (14) for non-negative optimal wealth levels.

Risk aversion Figure 4 plots the consumption (Panel A) and indirect utility (Panel B) Arrow-Pratt relative risk aversion indices for the three models.

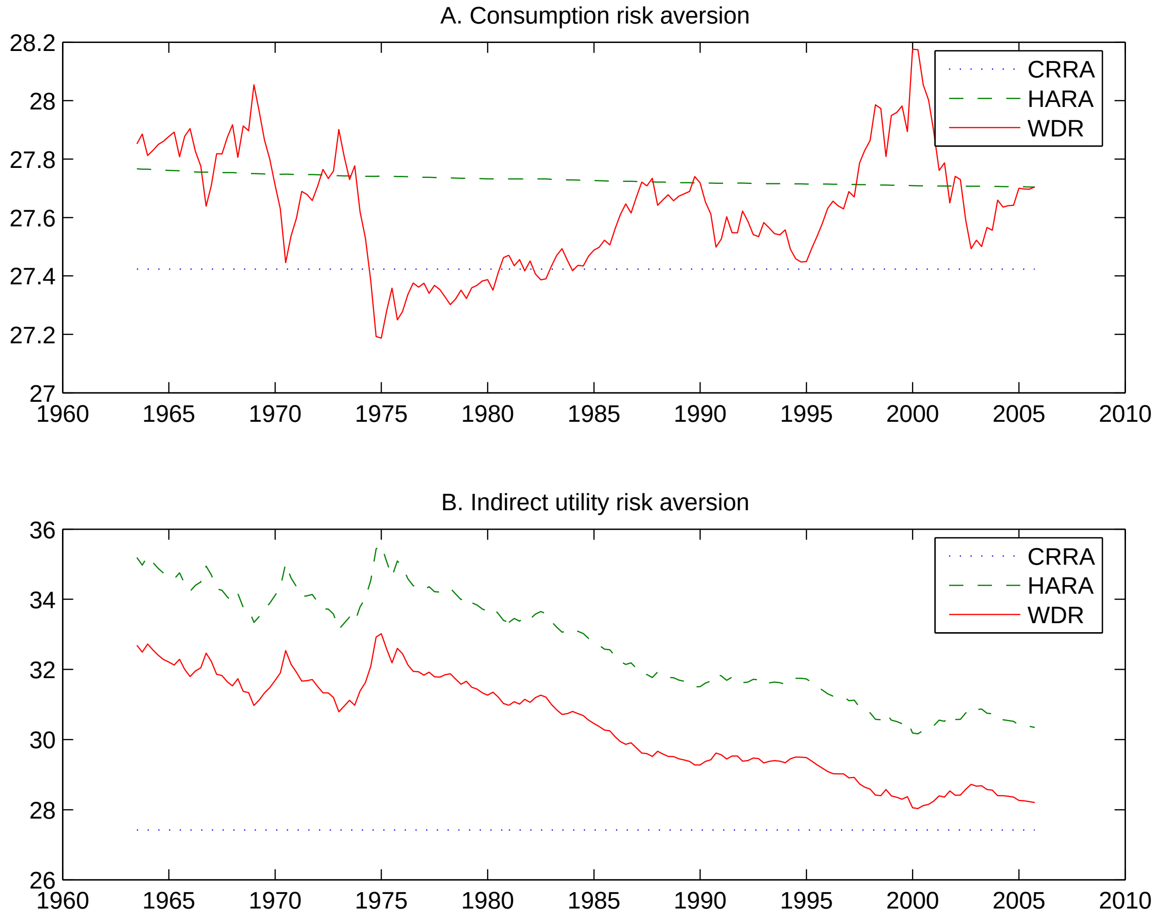

Figure 4: Consumption and indirect relative risk aversion, wealth–determined references

A first observation is that both consumption and wealth risk aversion appear high by the usual standards. Levels above 10 for relative risk aversion are usually considered excessive. Our results obtained over portfolio data confirm the ones obtained under estimation of pricing kernels. Representative–agent models require excessive risk aversion to reproduce the asset market dynamics.

Second, the consumption and indirect utility risk aversion under CRRA are of course equal and set to the estimated curvature index γ. This is not the case for both HARA and WDR. Under HARA preferences, the consumption risk aversion presents a slight downward trend and almost no perceptible short-term movements. Since the reference level of consumption is fixed at $X_t = \eta_0, \forall t$, the growth in aggregate non-durable consumption implies that risk aversion falls as consumption increases away from its reference level.

Conversely, under WDR preferences, both consumption movements around reference, and movements in reference itself caused by wealth movements cause fluctuations in consumption risk aversion which is much more volatile than under HARA or CRRA. In particular, since consumption relative risk aversion is given in (4) by γ/S_t, a counter-cyclical surplus in Panel B of Figure 3 translates in a pro-cyclical consumption risk aversion. As discussed earlier, downturns are associated with more important falls in reference consumption than in actual consumption. As a result, consumption risk aversion decreases. Note that, contrary to HARA, risk aversion under WDR is stationary as the share of reference consumption X_t/C_t was found to be stationary.

In Panel B, we can see that indirect utility risk aversion display parallel movements for HARA and WDR utility. The reason is that the value function are iso-morphic under both types of preferences. Moreover, the estimated parameters produce almost identical wealth reference levels under HARA and under WDR. Therefore the slightly lower risk aversion under WDR simply reflects the lower estimated curvature parameter γ. Unsurprisingly, both HARA and WDR predict counter-cyclical risk aversion at the optimum. Since reference level is constant, positive, and approximately equal, risk aversion increases in downturns as wealth falls toward its reference level. In comparison, CRRA predicts a-cyclical relative risk aversion under the iso-elastic restriction of zero reference.

4.4 Narrow definition of wealth

Numerous studies of either returns or portfolio dynamics rely on a narrow definition of wealth based on financial assets only (Epstein and Zin, 1991; Campbell et al., 2003, among others). To verify the robustness of our results, we therefore re-estimate the model using a subset of our previous assets consisting of financial assets only: $V_t = [CASH_t, BONDS_t, STOCK_t]$. Table 4 presents the estimation results for this specification.

Table 4: Estimation results: Financial wealth only

Model	CRRA		HARA		WDR	
parameter	estimate	T-stat	estimate	T-stat	estimate	T-stat
A. Preference parameters						
γ	8.3096	8.3353	8.4449	6.5439	10.269	5.1075
η_0			28.3	53.06	-384.99	-5.6712
η_w					0.0647	5.9141
B. Drifts of risky returns process						
μ_1	0.0065	25.176	0.0067	20.193	0.0074	20.745
μ_2	0.0170	9.8923	0.0178	19.192	0.0182	2.3096
C. Diffusions of risky returns process						
$Q_{1,1}$	0.0062	16.952	0.0061	17.353	0.006	12.077
$Q_{1,2}$	0.01	1.7415	0.0099	1.742	0.0107	1.8826
$Q_{2,2}$	0.0728	18.294	0.0728	18.312	0.0728	18.26
LLF	-5674.8		-5561.0		-5558.9	
nb. obs.	1190		1190		1190	

Estimation results for model (18)–(20) where assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Fixed parameter $\rho = (1 + 0.035)^{1/4} - 1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\boldsymbol{Q}_{pp} \equiv \mathrm{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process. Number of observation is number for full system. Estimation period is 1963:Q1–2005:Q3 (170 quarterly observations).

Overall, our earlier findings are maintained. Again, the CRRA model is strongly rejected when tested against the HARA and WDR alternatives, while the HARA restriction is strongly rejected when tested against the WDR alternative.[12] Reference levels for consumption are increased with $X_t \in [756, 3'034]$, mainly due to the larger estimated parameter η_w compared to

[12] These results may however depend on the time period under consideration. In an earlier version of this paper, using financial wealth only, a negative, and significant $\eta_w = -0.019$, t-stat $= 3.215$ was estimated for the sample period 1952:Q2–2000:Q4. Recall that the current sample is 1963:Q1–2005:Q3.

the larger definition of wealth. Wealth reference levels however remain virtually unchanged for both HARA (6'395\$) and WDR (6'383\$).

Interestingly, the curvature parameters are lower when a narrow definition of wealth is used, compared to the one which incorporated real estate. As a result, consumption risk aversion is also lower (averaging 8.46 for HARA, and 11.39 for WDR). This can be understood as a dual of the main pricing anomalies. For instance, the equity premium puzzle states that the observed equity premium is too high relative to its theoretical counterpart. In portfolio terms, this translates in observed portfolio shares that are too low compared to those predicted by the model. When a narrow definition of wealth is used, the denominator W_t in the definition of portfolio shares $v_{i,t} \equiv V_{i,t}/W_t$ is too low. Consequently, the observed shares are artificially higher, and the model is able to reproduce them at realistic risk aversion levels.[13]

5 Results: Internal habit–determined references

We mentioned earlier that an alternative time–varying specification for the reference point is suggested by the habit literature, whereby references depend upon past, instead of expected future consumption streams as under WDR. Habit can be controlled (internal) by the agent or not (external). Recent evidence on asset market dynamics suggests that the internal habit specification is more consistent with observed asset and bond returns than external habit, especially when long horizons of consumption are considered (Chen and Ludvigson, 2004; Grishchenko, 2005). We consequently focus on this specification as an alternative to WDR. Specifically,

[13] An alternative explanation can be related to the critique by Roll (1977). State-dependent models such as WDR imply multiple–beta pricing kernels, similar to the ones obtained under Non-Expected utility (Duffie and Epstein, 1992). In particular, a total wealth beta then supplements the standard consumption beta. As Kocherlakota (1996) points out, relying on narrow definition of total wealth artificially increases the covariance of individual returns with wealth. The low consumption beta is therefore augmented by a high wealth beta and the high premia can be reproduced at reasonable levels of risk aversion.

Constantinides (1990) considers the internal habit specification (7), while maintaining a power utility functional (3) over $C_t - X_t$.

In parallel with our earlier homoscedasticity–inducing transformation, and adapting the solutions in Constantinides (1990), Theorem 1, and Appendix A, reveals that the multivariate Brownian motion for the transformed optimal rules is characterized by:

$$\begin{pmatrix} d\hat{C}_t \\ d\hat{\boldsymbol{V}}_t \\ d\hat{W}_t \end{pmatrix} = \boldsymbol{\iota}\left[\left(\frac{n}{\boldsymbol{m}'\boldsymbol{\sigma}_p} - 0.5\boldsymbol{m}'\boldsymbol{\sigma}_p\right)dt + dZ_t\right] \tag{35}$$

$$\hat{C}_t \equiv \frac{\log(C_t - X_t)}{\boldsymbol{m}'\boldsymbol{\sigma}_p}, \quad \hat{\boldsymbol{V}}_t \equiv \frac{\log(\boldsymbol{V}_t)}{\boldsymbol{m}'\boldsymbol{\sigma}_p}, \quad \hat{W}_t \equiv \frac{\log[W_t - X_t/(r + a - b)]}{\boldsymbol{m}'\boldsymbol{\sigma}_p} \tag{36}$$

$$n \equiv \gamma^{-1}(r - \rho) + 0.5M(1+\gamma)/\gamma^2, \quad \boldsymbol{m} \equiv \gamma^{-1}\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r), \quad M \equiv (\boldsymbol{\mu}_p - r)'\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r) \tag{37}$$

Again the system (35) can be estimated without discretization bias by MLE using, applying the proper Jacobian term: $K_{c,t} = [\boldsymbol{m}'\boldsymbol{\sigma}_p(C_t - X_t)]^{-1}, K_{vi,t} = [\boldsymbol{m}'\boldsymbol{\sigma}_p V_{i,t}]^{-1}, K_{w,t} = [\boldsymbol{m}'\boldsymbol{\sigma}_p(W_t - X_t/(r + a - b))]^{-1}$.[14] We therefore estimate (35) using the same data set, and identification strategy as with WDR preferences.[15] Table 5 presents the estimation results.

First, all the preference parameters γ, X_0, a, b have the correct sign and are strongly significant. Hence, the nulls of CRRA ($X_0 = a = b = 0$) HARA ($a = b = 0$) are once again strongly rejected in favor of time–varying references. However, the theoretical restriction (Constanti-

[14] In constructing the reference X_t in (7), we use the discrete–time recursion:

$$X_t = e^{-a}(X_{t-1} + bC_{t-1})$$

where the initial observation X_0 is treated as a parameter and is estimated jointly with the other preference parameters a, b, γ.

[15] In particular, we used the parameters from the HARA specification (corresponding to the special case of $a = b = 0$) and the parameters in Constantinides (1990) as starting values. We verified global optimization and robustness by spanning the starting values set.

Table 5: Estimation results: Internal Habit model

Model parameter	Habit estimate	T-stat
A. Preference parameters		
γ	22.435	4.7987
X_0	7166	84.8310
b	0.9480	3.8658
a	0.7120	5.2996
B. Drifts of risky returns process		
μ_1	0.0068	9.6328
μ_2	0.0212	7.2667
μ_3	0.0113	4.9114
μ_4	0.0089	10.3650
C. Diffusions of risky returns process		
$Q_{1,1}$	0.0062	14.7300
$Q_{1,2}$	0.0031	0.4747
$Q_{1,3}$	-0.0121	-3.3176
$Q_{1,4}$	0.0068	12.8920
$Q_{2,2}$	0.0719	18.4690
$Q_{2,3}$	0.0027	0.7904
$Q_{2,4}$	0.0003	0.7812
$Q_{3,3}$	0.0408	18.4320
$Q_{3,4}$	-0.0004	-2.1738
$Q_{4,4}$	0.0031	25.0700
LLF	-6010.6	
nb. obs.	1870	

Estimation results for Habit model (35)–(36) where assets are financial and real estate assets: $V_t = [CASH, BONDS, STOCK, HOME, MRTG]$. Fixed parameter $\rho = (1 + 0.035)^{1/4} - 1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\boldsymbol{Q}_{pp} \equiv \mathrm{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process. Number of observation is number for full system. Estimation period is 1963:Q1–2005:Q3 (170 quarterly observations).

nides, 1990, eq. (8)):

$$r + a - b > 0$$

is satisfied at the 1%, but not at the 5% level, with a value (T-stat) of -0.2316 (-2.0873). Second, the curvature γ is slightly lower, although of the same order of magnitude than our earlier estimates. Third, X_0 is large and initializes a habit process close to minimum consumption for our sample.

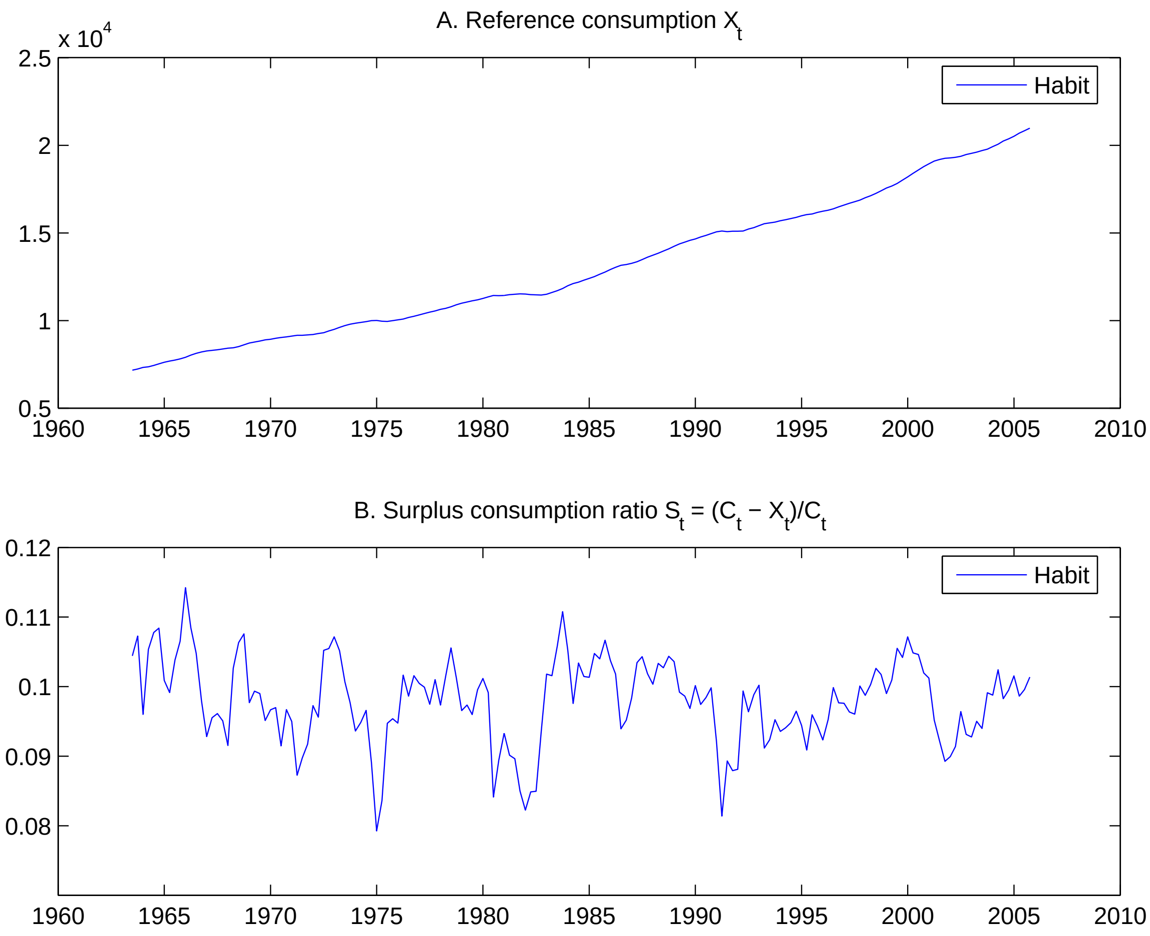

Figure 5: Reference and surplus consumption, habit–determined references

Next, we generate and plot the corresponding reference and surplus consumption series in Figure 5. First, the consumption reference X_t in Panel A is smoothly growing at an exponential rate similar to that of consumption. Consequently, the ratio of X_t/C_t is stationary and the surplus consumption $S_t = 1 - X_t/C_t$ in Panel B is an $I(0)$ process. Contrary to our WDR estimates, surplus consumption is now pro-cyclical. All adjustments to surplus are ascribed to movements in consumption. Since the habit–determined reference is a smooth process, downturns are associated with declines in consumption, but only minor adjustments to references. Importantly, compared to our earlier estimate under WDR of mean surplus equal to 93%, surplus consumption is much lower, with an average of 10%. This level is in the range usually reported in the Habit literature.[16]

We plot the consumption (Panel A) and indirect utility (Panel B) risk aversion series in Figure 6. Since the curvature parameter was virtually unchanged compared to our earlier estimates, the low level of surplus consumption has a strong impact on consumption risk aversion. Unsurprisingly, consumption risk aversion is clearly huge, averaging more than 8 times levels obtained under CRRA, HARA, or WDR.

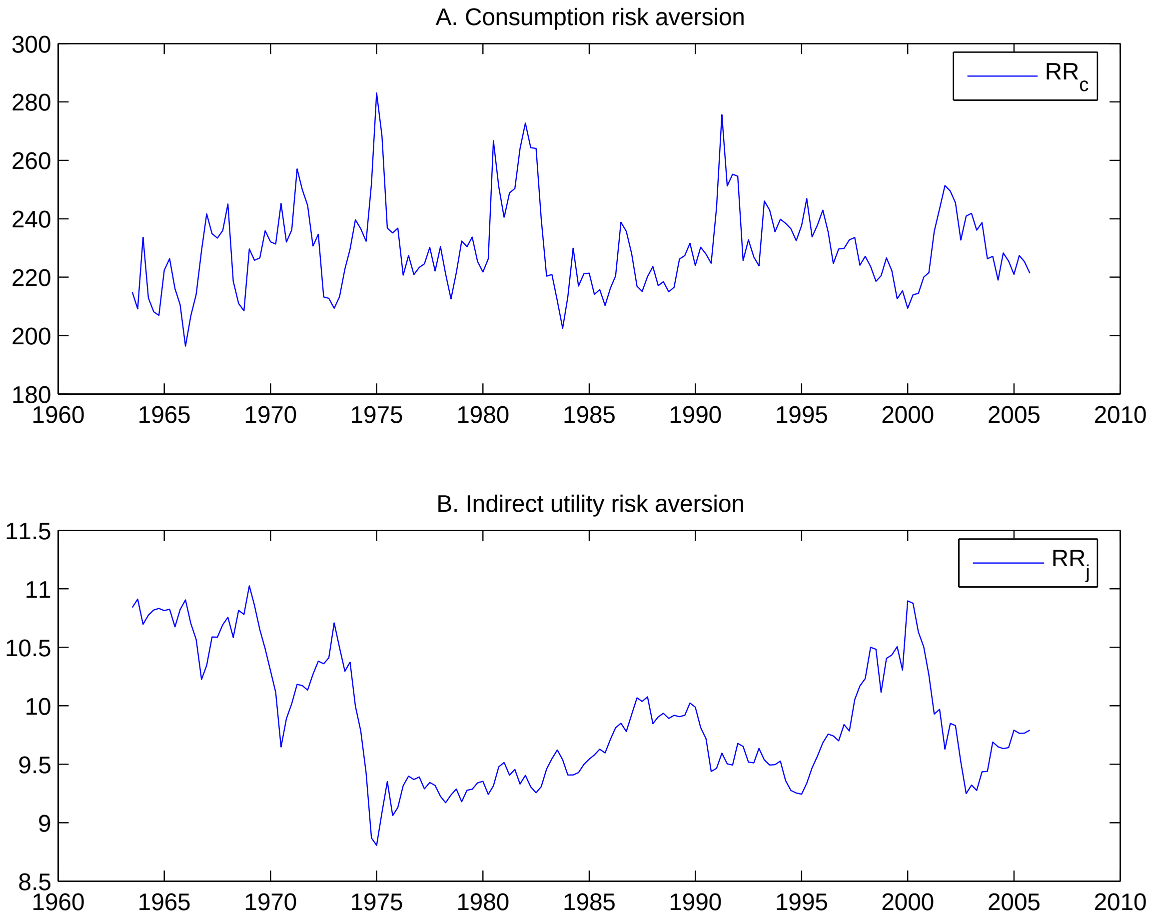

Figure 6: Consumption and indirect relative risk aversion, habit–determined references

[16]For example, (Campbell and Cochrane, 1999, p. 218) report implied surplus of 5.7% at the steady state, and a maximum of 9.4%; (Tallarini and Zhang, 2005, p. 14) estimate a steady state surplus ratio of only 4.7%, with maximum level of 7.7%. (Li, 2005, p. 232) reports estimates varying between 1.5% and 6.7%. Internal habit models often yield larger surplus estimates. Constantinides (1990) reports surplus ratios of 20%, while Grishchenko (2005) finds steady–state estimates of surplus varying between 16% and 32% depending on choice of instruments, and lags in the habit process.

The indirect utility level of risk aversion plotted in Panel B however is more realistic, although still excessive. This can be explained from the reference level of wealth, $X_w = X_t/(r + a - b)$ which is estimated to be negative, since $r + a - b$ was found to be negative. Consequently, the risk aversion level is lower than the curvature index. Although this last result is appealing, $X_w < 0$ is counter–intuitive and violates the condition for non–negative optimal wealth.

To summarize, habit–determined references imply estimates for consumption reference that are very high throughout our sample period. Since surplus consumption is low, and curvature remains similar to our other estimates, a consequence is that risk aversion is hugely in excess of admissible range. Conversely, since the reference level of wealth is negative, indirect utility risk aversion is more realistic, although still very high.

6 Conclusion

A main feature of Reference Dependent preferences is that agents gauge possible outcomes using deviations from benchmark levels. Regret Theory associates references with assets being held, whereas Habit models relate references to past consumption profiles.

This paper confronts the predictions of the two alternatives to the actual aggregate asset holdings of US households. We obtain closed-form expressions for optimal consumption and portfolio when references are an affine function of wealth. Substituting these into the budget constraint yields a multi-variate Brownian motion in consumption, asset values and wealth, which is estimated addressing discretization bias. For comparison purposes, we use a similar approach to estimate the Brownian motion under an internal habit specification.

Our results can be summarized as follows. First, the null hypotheses of CRRA and HARA utility are strongly rejected against WDR. Second, all models yield very similar curvature parameters. Third, WDR yield estimates of references that are (i) realistic compared to and (ii) co-integrated with consumption, such that references remain economically meaningful relative to actual consumption. Fourth, the relative risk aversion over consumption and for the indirect utility function remain excessive, and similar between CRRA, HARA and WDR.

Fifth, CRRA and HARA utility are again rejected when tested against internal Habit. Sixth, the parameter estimates under Habit point to a much higher reference level relative to consumption, while co-integration with consumption is maintained. Consequently, as the curvature parameter is virtually unchanged compared to other models, consumption risk aversion is hugely in excess of admissible range. Nonetheless, we estimate a negative reference level of wealth for the indirect utility function, such that indirect utility risk aversion is lower than curvature and more in line with acceptable levels.

Overall, our results highlight the relevance of a consumption reference that remains meaningful throughout the sample period. Future research should focus on comparing the asset pricing implications of both the WDR and Habit models.

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