Performance Measurement and Appraisal of Private Equity Investments relative to Public Equity Markets*

Working Paper
WARNING: preliminary and incomplete, do not quote

This Draft: April, 2005

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* All Private Equity data for this project were obtained from the CEPRES Center of Private Equity Research (www.cepres.de). Market quotes and exchange rates were obtained form Bloomberg and Datastream. We gratefully acknowledge Rüdiger Blank, Phillip Krohmer, Eric Nowak, Arno Poschik, Daniel M. Schmidt, Timo Stetter, and Michael Rudolf for helpful comments.

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Abstract

In this paper we investigate the risk return relationship of Private Equity (PE) relative to Public Market Equity (PM) investments to assess the adequateness of PE's return premium. We analyze cash flows of PE projects gross of fees and any other externalities. Our analysis is based on simulated PM investments, mimicking the cash flow patterns of the PE investments. The comparison of alternative cash flow based performance measures reveals a substantial impact of the reinvestment hypothesis. Prior to any risk adjustment, PE investments outperform their PM counterparts with varying levels depending on the chosen benchmark (broad, industry specific, local). Next we compare standard risk measures and find downside deviation and shortfall to better describe the characteristics of not normally distributed PE investment returns than standard deviation. Thus, it is not surprising to observe substantially higher Sharpe Ratios for PM relative to PE investments. We adjust the Sharpe Ratio measuring risk in terms of downside deviation and still observe underperformance of PE relative to PM investments, with very heterogeneous results regarding industry, stage and size of the investments. Last we introduce Omega as alternative risk adjusted performance measure, because its risk definition better suits the characteristics of PE investments. For our PE sample we observe adequate excess returns over public stock markets given the higher shortfall risk. Our findings question the existence of an illiquidity puzzle on the fund level. Overall PE returns are highly skewed and very heterogeneous. We find later stage to be more attractive than early stage investments due to higher risk adjusted returns. As the PE investment universe is on average of poor quality compared to public equity markets, investment selection ability is of crucial importance.

Keywords: Private Equity, Venture Capital, Internal Rate of Return, Public Market Equivalent, Reinvestment Hypothesis, Performance Appraisal, Shortfall, Sharpe Ratio, Omega

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1 Introduction

The performance of private equity funds and their investments has recently gained wide attention from the academic community. Besides raising quantity and quality of available data, interest of investors (mainly institutional) in private equity as a vehicle for diversification and return enhancement has triggered this increase in literature on the economics of this asset class. A key topic is the risk return relationship of private equity (PE), its performance relative to public markets (PM) and the attractiveness of the investment opportunities.

Early papers mainly address the performance on the fund level. *Gompers* and *Lerner* were the first to empirically analyze the return of private equity funds relative to investments in public equity (1997). Prominent examples for research tackling the returns of private equity on the partnership level are *Kaplan* and *Schoar* (2003), *Ljungqvist* and *Richardson* (2003) and *Jones* and *Rhodes-Kropf* (2003). *Cochrane* (2001) examined return properties on the investment/project level. They all conclude that private equity investments do outperform public markets gross of all fees on an aggregate level. Adding a risk-adjustment, as pursued by *Ljungqvist* and *Richardson*, supported these findings. *Jones* and *Rhodes-Kropf* explicitly assess the risk premium inherent in private equity investments. Some recent studies, such as *Schmidt* (2003) and *Gottschalg* et al. (2004) question the positive alpha returns of private equity investment. In this paper we investigate whether PE investments generate a return premium over public stock markets on the project level gross of all externalities. Furthermore we are interested to assess whether this premium is adequate on a risk adjusted level.

In comparison to public equity there are two major characteristics of private equity: No liquid secondary markets and very restrictive disclosure politics of market players. Investments are in practice frequently appraised in reference to the return of a chosen benchmark investment rather than via maximizing individual utility functions. Objective and unbiased returns of public market investment often serve as reference points for performance appraisal purposes. If the relation between public market returns and the investment's return is unclear, selecting adequate benchmarks and drawing unambiguous conclusions is difficult. In contrast to existing work we strive to explore the relation

between public markets and portfolio companies rather than on a fund level. Our focus is on "pure" investment returns prior to any pooling activity by PE funds as we are interested in project returns and the quality of the PE investment universe. We strive to assess whether PE projects generate adequate return premiums over PM investments.

We use a data sample of 5,991 PE projects derived from the records of the Center of Private Equity Research (CEPRES) for our empirical analysis.[1] Besides PE as an asset class we look at cross sectional difference from industry, geographic and stage perspectives. To answer the question whether private equity outperforms public equity we apply and compare the performance measure concepts of excess return, based on the Internal Rate of Return (IRR), and Public Market Equivalent (PME). We especially analyze the impact of the distinct reinvestment assumptions. In a second step we alter the public benchmark used, from broad to more specific indices and discuss effects for performance appraisal. Prior to introducing a risk adjustment to our performance measure we compare several risk measures and their suitability for PE investments on the project level. Based on these results we appraise performance using the established concept of the Sharpe ratio. We then introduce alternative risk adjusted performance measures that are tailored to the characteristics of private equity investments.

2 Related Literature

Kaplan and *Schoar* analyze private equity performance on the fund level. They calculate IRR, PME and TVPI for 746 private equity funds obtained from Venture Economics. They find a large heterogeneity in fund returns. Analyzing PMEs based on net cash flows they find a relative underperformance (outperformance) of PE investments to S&P 500 on an equally (value) weighted basis. Using average fees and carried interest figures they conclude that on average PE outperformed the S&P 500 gross of fees, but they can only approximate the impact of fees and do not analyze risk adjusted performance measures. Focusing on dynamics of fund returns, they find a strong

[1] We thank CEPRES for delivering the data. www.cepres.de

persistence of fund returns and improving returns with increasing experience of PE funds.[2]

Ljungqvist and *Richardson* study the returns to investments in 73 private equity funds by a large limited partner in funds raised between 1981 and 1993. They calculate IRR, TVPI, Excess IRR and a Profitability Index for investments on fund level. In their analysis they benchmark PE relative to S&P 500 and Nasdaq Composite Index and observe an outperformance of 6-8% for PE investments based on IRR. They calculate a risk-adjusted profitability index discounting cash inflows at the cost of capital. The cost of capital is estimated on fund level using *Fama* and *French's* Industry Cost of Equity figures.[3] On this risk adjusted basis they observe excess alpha returns on the PE fund level. Their focus is on a general analysis of private equity fund's cash flow patterns, draw down rates and performance determinants.[4] In their follow on paper, *Ljungqvist* and *Richardson*, analyze the investment behavior of PE fund managers. They discuss the impact of the competitive environment on timing of investment and divestments. Furthermore, they identify determinants of investment performance, measured in terms of a multiple (cash inflow/invested capital).[5]

Jones and *Rhodes-Kropf* use data from 1,245 funds to investigate whether and how idiosyncratic risk is priced in Venture Captial (VC) markets. They find that unavoidable principal-agent problems result in fund returns that are increasing in the amount of idiosyncratic risk. Thus, in a competitive model for VC funds, total risk rather than only systematic risk is priced. In their model funds are necessary intermediaries that identify NPV positive projects for their investors from a VC investment universe of questionable quality with average investments that are NPV negative. Our analysis strives to challenge this assumption looking at the performance of average investments. *Jones* and *Rhodes-Kropf* estimate "long-run" betas for VC (1.80) and Buyout (BO) (0.65) funds based on a time series regression of quarterly returns derived from Net Asset Values reported by the funds.[6]

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[2] see Kaplan/Schoar (2003)
[3] see Fama/French (1997)
[4] see Ljungqvist/Richardson (2002)
[5] see Ljungqvist/Richardson (2003)
[6] see Jones/Rhodes-Kropf (2003)

Cochrane focuses on the individual portfolio company level and infers the aggregate performance of private equity investments. He stresses the importance of adjusting for survivorship bias, which potentially arises due to the high failure rate of private equity investments. In his paper he measures the mean, standard deviation, alpha and beta of venture capital investments using a maximum likelihood estimate that corrects for selection bias. He finds that mean log returns of individual portfolio investments are around fifteen percent, though arithmetic mean returns are much higher and generate an arithmetic alpha of 32%.[7]

Schmidt analyzes the risk/-return characteristics of 642 U.S. private equity investments and investigates how this asset class can be used for diversification purposes. He finds that BO returns are less skewed than VC returns, indicating crucial importance of investment selection skills in the VC industry. To assess the relationship between PE and PM investments he generates cash flow streams of public market investments mimicking cash flow patterns of PE investments. In our paper we use a very similar simulation approach. *Schmidt* performs a bootstrap simulation to observe risk-/return characteristics of portfolios comprising PE investments. He shows that the average portfolio size of 20 to 28 PE investments eliminates over 80% of non-systematic risk and thus can be regarded as balanced. Furthermore, PE investments bare higher levels of non-systematic risk compared to PM investments, with the exception of mezzanine. He concludes that PE as an asset class offers diversification potential, as correlations factors with public markets are low. The different PE investments categories are very heterogeneous regarding risk, return and correlations with public market investments.

Gottschalg et al. analyze returns of more than 500 PE funds, derived from the records of Venture Economics. Based on net cash flows they find that realized funds underperform public stock-markets. As they find PE performance to be pro-cyclical relative to public markets, desirable hedging properties do not justify low return levels. Computing average CAPM-betas of 1.7 and 1.6 for BO and VC funds respectively, they argue that PE funds are exposed to non-negligible risk that should command a return premium over public markets rather than the observed discount. Besides a learning hypothesis, that PE funds still develop their investments selection and management skills,

[7] see Cochrane (2003)

they question the validity of the public benchmark and ignorance of fees in the pricing process as potential explanations for the illiquidity puzzle.[8] In our work we use a broad variety of public benchmarks and analyze the hedging probabilities on the PE project level to investigate the validity of their arguments. If we observe premium returns of PE over PM investments, fees on the fund level could be an explanation for the puzzle, as our work is based on gross cash flows.

The work closest to that presented in this paper is that of *Kaserer* and *Diller*, who analyze the risk-/return relationship of 794 European PE funds to assess the role of PE in the asset allocation. They measure performance of the PE funds relative to the MSCI Europe and the J.P. Morgan Government Bond Index in terms of IRR based excess returns and PMEs. Results reveal a slight underperformance of the average realized fund relative to the MSCI Europe (PME based). BO funds exhibit consistently higher performance figures than VC funds. To add a risk adjustment they calculate Sharpe Ratios and find significantly lower ratios for PE funds compared to the MSCI Europe. They approximate correlations between PE and the public benchmarks to be 0.8 (MSCI Europe) and 0.1 (Bond index) based on the PME and Bond Market Equivalent (BME) figures. Using these parameters, they show that adding PE to a portfolio comprising MSCI Europe and J.P. Morgan Government Bond Index shares triggers diversification effects. They conclude that private equity will have a substantial role in asset allocation.[9] In our paper we perform similar analysis on the project level and contrast our findings with their results for the fund level. As we have detailed information on the companies we benchmark our sample against specific and a variety of broad benchmarks to draw a sharper picture of their relation. Furthermore we challenge the appropriateness of the Sharpe Ratio for PE on the project level and introduce alternative risk adjusted measures.

[8] see Gottschalg et al. (2004)
[9] see Kaserer/Diller (2004)

3 Data

3.1 Data Description

The dataset we use provides information on 86 private equity companies, 243 private equity funds and their 5,991 investments in 4,819 different companies.[10] The investments span over a time of 28 years (1975 – 2003) and cover 51 Countries. Information is completely anonymous, but provides characteristics of both, the investing fund and the portfolio company. Names of neither funds nor firms are disclosed. The key advantage of the dataset is cash flow and write-off information provided for each individual investment. These cash streams between the portfolio company and the PE fund are reported gross of fees and thus are not biased by any externalities, especially management fess and carried interest. CEPRES contains all investments pursued by included private equity funds, thus we do not suffer a selection bias.[11]

The dataset used in this study is closely related to the one used by *Schmidt* (2003) and *Cummings et al.* (2004). Compared to these studies the dataset has grown since then. Our dataset comprises of fully (2,909), partially (1,052) and unrealized (2,030) investments. As we strive to explore the relationship between private equity and public market returns, it is crucial to only include unbiased returns. In this context returns are unbiased if their calculation is based on objective market values. For partially and unrealized investments the dataset includes the Net Asset Value (NAV) at the valuation date. The NAV is determined and communicated by the PE fund and thus a subjective value. We can therefore not generally include them in our analysis.

Dropping all partially and unrealized investments would bias the sample towards older investments. As the mean holding period for all realized investments is 46.5 months, very few recently pursued investments would be included. Several approaches have been discussed to expand the data sample while limiting the impact of subjective valuations. *Kaplan* and *Schoar* use the correlation between IRR of unrealized funds, based on NAV, and IRR of realized funds. They find the correlation for funds being at least 5 years old to be high (0.9) and include these funds in their sample. This approach is

[10] Our dataset is derived from the records of CEPRES. www.cepres.de

[11] As CEPRES data comprise mainly private equity-managers reporting performance over the last years, we might face a certain survivorship bias, because we have no information of managers which had no longer been in business in the mid 90ties.

not appropriate for our analysis as we focus on investments in portfolio companies rather than funds. On the investment level idiosyncratic aspects have a higher impact on returns than for funds of pooled investments. An alternative approach used by *Kaserer* and *Diller* focuses on the importance of the Residual Net Asset Value (RNAV) for the return. They include funds if their residual value is below 10% or 20% of the undiscounted sum of the absolute value of all previously accrued cash flows.[12] As this approach ensures that the impact of subjective valuations is limited, it can be applied for our analysis. To enhance comparability with other studies the RNAV is compared to Paid in Capital (PI) rather then the sum of all cash flows. Doing so, the established Residual Value to Paid in Capital ratio (RVPI) can be applied. Not fully realized investments are included in our sample if they meet the following condition:

$$RVPI_{it} = \frac{RV_{it}}{\sum_{n=1}^{t} \left| PI_{in} \right|} < q \qquad (1)$$

with:

$RVPI_{it}$ = Residual Value to Paid in Capital for investment *i* in final period *t*
RV_{it} = Residual Value of investment i in final period *t*
PI_{in} = total paid-in-capital for investment *i* in period *n*
q = critical value

The critical value q is set to 5% indicating that not fully realized investments are included in the sample if their RNAV is less than 5% of total paid in capital. As our yardstick is paid in capital and not the absolute value of all occurred cash flows, we are more restrictive including not fully realized than *Kaserer* and *Diller* in two aspects.

In a last step we eliminate all investments with missing or obviously incomplete data. Looking at the cash flow entries we require investments to cover at least half a year of information. Our assumption that investments with shorter investment horizons are incomplete entries is supported by missing information regarding most other characteristics of these investments. We eliminate 238 investments due to incomplete information.[13]

[12] see Kaserer / Diller (2004) pp. 45-47

[13] Results of following analysis for the larger sample including these investments do not differ significantly and are available upon request.

3.2 Summary Statistics

The resulting dataset comprises of 2,658 fully and 95 partially realized investments. Vintage Years of the investments span over 28 years (1975 to 2003). Exhibit 1 summarizes the distribution over time and indicates that the majority of the investments (93%) have a vintage year between 1986 and 2000. The dataset can be considered as a good sample of the overall market of realized PE investments taking into account our goal to include objective performance figures. As a matter of fact, only few investments pursued after the burst of the internet bubble have materialized sufficiently to be included in our sample. We will pick up this issue and how it impacts performance appraisal later in our analysis.

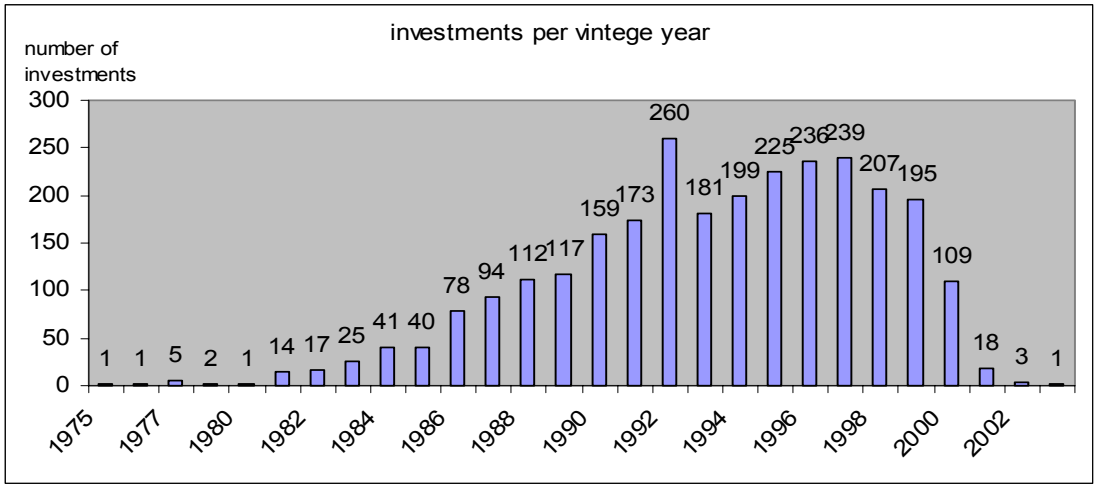

Exhibit 1: Number of PE investments per vintage year

Geographically the sample covers PE investments in four continents: North America, South America, Europe and Asia. Most transactions are from the U.S. (1,312), the United Kingdom (385), France (264) and Germany (116). The remaining investments were pursued in 28 other countries.[14] Overall, the geographic distribution of the investments roughly reflects the size of the corresponding PE market at that point in time. Developing Venture Capital markets, such as China, are not in the scope of our analysis.[15]

[14] Argentina, Austria, Belgium, Benelux, Brazil, Canada, China, Denmark, Finland, Iceland, Indonesia, Ireland, Israel, Italy, Japan, Korea, Luxemburg, Netherlands, Norway, Philippines, Portugal, Puerto Rico, Russia, Singapore, Spain, Sweden, Switzerland.

[15] see e.g. Lerner / Schoar (2004) who focus on PE transactions in developing markets

The dataset includes a variety of company, fund and investment specific characteristics. Most important for our analysis are investment stage, industry classification, investment horizon and real investment value.

PE investments are often clustered according to the stage of the private company. We further break-down the general Venture Capital / Buyout classification to get a more detailed picture of differences regarding realized investment returns. Stage information is available for 1,390 investments. The missing observations are not related to any other information or characteristic of the portfolio company. We cluster our sample into the following four distinct stages:

- Early stage: seed and early (initial) financing of private start-up companies mainly by Venture Capital funds
- Expansion: organic growth and acquisition financing of private companies by Venture Capital and Buyout funds
- Later stage: financing of established companies, especially Leveraged Buyouts, Management Buyouts/-ins and public to private transactions mainly by Buyout funds
- Turnaround: recapitalization and other turnaround investments mainly by Buyout funds

A prerequisite for measuring investment returns against specific rather than broad benchmarks is the classification according to industries. For our dataset we distinguish 9 industry clusters.[16] 590 investments can not be assigned to a cluster and are categorized as "other industries". Table 1 summarizes the distribution across the industries. Industry cluster need to have a certain size to smoothen idiosyncratic factors of individual investments. Thus, we only analyze industry clusters with at least 70 observations and drop Financial Services and Materials as separate industry clusters. The frequency distributions of investment stages (Table 1) per industry cluster reveal a larger share of early stage investments for high-tech driven industries. This relates to IT, Internet &

[16] Starting from 25 different industry classifications provided from CEPRES, we aggregate sub-cluster to form 9 larger and homogenous clusters. Following are our clusters with comprising sub-clusters in brackets: Consumer Discretionary (Hotel, Leisure, Retail, Textile), Financial Services, Healthcare, Industrial Production (Construction, Traditional Business), Information Technology (High-tech, Semiconductor, Software), Internet & Media, Materials (Natural Resources), Services (Environment, Logistics, Waste, Recycling), Telecommunication.

Media, Healthcare and Telecommunications. On the other hand low-tech industries are investment targets of later stage and turnaround investors. In our sample this is the case for the Consumer Discretionary, Industrial Production and Services industry clusters.

The time horizons of our investments range from 6 to 230 months. We calculate the investment horizon as time period between the initial and final cash flow entry in the CEPRES cash flow database. In case of write-offs we assume that the last entry marks the period of the actual write-off. Based on this, the average investment is held 50.3 months with a standard deviation of 33 months. Table 2 illustrates the distribution of investment horizons per industry cluster. As expected, there are only small differences regarding the share of short, medium and long term investments between industry clusters.[17]

The real investment amounts are calculated in terms of 2004 U.S. Dollars.[18] Investment amounts range from $1,393 to $ 356 mio. With an arithmetic mean of $ 117 Mio and a standard deviation of $ 245 mio the wide dispersion of investment amounts becomes apparent. Table 3 summarizes the investment amount for the different investment stages. Early stage investments have a larger share (28%) of small investments, while the other stages are pretty similar regarding their amount distribution.[19] This is in line with Venture Capital and Buyout Funds general investment strategy.

4 Excess Returns

4.1 Performance measures

Private Equity funds commonly measure and communicate their performance in terms of annualized IRR, which can be considered as industry standard. The key advantage of IRR as a performance measure is its intuitive interpretation: the IRR is the yield an investor receives on the capital currently invested. Mathematically the internal rate of return is the discount rate that would result in a net present value (NPV) of zero for a series of cash in- and outflows:

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[17] short term: up to 24 months; medium term: between 25 and 72 months; long term: at least 73 months

[18] The inflation adjustment is based on Consumer Price Index (CPI) data for all urban households and all items. Data is derived from the records of U.S. Department of Labor (www.bls.gov).

[19] small amount: up to $1 Mio; medium amount: between $1 Mio and $10 Mio; large amounts: exceed $10 Mio

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0 \qquad (2)$$

with:

CF_t = Net Cash Flow in period t

T = Investment Horizon

The calculation of the IRR can result in more than one solution or might not be solvable. We will show that this is a problem for PE investments. A disadvantage of the IRR concept is the complete ignorance of investment risk. As we focus on the portfolio company level, General Partners do have discretionary power regarding the timing of cash flows and value weighted performance measures like the IRR are applicable. A major issue is the reinvestment hypothesis. Intermediate cash flows are assumed to be reinvested in an alternative investment offering a return equal to the IRR. It is obvious that for private equity investments this is unrealistic, as investments vary widely and are not easily replicable. Furthermore, cash distributions usually occur almost uniformly over PE funds lifetime.[20] Investors are more likely to invest the distributions they receive in public markets instead of similar PE investments via comparable intermediaries.

IRR as a standalone measure indicates whether an investment generates a profit or a loss. To appraise performance relative to a benchmark investment, in our case public equity indices, excess return (ER) of investment i is calculated as follows:

$$ER_i = IRR_i - IRR_b \qquad (3)$$

with:

IRR_i = Internal Rate of Return of investment i

IRR_b = Internal Rate of Return of benchmark investment b

The PE investment outperforms the benchmark if the ER is positive. A key advantage of relative performance appraisal is the implicit reflection of market abnormalities, like the internet bubble. If extraordinary effects or shocks trigger higher (lower) market valuation we automatically reflect them via deducting higher (lower) public market returns.

[20] Kaserer/Diller show that the majority of the disbursements are paid out within the first half of an average PE funds lifetime. See Kaserer/Diller (2004), p. 33

To generate comparable IRR figures for the benchmark, amounts and cash flow timing needs to be matched with the private equity investment. Multiple financing rounds, which are common for PE investments, complicate this intention. We apply a simulation approach similar to the one introduced by *Schmidt*.[21] As we do have detailed cash flow information for the PE investments, a purchase (sale) of shares in the benchmark is simulated when a negative (positive) cash flow occurs for the underlying PE investment. Our simulation generates a cash flow pattern for benchmark investments that mimics the underlying investment in a specific PE portfolio company. With the simulated cash flow of the benchmark investment we can calculate the IRR_b and thus ER_b . Please refer to appendix A for a detailed discussion of the simulation approach.

An alternative measure to assess investment performance of PE relative to public benchmarks is the PME. Basically the PME determines how many dollars one would need to invest in the chosen benchmark to generate a return equal to that of a one dollar investment in PE on a present value basis. It assumes that intermediate cash flows are reinvested in the public benchmark and determines the value of the cash flows accordingly. There are several ways to calculate the PME.[22] We follow the definition of *Kaplan* / *Schoar* (2003), where PME is the ratio of the discounted value of all cash outflows (distributions) and the discounted value of all cash inflows. The discount rate is equal to the return of the benchmark investment. For investment *i* the PME is defined as:

$$PME_i = -\frac{\sum_{t=0}^{T}\left(CFO_{ti}\cdot\prod_{n=0}^{t}\left(1+r_{bn}\right)^{-1}\right)}{\sum_{t=0}^{T}\left(CFI_{ti}\cdot\prod_{n=0}^{t}\left(1+r_{bn}\right)^{-1}\right)} \qquad (4)$$

with:

CFO_{ti} = cash outflow (distribution) of investment *i* in period *t*

CFI_{ti} = cash inflow of investment *i* in period *t*

r_{bn} = total return of benchmark *b* in period *n*

[21] see Schmidt (2003), pp. 11

[22] Kaserer / Diller (2004) define PME as the "…ratio of the terminal wealth obtained under the (public market) reinvestment hypothesis when investing in a private equity fund compared to the terminal wealth obtained when investing the same amount of money in the given public market index."

If PME exceeds one, the private equity investment outperformed the public benchmark, while a PME of less than one reveals underperformance. As the PE cash flows in our sample and benchmark returns are both gross of fees, our relative performance appraisal is unbiased.

The PME concept resolves some of the deficits related to IRR. A key advantage of PME is the modified reinvestment hypothesis. It is at least feasible that investors re-invest cash distributions in public benchmark investments. In general it is possible to assign different reinvestment assumptions (benchmarks) to specified investment periods or cash flows. *Ljungqvist/Richardson* and *Gottschalg et al.* for example assume that cash inflows are debt financed and thus are discounted using risk-free rates, while cash outflows are invested in public equity.[23] We do not follow this approach as we are mainly interested to observe whether PE earns a return premium over PM we want the PME ratio to be unbiased by financing considerations. Discounting the Cash Inflows at the lower risk free rather than stock market return, results in lower PME figures and thus negatively biases results.

The PME algorithm generates unambiguous results. If all input data, PE cash flow stream and public benchmark returns, is available the calculation can be performed in all cases. It is assumed that investment risk of PE is equal to that of the public benchmark. We pick up the question of adding more differentiated risk adjustments later. To fully analyze the influence of the reinvestment hypothesis, excess returns and PMEs need to be comparable. As PMEs is a measure of total rather than periodic return, we transform PMEs into annual excess return figures using a model similar to the one introduced by *Kaserer* and *Diller*. The inputs are the investment horizon, total returns of the simulated PM investments and the PME ratios for all investments and benchmarks. For a detailed description please refer to Appendix B.

4.2 Summary Statistics

4.2.1 IRR based excess returns

The distribution of IRR figures is summarized in Table 4. In 68 cases the IRR could not be determined, due to the outlined iteration problem. Performance of the remaining

[23] See Ljungqvist/Richardsson (2002), p. 18-20. Gottschalg et al. (2004) p. 9-10. In both studies the resulting measure is labeled Profitablitiy Index rather than PME.

investments (2,685) ranges from -100% to 7,764% on an annualized basis. The distribution has an arithmetic mean of 46 % and a standard deviation of 335 %. 71% investments do have a positive IRR indicating a positive total return with median performance equal to 18.1 %.[24] Investment returns are highly skewed and not normally distributed. There are several investments with returns of thousands of percent and many which resulted in losses.

To measure performance on a relative basis we calculate excess returns over five broad indices: S&P 500, Nasdaq Composite, Russel 2000, Dow Jones Industrial Average and MSCI World. To achieve comparability we only include investments for which all excess returns and IRR can be determined unambiguous.[25] This reduces our sample to 2,380 investments. It is important to keep in mind that we measure excess returns on the project (single investment) level and then aggregate our findings to appraise PE as an asset class. Doing so, we contrast investments in a single PE project with alternative investments in a diversified public stock index. As excess return figures assume equal risk for both investments, standard deviations are likely to overstate PE risk and need to be analyzed cautiously.[26] We address the question of appropriate risk measurement later in this paper. Table 5 summarizes the results for the defined clusters.

As returns of PE investments are highly skewed we need to distinguish two perspectives in our analysis: average return generated by the asset class or cluster and the return generated by the average investment. In contrast to normally distributed returns, mean and median returns of PE investments tend to deviate widely from each other and thus the two perspectives have different results. To assess the characteristics of PE as an asset class and of our investment clusters, we look at mean returns. The second perspective turns our focus on the return of the "average PE investment" and thus on median returns. We are interested in the specific return characteristics of this investment, as this helps to describe the quality of available investment opportunities.

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[24] This is in line with Cochrane's results after he corrected for the survivorship bias of his data sample.

[25] The iteration problem inherent the IRR calculation algorithm materializes for both, the PE's and the simulated benchmark investment's return. Furthermore, index quotes of the benchmarks are not available for some of the early investments.

[26] We anyhow provide standard deviation figures for illustration purposes.

Looking at all investments, arithmetic mean excess returns range from 29.8% to 41.6%. This indicates substantial outperformance of PE as an asset class relative to public markets. Looking at median excess returns alters the results. Median excess returns lie between -3.4% and 8.0%. Now the question of whether PE outperforms PM depends on the chosen benchmark and can be negatively answered relative to the Nasdaq Composite Index. Very broad unbiased indices such as Russel 2000, MSCI World, S&P 500 and Dow Jones Industrial Average result in higher excess returns. The Nasdaq Composite on the other hand generated higher returns for the simulated benchmark investments causing lower excess returns. Taking into account the timely distribution of our sample, with most investments being pursued between 1986 and 2000, this is not surprising. But what is surprising is the underperformance of the median PE investment relative to a mimicked investment in the Nasdaq Composite Index. It is at least questionable whether the positive outliers still adequately compensate investors for additional risks of PE investments if we alter the reinvestment hypothesis and add a risk adjustment.

The industry clusters exhibit large deviations regarding excess returns. While mean excess returns are positive for all clusters and benchmarks, they fluctuate widely between 3.5% and 89.3%. Industrial production and IT are the negative and positive leaders, respectively. Median excess returns draw a different picture. Consumer Discretionary is the only cluster to maintain its relative outperformance for both, asset class and average investment perspective, in all cases, while IT is now the opposite extreme with negative medians for all benchmarks. For all other clusters it depends on the chosen benchmark.

For the stage perspective, only early stage investments exhibit a negative mean excess return (-1.3%) relative to the Nasdaq Composite Index. Mean excess returns are positive for all other stages and benchmarks, fluctuating between 3.7% and 79.9%. It is not the fact that early stage investments do have lower medians, but the extent of underperformance that strikes us. Median excess returns indicate substantial underperformance of the average early stage investment relative to broad public indices, ranging from -33.2% to -21.2%. The bet character of early stage PE investments materializes in a highly skewed return distribution with the majority of the investments significantly underperforming public markets and a few generating enormous returns.

Therefore, “Investment selection ability” seems to be of crucial importance for a VC fund manager’s success.

The results for the size (investment amount) perspective are in line with the previous findings. Mean excess returns are positive and substantially larger than median figures indicating outperformance of PE as an asset class. Small investments’ excess returns exhibit by far the largest dispersion. Considering the large share of early stage investments of this cluster this is not surprising. The large investments cluster is somehow extraordinary as its mean and median excess returns are both positive for all benchmarks and only deviate slightly. Taking into account that transactions of this size (exceeding $10,000,000) often involve public markets via Buyout structures, narrower return distributions with less biases are explainable.

Our results indicate very distinct risk return profiles for both investment stage and amount, which correspond with different investment universes of VC and BO funds respectively. The choice of the reference (market-) portfolio impacts relative performance appraisal. In the next step we drop the critical reinvestment assumption of the IRR concept and analyze its impact on performance appraisal.

4.2.2 Reinvestment Hypothesis

To analyze the impact of the reinvestment assumption, we now apply the PME concept and assume a reinvestment in public benchmark indices, gross of fees. We expect to see “smoothened” PE investment returns. The early distributions of very successful investments no longer generate skyrocket returns via reinvestment in equally well performing projects, but meaningful public market returns. On the other hand, bad investments generating at least some cash distributions prior to realization do perform better as cash is assumed to be reinvested in public markets rather than another equally weak performing PE investment. Table 6 summarizes PME results and corresponds with our prior analysis of excess returns, regarding perspectives, clusters and sample structure.

The first thing to notice is the increase of observations by 90 to 2,470 due to the unambiguous calculation algorithm of the PME concept. More than 4% of the PE investments that could not be appraised based on excess returns can now be analyzed. Generally, our prior results are confirmed: arithmetic mean PMEs exceed one, indicating

relative outperformance of PE as an asset class, but median PMEs reveal relative underperformance of the average investment in many instances. Our results are in line with the findings and conclusions of *Kaplan/Schoar* on the fund level.[27] With a median PME of 1.05 relative to S&P 500, the average PE investments slightly outperforms the index gross of fees and carried interest. But it needs to be noted that this is not the case for the Nasdaq Composite Index and that we still assume equal risks for PE and PM investments.

Next we analyze whether the reinvestment assumptions alters the direction of relative outperformance, switching from PE's outperformance to underperformance or vice versa. This is the case if excess returns are positive (negative) and PMEs is below (exceeds) one. As we can only include investments with unambiguous IRR and PME figures, our sample is the reduced sample used for excess return analysis (comprising 2,380 investments). It can be observed that the reinvestment assumption rarely (in less than 1.2% of all cases) alters the conclusion about the direction of relative outperformance. For only 28 out of 2,380 investments the performance appraisal of PE investments relative to the Nasdaq Composite Index changes with the reinvestment hypothesis. In most of these cases excess return indicates underperformance while PME exceeds one. Thus, the assumption of cash flows being reinvested in PE not only has little impact on the direction of relative outperformance, but also puts PE investments at a disadvantage in most cases. More disputable than the direction is the degree of relative performance. We compare excess returns derived from PME figures with excess returns based on IRR.[28] A difference is noted if the excess return based on PME deviates by more than 10%, 25% or 50% from the investment's IRR based excess return on an absolute basis.[29]

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[27] *Kaplan/Schoar* (2003) find a PME of 0.96 relative to the S&P 500 based on net cash flows. Adding average margins for fees and carried interest they conclude a slight outperformance of the average PE investment.

[28] See appendix B for detailed description of the underlying model used to derive excess returns from PME figures.

[29] Difference is noted if $ER_{IRR} - ER_{PME} > |x|$ for $|x| =$ 0.1, 0.25, 0.5. We also analyzed relative differences of return percentages. Results are in line with the absolute perspective and are available upon request.

Table 8 reveals a substantial impact of the reinvestment hypothesis for the appraisal of PE investments relative to our five broad benchmarks: for up to 29% of the PE investments excess returns deviate by more than 10% (absolute return points) depending on the reinvestment hypothesis. If the significance level is raised to 50% still more than 7% of our investments trigger a deviation notation relative to the Nasdaq Composite Index. IRR based excess returns tend to overstate the performance of PE investments, as in more than 64% of the deviations (for the 10% significance level relative to S&P 500), the assumed reinvestment in equivalent PE ventures significantly increases excess returns over the PME based return figures. This share increases with the significance level and reaches 90% for deviations exceeding 50 absolute percent points (again relative to S&P 500). Regarding performance reporting of PE funds this puts high IRR figures and resulting excess returns in a different light. If a fund would report performance with an assumed reinvestment in public rather than private equity markets (based on the PME measure), the figures are likely to substantially decline. The value weighted aggregation of returns on the fund level can either de- or increase this effect depending on the investment's size relative to the other portfolio investments. Our results are stable over the chosen benchmarks and defined investment clusters.[30]

It can be concluded that the reinvestment hypothesis has a substantial impact on the performance appraisal of PE investments. While the direction of relative performance to a public benchmark is rarely impacted, the degree of excess returns exhibits a strong positive bias if IRR rather than the PME concept is applied. As we believe PME implies a realistic reinvestment scenario, we focus on PME based excess returns as the appropriate performance measure for PE investment going forward in our analysis. To illustrate the impact on performance appraisal we first restate the excess return table (table 5) based on PMEs for all clusters.

4.2.3 PME based excess returns

Table 9 summarizes the PME based excess returns for our industry, stage and investment amount clusters. As expected, the excess return distribution exhibits less dispersion than its IRR based counterpart. We still observe the surprisingly high share of

[30] Results for the other benchmarks are available upon request.

negative excess returns, but the positive outliers reach distinctively lower percentages. Table 10 compares the excess return distributions relative to Nasdaq Composite Index. Maximum excess return drops from 7658% (IRR based) to 5398% (PME based). Looking at all investments, arithmetic mean excess returns are now dramatically lower for all benchmarks. Not the direction, but the magnitude of outperformance decreases heavily. As mean excess returns remain positive, well performing investments can still over-compensate investors for negative returns of poor ventures, but to a considerably lower extent. PE as an asset class still outperforms PM. Excess returns of the average investment decline substantially. These results fuel our suspicion regarding the attractiveness of PE investments on a risk adjusted basis. While median excess return is negative relative to Nasdaq Composite Index it stays positive relative to all other benchmarks.

The industry perspective shows the same pattern: mainly positive mean returns and altering outperformance based on medians. Striking are the alterations for the IT, Telecommunication, Healthcare and Internet & Media clusters compared to their IRR based excess return. Means drop dramatically, e.g. from 77% to 3.75% (IT relative to Nasdaq Comp.). An analysis of the stage perspective gives further insights into performance characteristics of VC and BO funds. Later stage and turnaround investments maintain their good performance and do substantially outperform the Nasdaq Composite Index for cluster- and average investment perspective. Notably is again the devastating performance of early stage investments. Although the medians do improve relative to IRR based excess returns, they still fluctuate between -25% and -15%. As mean excess returns decline dramatically to -23% relative to Nasdaq Composite, well performing investments consistently could not compensate investors for the losses generated by the majority of the investments. Our results are in line with the findings of *Schmidt* and *Gottschalg et al.*, but contrast results of *Kaplan/Schoar* and *Ljungqvist/Richardson,* who observe a better performance of VC relative to BO funds based on Venture Economics data.[31]

[31] See *Kaplan/Schoar* (2003), p. 10 and *Ljungqvist/Richardson* (2003), p. 38. Our share of first time funds for VC (33%) and BO (51%) investments favors VC performance rather BO and can not be the cause for the deviating results.

Results for the investment amount clusters alter as expected. Small investments relative to Nasdaq Composite Index exhibit a sharp decline of mean excess return (from 71% to 18%) while the median improves (from -12% to 10%). For medium and large size investments, excess returns decline but maintain, although on a lower level, their outperformance compared to all benchmarks.

Overall PME based excess returns deviate strongly for the respective industry, stage and size clusters. While most clusters as a group of investments outperform PM investments, average investments, to say the least, perform weaker. To better reflect industry characteristics we appraise performance relative to industry specific benchmarks in the next step.

4.2.4 Specific Benchmarks

The necessity for specific benchmarks is frequently mentioned in discussions about performance appraisal of PE investments.[32] The reasoning is the individualistic character of PE as an asset class. But without liquid secondary markets and continuous market quotes, PE specific indices are rare and the appropriateness of NAV based indices is questionable.[33] To anyhow examine whether performance measurement relative to specific benchmarks leads to different conclusions we appraise our PE investments relative to five public industry and two local indices.[34]

As some of the benchmarks have been constructed and initially quoted during the nineties, excess returns can not be determined for earlier investments of our sample. To ensure comparability we only include investments for which all benchmark quotes are available. Table 11 illustrates the reduced samples and results. For a more intuitive analysis we immediately look at PME based excess returns rather than PMEs.

We observe a mixed picture of the industry cluster's performance relative to specific benchmarks. All PE investment clusters exhibit positive mean excess returns over their industry specific benchmark, but deviating strongly. Healthcare and telecommunication investments are best performing with 57% and 21% mean excess return, respectively.

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[32] See for example "Beyond IRR" in Private Equity International, November 2002, pp. 28

[33] Due to the subjective character of reported Net Asset Values (NAV).

[34] Industry specific Benchmarks: Nasdaq Computer, S&P Consumer, S&P Industrials, Nasdaq Telecommunication, S&P Healthcare. Local Benchmarks: DAX, FTSE UK 100.

Appraisal of average investments show outperformance of PE for the consumer (6.5%) and industrial production (3.7%) cluster, while medians for the telecommunication, healthcare and IT underperform in descending order. With the exception of the IT investments, mean excess return of our industry clusters over specific benchmarks are larger than for the Nasdaq Composite Index. As the direction of outperformance changes for industrial production investments from out- to underperformance relative to the Nasdaq Composite, effects are noticeable.

Assuming that specific benchmarks better reflect our PE investments' characteristics, we overweigh the specific benchmark perspective and conclude a relative outperformance at this point. It should be noted that the choice of the benchmark has a significant impact on performance appraisal and that with the exception of IT PE investments overall outperformed industry specific benchmarks. Median figures reveal underperformance of average PE investments in high-tech related industries, while more saturated industries exhibit a vice versa pattern.

For the geographic perspective we appraise PE investments in German and UK based companies relative to local benchmarks.[35] Both, German and UK investments, outperform their local public markets regarding the overall clusters (mean excess returns) and the average investment (median excess returns). Switching to Nasdaq as U.S. based benchmark reduces mean and median excess returns for both clusters. The UK cluster suffers more, but both maintain their outperformance. The results show that considering local stock market conditions, which are important exit channels for PE investors, put PE investments returns in a different, in our case more positive, light.

As benchmark selection impacts relative performance appraisal, adequateness of chosen benchmarks becomes an issue. Explanatory power of the benchmark for observed returns could be used as a yardstick. The approximation of cross sectional correlation factors is not in the scope of this paper and will be addressed in follow on work.

[35] The local indices are the DAX (Germany) and the FTSE UK 100 (U.K.).Clusters of other regions, especially Asia, are not large enough to derive stable results. We nevertheless performed matching analysis for these cluster, which are available upon request.

5 Risk Adjustment

Investment decisions involve a trade-off between risks and opportunities. A conclusion, whether to invest in a project or not, can be based on either maximizing the investor's utility function or by comparing the investment's characteristics with other investment opportunities. As the quantification of a utility function is challenging and ambiguous, investment projects are commonly appraised relative to a reference point. Risk is in this context the danger of realizing a lower return than the reference investment. The chance of outperforming the reference investment describes the project's opportunities. Projects can thus be evaluated (ex ante) and appraised (ex post) using performance measures that pick-up this trade-off. [36] In this context we will discuss the adequateness of variance, downside deviation and shortfall as risk measures for PE investments. In a second step we incorporate our results to calculate risk adjusted performance measures, most prominently the Sharpe-Ratio. Doing so, we can appraise whether the observed return premium of PE over PM is adequate to compensate investors for additional risks.

5.1 Risk Measurement

Standard Deviation of returns is the most prominent measure for investment risk. It measures the dispersion of investment returns relative to the mean return. Dispersion in this context is both, a positive or negative deviation from the distribution mean. Besides probability, the magnitude of a deviation is reflected. The standard deviation has the same dimension as the underlying distribution and is calculated as follows:

$$\sigma = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} \left(r_i - \bar{r}\right)^2} \qquad (5)$$

with:

n = number of observations

r_i = return of investment i

$\bar{r}$ = mean return

[36] See Albrecht, et al. (1998)

For our prior analysis of excess returns, the standard deviation thus measures the deviation of excess returns relative to the mean excess return. This triggers several issues regarding the adequateness of the concept to measure investment risk for PE investments. Standard deviation assumes a normal distribution of underlying returns. As illustrated above, this is not the case for our sample. Assuming that we appraise PE relative to a benchmark rather than to maximize a utility function, positive deviations from the distribution mean should be considered as an opportunity rather than a risk. The standard deviation does not differentiate between positive and negative deviations and only measures the level of dispersion. In a PE context, where positive outliers are generating returns of several hundred or thousand percent return compensate investors for the high probability of write-offs, standard deviations are misleading. For our analysis standard deviation thus most likely overstates investment risk and negatively impacts the risk adjusted appraisal of well performing investment clusters.

Downside deviation is closely related to the standard deviation concept but resolves some of its deficits related to measuring risk of PE investments. It measures the dispersion of a distribution, but solely of observations below a pre-defined reference point. Thus, downside deviation distinguishes between investment risk, of realizing a return below the reference point, and opportunities, realizing returns exceeding the reference point. As it measures dispersion it takes into account both, occurrence and magnitude of negative deviations. Furthermore, it is flexible with regard to the reference point.[37] If the reference return is equal to the distribution's mean, the downside deviation is called Semi-standard deviation. Downside deviation (DD) is defined as follows:

$$DD = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(R - r_i\right)^2 \middle| r_i < R} \qquad (6)$$

with:

R = reference return

Shortfall is a general applicable risk measure and does not require a specific probability distribution. Like downside deviation it splits the distribution into an excess- and shortfall area relative to a reference point. Risk is defined as probability of realizing a

[37] see Farinelli/Tibiletti (2002) for a detailed discussion of one-sided-volatility measures.

shortfall investment return. Unlike the other measures, shortfall solely accounts for the probability and not for the magnitude of realizing a loss:

$$shortfall = P(r < R) \tag{7}$$

We already calculated standard deviations in our previous analysis. To empirically illustrate differences between these risk measures we calculate downside deviation and shortfall for our sample investments. Table 12 exhibits the results for all clusters. We base our calculations on IRR figures. Downside deviation is calculated relative to mean IRR of our PE investments[38] and relative to mean IRR of simulated investments in the Nasdaq Composite Index. Shortfall probability is also referenced to IRR figures of corresponding investments in the Nasdaq Composite Index. Comparing standard deviations with shortfall probabilities and downside deviations reveals substantial overstatement of some clusters' risk by standard deviation, namely healthcare and later stage investments. High shortfall probability in combination with low downside deviation indicates a relatively low magnitude of losses, as in case for the service cluster.

5.2 Risk adjusted Performance Measurement

5.2.1 Sharpe Ratio

One of the most prominent risk adjusted measures of investment performance is the Sharpe Ratio.[39] It relates an investment's excess return over the risk free rate to the standard deviation of the investment's return. We apply the following formula:

$$SR_i = \frac{r_i - r_f}{\sigma_{ci}} \tag{8}$$

with:

$r_i =$ total annualized IRR of investment *i* over the investment

$r_f =$ total annualized IRR of a risk free investment

$\sigma_{ci} =$ standard deviation of cluster *c* that investment *i* belongs to

The appropriateness of the Share Ratio to appraise PE projects relative to PM investments is questionable. As it applies standard deviation to measure investment risk, all of the related and previously discussed deficits do materialize.

[38] This special case of the downside deviation measure is referred to as semi-standard deviation.
[39] see Sharpe (1994)

We again use our introduced simulation approach to solve the problem of missing time series data for the PE investments. To generate corresponding risk free returns, we now simulate investments in a risk free asset that mimics the PE investments with regard to timing and amount of invested cash flows. Instead of shares on a stock index as in our previous analysis, we now purchase and sell shares in a bond index derived from monthly market returns of 2 years US Government bonds. For the standard deviation of the PE investments' return we apply our previously calculated cross sectional values for all clusters based on IRR (refer to table 12).

In the first step we calculate the Sharpe Ratio for our entire sample of PE and simulated PM investments. The PM investments are in this case the returns (IRR) of simulated investments in the Nasdaq Composite Index. One needs to keep in mind that we contrast investments in a single PE project with alternative investments in a diversified public stock index. Thus, standard deviations of PM investments are that of a diversified index rather than single investments in a public stock and put PM investments at an advantage. Table 13 summarizes our findings. As the PE investments exhibit substantially higher standard deviations, we expect to observe on average lower Sharpe Ratios in comparison to PM investments. Indeed, mean and median ratios of PM investments significantly exceed their PE counterparts for all analyzed clusters. Based on the Sharpe Ratio, PE investments thus on average underperform relative to PM investments. This conclusion is the same for all of our broad and specific benchmark indices and both perspectives, mean and median ratios.[40] Our results are in line with the observations of *Kaserer*/*Diller* who find distinctively lower Sharpe ratios for European PE funds compared to an investment in the MSCI Europe.[41]

It is important to adequately interpret these results. As previously discussed, standard deviation tends to overstate investment risk of PE for our relative analysis and we therefore alter the risk measure in the next step.

[40] Results for the S&P500, Russel 2000, Dow Jones Industrial Average and MSCI World are available upon request.

[41] See Kaserer/Diller (2003), pp. 56

5.2.2 Modified Sharpe Ratio

Excess return as defined by the Sharpe ratio is generally applicable for all risky asset classes. Thus we only need to modify the risk measure used in the denominator to better reflect PE's return characteristics. Doing so, we substitute return's standard deviation by downside deviation.[42] To enable comparability between public and private equity investments we chose our simulated risk-free returns as reference point to calculate downside deviations. Our modified Sharpe Ratio defines risk as the distribution's dispersion given the risky investment's return (PE and PM) is below that of the risk-free investment. It is calculated as follows:

$$mSR_i = \frac{r_i - r_f}{DD_{ci}} \tag{9}$$

with:

r_i = total annualized IRR of investment i over the investment

r_f = total annualized IRR of a risk free investment

DD_{ci} = downside deviation of cluster *c* that investment *i* belongs to relative to r_f

The modified Sharpe Ratio accounts for both, probability and magnitude of the shortfall relative to the return of the 2 year US Government bonds. As we are interested in cross sectional differences we assign the cluster's downside deviation of excess returns to all investments belonging to the cluster. For the PM returns we use the IRRs of the simulated investments in the Nasdaq Composite Index and corresponding excess returns over the Government bonds.

In contrast to the results for the Sharpe Ratios we expect to see a more differentiated picture of PE performance. While excess returns remain unchanged, risk measures should be significantly lower for the PE investments as positive outliers are now flagged as opportunities rather than tracking error risk. As we are still comparing single PE projects with a diversified index, we are more interested in changes compared to the Sharpe Ratio rather than the absolute amount of the modified Ratio to assess the impact of the modified risk definition. Assuming normally distributed returns, risks of PM investments are cut in half while the risk reduction effect for the highly skewed PE investments

[42] For an overview of illiquidity and non-normality adjustments to the Sharpe Ratio see e.g. Berényi (2001).

should be much larger. But as PE investments are far more likely to underperform or even completely fall out, they still need to generate significantly larger excess returns compared to PM investments to achieve a larger modified Sharpe Ratio.

Table 14 summarizes the results for the modified Sharpe Ratio. Our adjustment of the Sharpe Ratio triggered a substantial increase of mean values, 680% and 119% for PE and PM investments respectively. The substantially larger increase for PE supports our hypothesis that standard deviation overstates PE's investment risk in our relative analysis. Nevertheless, mean and median ratios of PE investments are below that of investments in Nasdaq Composite Index, indicating underperformance. For the outlined risk definition, PE investments overall thus did not generate sufficient excess returns to compensate investors for additional risk they bare compared to putting funds in the Nasdaq Composite Index.

Looking at our industries clusters, approximately half of them beat the Nasdaq Composite on a mean basis while the other half underperforms. High-tech related investments in IT, Telecommunication and Internet&Media companies performed well on a risk adjusted basis. Even more interesting is the stage perspective. While the mean ratio of later stage PE investments matches its PM counterpart, early stage and expansion PE investments are dominated by PM investments (-0.64). For these clusters, typically defining the investment universe of VC funds, investment selection ability is of crucial importance due to the inadequate returns of the majority of the investment opportunities. On a risk adjusted basis only the funds performing substantial above average are attractive for return enhancement purposes. If we assume that these are funds only open to invited investors, the investment rationale for left-out investors is diversification rather than return enhancement.

Analyzing the above results, one needs to take into account several issues related to our modified Sharpe Ratio. Most important, investments are equally rather than value weighted. Assuming that bad performing PE investments, especially in the VC industry, are unlikely to find follow-on financing, these investments are over-weighted in the analysis and trigger a negative bias. To test the impact of the magnitude of shortfall, we substitute downside deviation by shortfall probability and re-run our analysis. Table 15 now indicates outperformance of our PE investments over the Nasdaq Composite Index

on a risk adjusted basis. Thus, it is important to appraise PE relative to PM investments on a value weighted basis. Another aspect is that we measured returns in terms of IRR so far, with the discussed unrealistic reinvestments hypothesis.

5.2.3 Omega

The Sharpe Ratio and the modified Sharpe Ratio only allow for an indirect comparison of PE and PM investments relative to a third benchmark, in our analysis the return of risk free investments in US Government Bonds. To perform a direct comparison we use the universal performance measure Omega, first introduced by *Keating/Shadwick*.[43] It is applicable for non-normal return distributions and has been used to appraise performance (ex post) and to construct portfolios (ex ante) of other alternative assets classes, especially Hedge Funds.[44]

Omega is flexible regarding the point of reference and basically relates total excess chances (*EC*) to total shortfall risk (*SR*). Total chance and risk are measured in absolute terms, in our case USD. *EC* and *SR* are the areas above and below reference point *z* of the return distribution as illustrated in exhibit 5:

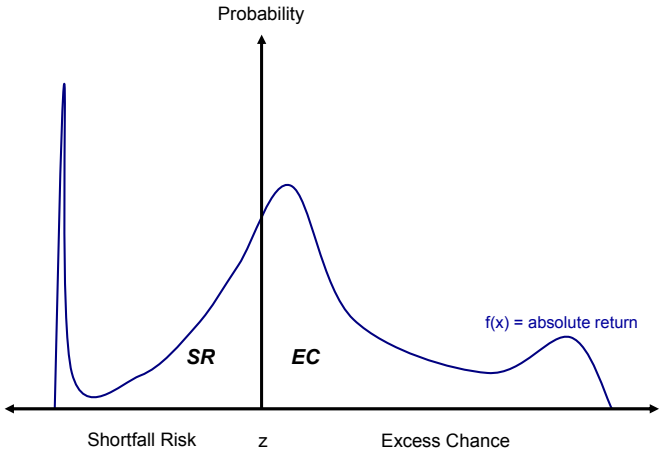

Exhibit 2: Shortfall and excess areas of a PE investment (illustrative) relative to reference return z

Omega is calculated as follows:

[43] See Keating/Shadwick (2002)
[44] see Favre-Bulle/Pache (2003), Roland/Xiang (2004)

$$\Omega(z) = \frac{\int_{z}^{\infty} [1 - F(x)]dx}{\int_{-\infty}^{z} F(x)dx} = \frac{EC(z)}{SR(z)} \tag{10}$$

with:

$z =$ reference return
$F(x) =$ probability distribution of total absolute returns for chosen investments

The Omega accounts for probability and magnitude of both, excess chance and shortfall probability. It can be calculated for an arbitrary group of investments, but not for individual investments. Thus, we can appraise clusters of PE investments but not the average investment (median). In contrast to the previously discussed measures Omega value weights rather than equally weights all investments. While it is a nice framework to conclude our relative ex post performance appraisal, Omega is more effective in ex ante analysis. When for example constructing portfolios, it accounts for the entire historic distributions of investment opportunities. Compared to traditional portfolio theory, where distributions are described by distribution mean and standard deviation only, this considers all distribution momentums. In case of PE this can be of interest since returns are not normally distributed.

As reference point we use the return of our simulated investments in the Nasdaq Composite in a first step before switching to the S&P 500. The excess chance comprises of all PE investments with PME exceeding one, whereas the shortfall risk consist of all PE investments with PME below 1. We calculate total annualized return of investment *i* as PME based annualized excess return multiplied with the real investment amount (in terms of 2004 U.S. Dollars). Excess Chance and Shortfall Risk are thus denominated in 2004 U.S. Dollars, while we assume a reinvestment of intermediate cash flows in the Nasdaq Composite. Doing so, Omega resolves all of the deficits related to the Modified Sharpe Ratio and allows an unbiased appraisal of our PE sample relative to the chosen benchmark.

The reference investments in the public stock market index neither bare an excess chance nor a shortfall risk. PE investments do outperform their PM counterparts if chances exceed risks, resulting in an Omega above one. If Omega drops below one, PM

investments outperform PE on a risk adjusted basis. As we move the reference point up from risk free returns, used in the Sharpe Ration analysis, to Nasdaq Composite returns, shortfall risk will increase while excess chances are likely to decrease. If our previous assumption, that bad performing PE investments do not find follow-on financing and thus do have overall lower invested values, proves to be right, value weighting will improve relative performance appraisal of PE as an asset class.

Table 16 summarizes the Omegas for our PE sample. For our entire sample we observe an Omega of 0.99. This result indicates that investors in PE face matching total risk to realize a shortfall or an excess return relative to the Nasdaq Composite Index. One needs to keep in mind that we are not considering a utility maximization but focus on comparison between two investment opportunities in terms of absolute return. In this context, the return premium of PE over PM adequately compensates investors for additional risk on a gross of fee basis. As the margin of PM's Omega over PE's Omega is minimal we expect to see mixed results for our clusters. For the industry clusters, solely investments in Internet & Media and Telecommunication companies exhibit an omega exceeding one (1.89 and 1.24). PE investments in all other industries are below 1, although deviating widely.

Analyzing stage clusters reveals a familiar picture. While later stage and expansion investments exhibit Omegas considerably exceeding one (1.40 and 1.27), early stage investment are the overall worst performing cluster with an Omega of 0.27. Although early stage investors usually provide small funds to their portfolio companies, total excess chances are tiny relative to total shortfall risk. This emphasizes the importance of investment selection ability of VC fund managers, as the universe of existing opportunities consists to a large extent of underperforming ideas and companies. Of the investment amount clusters, the cluster of small and medium investments benefit most from the value weighting with Omegas exceeding one (1.20 and 1.04). For large investments we observe Omegas marginally below one (0.98).

If we use the S&P 500 index as reference point, we observe consistently higher Omegas. The Omega value for our entire sample rises to 1.27, underpinning the adequacy of return premiums for PE investment risks. Early stage investments maintain their poor performance (0.32) while later stage investments outperform the S&P 500 significantly

(1.70). We conclude that PE investments generate an adequate relative return premium over PM investments gross of fess, as Omegas match one relative to Nasdaq Composite Index and exceed one for all other benchmarks. This supports the findings of *Kaplan/Schoar* and *Ljungvist/Richardsson,* who on the fund level observe gross of fees outperformance of value weighted PE investments and underperformance if investments are value weighted. While their analysis is based on approximated gross of fees cash flows, we use observed "pure" cash flows.

Last we recalculate Omegas relative to Nasdaq Composite Index using IRR based annualized excess returns to again analyze the impact of the reinvestment hypothesis (Table 17). In line with our prior findings we observe higher excess chances while shortfall risks decrease, resulting in overall substantially higher Omega values. Thus, assuming a reinvestment of disbursements in identical PE projects leads to the conclusion that PE outperforms PM. It should be noted that early stage investments are the exception and still underperform relative to the Nasdaq Composite Index.

6 Summary and Implications

In this paper we examine the relationship between returns of PE and PM investments. The dataset we use is derived from the records of CEPRES and covers 5,991 representative investments. Our analysis is based on monthly realized cash flows between PE funds and portfolio companies. In contrast to other studies based on Venture Economics data, cash flows are gross of fees and any other externalities. This is important as we are interested in the quality and structure of the available investment opportunities in the PE industry prior to any external effects.

To generate comparable PM returns for our sample, we simulate investments in public indices mimicking the cash flow patterns of the PE investments. Starting from these comparable cash flow pairs, we calculate IRRs and excess returns of PE investments over various public benchmark indices. We find that PE as an asset class substantially outperforms PM investments, but observe that this does not hold true for the average investments. The most important structural differences relate to early and later stage investments. While later stage investments generate large positive excess returns, early stage investments underperform relative to the Nasdaq Composite index. As cash

distributions occur almost uniformly over a funds lifetime, it is more realistic to assume that investors reinvest them into public benchmark indices. Doing so, we calculate PMEs rather than IRRs. We show that the different reinvestment hypothesis significantly reduces the level of excess returns, although it rarely impacts the direction of relative performance. PE investments only maintain a marginal outperformance over broad public indices, with the exception of the Nasdaq Composite Index that performs slightly better. Relative to specific and local benchmarks, PE investments perform better.

We then compare alternative risk measures for PE investments and find that downside deviation and shortfall better describe the relative risk of PE than standard deviation, as positive deviations from the mean are opportunities rather than tracking error risk and returns are not normally distributed. It is not surprising to observe substantially higher Sharpe Ratios for PM relative PE investments, as standard deviations in this analysis overstates PE's risk. As we show that traditional concepts of risk adjusted performance appraisal for PE are inappropriate, we modify the Sharpe Ratio measuring risk in terms of downside deviation and shortfall. We find that PE overall underperforms relative to PM if downside deviation is applied, but that there are differing results regarding industry, stage and size of the PE investments. Our prior findings are underpinned as early stage investments substantially underperform against PM, while later stage investments match PM performance. Last we apply Omega as alternative risk adjusted performance measure to appraise PE investments in absolute terms and on a value weighted basis. Omega relates total excess chance to total shortfall risk and reveals overall a nearly matching performance between PE and Nasdaq Composite Index investments. Later stage investments outperform PM, while early stage investments form the worst performing cluster. If we use investments in the S&P 500 as reference point, we observe consistently larger Omega values. Overall we conclude that PE investments earn adequate excess returns over public stock markets on a value-weighted and gross of fee basis. Our findings question the existence of an illiquidity puzzle on the fund level but can not fully exclude it for investors in PE funds, as fees and other externalities reduce excess returns.

Based on our findings the performance of PE investments should be appraised applying PME rather than IRR. The reinvestment hypothesis of the IRR concepts tends to

overstate the highly skewed PE returns, especially of extraordinary well performing investments. Risk adjustments need to be added cautiously, as traditional concepts, especially if they use standard deviation as risk measure, tend to overstate investment risk of PE investments in relative analysis.

Our findings on the relation between PE and PM returns are valuable with regards to investment strategies in the PE sector. As we only look at investments pursued by "knowledgeable" PE fund managers we can conclude that the universe of PE investment opportunities is of limited quality and that "stock selection" is of crucial importance. Overall PE investments selected by specialized investment professionals can hardly outperform PM investments, while average investments consistently underperform. *Kaplan/Schoar's* results on the persistence of PE fund returns underpin the importance of experience and investment manager's skills.[45] Assuming that intermediaries already sorted out the worst PE investment opportunities, our findings justify their existence to prevent investors from substantial underperformance and support the assumption *Jones/Rhodes-Kropf* base their work on.[46]

Early stage investments opportunities overall are of poor quality relative to public market investments. Finding the "rare jewels" requires excellent investment selection skills and access to top opportunities. As both of these characteristics are very scarce, VC funds combining them with a proven exceptional track record are more likely to be accessible for invited investors only. Left out investors can either try to get access via fund-to-fund intermediaries or need to be aware of investing in leftover funds with assumingly lower probability of outperforming public markets. Although the average BO investment still underperforms, overall these investments substantially outperform PM on a risk adjusted basis. Investment selection is still very important, but as investment targets already have a decent history and track record, shortfall risk is lower in comparison to Venture Capital investments. We find that investors hunting for the bigger maximum returns of VC investments missed out solid excess returns generated by BO investments.

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[45] Kaplan/Schoar (2003) show that fund returns improve with partnership experience. Gottschalg et al. (2004) come to the same conclusion.

[46] Jones/Rhodes-Kropf (2003) assume a negative NPV for average VC investment opportunities.

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Tables

Table 1

Industry		Investment Stage					
		early stage	expansion	later stage	turnaround	no observation	total
IT	*frequency*	151	34	64	7	173	429
	row [%]	35,20	7,93	14,92	1,63	40,33	100,00
	column [%]	29,61	11,49	12,50	9,72	12,69	15,58
consumer discretionary	*frequency*	4	30	102	20	156	312
	row [%]	1,28	9,62	32,69	6,41	50,00	100,00
	column [%]	0,78	10,14	19,92	27,78	11,45	11,33
industrial production	*frequency*	0	39	112	21	195	367
	row [%]	0,00	10,63	30,52	5,72	53,13	100,00
	column [%]	0,00	13,18	21,88	29,17	14,31	13,33
financial services	*frequency*	2	6	9	1	5	23
	row [%]	8,70	26,09	39,13	4,35	21,74	100,00
	column [%]	0,39	2,03	1,76	1,39	0,37	0,84
materials	*frequency*	16	13	18	6	13	66
	row [%]	24,24	19,70	27,27	9,09	19,70	100,00
	column [%]	3,14	4,39	3,52	8,33	0,95	2,40
telecommunication	*frequency*	80	14	23	1	66	184
	row [%]	43,48	7,61	12,50	0,54	35,87	100,00
	column [%]	15,69	4,73	4,49	1,39	4,84	6,68
services	*frequency*	15	20	52	3	98	188
	row [%]	7,98	10,64	27,66	1,60	52,13	100,00
	column [%]	2,94	6,76	10,16	4,17	7,19	6,83
healthcare	*frequency*	109	49	32	4	140	334
	row [%]	32,63	14,67	9,58	1,20	41,92	100,00
	column [%]	21,37	16,55	6,25	5,56	10,27	12,13
internet & media	*frequency*	85	60	46	3	66	260
	row [%]	32,69	23,08	17,69	1,15	25,38	100,00
	column [%]	16,67	20,27	8,98	4,17	4,84	9,44
other industries	*frequency*	48	31	54	6	451	590
	row [%]	8,14	5,25	9,15	1,02	76,44	100,00
	column [%]	9,41	10,47	10,55	8,33	33,09	21,43
total	*frequency*	510	296	512	72	1.363	2753
	row [%]	18,53	10,75	18,60	2,62	49,51	100,00
	column [%]	100,00	100,00	100,00	100,00	100,00	100,00

Table 2

Industry		Investment Horizon			
		short	medium	long	total
IT	*frequency*	88	259	82	429
	row [%]	20,51	60,37	19,11	100,00
consumer discretionary	*frequency*	61	189	62	312
	row [%]	19,55	60,58	19,87	100,00
industrial production	*frequency*	65	212	90	367
	row [%]	17,71	57,77	24,52	100,00
financial services	*frequency*	6	8	9	23
	row [%]	26,09	34,78	39,13	100,00
materials	*frequency*	13	32	21	66
	row [%]	19,70	48,48	31,82	100,00
telecommunication	*frequency*	43	108	33	184
	row [%]	23,37	58,70	17,93	100,00
services	*frequency*	44	108	36	188
	row [%]	23,40	57,45	19,15	100,00
healthcare	*frequency*	50	187	97	334
	row [%]	14,97	55,99	29,04	100,00
internet & media	*frequency*	55	160	45	260
	row [%]	21,15	61,54	17,31	100,00
other industry	*frequency*	153	320	117	590
	row [%]	25,93	54,24	19,83	100,00
Total	*frequency*	578	1.583	592	2.753
	row [%]	21,00	57,50	21,50	100,00

Investment horizons are considered to be short if they span over 24 or less months. Medium investment horizons lie between 25 and 72 months. Long investments horizons exceed 72 month.

Table 3

Stage		Investment Amount			
		small	medium	large	Total
early stage	*frequency*	145	306	59	510
	row [%]	28,43	60,00	11,57	100,00
expansion	*frequency*	18	146	132	296
	row [%]	6,08	49,32	44,59	100,00
later stage	*frequency*	28	281	203	512
	row [%]	5,47	54,88	39,65	100,00
turnaround	*frequency*	4	38	30	72
	row [%]	5,56	52,78	41,67	100,00
no observation	*frequency*	291	712	360	1363
	row [%]	21,35	52,24	26,41	100,00
Total	*frequency*	486	1.483	784	2.753
	row [%]	17,65	53,87	28,48	100,00

Investment amounts are in terms of 2004 US Dollar. Investment amounts are considered small if they are below $1,000,000. Medium investment amounts are between $1,000,000 and $10,000,000. Large investment amounts exceed $10,000,000. The arithmetic mean is $ 117 Mio. The large standard deviation of $ 245 Mio indicates the wide dispersion of investment amounts.

Table 4

Internal Rate of Return (IRR) of PE investments

	Percentiles	Smallest		
1%	-100	-100		
5%	-100	-100		
10%	-100	-100	Obs	2685
25%	-27.76	-100	Sum of Wgt.	2685
50%	18.13		Mean	45.7554
		Largest	Std. Dev.	334.894
75%	45.74	6769		
90%	106.88	6798.24	Variance	112153.8
95%	211.97	6879.61	Skewness	15.5048
99%	911.69	7763.87	Kurtosis	304.772

In this table the IRR figures of our sample used for following analysis are described. Besides mean (45.75%), median (18.13%) and standard deviation (334.89%) percentile characteristics are provided in the left columns. Skewness and kurtosis values reveal a non-normal distribution of IRR figures.

Table 5

Excess Return *IRR based*		**Benchmark Index**														
		S&P 500			Nasdaq Composite			Russel2000			Dow Jones Ind. Avg.			MSCI World		
	obs	mean *[%]*	std dev *[%]*	median *[%]*	mean *[%]*	std dev *[%]*	median *[%]*	mean *[%]*	std dev *[%]*	median *[%]*	mean *[%]*	std dev *[%]*	median *[%]*	mean *[%]*	std dev *[%]*	median *[%]*
All investments	2380	36.63	353.81	2.99	29.79	350.96	-3.41	39.74	353.60	6.09	36.73	354.34	3.32	41.63	353.43	8.00
Industry																
IT	361	84.22	597.28	-6.54	77.21	591.26	-10.13	88.23	596.78	-2.03	84.64	598.10	-6.24	89.32	596.52	-1.05
consumer discretionary	273	14.00	97.71	8.00	7.88	98.25	3.03	18.02	97.86	11.75	14.03	98.03	8.25	19.47	97.93	13.68
industrial production	323	11.51	103.55	7.13	3.49	104.05	-0.39	15.37	104.06	9.19	11.71	103.45	7.52	16.72	103.19	13.56
telecommunication	151	66.53	269.98	2.45	57.52	260.77	-2.84	69.01	270.43	11.35	66.53	271.72	3.95	71.06	269.26	6.68
services	172	11.39	201.38	-0.20	4.75	204.78	-9.40	14.78	202.57	3.38	11.48	201.07	-0.94	16.81	202.34	2.23
healthcare	288	38.76	453.87	-2.54	31.72	449.84	-10.00	41.76	452.02	0.77	38.64	454.58	-3.21	43.65	453.14	1.69
internet & media	232	63.26	509.19	2.57	55.81	506.81	-3.06	66.06	510.46	6.44	63.45	509.50	2.77	68.16	508.79	7.23
other industry	580	18.74	205.99	3.23	13.27	205.17	-3.38	20.73	205.22	4.98	18.70	206.39	2.51	23.43	205.69	7.83
Stage																
early stage	426	4.10	215.75	-27.06	-1.28	214.69	-33.21	7.43	217.44	-23.45	3.68	216.63	-26.81	9.13	215.85	-21.12
expansion	263	21.01	116.12	7.55	14.40	114.61	0.52	25.13	115.46	12.71	21.22	116.73	7.64	26.77	115.61	13.72
later stage	448	74.88	424.69	14.05	66.15	420.94	7.05	78.62	423.99	17.92	75.39	425.37	14.13	79.89	424.22	18.59
turnaround	68	67.11	437.34	14.83	60.79	437.88	8.72	70.93	437.88	14.45	67.01	437.16	12.83	71.50	438.05	18.94
no observation	1175	35.57	391.38	3.07	28.83	388.08	-3.28	38.09	390.93	5.24	35.70	391.83	2.88	40.41	390.84	7.30
Investment Amount																
small	373	76.70	591.98	-8.71	71.17	588.63	-12.22	80.36	592.10	-4.64	72.85	592.50	-9.12	81.81	591.39	-3.49
medium	1300	39.32	362.06	1.15	32.08	358.49	-5.31	42.07	361.72	4.47	39.17	362.74	0.64	44.07	361.66	5.84
large	707	13.95	102.70	9.17	6.08	101.10	2.56	17.26	101.94	11.92	14.42	103.17	10.03	19.25	102.55	14.84

In this table we summarize the appraisal of PE investments relative to five different broad public equity benchmarks. Excess returns of PE investments over the benchmarks are based on IRR differences between the PE and simulated investments in the benchmark. We distinguish industries, stages and size to analyze structural differences. Market quotes are obtained from the records of Bloomberg and Datastream.

Table 6

PME		**Benchmark Index**														
		S&P 500			Nasdaq Composite			Russel2000			Dow Jones Ind. Avg.			MSCI World		
	obs	mean	std dev	median	mean	std dev	median	mean	std dev	median	mean	std dev	median	mean	std dev	median
All investments	2470	3.01	22.29	1.05	2.75	22.44	0.93	3.40	25.85	1.13	3.05	22.66	1.05	3.35	24.63	1.17
Industry																
IT	382	2.96	8.18	0.78	2.58	7.50	0.66	3.32	9.49	0.85	3.05	8.63	0.76	3.33	9.25	0.94
consumer discretionary	290	1.98	7.48	1.17	1.82	6.71	1.06	2.15	7.51	1.29	1.96	7.34	1.17	2.21	7.61	1.31
industrial production	333	2.03	7.74	1.13	1.75	6.84	0.99	2.20	7.74	1.22	2.00	7.17	1.14	2.25	7.25	1.29
telecommunication	163	3.19	10.10	1.00	2.58	8.22	0.87	3.82	13.67	1.04	3.31	10.68	0.97	3.48	10.78	1.05
services	180	1.28	1.37	0.99	1.24	1.57	0.84	1.40	1.47	1.06	1.26	1.32	0.98	1.45	1.56	1.04
healthcare	293	7.00	46.05	0.88	7.00	48.43	0.73	8.02	54.32	1.05	7.18	48.28	0.86	7.48	47.90	1.10
internet & media	239	2.82	10.75	1.07	2.19	7.37	0.94	3.34	14.17	1.12	2.90	11.48	1.08	3.15	12.44	1.19
other industry	590	2.68	28.85	1.06	2.49	28.34	0.95	2.94	32.57	1.11	2.65	28.10	1.05	3.10	34.28	1.16
Stage																
early stage	442	2.52	9.87	0.40	2.08	7.69	0.35	3.11	13.51	0.45	2.63	10.64	0.40	2.90	11.43	0.45
expansion	276	1.87	3.75	1.11	1.63	2.78	0.99	2.12	4.75	1.26	1.87	3.92	1.13	2.15	4.08	1.29
later stage	469	2.42	7.00	1.32	2.16	6.05	1.14	2.61	7.18	1.46	2.42	6.92	1.32	2.64	7.10	1.49
turnaround	71	11.31	83.34	1.31	11.02	81.93	1.13	12.68	94.09	1.32	11.03	81.16	1.28	13.31	99.05	1.39
no observation	1212	3.20	23.43	1.04	2.99	24.43	0.93	3.55	27.45	1.12	3.25	24.47	1.04	3.47	24.35	1.17
Investment Amount																
small	385	8.95	55.89	0.85	8.70	56.70	0.74	10.28	64.73	0.91	9.10	56.77	0.83	9.79	61.83	0.97
medium	1343	2.24	6.40	1.00	1.90	5.32	0.84	2.50	7.67	1.07	2.27	6.66	0.98	2.50	6.94	1.12
large	742	1.56	1.67	1.20	1.42	1.63	1.05	1.71	1.85	1.28	1.55	1.63	1.20	1.79	1.91	1.34

Here we appraise PE investments relative to five different broad public equity benchmarks based on the Public Market Equivalent (PME) concept. In contrast to IRR, the PME assumes a reinvestment of intermediate cash flows in the public benchmark. PMEs exceeding (below) one indicate a relative outperformance (underperformance) of PE.

Table 7

	Benchmark Index				
	S&P 500	Nasdaq Composite	Russel2000	Dow Jones Ind. Avg.	MSCI World
observations	2,380	2,380	2,380	2,380	2,380
different outperformance conclusion [%]	0.42	1.18	0.21	0.42	0.21
excess return > 0 & PME < 1	3	2	2	3	1
excess return < 0 & PME > 1	7	26	3	7	4

In this table we investigate the impact of the reinvestment hypothesis on the conclusion whether PE outperforms the public benchmark or not. Excess return and PME are used as performance measures respectively. If excess returns are positive (negative) and PMEs is below (exceeds) one, a different outperformance conclusion is noted. The table solely tackles the direction of outperformance, not the degree. It can be observed that the reinvestment assumption rarely (in less than 1.1% of all cases) alters the conclusion about the direction of outperformance.

Table 8

Deviation of excess return		Benchmark Index														
		S&P 500			Nasdaq Composite			Russel2000			Dow Jones Ind. Avg.			MSCI World		
	obs	dev	share [%]	*IRR > PME*	dev	share [%]	*IRR > PME*	dev	share [%]	*IRR > PME*	dev	share [%]	*IRR > PME*	dev	share [%]	*IRR > PME*
All investments																
10%	2380	665	28	428	632	27	371	694	29	463	668	28	431	688	29	474
25%	2380	320	13	246	306	13	219	318	13	254	319	13	249	323	14	259
50%	2380	175	7	157	171	7	149	181	8	163	177	7	159	183	8	165

Absolute differences in percentage points between excess return based on IRR and excess return based on PME are in the scope of this table. For five broad benchmarks a deviation (dev) is noted if the return difference exceeds the significance level (10%, 25%, 50%). Our results reveal substantial impact of the reinvestment hypothesis for the appraisal of PE investments. For more than 27% of the PE investments excess returns deviate by more than 10% depending on the reinvestment hypothesis. If the significance level is raised to 50% still more than 7% of the investments exceed this level. IRR based excess returns substantially overstate the performance of PE investments. In more than 64% of all deviations (for the 10% significance level relative to S&P 500) the assumed reinvestment in equivalent PE ventures of the IRR concept significantly increases excess returns in comparison to a reinvestment in the public index (PME). This share increases with the significance level and reaches 90% for deviations exceeding 50% (again relative to S&P 500).

Table 9

Excess Return *PME based*		**Benchmark Index**														
		S&P 500			Nasdaq Composite			Russel2000			Dow Jones Ind. Avg.			MSCI World		
	obs	mean [%]	std dev [%]	median [%]	mean [%]	std dev [%]	median [%]	mean [%]	std dev [%]	median [%]	mean [%]	std dev [%]	median [%]	mean [%]	std dev [%]	median [%]
All investments	2380	10.14	166.47	2.00	5.71	167.54	-1.83	12.40	166.35	4.05	10.06	166.51	2.31	13.88	165.83	5.56
Industry																
IT	361	7.59	145.91	-4.41	3.75	148.71	-8.33	10.27	145.84	-1.44	7.58	146.04	-4.77	11.56	145.24	-0.65
consumer discretionary	273	10.02	88.38	5.83	5.74	89.07	2.35	13.08	88.42	7.71	9.94	88.47	5.35	14.02	88.46	8.91
industrial production	323	4.22	77.88	4.42	-1.43	79.44	0.29	6.84	77.12	6.76	4.38	78.08	5.82	8.07	77.29	8.20
telecommunication	151	23.51	151.82	1.68	19.21	152.33	-2.35	25.47	151.98	8.08	23.00	152.73	3.07	27.07	151.34	4.15
services	172	-9.62	57.47	-0.14	-15.20	60.37	-6.40	-7.39	57.56	2.23	-9.57	57.94	-0.45	-5.96	56.50	1.62
healthcare	288	19.73	334.99	-2.24	14.79	336.50	-7.36	22.59	335.60	0.69	19.47	334.20	-2.48	23.73	334.21	1.39
internet & media	232	19.60	157.25	1.66	15.49	158.00	-1.93	21.58	157.28	5.49	19.63	157.75	1.87	23.54	156.73	5.40
other industry	580	8.23	143.90	2.24	4.25	143.51	-1.43	9.59	142.79	2.88	8.07	144.34	2.13	11.55	142.82	5.24
Stage																
early stage	426	-19.53	96.69	-21.66	-23.01	98.55	-25.92	-17.26	97.14	-17.51	-20.18	96.39	-21.19	-15.50	95.93	-16.40
expansion	263	11.74	94.36	4.77	7.09	93.22	0.26	14.68	93.90	8.22	11.89	94.90	5.67	15.88	93.71	9.12
later stage	448	40.91	290.90	9.57	36.14	295.11	4.64	43.52	291.53	12.28	41.10	290.43	9.65	44.48	290.54	13.28
turnaround	68	34.51	238.28	7.46	30.67	238.85	4.86	36.41	236.48	10.14	34.06	238.73	8.11	36.70	235.64	11.19
no observation	1175	7.41	120.83	1.89	2.87	120.50	-1.67	9.38	120.02	3.72	7.39	121.09	2.17	11.10	120.07	5.08
Investment Amount																
small	373	21.58	211.25	-6.21	17.90	210.32	9.95	24.46	209.94	-4.27	21.53	211.69	-7.15	25.80	209.73	-2.45
medium	1300	9.00	189.75	0.88	4.92	192.24	-3.42	10.98	190.17	3.19	8.68	189.59	0.54	12.65	189.30	4.25
large	707	6.55	83.77	5.50	1.11	81.99	1.85	8.88	82.85	7.89	6.82	84.15	5.96	10.20	83.22	9.12

In this table we summarize the appraisal of PE investments relative to five different broad public equity benchmarks. Excess returns of PE investments over the benchmarks are derived from PME figures according to the model illustrated in appendix B. We distinguish industries, stages and size to analyze structural differences.

Table 10

Excess return distribution realtive to Nasday Composite Index

IRR based computation

	Percentiles	Smallest		
1%	-152.5541	-270.8378		
5%	-126.9742	-230.7748		
10%	-108.7087	-225.1567	Obs	2380
25%	-44.79809	-209.3528	Sum of Wgt.	2380
50%	-3.412843		Mean	29.7933
		Largest	Std. Dev.	350.9595
75%	26.41874	6699.065		
90%	88.71028	6742.072	Variance	123172.6
95%	204.4509	6833.965	Skewness	14.82416
99%	962.2832	7658.873	Kurtosis	276.0529

PME based computation

	Percentiles	Smallest		
1%	-143.9208	-265.9287		
5%	-117.8298	-221.9666		
10%	-92.04899	-205.9667	Obs	2380
25%	-28.53416	-184.4794	Sum of Wgt.	2380
50%	-1.82974		Mean	5.71054
		Largest	Std. Dev.	167.542
75%	18.97827	1369.327		
90%	61.99697	1953.427	Variance	28070.34
95%	110.565	2119.202	Skewness	17.12159
99%	535.6398	5398.333	Kurtosis	480.2083

Table 11

Excess Returns *specific benchmarks* *PME based*	Specific benchmark	obs	**Specific Benchmarks** PME based excess return mean *[%]*	std dev *[%]*	median *[%]*	**Nasdaq Composite** PME based excess return mean *[%]*	std dev *[%]*	median *[%]*
Industry								
IT	S&P IT	228	10.64	180.51	-14.59	15.6	180.2	-8.3
consumer discretionary	S&P Consumers	116	6.88	58.71	6.53	2.4	60.2	2.3
industrial production	S&P Industrials	322	2.69	74.75	3.68	-2.8	75.9	0.2
telecommunication	Nasdaq Teleco.	150	21.39	152.79	-1.22	19.4	152.8	-1.7
healthcare	S&P Healthcare	129	57.01	498.25	-3.70	52.2	498.5	-9.8
Geography								
Germany	DAX	104	10.57	117.57	2.44	10.16	120.34	1.15
UK	FTSE UK 100	342	9.10	122.58	7.65	1.01	125.35	1.53

Here we compare excess returns of PE investments relative to specific (industry and local) and broad (Nasdaq Composite) indices. As we reduce our sample to ensure comparability, the second column contains comparable excess returns of the PE investments over the Nasdaq Composite Index. Excess returns are derived from PME figures according to the model illustrated in appendix B. The choice of benchmark rarely impacts direction but substantially the degree of relative performance.

Table 12

Risk measures *IRR based*	obs	std. dev. *[%]*	semi-std. dev. *relative to mean [%]*	downside dev. *relative to Nasdaq Comp. [%]*	shortfall *relative to Nasdaq Comp. [%]*
All investments	2380	354.15	72.39	52.76	53.57
Industry					
IT	361	598.76	123.54	66.32	58.17
consumer discretionary	273	99.69	44.32	40.83	44.69
industrial production	323	105.69	49.62	48.81	50.46
telecommunication	151	273.57	98.50	57.23	50.99
services	172	204.66	60.19	57.91	58.14
healthcare	288	452.51	78.82	57.04	58.68
internet & media	232	509.73	88.00	47.69	53.45
other industry	586	206.23	57.30	47.26	53.92
Stage					
early stage	426	217.73	71.21	71.00	70.19
expansion	263	115.24	50.74	43.70	49.43
later stage	448	424.48	91.06	49.23	42.19
turnaround	68	435.42	75.59	33.68	36.76
no observation	1,175	391.29	67.93	48.93	53.79
Investment Amount					
small	373	591.42	109.90	54.36	62.40
medium	1,300	362.59	77.95	55.90	55.71
large	707	102.76	51.65	48.83	45.81

As returns of PE investments are highly skewed we compare alternative risk measures (standard deviation, semi-standard deviation, downside deviation, shortfall) to analyze their adequateness to measure PE investment risk. We base our calculations on IRR figures. Semi-standard deviation is calculated relative to mean IRR of our PE investments, while downside deviate is measured relative to mean IRR of simulated investments in the Nasdaq Composite Index. Shortfall probability is also referenced to IRR figures of corresponding investments in the Nasdaq Composite Index.
Our results reveal that risks of some clusters, for example healthcare and later stage investments, are overstated by standard deviation, because returns are not normally distributed and positive outliers are treated as “risk”.

Table 13

Sharpe Ratios *IRR based*	obs	PE investments std. dev.	Sharpe Ratio *mean*	*median*	*max*	PM investments std. dev.	Sharpe Ratio *mean*	*median*	*max*	Differences (mean) absolute difference *PE- PM*	Outperformer
All investments	2380	354.15	0.13	0.04	21.90	21.53	0.70	0.69	7.65	-0.58	PM
Industry											
IT	*361*	598.76	0.15	0.00	12.98	24.22	0.61	0.63	6.15	-0.46	PM
consumer discretionary	*273*	99.69	0.24	0.18	11.24	17.99	0.86	0.85	7.08	-0.62	PM
industrial production	*323*	105.69	0.20	0.16	9.29	17.74	0.98	0.88	6.13	-0.79	PM
telecommunication	*151*	273.57	0.27	0.05	7.46	31.15	0.51	0.64	5.28	-0.24	PM
services	*172*	204.66	0.10	0.05	11.55	23.21	0.68	0.68	5.24	-0.58	PM
healthcare	*288*	452.51	0.10	0.01	14.98	19.32	0.78	0.76	6.11	-0.68	PM
internet & media	*232*	509.73	0.14	0.02	13.47	27.55	0.56	0.57	5.75	-0.42	PM
other industry	*580*	206.23	0.13	0.06	17.26	17.31	0.79	0.80	4.29	-0.66	PM
Stage											
early stage	426	217.73	0.05	-0.08	13.56	27.93	0.43	0.57	5.89	-0.38	PM
expansion	263	115.24	0.26	0.14	7.82	20.54	0.77	0.84	4.98	-0.51	PM
later stage	448	424.48	0.20	0.06	15.98	22.60	0.80	0.81	7.00	-0.60	PM
turnaround	68	435.42	0.17	0.05	8.13	16.89	0.80	0.81	3.41	-0.64	PM
no observation	1175	391.29	0.11	0.03	19.82	18.58	0.81	0.74	8.52	-0.70	PM
Investment Amount											
small	373	591.42	0.14	0.00	11.61	19.81	0.69	0.68	7.58	-0.55	PM
medium	1300	362.59	0.13	0.03	21.38	22.78	0.63	0.65	7.23	-0.50	PM
large	707	102.76	0.23	0.19	10.39	20.75	0.83	0.76	7.63	-0.60	PM
Geogrpahy											
Germany	104	782.68	0.15	0.02	9.91	18.98	0.88	0.99	5.22	-0.73	PM
UK	342	207.69	0.12	0.08	17.16	20.56	0.79	0.75	7.68	-0.67	PM

To appraise PE investments relative PM on a risk adjusted basis we calculate the Sharpe Ratio for our entire sample and all clusters. As standard deviation overstates investment risks of PE, we are not surprised to observe substantially higher Sharpe Ratios for PM investments. Based on these results, PM outperform PE investment by a considerable margin.

Table 14

Modified Sharpe Ratios *IRR based* *downside deviation (dwn.dev.)* *relative to 2 yrs US Gov*	obs	PE investments dwn. dev. *[%]*	mod. Sharpe Ratio *mean*	*median*	*max*	PM investments (Nasdaq Comp. dwn. dev. *[%]*	mod. Sharpe Ratio *mean*	*median*	*max*	Differences (mean) absolute difference *PE- PM*	Outperformer
All investments	2380	45.74	0.98	0.29	169.61	9.77	1.54	1.52	16.85	-0.56	PM
Industry											
IT	361	57.98	1.59	0.05	133.88	11.75	1.26	1.33	12.79	0.33	PE
consumer discretionary	273	33.87	0.69	0.51	32.36	6.99	2.21	2.18	18.22	-1.52	PM
industrial production	323	39.67	0.51	0.41	24.12	6.73	2.47	2.31	16.15	-1.97	PM
telecommunication	151	51.03	1.44	0.27	39.93	14.03	1.14	1.42	11.74	0.30	PE
services	172	48.25	0.42	0.20	48.80	9.03	1.74	1.74	13.47	-1.31	PM
healthcare	288	49.10	0.95	0.11	138.32	7.79	1.93	1.88	15.16	-0.98	PM
internet & media	232	42.10	1.69	0.27	163.29	12.99	1.18	1.21	12.20	0.51	PE
other industry	580	42.98	0.63	0.31	82.94	9.09	1.51	1.52	8.17	-0.88	PM
Stage											
early stage	426	63.67	0.17	-0.28	46.63	15.00	0.81	1.05	10.98	-0.64	PM
expansion	263	35.38	0.85	0.47	25.52	9.91	1.60	1.75	10.32	-0.74	PM
later stage	448	40.29	2.08	0.59	168.57	8.52	2.11	2.14	18.56	-0.03	PM
turnaround	68	30.04	2.47	0.72	118.66	8.17	1.66	1.67	7.06	0.81	PE
no observation	1175	42.63	1.03	0.28	181.93	7.63	1.97	1.81	20.76	-0.94	PM
Investment Amount											
small	373	49.00	1.73	0.02	140.29	8.78	1.57	1.54	17.12	0.17	PE
medium	1300	48.94	0.96	0.23	158.52	11.19	1.28	1.33	14.71	-0.32	PM
large	707	40.32	0.58	0.49	26.50	7.78	2.21	2.02	20.36	-1.63	PM
Geogrpahy											
Germany	104	52.27	2.25	0.33	148.42	9.30	1.80	2.03	10.65	0.44	PE
UK	342	43.34	0.58	0.39	82.25	7.06	2.30	2.17	22.40	-1.72	PM

To better suit the characteristics of PE returns, we modify the Sharpe Ratio and measure risk in terms of downside deviation rather than standard deviation. The modified Sharpe Ratio accounts for both, probability and magnitude of the shortfall relative to the return of the 2 year US Government bonds. As we are interested in cross sectional differences we assign the cluster's downside deviation of returns to all investments belonging to the cluster. For the PM returns we use the IRRs of the simulated investments in the Nasdaq Composite Index and corresponding excess returns over the Government bonds.
Our adjustment of the Sharpe Ratio triggered a substantial increase of mean values, 680% and 119% for PE and PM investments respectively. The substantially larger increase for PE supports our hypothesis that standard deviation overstates PE's investment risk in our relative analysis. Mean and median ratios indicate underformance of PE as an asset class relative to investments in the Nasdaq Composite for the overall sample and the average investment. The ratios of the PE clusters reveal very heterogeneous risk adjusted performance. While turnaround and later stage investment outperform or match the performance of the Nasdaq Composite Index, early stage and expansion investments substantially underperform.

Table 15

Modified Sharpe Ratios *IRR based* *Shortfall* *relative to 2 yrs US Gov*	obs	PE investments shortfall	mod. Sharpe Ratio			PM investments shortfall	mod. Sharpe Ratio			Differences (mean) absolute difference	Outperformer
		[%]	*mean*	*median*	*max*	*[%]*	*mean*	*median*	*max*	*PE- PM*	
All investments	2380	35.97	1.25	0.37	215.68	13.66	1.11	1.09	12.05	0.14	PE
Industry											
IT	361	46.81	1.97	0.06	165.73	17.73	0.84	0.88	8.47	1.13	PE
consumer discretionary	273	21.61	1.08	0.80	50.73	10.99	1.41	1.39	11.59	-0.33	PM
industrial production	323	32.82	0.61	0.50	29.14	9.29	1.79	1.68	11.70	-1.18	PM
telecommunication	151	35.76	2.05	0.38	56.98	21.85	0.73	0.91	7.54	1.33	PE
services	172	40.70	0.50	0.24	57.85	16.28	0.96	0.96	7.47	-0.46	PM
healthcare	288	44.10	1.06	0.12	154.01	15.28	0.98	0.96	7.72	0.08	PE
internet & media	232	38.79	1.83	0.29	177.22	15.95	0.96	0.99	9.93	0.87	PE
other industry	580	32.25	0.84	0.41	110.53	10.07	1.36	1.38	7.38	-0.53	PM
Stage											
early stage	426	59.39	0.18	-0.30	49.76	23.24	0.52	0.68	7.08	-0.34	PM
expansion	263	29.28	1.03	0.57	30.84	11.79	1.34	1.47	8.67	-0.31	PM
later stage	448	26.12	3.22	0.90	260.02	12.72	1.41	1.43	12.43	1.81	PE
turnaround	68	22.06	3.37	0.99	161.59	13.24	1.03	1.03	4.36	2.35	PE
no observation	1175	33.53	1.31	0.36	231.37	10.98	1.37	1.26	14.43	-0.06	PM
Investment Amount											
small	373	28.49	2.98	0.03	241.28	15.04	0.91	0.90	9.99	2.07	PE
medium	1300	38.63	1.22	0.29	200.83	15.16	0.94	0.98	10.86	0.27	PE
large	707	28.49	0.82	0.69	37.51	11.01	1.56	1.43	14.39	-0.74	PM
Geogrpahy											
Germany	104	32.69	3.59	0.52	237.31	8.65	1.94	2.18	11.46	1.65	PE
UK	342	28.65	0.88	0.59	124.42	11.99	1.35	1.28	13.46	-0.48	PM

To test the impact of the magnitude of shortfall, we substitute downside deviation by shortfall probability and re-run our modified Sharpe Ratio analysis. The modified Sharpe Ratio now accounts for shortfall probability relative to the return of the 2 year US Government bonds only. As we are interested in cross sectional differences we assign the cluster's shortfall probability to all investments belonging to the cluster. For the PM returns we use the IRRs of the simulated investments in the Nasdaq Composite Index and corresponding excess returns over the Government bonds.

Mean ratios now indicate an overall outperformance of PE as an asset class, while median ratios are lower compared to their PM counterparts revealing underperformance of the average investment. The ratios of the PE clusters reveal very heterogeneous risk adjusted performance. While turnaround and later stage investment substantially outperform the Nasdaq Composite Index, early stage and expansion investments still underperform.

Table 16

Omega *PME based* *RT > Benchmark*	obs	**Nasdaq Composite Index** G *[mio US$]*	L *[mio US$]*	Omega	Difference to PM *(Omega = 1)* *Outperformer*		**S&P 500** G *[mio US$]*	L *[mio US$]*	Omega	Difference to PM *(Omega = 1)* *Outperformer*	
All Investments	2,380	5,630	-5,670	0.99	-0.01	PM	6,310	-4,970	1.27	0.27	PE
Industry											
IT	361	831	-1,070	0.78	-0.22	PM	859	-1,010	0.85	-0.15	PM
consumer discretionary	273	664	-735	0.90	-0.10	PM	734	-604	1.22	0.22	PE
industrial production	323	681	-758	0.90	-0.10	PM	769	-606	1.27	0.27	PE
telecommunication	151	392	-315	1.24	0.24	PE	425	-308	1.38	0.38	PE
services	172	222	-435	0.51	-0.49	PM	253	-363	0.70	-0.30	PM
healthcare	288	609	-759	0.80	-0.20	PM	658	-661	1.00	0.00	PM
internet & media	232	1,040	-550	1.89	0.89	PE	1,290	-492	2.62	1.62	PE
other industry	580	1,100	-1,070	1.03	0.03	PE	1,250	-957	1.31	0.31	PE
Stage											
early stage	426	323	-1,180	0.27	-0.73	PM	358	-1,110	0.32	-0.68	PM
expansion	263	758	-598	1.27	0.27	PE	898	-540	1.66	0.66	PE
later stage	448	2,090	-1,490	1.40	0.40	PE	2,280	-1,340	1.70	0.70	PE
turnaround	68	61	-119	0.51	-0.49	PM	76	-112	0.68	-0.32	PM
no observation	1,175	2,400	-2,280	1.05	0.05	PE	2,700	-1,870	1.44	0.44	PE
Investment Amount											
small	373	74	-62	1.20	0.20	PE	78	-59	1.32	0.32	PE
medium	1,300	1,430	-1,380	1.04	0.04	PE	1,520	-1,240	1.23	0.23	PE
large	707	4,130	-4,230	0.98	-0.02	PM	4,720	-3,680	1.28	0.28	PE
Geogrpahy											
Germany	104	268	-166	1.61	0.61	PE	270	-149	1.81	0.81	PE
UK	342	856	-1,080	0.79	-0.21	PM	985	-882	1.12	0.12	PE

Omega relates total excess chance to total shortfall risk. It is flexible regarding the reference point. We choose returns of investments in the Nasdaq Composite Index (left columns) and S&P 500 (right columns) as reference points, and consider all investments with a PME > 1 (PME < 1) as excess chance (shortfall risk). We calculate total annualized return of investment *i* as PME based annual excess return multiplied with the real investment amount (in terms of 2004 U.S. Dollars). Omega value weights all investments. Our results show a balanced shortfall risk and excess chance for PE investors relative to the Nasdaq Composite Index. As the outperformance of PM relative to PE is only minimal we are not surprised to see mixed results for out clusters. While later stage investments exhibit Omegas considerably exceeding one, early stage investments are the overall worst performing cluster. Switching to S&P 500 we observe higher Omegas indicating risk-adjusted outperformance of PE over public index. We conclude that PE investments generate an adequate return premium over PM investments gross of fess, as Omegas match one relative to the Nasdaq Composite Index and exceed one for all other benchmarks.

Table 17

Omega *IRR based* *RT > Nasdaq Composite*	obs	PE investments G *[mio US$]*	 L *[mio US$]*	Omega	Difference to PM *(Omega = 1)* *Outperformer*
All Investments	2,380	9,700	-6,620	1.47	0.47	PE
Industry						
IT	361	2,200	-1,150	1.91	0.91	PE
consumer discretionary	273	1,130	-865	1.31	0.31	PE
industrial production	323	990	-939	1.05	0.05	PE
telecommunication	151	659	-336	1.96	0.96	PE
services	172	442	-520	0.85	-0.15	PM
healthcare	288	886	-853	1.04	0.04	PE
internet & media	232	1,450	-626	2.32	1.32	PE
other industry	580	1,790	-1,360	1.32	0.32	PE
Stage						
early stage	426	651	-1,260	0.52	-0.48	PM
expansion	263	1,220	-734	1.66	0.66	PE
later stage	448	2,940	-1,780	1.65	0.65	PE
turnaround	68	108	-174	0.62	-0.38	PM
no observation	1,175	4,780	-2,670	1.79	0.79	PE
Investment Amount						
small	373	174	-68	2.56	1.56	PE
medium	1,300	3,230	-1,570	2.06	1.06	PE
large	707	6,300	-4,980	1.27	0.27	PE
Geogrpahy						
Germany	104	1,250	-211	5.92	4.92	PE
UK	342	1,310	-1,250	1.05	0.05	PE

We recalculate Omegas using IRR based annualized excess returns over the Nasdaq Composite Index. The results indicate substantially higher Omegas (compared to the PME based results in Table 16) triggered by larger excess chances and smaller shortfall risks. This is in line with our prior findings of substantially larger excess returns if a reinvestment in identical PE projects is assumed. Thus, assuming a reinvestment of disbursements in identical PE projects leads to the conclusion that PE outperforms PM on a risk adjusted basis. It should be noted that early stage investments are the exception and still underperform relative to the Nasdaq Composite Index. As this reinvestment hypothesis is especially unrealistic for PE projects, PME based returns should be used for an unbiased assessment of project returns.

Appendix A

We strive to generate cash flow patterns for public market investments that mimic our private equity investments. As we do have detailed cash flow information for the PE investments, a purchase (sale) of shares in the benchmark is simulated when a negative (positive) cash flow occurs for the underlying PE investment. Doing so we achieve pairs of public and private equity returns that are matched and thus comparable. To illustrate our approach we use sample investment *i* of a private equity fund.

The monthly cash flow information forms the starting point for our model. In a first step we look at the shares we need to purchase in the public benchmark investment *b* in period *t*. When cash flows into the private equity investment we simulate an investment in the public benchmark by purchasing an equivalent number of shares:

$$SP_{tb} = \frac{CFI_{ti}}{P_{tb}}$$

with:

SP_{tb} = shares purchased of public benchmark investment *b* in period *t*

CFI_{ti} = cash inflow of investment *i* in period *t*

P_{tb} = price per share of benchmark investment *b* in period *t*

The total number of shares bought SP_b over the entire holding period *T* then is:

$$SP_b = \sum_{t=0}^{T} SP_{tb}$$

We now add the sale of shares to our simulation. A sale occurs with a cash outflow (distribution) from the private equity investment. To determine the number of shares to be sold in period *t* we use the ratio of cash outflows in period *t* and total cash outflows over *T*.

$$SS_{tb} = \frac{CFO_{ti}}{CFO_i} \cdot SP_b$$

with:

SS_{tb} = shares sold of public benchmark investment *b* in period *t*

CFO_{ti} = cash outflow of investment *i* in period *t*

CFO_i = total cash outflows of investment *i* over holding period *T*

SP_b = total number of shares in *b* bought over holding period T

Extreme cash flow patterns can trigger a simulated sale of more shares than have been purchased previously. If for example a substantial cash outflow is followed by cash inflows and a write-off, all shares would be sold when the cash outflow occurs and latter "re-purchased" with the cash inflow. To avoid these misleading short sales we introduce the following boundaries for the number of shares sold in period *t*:

$$SS_{tb} = \frac{CFO_{ti}}{CFO_i} \cdot SP_b \qquad \forall tb : S_{t-1,b} \geq \frac{CFO_{ti}}{CFO_i} \cdot SP_b$$

$$SS_{tb} = S_{t-1,b} \qquad \forall tb : S_{t-1,b} < \frac{CFO_{ti}}{CFO_i} \cdot SP_b$$

with:

$S_{t-1,p}$ = shares held in benchmark investment *b* at the end of period *t-1*

In addition, a sale needs to be simulated if the private equity investment is completely written off. We are not interested in partial write-offs during the holding period, but solely in final write-offs marking the termination of investment *i*. After this point in time no further cash in- or outflows occur. To appropriately reflect this fact, we simulate a sale of all remaining shares of *b* in the period after the write-off. :

$$SS_{tb} = S_{t-1,b} \qquad \forall t : CF_{ti} = WO_i$$

with:

WO_i = final and complete write-off of investment *i*

We can now track the number of shares held at the end of each period t. As we analyze net cash flows, timing of share purchases and sales within a period does not matter. The number of shares held at the end of period *t* is:

$$S_{tb} = S_{t-1,b} + SP_{tb} - SS_{tb}$$

The last step is to transform the time series of shares held into a cash flow stream. To do so we multiply the change of shares held with the share price for each period *t*:

$$CF_{tb} = \left(S_{t-1,b} - S_{tb}\right) \cdot P_{tb}$$

with:

CF_{tb} = simulated cash flow of mimicked investment in benchmark *b* for private equity investment *i* in period *t*

We have now generated the cash flow of an investment in a public benchmark that mimics the corresponding investment in a private equity project. These cash flow pairs enable us to perform cross sectional analysis between public and private equity. The model is flexible with regard to both key parameters, the public benchmark and the source of the cash flow stream. Besides cash flows of private equity projects, other alternative asset classes can be used to enable a cross sectional analysis.

Appendix B

In order to transform the PME ratio into return figures we start from the benchmark index's (public market) return:

$$y_{it} = \frac{IV_t}{IV_{t-1}} - 1$$

with:

y_{it} = return of benchmark index *i* in period *t*

IV_t = Index value in period *t*

If we assume discrete compounding and an investment horizon of *T* years, the total public benchmark return Y_i is:

$$Y_i = \prod_{t=1}^{T} (1 + y_{it}) - 1$$

As we analyze realized investments all inputs are known and certain. In a next step we annualize the benchmark return:

$$y_i = \left[\prod_{t=1}^{T} (1 + y_{it})\right]^{\frac{1}{T}} - 1$$

The total return of PE investments *j* is:

$$X_i = \frac{V_{jT}}{V_{j0}} - 1$$

with:

V_{j0} = value of private equity investment *j* in period *0*

V_{jT} = value of private equity investment *j* in period *T*

Based on our PME definition, the following relationship must hold:

$$\frac{V_{jT}}{V_{j0}} = PME_{ji} \cdot \prod_{t=1}^{T} (1 + y_{it})$$

with:

PME_{ji} = Public Market Equivalent ratio of investment *j* relative to benchmark *i*

The terminal value of 1 $ investment in PE project *j* is equal to the terminal value of PME_{ji} $ in public benchmark *i*. If we annualize the PE return we get:

$$x_i = \left(\frac{V_{jT}}{V_{j0}}\right)^{\frac{1}{T}} - 1 = \left(PME_{ji} \cdot \prod_{t=1}^{T}(1+y_{it})\right)^{\frac{1}{T}} - 1$$

The final step is to deduct the benchmark return as we are interested in excess returns over the chosen benchmark. For the PME based excess return ER_{PME} we get:

$$ER_{PMEj} = x_i - y_i = \left[\left(PME_{ji} \cdot \prod_{t=1}^{T}(1+y_{it})\right)^{\frac{1}{T}} - 1\right] - \left[\left(\prod_{t=1}^{T}(1+y_{it})\right)^{\frac{1}{T}} - 1\right]$$

$$ER_{PMEj} = \left[\left(PME_{ji}\right)^{\frac{1}{T}} - 1\right] \cdot \left(\prod_{t=1}^{T}(1+y_{it})\right)^{\frac{1}{T}}$$

All inputs are available, as we can calculate y_{it} for all investments and all benchmarks based on observable market quotes. We have also calculated the PME_{ji} figures (please refer to Table 6).